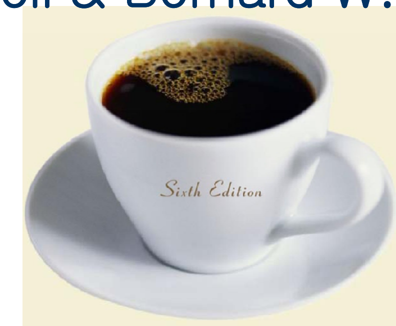
Chapter 13

Inventory Management

Operations Management - 6th Edition

Roberta Russell & Bernard W. Taylor, III



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Lecture Outline

- Elements of Inventory Management
- Inventory Control Systems
- Economic Order Quantity Models
- Quantity Discounts
- Reorder Point
- Order Quantity for a Periodic Inventory System

What Is Inventory?

- Stock of items kept to meet future demand
- Purpose of inventory management
  - how many units to order
  - when to order

Inventory and Supply Chain Management

- Bullwhip effect
  - demand information is distorted as it moves away from the end-use customer
  - higher safety stock inventories to are stored to compensate
- Seasonal or cyclical demand
- Inventory provides independence from vendors
- Take advantage of price discounts
- Inventory provides independence between stages and avoids work stoppages

Inventory and Quality Management in the Supply Chain

- Customers usually perceive quality service as availability of goods they want when they want them
- Inventory must be sufficient to provide high-quality customer service in QM

Types of Inventory

- Raw materials
- Purchased parts and supplies
- Work-in-process (partially completed) products (WIP)
- Items being transported
- Tools and equipment

Two Forms of Demand

- Dependent
  - Demand for items used to produce final products
  - Tires stored at a Goodyear plant are an example of a dependent demand item
- Independent
  - Demand for items used by external customers
  - Cars, appliances, computers, and houses are examples of independent demand inventory

Inventory Costs

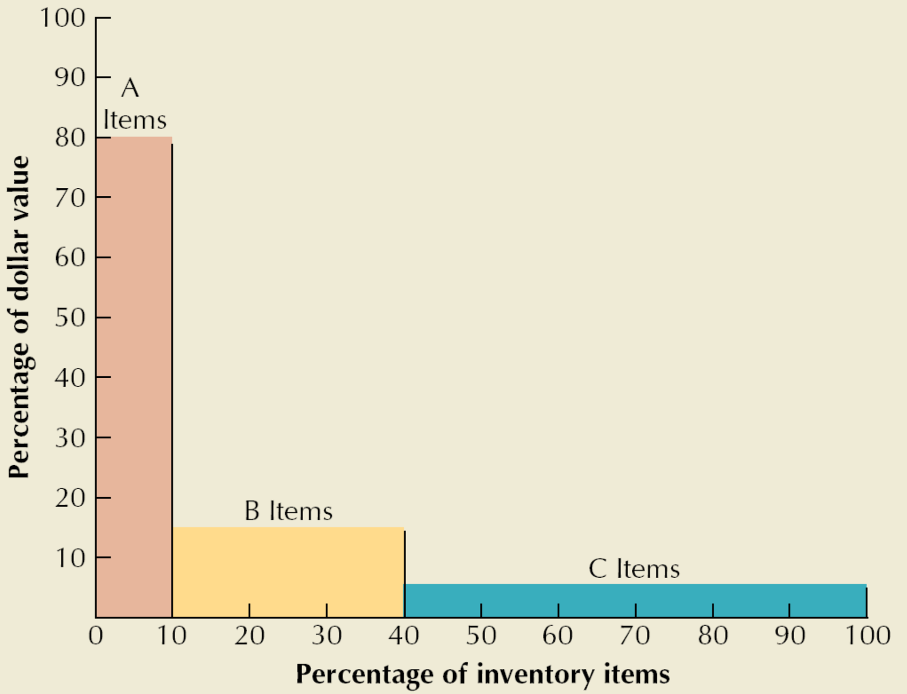
- Carrying cost
  - cost of holding an item in inventory
- Ordering cost
  - cost of replenishing inventory
- Shortage cost
  - temporary or permanent loss of sales when demand cannot be met

Inventory Control Systems

- Continuous system (fixed-order-quantity)
    - constant amount ordered when inventory declines to predetermined level
- Periodic system (fixed-time-period)
    - order placed for variable amount after fixed passage of time

ABC Classification

- Class A
  - 5 – 15 % of units
  - 70 – 80 % of value
- Class B
  - 30 % of units
  - 15 % of value
- Class C
  - 50 – 60 % of units
  - 5 – 10 % of value



ABC Classification: Example

PART	UNIT COST	ANNUAL USAGE
1	$ 60	90
2	350	40
3	30	130
4	80	60
5	30	100
6	20	180
7	10	170
8	320	50
9	510	60
10	20	120

ABC Classification: Example (cont.)

PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY		% CUMMULATIVE
9	$30,600	35.9	6.0		6.0
8	16,000	18.7	5.0	A	11.0
2	14,000	16.4	4.0		15.0
1	5,400	6.3	9.0		24.0

CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
A	9, 8, 2	71.0	15.0
B	1, 4, 3	16.5	25.0
C	6, 5, 10, 7	12.5	60.0

