[2008] [01-2][02-1][02-2]

# Planning Procedure of Naval Architecture & Ocean Engineering

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# Part 1. Stability & Trim

## [01-2],[02-1],[02-2] Hydrostatic Pressure and Force/Moment on a floating body

# 부력(Buoyancy) “Archimedes Principle”

- **Archimedes’ Principle**
  - **“유체 속에 있는 물체가 받는 부력의 크기는 그 물체가 밀어낸 유체의 무게와 같고 그 방향은 중력과 반대 방향이다”**

① 아르키메데스의 원리

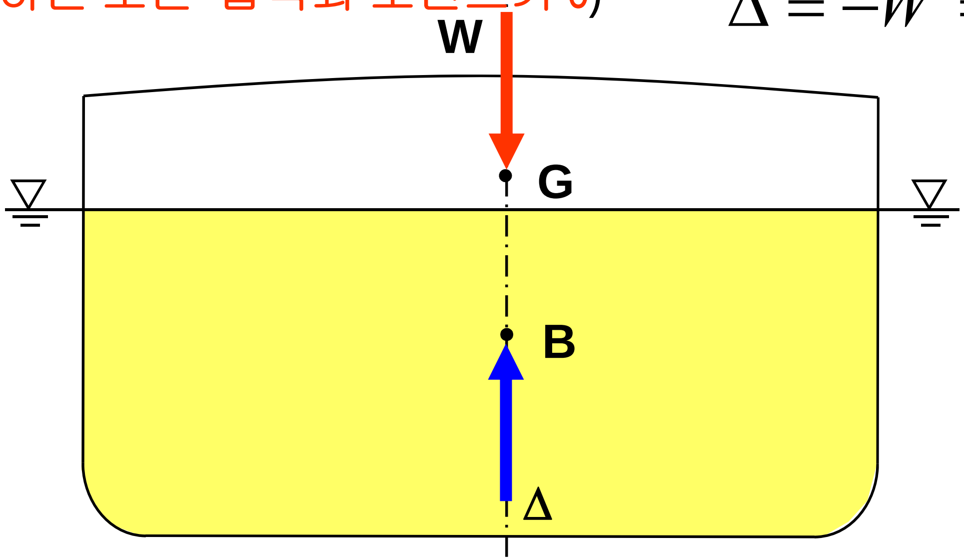
물체의 부력 = 물체가 밀어낸 유체의 중량(배수량; **Displacement**)

② 평형 상태 (이 물체에 작용하는 모든 합력과 모멘트가 0)

$$\Delta = -W = \rho g V$$

물체의 부력 - 물체의 중량=0

③∴배수량 = 물체의 중량



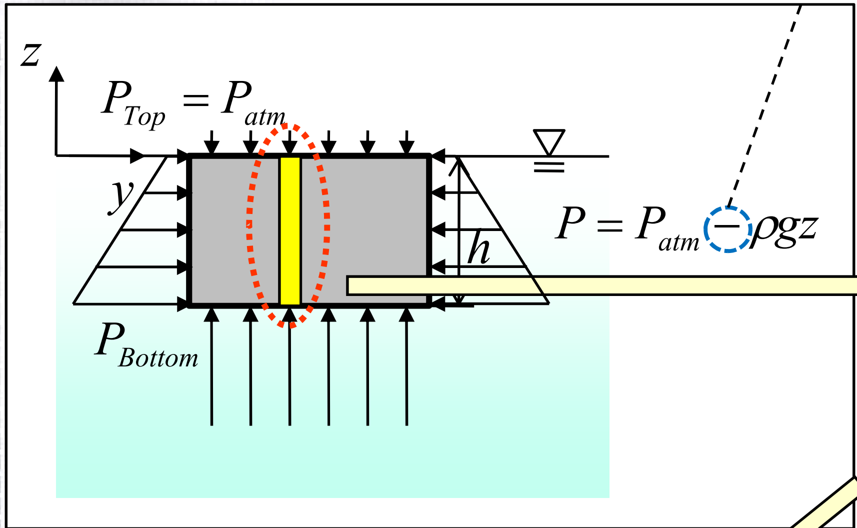

G: 중심, B: 부심
W: 중력, Δ: 부력
ρ: 유체의 밀도
V: 유체 속에 잠겨있는 물체의 부피

# 압력의 개념을 도입한 부력계산

※ 압력(Pressure) : 단위 면적에 수직으로 작용하는 힘
즉, 힘을 구하기 위해서는 압력에 면적과
그 작용면의 법선 벡터(Normal Vector)를 곱해야 함

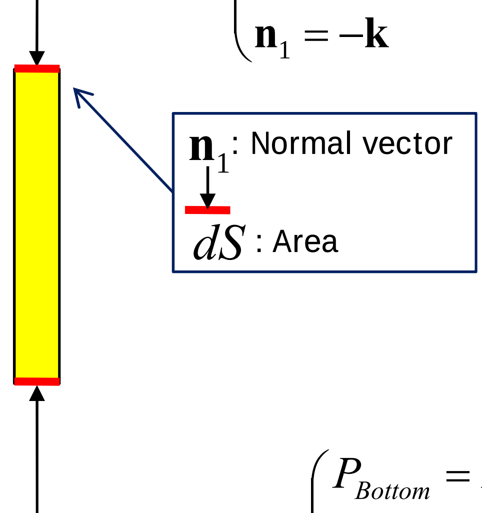
좌표계 설정에 따라 z값이 (-)이므로( -)를 붙임
→ 아래쪽으로 갈수록 압력이 커짐

✓ 아래 물체에 작용하는 수직방향의 정적인 힘은?




