

Chapter 13

Inventory Management

Operations Management - 6th Edition

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Lecture Outline

- Elements of Inventory Management
- Inventory Control Systems
- Economic Order Quantity Models
- Quantity Discounts
- Reorder Point
- Order Quantity for a Periodic Inventory System

What Is Inventory?

- Stock of items kept to meet future demand
- Purpose of inventory management
 - how many units to order
 - when to order

Inventory and Supply Chain Management

- Bullwhip effect
 - demand information is distorted as it moves away from the end-use customer
 - higher safety stock inventories to are stored to compensate
- Seasonal or cyclical demand
- Inventory provides independence from vendors
- Take advantage of price discounts
- Inventory provides independence between stages and avoids work stoppages

Inventory and Quality Management in the Supply Chain

- Customers usually perceive quality service as availability of goods they want when they want them
- Inventory must be sufficient to provide high-quality customer service in QM

Types of Inventory

- Raw materials
- Purchased parts and supplies
- Work-in-process (partially completed) products (WIP)
- Items being transported
- Tools and equipment

Two Forms of Demand

Dependent

- Demand for items used to produce final products
- Tires stored at a Goodyear plant are an example of a dependent demand item
- Independent
 - Demand for items used by external customers
 - Cars, appliances, computers, and houses are examples of independent demand inventory



Inventory Costs

- Carrying cost
 - cost of holding an item in inventory
- Ordering cost
 - cost of replenishing inventory
- Shortage cost
 - temporary or permanent loss of sales when demand cannot be met



Inventory Control Systems

- Continuous system (fixedorder-quantity)
 - constant amount ordered when inventory declines to predetermined level
- Periodic system (fixed-timeperiod)
 - order placed for variable amount after fixed passage of time



ABC Classification

- Class A
 - 5 15 % of units
 - 70 80 % of value
- Class B
 - 30 % of units
 - 15 % of value
- Class C
 - 50 60 % of units
 - 5 10 % of value



ABC Classification: Example

PART	UNIT COST	ANNUAL USAGE
1	\$ 60	90
2	350	40
3	30	130
4	80	60
5	30	100
6	20	180
7	10	170
8	320	50
9	510	60
10	20	120

ABC Classification: Example (cont.)

PAF	TOTAL RT VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY	% CUMMULATIVE
9	\$30,600	35.9	6.0	6.0
8	3 16,000	18.7	5.0	11.0
2	2 14,000	16.4	4.0	A 15.0
1	5,400	6.3	9.0	24.0
_4	1 900	5.6	6.0	R 20.0
_	CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
-	CLASS A	ITEMS 9, 8, 2	% OF TOTAL VALUE 71.0	% OF TOTAL QUANTITY 15.0
-	CLASS A B	ITEMS 9, 8, 2 1, 4, 3	% OF TOTAL VALUE 71.0 16.5	% OF TOTAL QUANTITY 15.0 25.0
1	CLASS A B C	ITEMS 9, 8, 2 1, 4, 3 6, 5, 10, 7	% OF TOTAL VALUE 71.0 16.5 12.5	% OF TOTAL QUANTITY 15.0 25.0 60.0

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Economic Order Quantity (EOQ) Models



- optimal order quantity that will minimize total inventory costs
- Basic EOQ model
- Production quantity model

Assumptions of Basic EOQ Model

- Demand is known with certainty and is constant over time
- No shortages are allowed
- Lead time for the receipt of orders is constant
- Order quantity is received all at once

Inventory Order Cycle





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EOQ Cost Model

Deriving Q_{opt} $TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$ $\frac{\partial \mathrm{TC}}{\partial \mathbf{Q}} = -\frac{\mathbf{C}_o \mathbf{D}}{\mathbf{Q}^2} + \frac{\mathbf{C}_c}{2}$ $0 = -\frac{C_0 D}{Q^2} + \frac{C_c}{2}$ **Q**_{opt}



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EOQ Example

$$C_c = \$0.75 \text{ per gallon}$$
 $C_o = \$150$
 $D = 10,000 \text{ gallons}$
 $Q_{opt} = \sqrt{\frac{2C_oD}{C_c}}$
 $TC_{min} = \frac{C_oD}{Q} + \frac{C_cQ}{2}$
 $Q_{opt} = \sqrt{\frac{2(150)(10,000)}{(0.75)}}$
 $TC_{min} = \frac{(150)(10,000)}{2,000} + \frac{(0.75)(2,000)}{2}$
 $Q_{opt} = 2,000 \text{ gallons}$
 $TC_{min} = \$750 + \$750 = \$1,500$

 Orders per year = D/Q_{opt}
 Order cycle time = 311 days/ (D/Q_{opt})
 $= 10,000/2,000$
 $= 311/5$
 $= 5 \text{ orders/year}$
 $= 62.2 \text{ store days}$

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Production Quantity Model

- An inventory system in which an order is received gradually, as inventory is simultaneously being depleted
 - AKA non-instantaneous receipt model
 - assumption that Q is received all at once is relaxed
- p daily rate at which an order is received over time, a.k.a. *production rate*
- d daily rate at which inventory is demanded Copyright 2009 John Wiley & Sons, Inc.