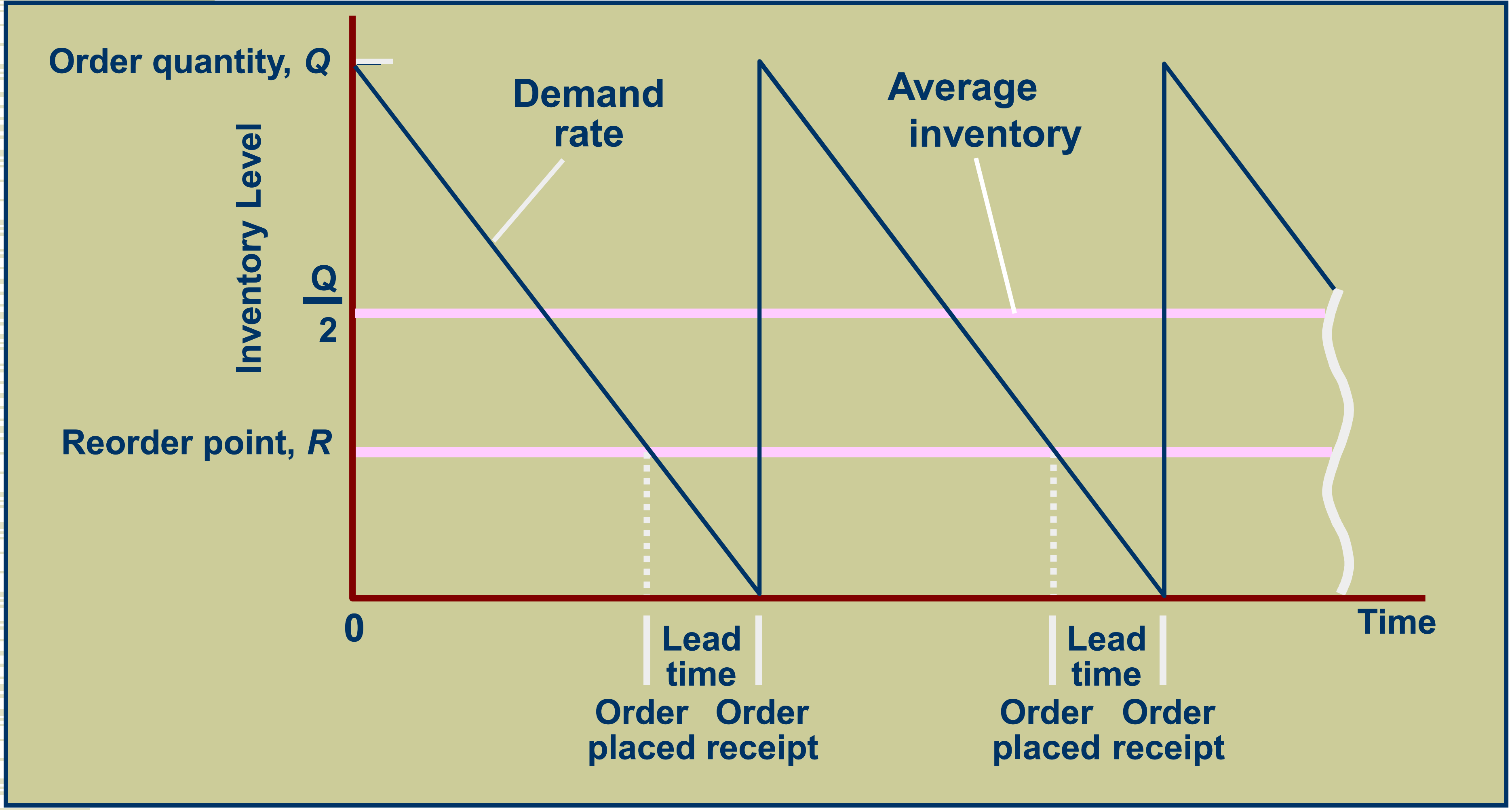
Economic Order Quantity (EOQ) Models

- EOQ
  - optimal order quantity that will minimize total inventory costs
- Basic EOQ model
- Production quantity model

Assumptions of Basic EOQ Model

- Demand is known with certainty and is constant over time
- No shortages are allowed
- Lead time for the receipt of orders is constant
- Order quantity is received all at once

Inventory Order Cycle



EOQ Cost Model

C_o - cost of placing order

C_c - annual per-unit carrying cost

D - annual demand

Q - order quantity

$$\text{Annual ordering cost} = \frac{C_o D}{Q}$$

$$\text{Annual carrying cost} = \frac{C_c Q}{2}$$

$$\text{Total cost} = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

EOQ Cost Model

Deriving Q_{opt}

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$\frac{\partial TC}{\partial Q} = -\frac{C_o D}{Q^2} + \frac{C_c}{2}$$

$$0 = -\frac{C_0 D}{Q^2} + \frac{C_c}{2}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}}$$

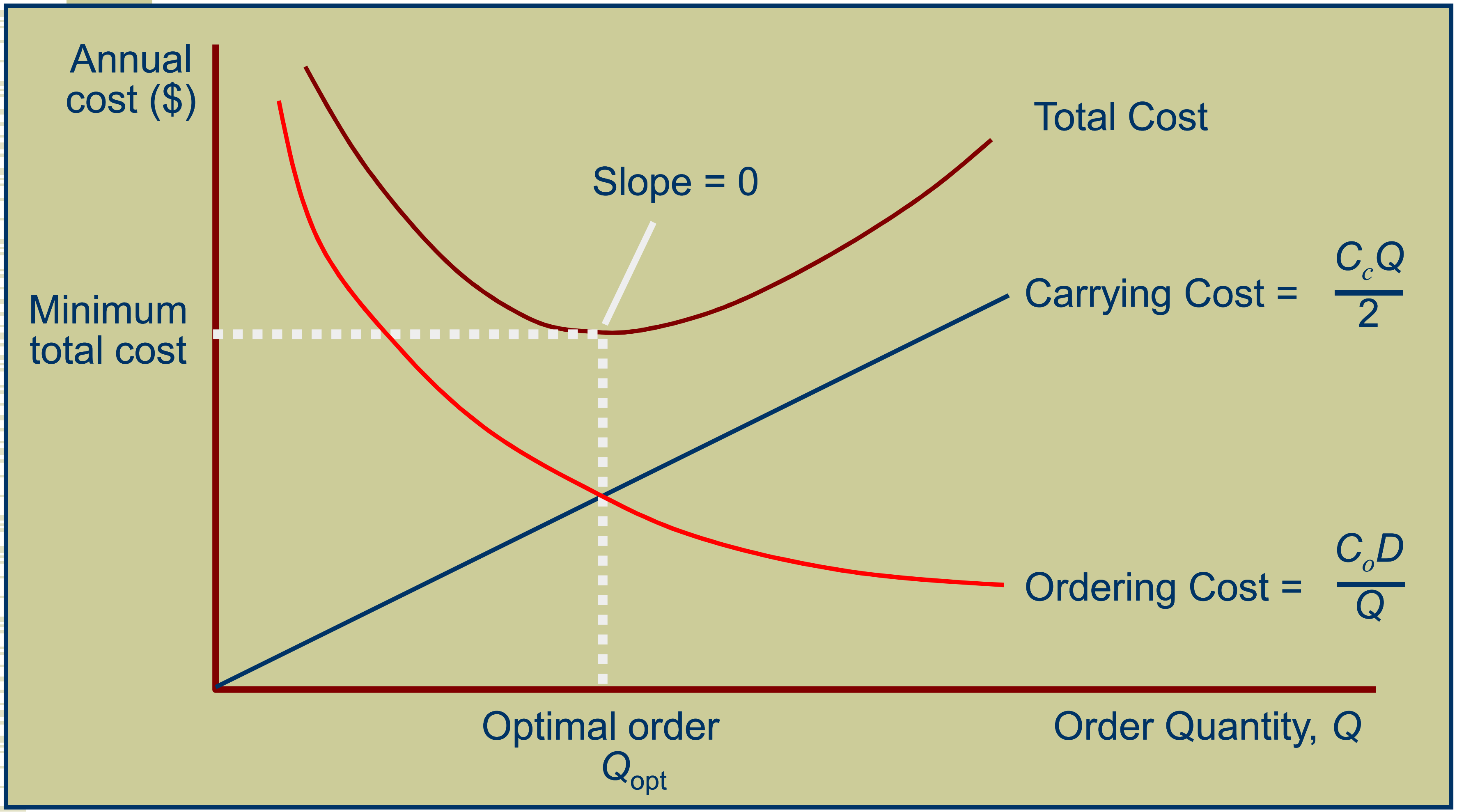
Proving equality of costs at optimal point

$$\frac{C_o D}{Q} = \frac{C_c Q}{2}$$

$$Q^2 = \frac{2C_o D}{C_c}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}}$$

EOQ Cost Model (cont.)