: 물체 윗면의 미소 면적에 작용하는 힘

$$d\mathbf{F}_{Top} = P_{Top} \cdot \mathbf{n}_1 dS \quad \begin{pmatrix} P_{Top} = P_{atm} - \rho g \cdot 0 \\ \mathbf{n}_1 = -\mathbf{k} \end{pmatrix}$$




$$d\mathbf{F}_{Bottom} = P_{Bottom} \cdot \mathbf{n}_2 dS \quad \begin{pmatrix} P_{Bottom} = P_{atm} - \rho g h \\ \mathbf{n}_2 = \mathbf{k} \end{pmatrix}$$

: 물체 아랫면의 미소 면적에 작용하는 힘

$$\begin{aligned} d\mathbf{F} &= d\mathbf{F}_{Top} + d\mathbf{F}_{Bottom} \\ &= P_{Top} \cdot \mathbf{n}_1 dS + P_{Bottom} \cdot \mathbf{n}_2 dS \\ &= \cancel{P_{atm}(-\mathbf{k})dS} + (\cancel{P_{atm}} - \rho g h)\mathbf{k} dS \\ &= -\rho g h \mathbf{k} dS = \mathbf{k}(-\rho g h \cdot dS) \end{aligned}$$

: 대기압에 의한 힘이 서로 상쇄됨

$$\mathbf{F} = \int d\mathbf{F} = \iint_{S_B} P \mathbf{n} dS \quad , (P = \underline{P_{static} = -\rho g z})$$

전체 정적 압력에서 대기압을 
뺀 순수 유체 정압력

$$= -\rho g \iint_{S_B} \mathbf{n} z dS$$

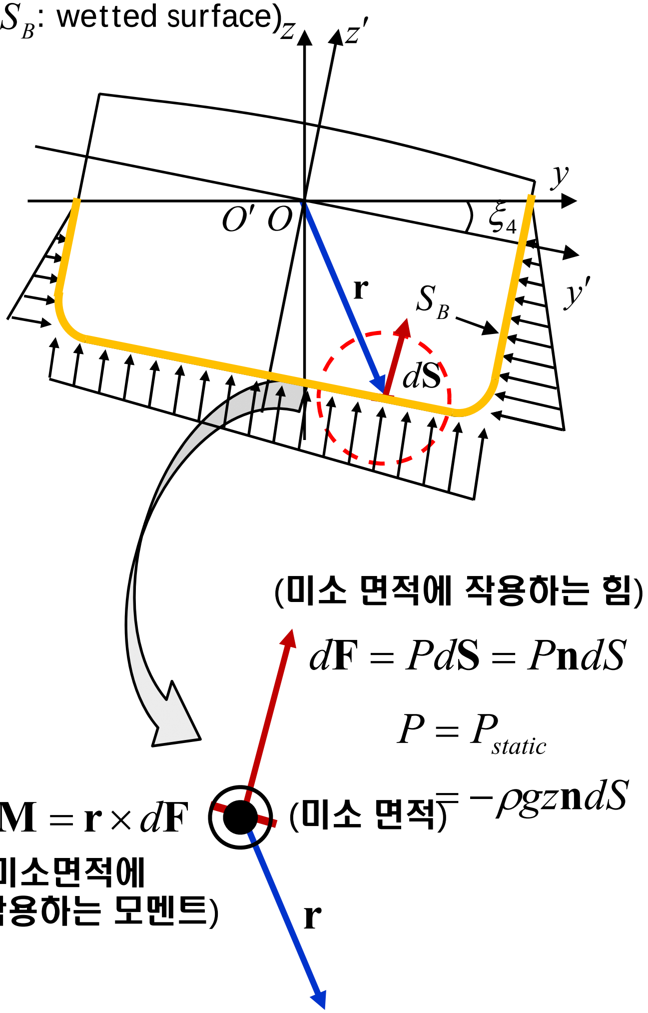
$$(P_{atm} - \rho g z) - P_{atm} = -\rho g z$$

선형화 한 Bernoulli eq.

$$P = \underbrace{-\rho g z}_{P_{static}} \underbrace{- \rho \frac{\partial \Phi}{\partial t}}_{P_{dynamic}}$$

# 선박에 작용하는 유체의 정적 압력(Hydrostatic pressure)과 부력(Buoyancy) -수학적 접근

**좌현으로 기울어진 상태**
**(선박을 정면에서 바라봄)**
( $S_B$: wetted surface)




**왜 r이 먼저 오는가? (좌표축에서 양의 방향을 고려함)**

- Hydrostatic force (Surface force)