Production Quantity Model (cont.)



Production Quantity Model (cont.)

p = production rate d = demand rate Maximum inventory level = $Q - \frac{Q}{d}$ $\frac{2C_o D}{\frac{d}{p}}$ $= Q\left(1 - \frac{d}{p}\right)$ $Q_{\text{opt}} = \mathcal{C}_c$ Average inventory level = $\frac{Q}{2} \left(1 - \frac{d}{p} \right)$ $TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} \left[1 - \frac{d}{p} \right]$

Production Quantity Model: Example

 $C_c = 0.75 per gallon $C_o = 150 D = 10,000 gallonsd = 10,000/311 = 32.2 gallons per day p = 150 gallons per day

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c \left(1 - \frac{d}{p}\right)}} = \sqrt{\frac{2(150)(10,000)}{0.75 \left(1 - \frac{32.2}{150}\right)}} = 2,256.8 \text{ gallons}$$

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} \left(1 - \frac{d}{p}\right) = $1,329$$

Production run = $\frac{Q}{p} = \frac{2,256.8}{150} = 15.05$ days per order

Production Quantity Model: Example (cont.)

Number of production runs = $\frac{D}{Q} = \frac{10,000}{2,256.8} = 4.43$ runs/year Maximum inventory level = $Q\left(1 - \frac{d}{p}\right) = 2,256.8\left(1 - \frac{32.2}{150}\right)$ = 1,772 gallons

Solution of EOQ Models with Excel

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Solution of EOQ Models with Excel (Con't)

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Solution of EOQ Models with OM

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Quantity Discounts

Price per unit decreases as order quantity increases

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

where

P = per unit price of the item *D* = annual demand





Quantity Discount: Example

QUANTITY	PRICE
1 - 49	\$1,400
50 - 89	1,100
90+	900

 $C_o = $2,500$ $C_c = 190 per TV D = 200 TVs per year

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ TVs}$$

For Q = 72.5 $TC = \frac{C_o D}{Q_{opt}} + \frac{C_c Q_{opt}}{2} + PD = $233,784$ For Q = 90

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD = $194,105$$

Quantity-Discount Model Solution with Excel

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Reorder Point

Level of inventory at which a new order is placed

R = dL

where

d = demand rate per period *L* = lead time

Reorder Point: Example

Demand = 10,000 gallons/year Store open 311 days/year Daily demand = 10,000 / 311 = 32.154 gallons/day Lead time = L = 10 days

R = dL = (32.154)(10) = 321.54 gallons

Safety Stocks

- Safety stock
 - buffer added to on hand inventory during lead time
- Stockout
 - an inventory shortage
- Service level
 - probability that the inventory available during lead time will meet demand

Variable Demand with a Reorder Point Q **nventory** level Reorder point, R 0 LT LT Time



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Reorder Point With Variable Demand

$$R = \overline{dL} + z\sigma_d \sqrt{L}$$

where

- *d* = average daily demand
- L = lead time
- σ_d = the standard deviation of daily demand
 - z = number of standard deviations corresponding to the service level probability

probability $z\sigma_d\sqrt{L}$ = safety stock

Reorder Point for a Service Level



Reorder Point for Variable Demand

The paint store wants a reorder point with a 95% service level and a 5% stockout probability

d = 30 gallons per day L = 10 days $\sigma_d = 5$ gallons per day

For a 95% service level, z = 1.65

$$R = \overline{dL} + z \,\sigma_d \sqrt{L}$$

= 30(10) + (1.65)(5)($\sqrt{10}$)

= 326.1 gallons

Safety stock = $z \sigma_d \sqrt{L}$ = (1.65)(5)($\sqrt{10}$) = 26.1 gallons

Determining Reorder Point with Excel

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Order Quantity for a Periodic Inventory System

$$Q = \overline{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I$$

where

- *d* = average demand rate
- t_b = the fixed time between orders
- L = lead time
- σ_d = standard deviation of demand

$$z\sigma_d \sqrt{t_b + L}$$
 = safety stock

I = inventory level

Periodic Inventory System



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Fixed-Period Model with Variable Demand

- d = 6 packages per day
- σ_d = 1.2 packages
- *t_b* = 60 days
- L = 5 days
 - I = 8 packages
- *z* = 1.65 (for a 95% service level)

$$Q = \overline{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I$$

= (6)(60 + 5) + (1.65)(1.2) $\sqrt{60 + 5}$ -

= 397.96 packages

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Fixed-Period Model with Excel

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