EOQ Example

C_c = \$0.75 per gallon $\quad C_o$ = \$150 $\quad D$ = 10,000 gallons

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} \qquad TC_{min} = \frac{C_oD}{Q} + \frac{C_cQ}{2}$$

$$Q_{opt} = \sqrt{\frac{2(150)(10{,}000)}{(0.75)}} \qquad TC_{min} = \frac{(150)(10{,}000)}{2{,}000} + \frac{(0.75)(2{,}000)}{2}$$

$$Q_{opt} = 2{,}000 \text{ gallons} \qquad TC_{min} = \$750 + \$750 = \$1{,}500$$

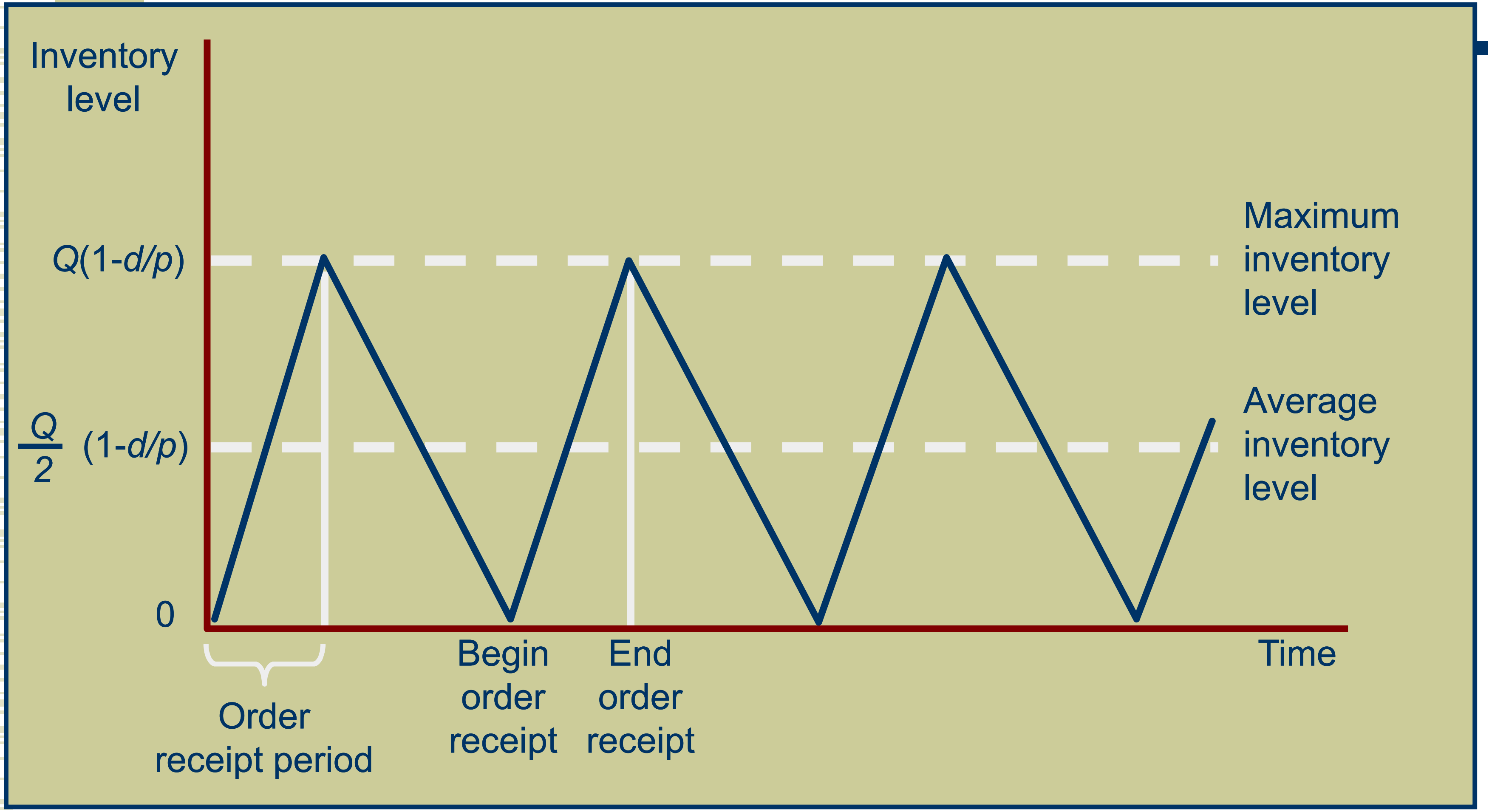
Orders per year = D/Q_{opt}
= 10,000/2,000
= 5 orders/year

Order cycle time = 311 days/(D/Q_{opt})
= 311/5
= 62.2 store days

Production Quantity Model

- An inventory system in which an order is received gradually, as inventory is simultaneously being depleted
  - AKA non-instantaneous receipt model
  - assumption that *Q* is received all at once is relaxed
- *p* – daily rate at which an order is received over time, a.k.a. *production rate*
- *d* – daily rate at which inventory is demanded

Production Quantity Model (cont.)



Production Quantity Model (cont.)

p = production rate

d = demand rate

$$\text{Maximum inventory level} = Q - \frac{Q}{p}d = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Average inventory level} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)$$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2}\left(1 - \frac{d}{p}\right)$$

$$Q_{\text{opt}} = \sqrt{\frac{2C_oD}{C_c\left(1 - \frac{d}{p}\right)}}$$

Production Quantity Model: Example

C_c = \$0.75 per gallon $\quad C_o$ = \$150 $\quad D$ = 10,000 gallons

d = 10,000/311 = 32.2 gallons per day $\quad p$ = 150 gallons per day

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c\left(1 - \frac{d}{p}\right)}} = \sqrt{\frac{2(150)(10{,}000)}{0.75\left(1 - \frac{32.2}{150}\right)}} = 2{,}256.8 \text{ gallons}$$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2}\left(1 - \frac{d}{p}\right) = \$1{,}329$$

$$\text{Production run} = \frac{Q}{p} = \frac{2{,}256.8}{150} = 15.05 \text{ days per order}$$