: **표면에 작용하는 모든 힘을 적분하여 구함**

✓ **미소 면적에 작용하는 힘** :

$$d\mathbf{F} = P \cdot d\mathbf{S} = P \cdot \mathbf{n}dS$$

**여기서** $P$**는** hydro static pressure, $P_{\text{static}}$**이다**.

$$P = P_{static} = -\rho gz$$

$$d\mathbf{F} = P_{static} \cdot \mathbf{n}dS = -\rho gz \cdot \mathbf{n}dS$$

✓ Total force : ( $S_B$: wetted surface)

$$\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \Rightarrow \boxed{\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS}$$

- Hydrostatic Moment : **(모멘트)=(거리)** X **(힘)**

✓ **미소 면적에 작용하는 모멘트** :

$$d\mathbf{M} = \mathbf{r} \times d\mathbf{F} = \mathbf{r} \times P\mathbf{n}dS = P(\mathbf{r} \times \mathbf{n})dS$$

✓ Total moment :

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r} \times \mathbf{n})dS \Rightarrow \boxed{\mathbf{M} = -\rho g \iint_{S_B} z(\mathbf{r} \times \mathbf{n})dS}$$

# 선박에 작용하는 유체의 정적 압력(Hydrostatic pressure)과 부력(Buoyancy) -수학적 접근

✓ Hydrostatic force (Surface force)

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(p458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(p414~417)

$$\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS$$

( $S$ : wetted surface)

divergence theorem[1] 사용하면,

$$\left( \iint_S f \cdot \mathbf{n}dA = \iiint_V \nabla f dV \right)$$

$$\mathbf{F} = \rho g \iiint_V \nabla z dV \qquad \left( \nabla z \overset{2)}{=} \frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} + \frac{\partial z}{\partial z}\mathbf{k} = \mathbf{k} \right)$$

$$= \mathbf{k}\rho g \iiint_V dV$$

$$= \mathbf{k}\rho g V(t)$$

선박의 운동 시, 시간에 따라 배수 용적 $V$가 달라지므로, $V$는 시간에 대한 함수 $V(t)$임

: 물에 잠긴 물체의 부피에 해당하는 물의 무게를 수면과 수직 방향으로 받음

(≒아르키메데스의 원리)

※ (-)부호가 사라진 이유

: Divergence theorem은 면의 외향 단위 벡터를 기준으로 한다.
부력 계산시 사용하는 Normal vector는 내향 단위 벡터이므로,
(–)를 곱한 뒤, Divergence theorem을 적용해야 한다.

# 유체입자에 작용하는 압력/힘

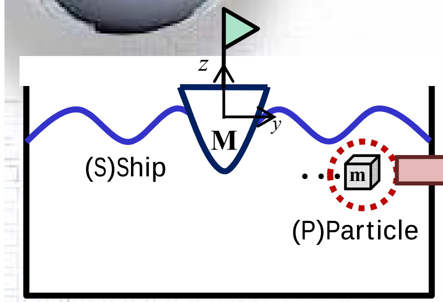
$\mathbf{x}$: 유체입자의 시간에 따른 변위

$\mathbf{V} = \frac{d\mathbf{x}}{dt}, \mathbf{a} = \frac{d^2\mathbf{x}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_{D}$: Diffraction force
$\mathbf{F}_{R}$: Radiation force

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bendidng Moment

✓ Assumption
① 뉴턴 유체 (Newtonian fluid)
② Stokes Assumption
③ 비점성 유동 (Invicid flow)
④ 비회전 유동 (Irrotational flow)




미시적 유도/
거시적 유도(RTT[1])

✓유체입자의 운동방정식

Newton's 2nd Law

$$m\ddot{\mathbf{x}} = \sum \mathbf{F}_P = \mathbf{F}_{body,P} + \mathbf{F}_{surface,P}$$

유체입자 6자유도

Lagrangian & Eulerian Description

Cauchy equation → ①, ② → Navier-Stokes equation (전단 응력) → ③ → Euler equation → ④ → Bernoulli equation

유체입자 3자유도 (회전 3자유도 X)

Curl과 회전

$$\frac{\partial \Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = F(t)$$

Mass Conservation Law → ②④ → Laplace Equation

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

Velocity potential

# 유체 입자의 운동 방정식 정리
# (Cauchy eq. ~ Bernoulli eq.)

1) 전단응력이 전단변형율(의 시간변화율)에 비례하는 유체
2) 선형 변형과 등방 팽창에 의한 점성 계수 $\mu,\lambda$의 관계식을 정의함

Cauchy Equation : $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}+\nabla\bullet\sigma \quad ,\left(\mathbf{V}=[u,v,w]^T\right)$

① 뉴턴 유체[1] (Newtonian fluid)
② Stokes assumption[2]

Navier-Stokes Equation : (in general form) $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}-\nabla P+\mu\left(\frac{1}{3}\nabla(\nabla\bullet\mathbf{V})+\nabla^2\mathbf{V}\right)$

$(\mu=0)$ ③ 비점성 유동 (Invicid flow)

Euler Equation : $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}-\nabla P$

$\rho=\rho(P)$ ④ barotropic flow

Euler Equation : (Another form) $\frac{\partial\mathbf{V}}{\partial t}+\nabla B=\mathbf{V}\times\boldsymbol{\omega} \quad ,\left(B=\frac{1}{2}q^2+gy+\int\frac{dP}{\rho} \;,\; q^2=u^2+v^2+w^2\right)$

$\left(\frac{\partial\mathbf{V}}{\partial t}=0\right)$ ⑤ Steady flow

Bernoulli equation (case1) $B=\text{Constant}$

$\left(\frac{1}{2}q^2+gy+\int\frac{dP}{\rho}=C\right)$

along streamlines and vortex lines

$\left(\mathbf{V}=\nabla\Phi \;\; q^2=|\nabla\Phi|^2,\boldsymbol{\omega}=0\right)$ ⑥ Unsteady, irrotational flow

Bernoulli equation (case2) $\frac{\partial\Phi}{\partial t}+\frac{1}{2}|\nabla\Phi|^2+gy+\int\frac{dP}{\rho}=F(t)$

$(\rho=constant)$ ⑦ Incompressible flow

Bernoulli equation (case3) $\frac{\partial\Phi}{\partial t}+\frac{1}{2}|\nabla\Phi|^2+gy+\frac{P}{\rho}=F(t)$

Continuity Equation(질량보존) $\frac{\partial\rho}{\partial t}+\nabla\bullet\rho\mathbf{V}=0$

⑦ incompressible flow

$\rho=constant\left(\frac{\partial\rho}{\partial t}=0\right)$

$\nabla\bullet\mathbf{V}=0$

⑥ irrotational flow

$(\mathbf{V}=\nabla\Phi)$

Laplace Equation $\nabla^2\Phi=0$

Newtonian fluid
Stokes assumption
Invicid flow
Unsteady flow
Irrotational flow
Incompressible flow

# Bernoulli equation의 F(t)의 의미와 계기 압력[1]

**Bernoulli Equation**

$$\cancel{\frac{\partial \Phi}{\partial t}} + \frac{P}{\rho} + \cancel{\frac{1}{2}|\nabla\Phi|^2} + g\,z = F(t)$$

☞ **항등식**

✓ **만약 정적인 상태인 경우를 고려하면**

$$\Phi = 0 \quad \left( \frac{\partial \Phi}{\partial t} = 0, \nabla\Phi = 0 \right)$$

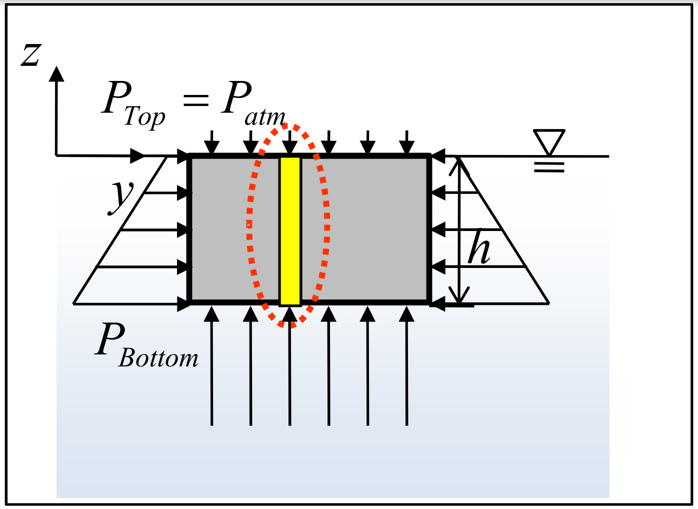
$$\frac{P}{\rho} + g\,z = F(t)$$

**항등식 이므로,** $z = 0$ **일 때도 성립**

$$\frac{P_{atm}}{\rho} = F(t)$$

$z=0$일 때 압력 = 대기압($P_{atm}$)

$$\therefore \frac{\partial \Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + g\,z = \frac{P_{atm}}{\rho}$$




✓ **물체 바닥에서의 압력은?**

$$\frac{\partial \Phi}{\partial t} + \frac{P_{Bottom}}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = \frac{P_{atm}}{\rho}$$

$$\frac{\partial \Phi}{\partial t} + \frac{\cancel{P_{atm}} + P_{Fluid}}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = \frac{\cancel{P_{atm}}}{\rho}$$

$$\therefore \frac{\partial \Phi}{\partial t} + \frac{\boxed{P_{Fluid}}}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = 0$$

**'계기 압력'**

※ Bernoulli equation**에서 우변** $F(t)=0$ **으로 표기한 경우,**
**압력 P는 대기압을 뺀 유체만의 압력임을 의미함**

# 유체입자에 작용하는 압력/힘

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

✓ Assumption
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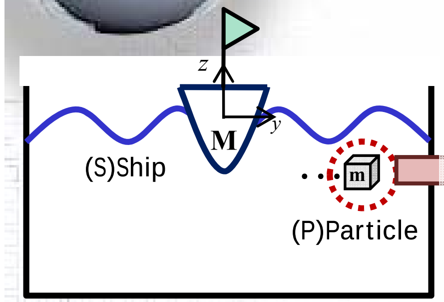


✓유체입자의 운동방정식

Lagrangian & Eulerian Description

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$m\ddot{\mathbf{x}} = \sum \mathbf{F}_P$
$= \mathbf{F}_{body,P} + \mathbf{F}_{surface,P}$

유체입자 6자유도

Cauchy equation ①,②

전단 응력

Navier-Stokes equation ③

유체입자 3자유도 (회전 3자유도 X)

Euler equation ④

Curl과 회전

Bernoulli equation

미시적 유도/ 거시적 유도(RTT 1))