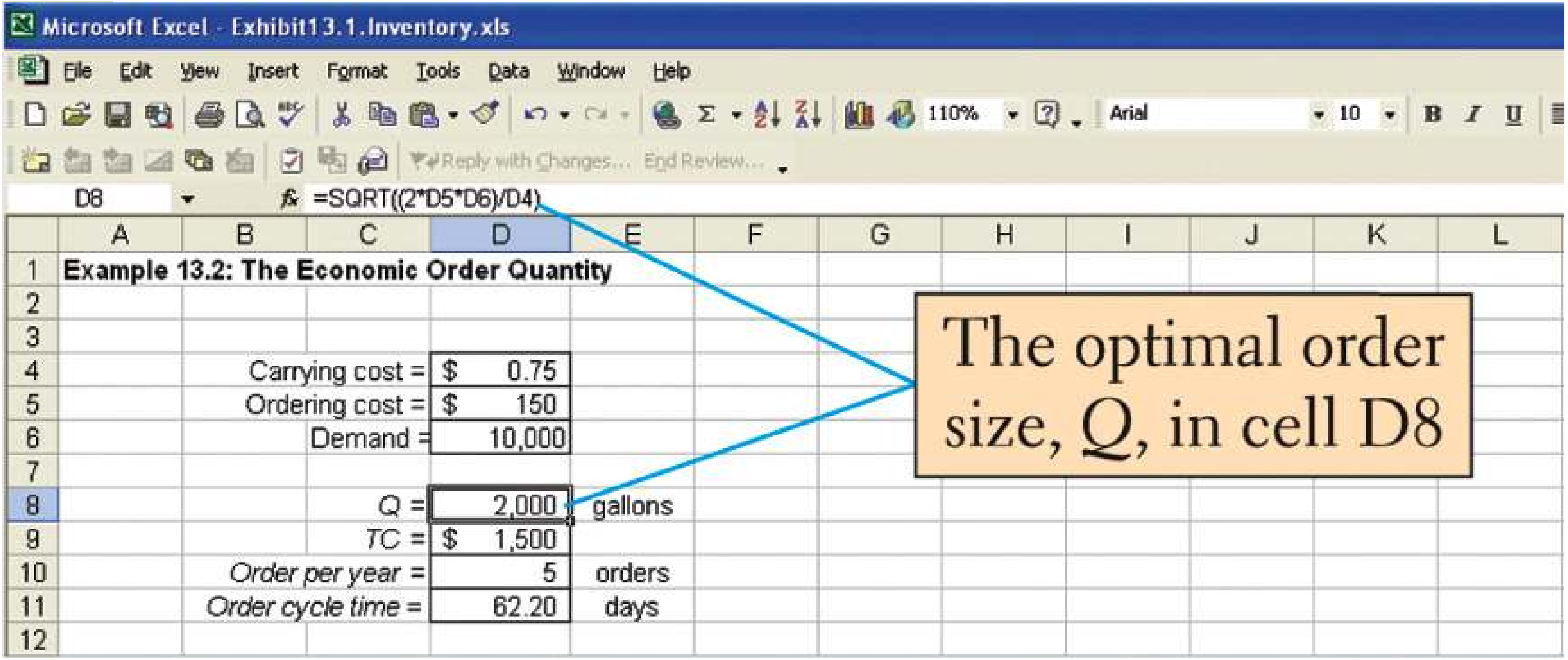
Production Quantity Model: Example (cont.)

$$\text{Number of production runs} = \frac{D}{Q} = \frac{10{,}000}{2{,}256.8} = 4.43 \text{ runs/year}$$

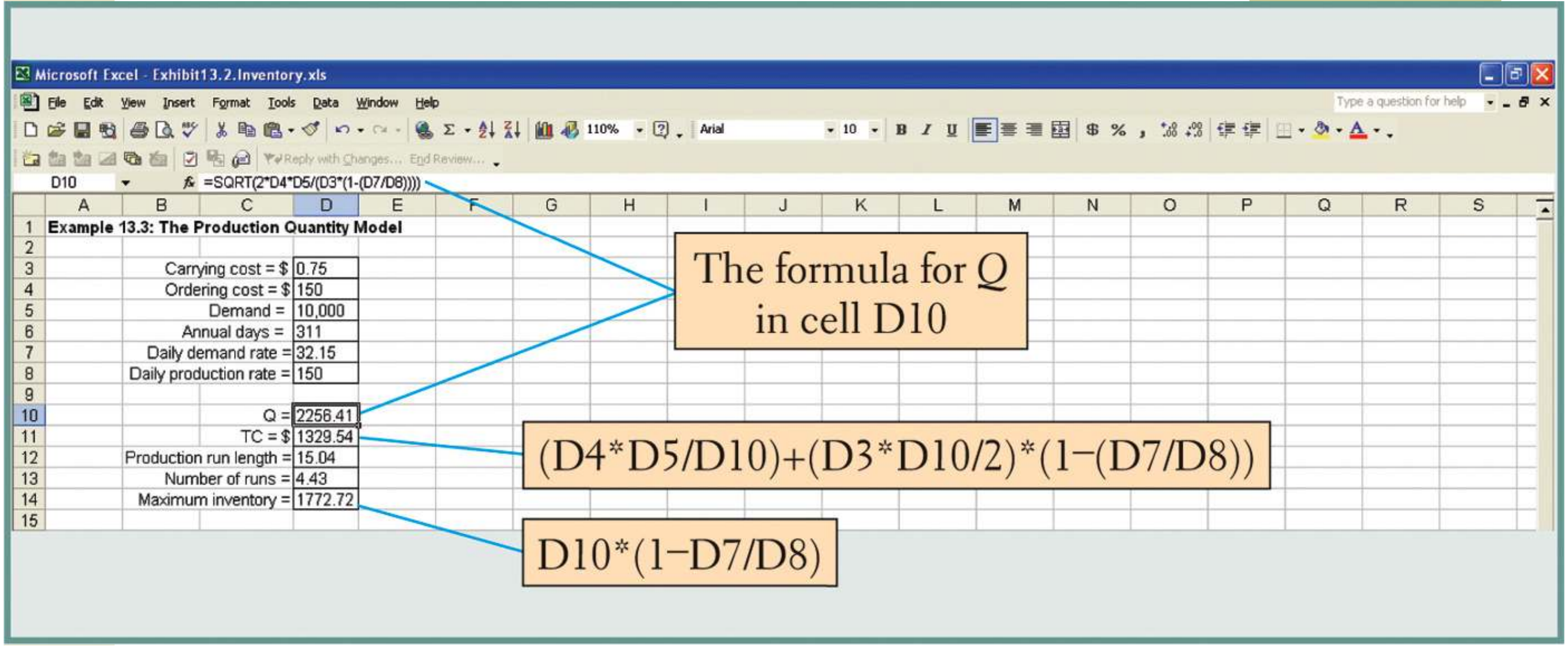
$$\text{Maximum inventory level} = Q\left(1 - \frac{d}{p}\right) = 2{,}256.8\left(1 - \frac{32.2}{150}\right)$$

$$= 1{,}772 \text{ gallons}$$

Solution of EOQ Models with Excel



Solution of EOQ Models with Excel (Con't)



Solution of EOQ Models with OM Tools

Microsoft Excel - Exhibit13.3.xls

D10 =SQRT((2*D5*D6)/D4)

The Economic Order Quantity Model

Input:

Carrying cost, Cc =	$0.75
Ordering cost, Co =	$150.00
Demand =	10,000
Days per year =	311

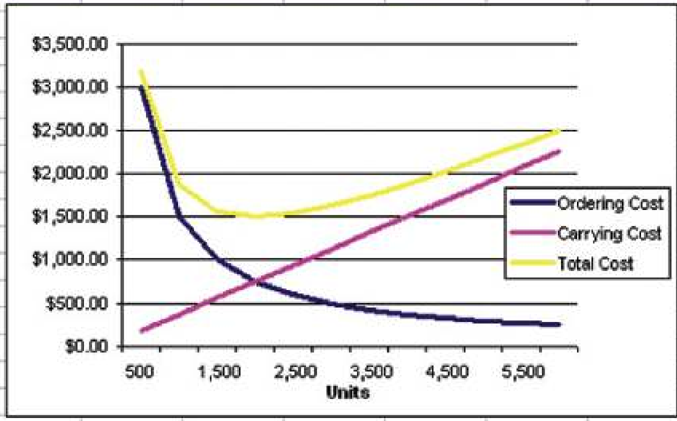
Output:

Qopt =	2,000	
Total Cost, TC =	$1,500.00	
Orders per year =	5	
Order cycle time =	62.20	days

Graph:

Q	Co * D/Q	Cc *Q/2	TC
500	$3,000.00	$187.50	$3,187.50
1,000	$1,500.00	$375.00	$1,875.00
1,500	$1,000.00	$562.50	$1,562.50
2,000	$750.00	$750.00	$1,500.00
2,500	$600.00	$937.50	$1,537.50
3,000	$500.00	$1,125.00	$1,625.00
3,500	$428.57	$1,312.50	$1,741.07
4,000	$375.00	$1,500.00	$1,875.00
4,500	$333.33	$1,687.50	$2,020.83
5,000	$300.00	$1,875.00	$2,175.00
5,500	$272.73	$2,062.50	$2,335.23
6,000	$250.00	$2,250.00	$2,500.00

Input costs, demand and days per year. Excel will calculate the optimal order quantity, total cost, the number of orders per year, and the order cycle time. Costs are graphed for various quantities ordered.



$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}}$$

Quantity Discounts

Price per unit decreases as order quantity increases

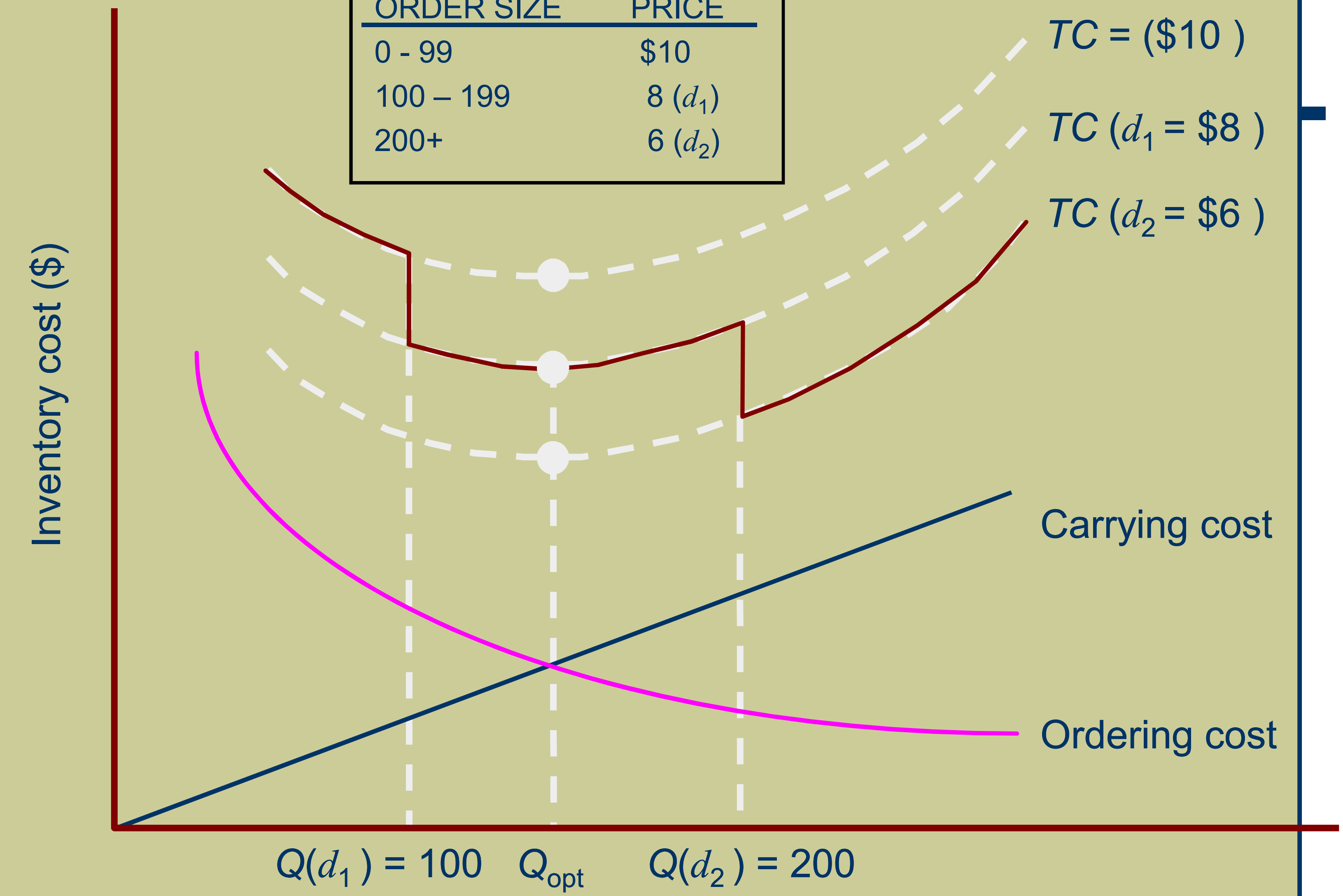
$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

where

P = per unit price of the item
D = annual demand

Quantity Discount Model (cont.)

ORDER SIZE	PRICE
0 - 99	$10
100 – 199	8 (d_1)
200+	6 (d_2)