Mass Conservation Law ②④

Laplace Equation

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\frac{\partial\Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = F(t)$$

Velocity potential

✓유체력 계산

$\Phi_T = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction velocity potential)
$+\Phi_R$ (Radiation velocity potential)

Linearization

$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho\frac{\partial\Phi_T}{\partial t}$$

( $S_B$: wetted surface)

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

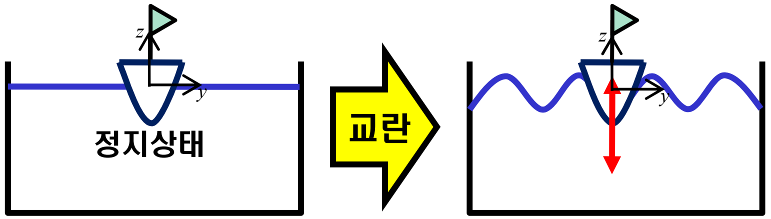
1) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 12.1 (pp 535~538)

# 파랑 중 선박이 받는 힘

✓ Laplace Equation

$$\nabla^2\Phi = 0 \quad ,(\mathbf{V} = \nabla\Phi)$$

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

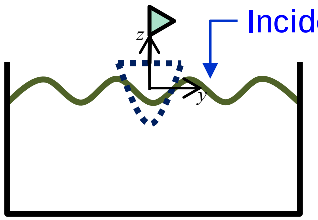



✓ **파랑 중 선박 주위 유체의 운동**

: 유체장의 운동으로 인해 유체 입자의 속도,가속도,압력이 변하게 되고, 선박 표면의 유체 입자가 선박에 가하는 압력도 변하게 된다.

**선형화[1]된 wave로 분해**

Incident wave velocity potential $(\Phi_I)$

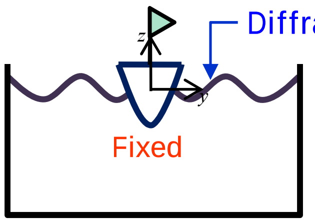


✓ 입사파가 선박에 의해 교란되지 않는다고 가정함

➔ 입사파에 의한 velocity potential

+

Diffraction velocity potential $(\Phi_D)$

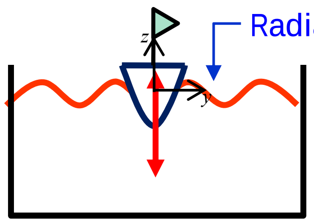



✓ 선박의 존재로 인하여 교란된 파(wave). 물체 고정

➔ 산란파에 의한 velocity potential

+

Radiation velocity potential $(\Phi_R)$



✓ 정수 중에서 선박의 강제 진동으로 인해 발생하는 파 (wave)

➔ 기진력에 의한 velocity potential

✓ Total Velocity Potential

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

Superposition Theorem

Laplace equation은 선형 방정식이므로, 각의 해를 더한 것 (superposition)도 해가 된다[1].

$$\cancel{\frac{\partial^2\Phi}{\partial x^2}}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

$$\Rightarrow \frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

# 선박에 작용하는 유체력

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

$\Phi_I$ : Incident wave velocity potential
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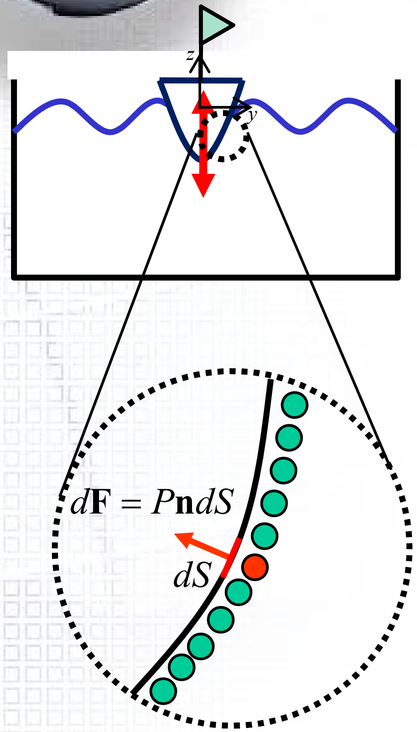


✓ Bernoulli Equation

$$\rho\frac{\partial\Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho\left|\nabla\Phi\right|^2} + \rho gz = 0$$

Linearization

✓ Laplace Equation

$$\nabla^2\Phi = 0$$

$$\Phi = \Phi_I + \Phi_D + \Phi_R$$

Linear combination of Basic solutions

Basic solutions

$$P_{Fluid} = -\rho gz - \rho\frac{\partial\Phi}{\partial t} = \boxed{-\rho gz} - \rho\left(\frac{\partial\Phi_I}{\partial t} + \frac{\partial\Phi_D}{\partial t} + \frac{\partial\Phi_R}{\partial t}\right)$$

유체입자가 표면에 작용하는 압력

$$= \boxed{P_{static}} + \underbrace{P_{F.K} + P_D + P_R}_{P_{dynamic}}$$

$$\mathbf{F}_{Fluid} = \iint_{S_B} P_{Fluid}\,\mathbf{n}dS = \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

( $S_B$: wetted surface)

$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

$dS$ : 미소 면적

$\mathbf{n}$ : 미소 면적의 Normal 벡터

선박의 침수 표면 전체에 대하여 적분
(유체입자가 선박에 작용하는 힘과 모멘트)

✓ 유체입자 하나에 작용하는 Body force 와 Surface force로부터 구한 압력을 선박의 침수 표면 전체에 대해 적분하여 선박에 작용하는 유체력을 계산함

# 선박의 6자유도 운동 방정식 유도

$F_{F.K}$: Froude- krylov force
$F_D$: Diffraction force
$F_R$: Radiation force

✓ Assumption
① 뉴턴 유체 (Newtonian fluid)
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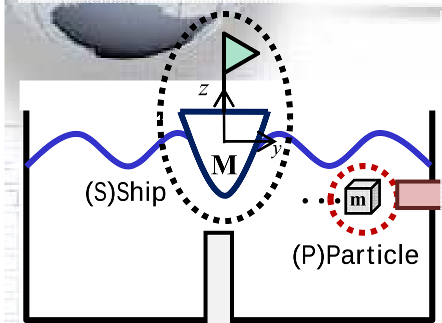
③ 비점성 유동 (Invicid flow)
④ 비회전 유동 (Irrotational flow)

$\mathbf{x}$: 유체입자의 시간에 따른 변위

$\mathbf{V} = \frac{d\mathbf{x}}{dt}, \mathbf{a} = \frac{d^2\mathbf{x}}{dt^2}$

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bendidng Moment




✓유체입자의 운동방정식

Lagrangian & Eulerian Description

Newton's 2nd Law

$$m\ddot{\mathbf{x}} = \sum \mathbf{F}_P = \mathbf{F}_{body,P} + \mathbf{F}_{surface,P}$$

유체입자 6자유도

Cauchy equation ①, ②

전단 응력

Navier-Stokes equation ③

유체입자 3자유도 (회전 3자유도 X)

Euler equation ④

Curl과 회전

Bernoulli equation

미시적 유도/ 거시적 유도(RTT[1])

Mass Conservation Law ②④

Laplace Equation

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\frac{\partial\Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = F(t)$$

## ✓선박의 6자유도 운동 방정식

① Coordinate system 정의
(Global & Body-fixed coordinate)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F}_S = \mathbf{F}_{body,S} + \mathbf{F}_{surface,S}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{Fluid}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$
$$= \mathbf{F}_{restoring} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

$$\boxed{(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} = \mathbf{F}_{restoring} + \mathbf{F}_{exciting}}$$

cf) 선형화 된 복원력 ($\mathbf{F}_{restoring} = -\mathbf{C}\mathbf{x}$)

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

z방향 성분

Velocity potential

$\Phi_T = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction velocity potential)
$+\Phi_R$ (Radiation velocity potential)

✓유체력 계산

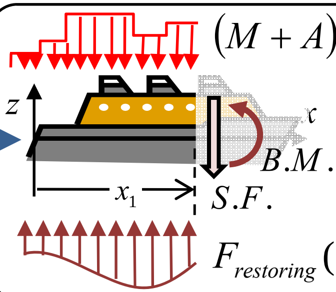
Linearization

$$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$$

$$P = -\rho g z - \rho\frac{\partial\Phi_T}{\partial t}$$

($S_B$: wetted surface)

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$




$(M+A)a_z(x)$

($a_z$ : z방향 가속도)
($v_z$ : z방향 속도)

$F_{restoring}(x) + F_{exciting}(x) - Bv_z(x)$

✓Shear force(S.F.) 및 bending moment(B.M.)

Shear force(S.F.) 계산

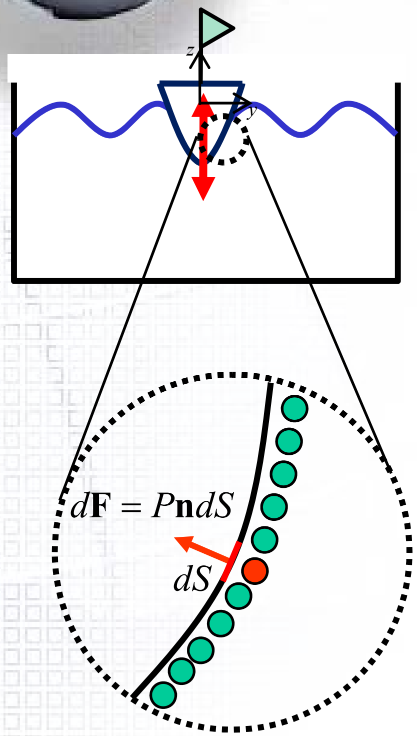
🡻(적분)

Bending moment(B.M.)