Quantity Discount: Example

QUANTITY	PRICE
1 - 49	$1,400
50 - 89	1,100
90+	900

C_o = $2,500
C_c = $190 per TV
D = 200 TVs per year

$$Q_{\text{opt}} = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ TVs}$$

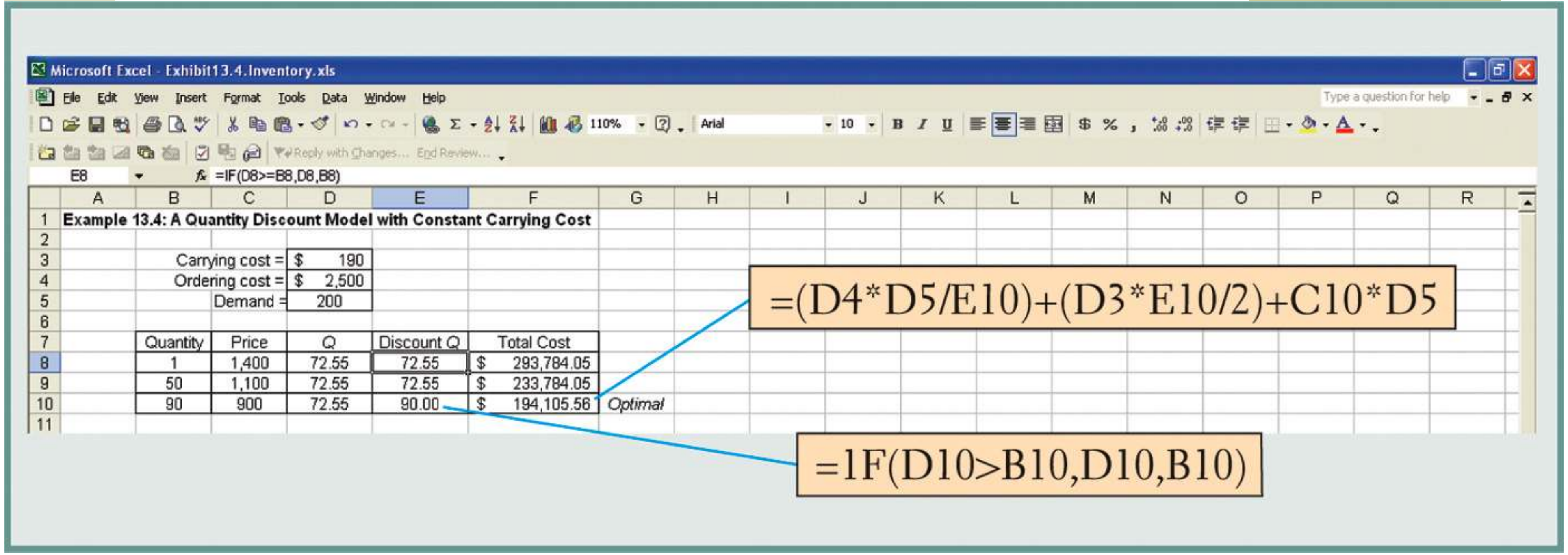
For $Q = 72.5$

$$TC = \frac{C_oD}{Q_{\text{opt}}} + \frac{C_cQ_{\text{opt}}}{2} + PD = \$233{,}784$$

For $Q = 90$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2} + PD = \$194{,}105$$