# 선박의 6자유도 운동 방정식 유도

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_{D}$: Diffraction force
$\mathbf{F}_{R}$: Radiation force

$\Phi_I$ : Incident wave velocity potential
$\Phi_D$ : Diffraction potential
$\Phi_R$ : Radiation potential




$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

$dS$ : 미소 면적

$\mathbf{n}$ : 미소 면적의 Normal 벡터

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi]^T$$

$x$: surge  $\phi$: roll
$y$: sway  $\theta$: pitch
$z$: heave  $\psi$: yaw

$\mathbf{M}_A$: 6×6 added mass matrix
$\mathbf{B}$: 6×6 damping coeff. matrix
$\mathbf{C}$: 6×6 restoring coeff. matrix

### ✓ 선박에 작용하는 유체력

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

: 유체입자 하나에 작용하는 (Body force) 와 (Surface force)로부터 구한 압력을 선박의 침수 표면 전체에 대해 적분하여 선박에 작용하는 유체력을 계산함

### ✓ 선박의 6자유도 운동방정식

Newton's 2nd Law

선박의 Surface force로 작용

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \text{(Body Force)} + \text{(Surface Force)}$$

$$= \mathbf{F}_{gravity} + \boxed{\mathbf{F}_{Fluid}} + \mathbf{F}_{external}$$

Body force  Surface force  기타 외부에서 작용하는 외력

$$\mathbf{M}\ddot{\mathbf{x}} = \boxed{\mathbf{F}_{Gravity}} + \boxed{\mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R} \boxed{+\mathbf{F}_{external,dynamic} + \mathbf{F}_{external,static}}$$

$\mathbf{F}_{\text{Restoring}}$  $\mathbf{F}_{\text{wave exciting}}$

$$\mathbf{F}_R = -\mathbf{M}_A\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

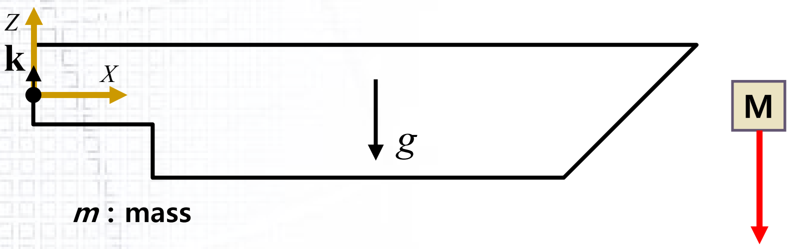
added mass  Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = (\mathbf{F}_{gravity} + \mathbf{F}_{static}) + \mathbf{F}_{wave\ exciting} - \mathbf{M}_A\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}} + \mathbf{F}_{external,dynamic} + \mathbf{F}_{external,static}$$

Linearization $,(F_{restoring} = (\mathbf{F}_{gravity} + \mathbf{F}_{static}) \approx -\mathbf{C}\mathbf{x})$

$$(\mathbf{M} + \mathbf{M}_A)\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{wave\ exciting} + \mathbf{F}_{external,dynamic} + \mathbf{F}_{external,static}$$

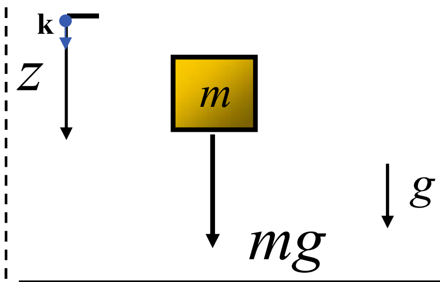
# Ex) Heave motion of ship – step 1




$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

✓ **Mass-Spring-Damper system**

①



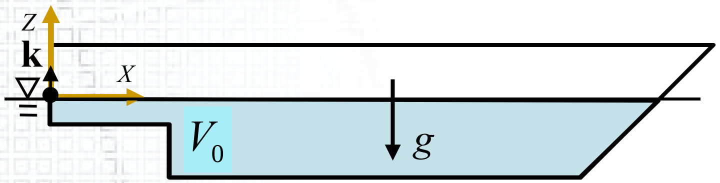

**By Newton's 2nd law,**

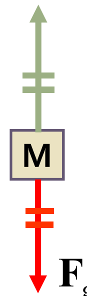
$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k}$$

# Ex) Heave motion of ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$




$\mathbf{F}_{gravity}$

$= -mg\mathbf{k}$

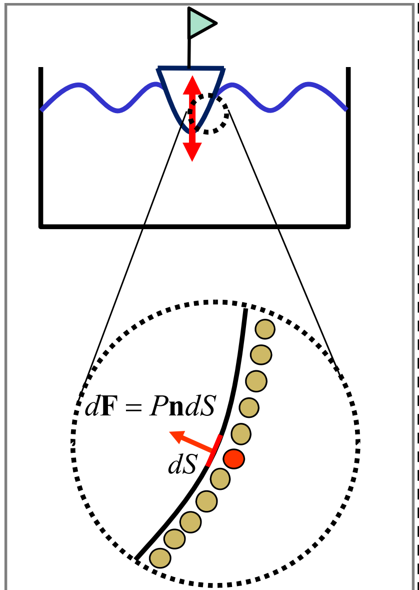
*m* : mass

$\rho$ : density of sea water

$V_0$: submerged volume

$S_B$ : submerged surface area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k}$$
$$= 0 \ (\because \ddot{z} = 0)$$
: static equilibrium