Quantity-Discount Model Solution with Excel



Reorder Point

Level of inventory at which a new order is placed

$$R = dL$$

where

d = demand rate per period
L = lead time

Reorder Point: Example

Demand = 10,000 gallons/year

Store open 311 days/year

Daily demand = 10,000 / 311 = 32.154 gallons/day

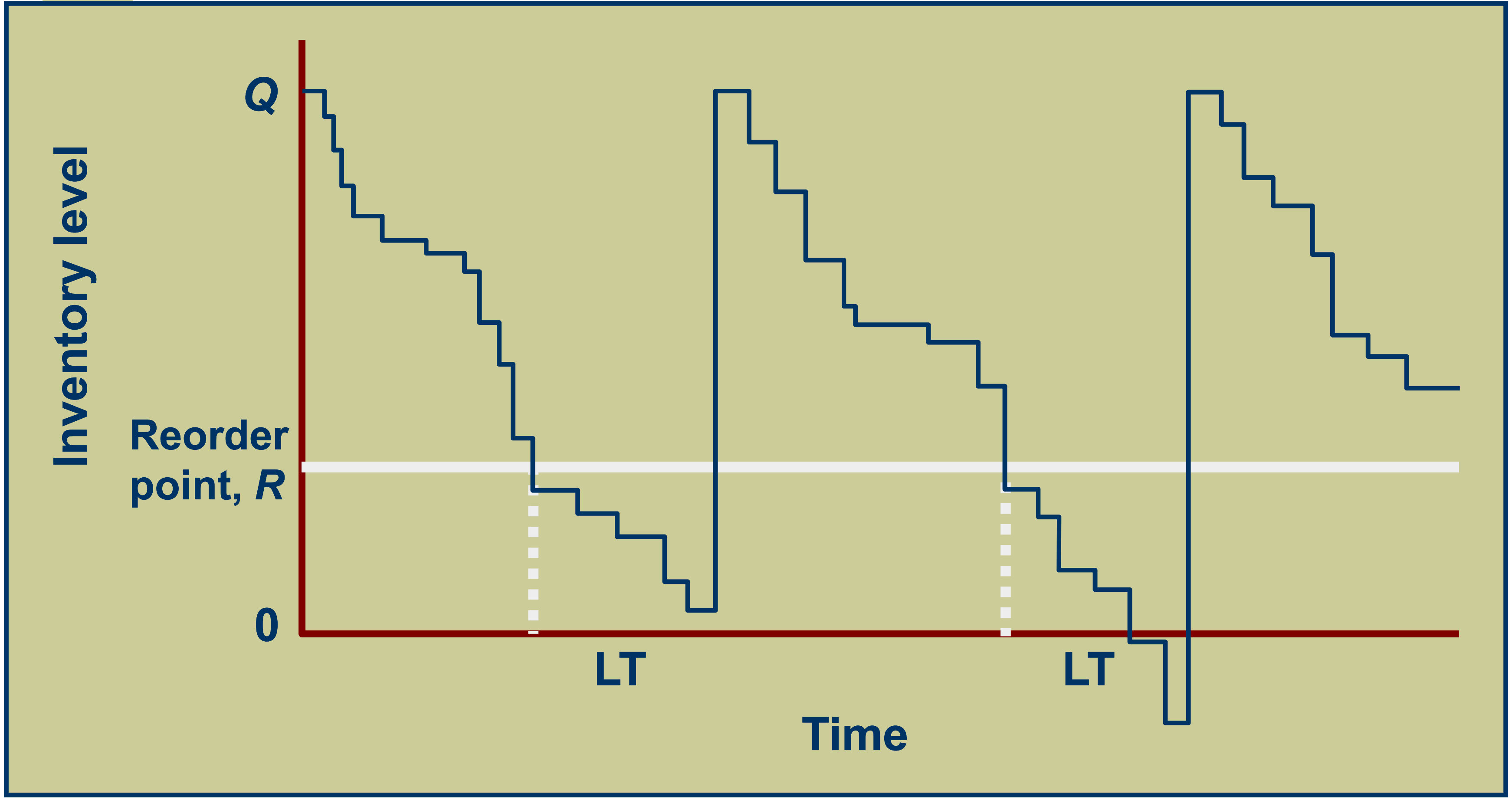
Lead time = L = 10 days

R = dL = (32.154)(10) = 321.54 gallons

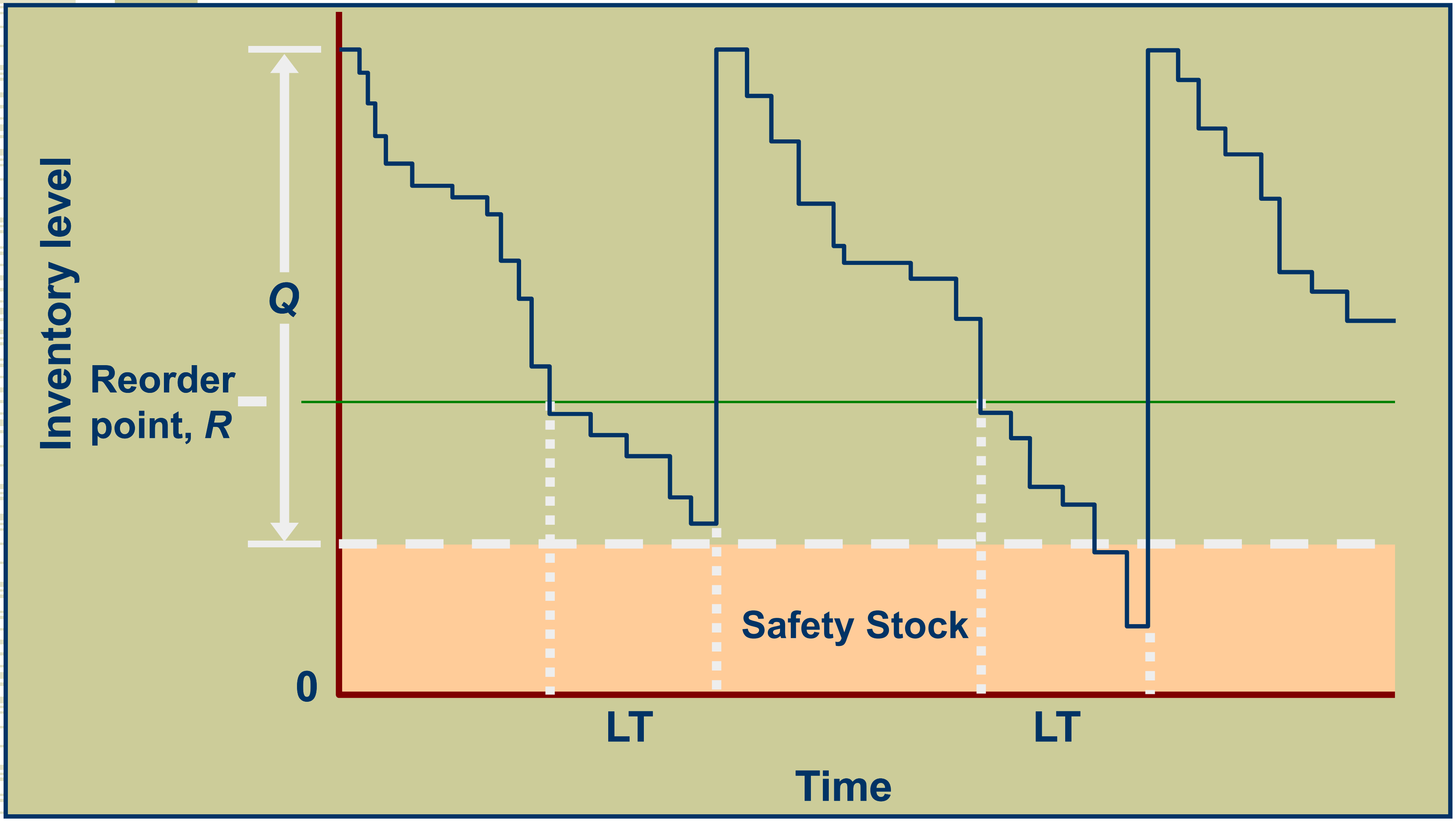
Safety Stocks

- Safety stock
  - buffer added to on hand inventory during lead time
- Stockout
  - an inventory shortage
- Service level
  - probability that the inventory available during lead time will meet demand

Variable Demand with a Reorder Point



Reorder Point with a Safety Stock



Reorder Point With Variable Demand

$$R = \bar{d}L + z\sigma_d\sqrt{L}$$

where

$\bar{d}$ = average daily demand

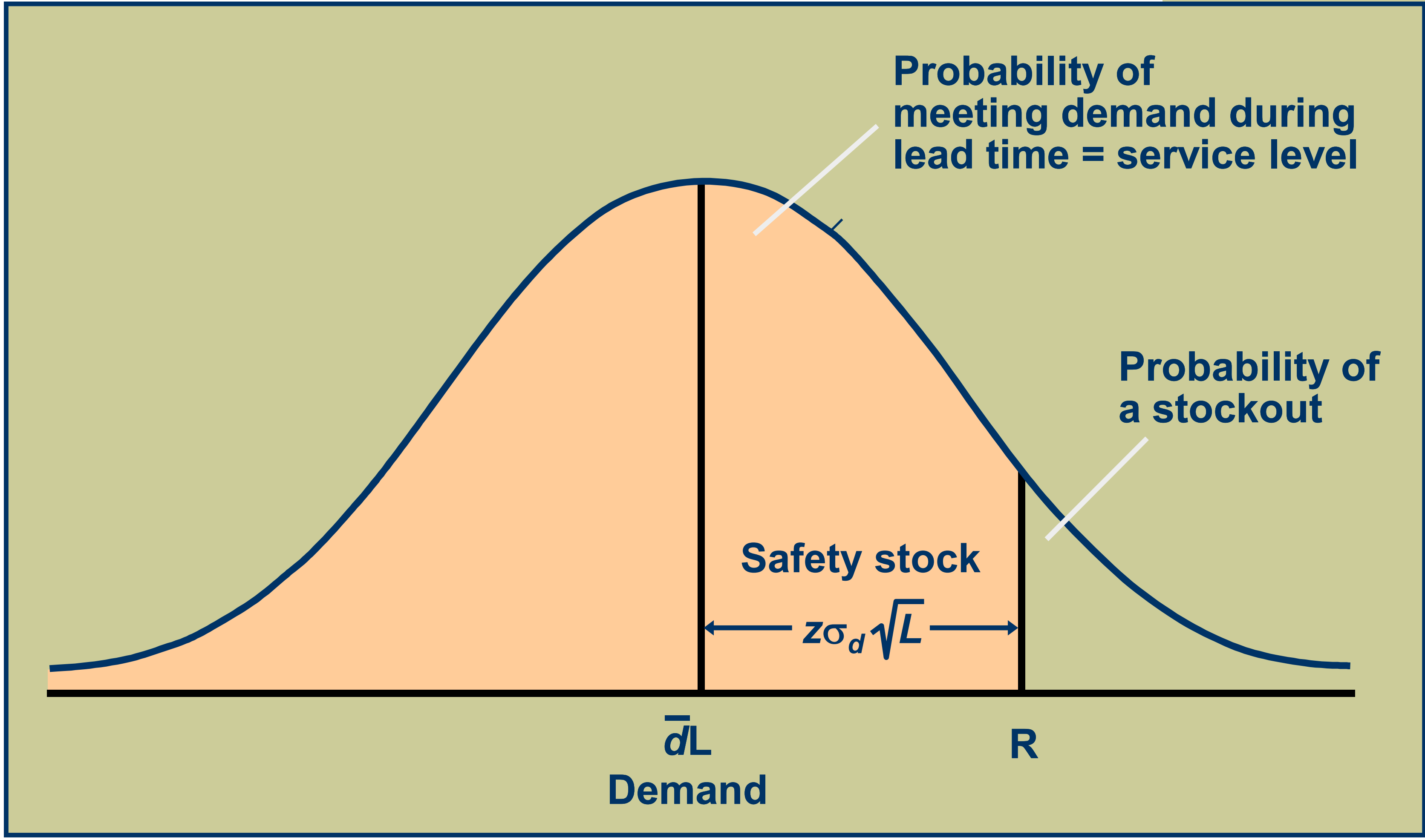
L = lead time

σ_d = the standard deviation of daily demand

z = number of standard deviations corresponding to the service level probability

$z\sigma_d\sqrt{L}$ = safety stock

Reorder Point for a Service Level



Reorder Point for Variable Demand

The paint store wants a reorder point with a 95% service level and a 5% stockout probability

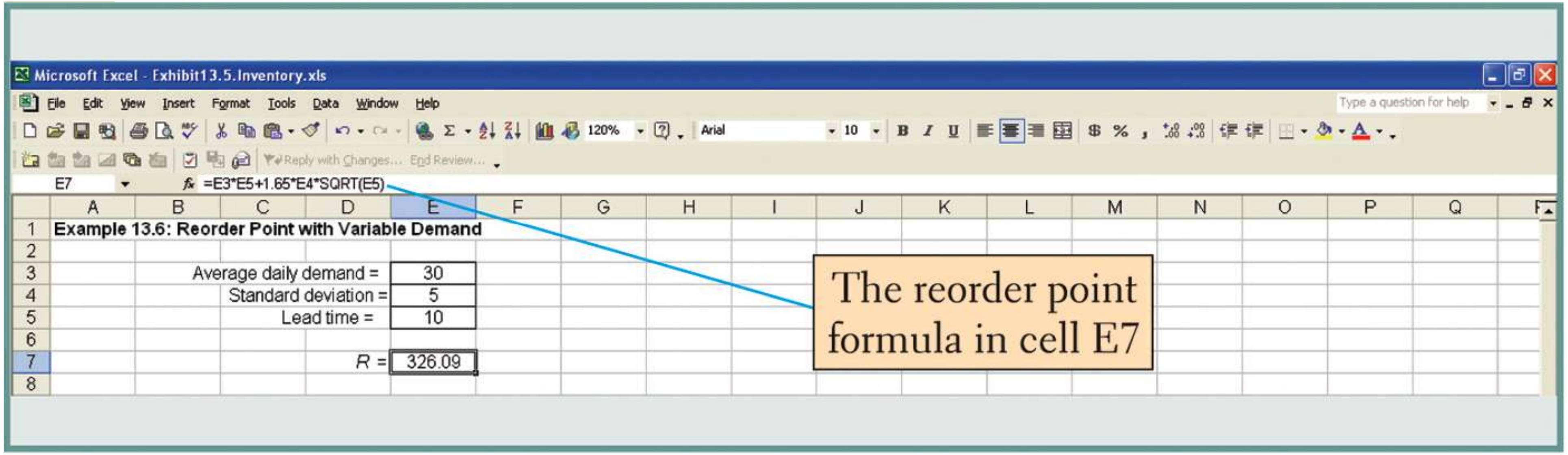
$\bar{d}$ = 30 gallons per day
L = 10 days
σ_d = 5 gallons per day

For a 95% service level, z = 1.65

$$R = \bar{d}L + z\,\sigma_d\sqrt{L}$$
$$= 30(10) + (1.65)(5)(\sqrt{10})$$
$$= 326.1 \text{ gallons}$$

$$\text{Safety stock} = z\,\sigma_d\sqrt{L}$$
$$= (1.65)(5)(\sqrt{10})$$
$$= 26.1 \text{ gallons}$$

Determining Reorder Point with Excel



Order Quantity for a Periodic Inventory System

$$Q = \bar{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I$$

where

d = average demand rate

t_b = the fixed time between orders

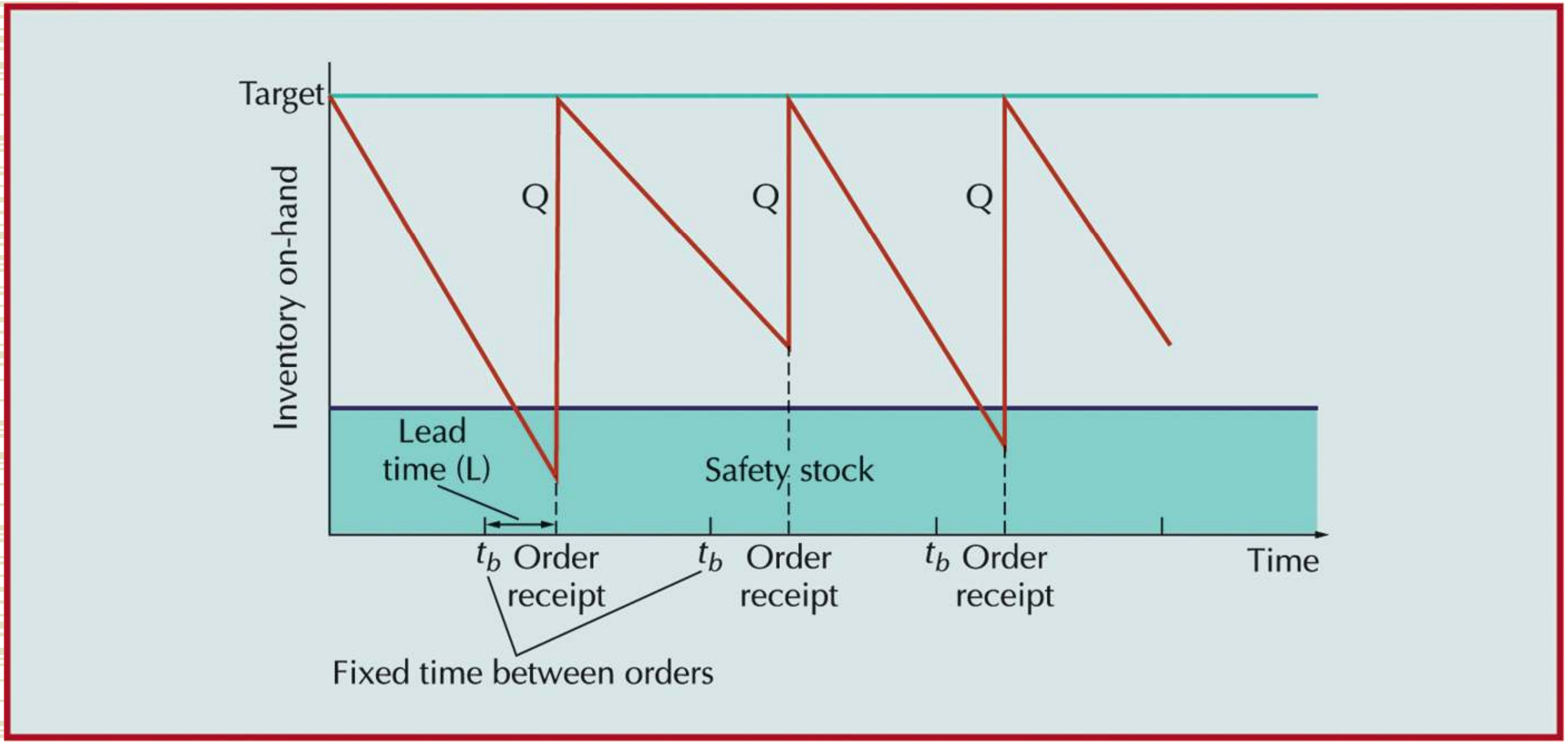
L = lead time

σ_d = standard deviation of demand

$z\sigma_d \sqrt{t_b + L}$ = safety stock

I = inventory level

Periodic Inventory System



Fixed-Period Model with Variable Demand

$\bar{d}$ = 6 packages per day
σ_d = 1.2 packages
t_b = 60 days
L = 5 days
I = 8 packages
z = 1.65 (for a 95% service level)

$$Q = \bar{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I$$

$$= (6)(60 + 5) + (1.65)(1.2)\sqrt{60 + 5} - 8$$

$$= 397.96 \text{ packages}$$

Fixed-Period Model with Excel