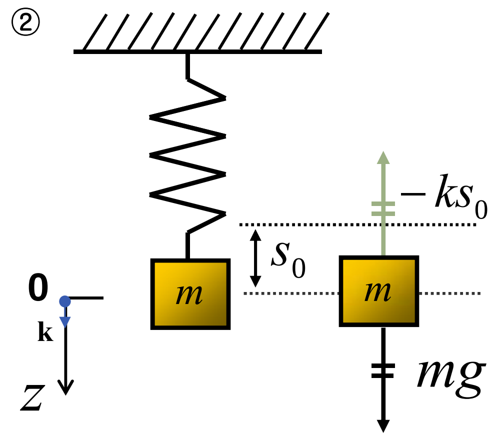



$dS$ : infinitesimal submerged surface area

$d\mathbf{F}$ : force exerted by the infinitesimal fluid element on $dS$

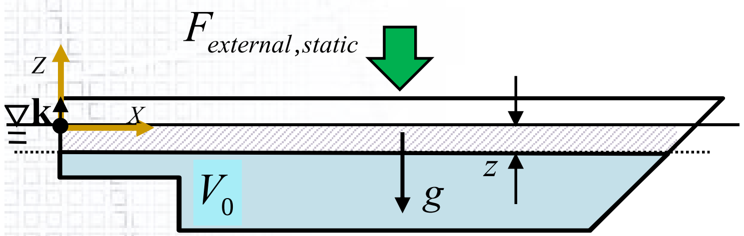
$\mathbf{n}$ : normal vector of $dS$

✓ Mass-Spring-Damper system

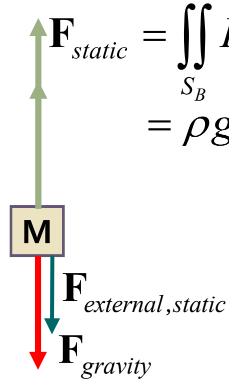



$$mz'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \ (\because z'' = 0)$$
: static equilibrium

# Ex) Heave motion of ship – step 3




**m** : mass
**$V_0$**: submerged volume
**$S_B$** : submerged surface area
**Awp** : waterplane area
**ρ** : density of sea water




$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\ bouyancy}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

if, z is small

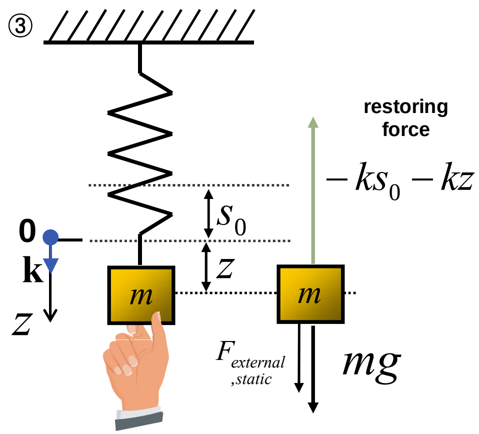
$$\mathbf{F}_{addtional\ bouyancy}$$
$$= -\rho g A_{WP}\mathbf{z}$$
$$= -k\mathbf{z}$$
$$, k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -k\mathbf{z} + \mathbf{F}_{external,static}$$
$$= 0 \ (\because \ddot{z} = 0)$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

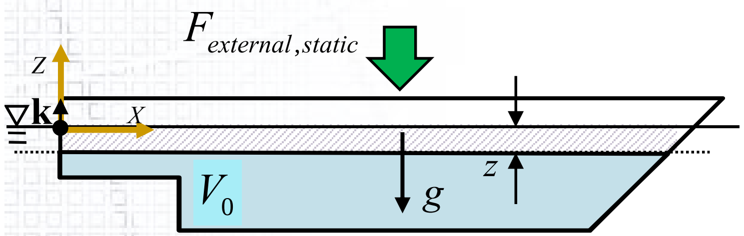
✓ **Mass-Spring-Damper system**

③

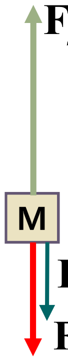



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= 0 \quad (\because z'' = 0)$$

# Ex) Heave motion of ship – step 4




**$m$** : mass  **$\rho$** : density of sea water
**$V_0$**: submerged volume
**$S_B$** : submerged surface area
**$Awp$** : waterplane area




$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\, bouyancy}$$

additional buoyancy caused by additional displacement z

$\mathbf{F}_{external,static}$

$\mathbf{F}_{gravity}$

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\, bouyancy}$$

$$= -\rho g A_{WP}\mathbf{z}$$

$$= -k\mathbf{z}$$

$$, k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$

$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$

$$= -k\mathbf{z} + \mathbf{F}_{external,static} \quad , k = \rho g A_{WP}$$

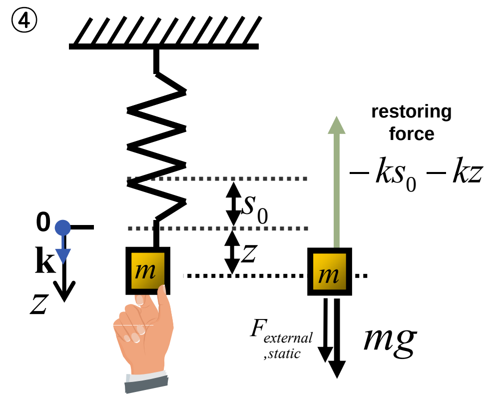
$$= 0 \ (\because \ddot{z} = 0)$$

**Linearized Restoring Force**

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

**✓ Mass-Spring-Damper system**

④




$$m\mathbf{z}'' = \mathbf{F}$$

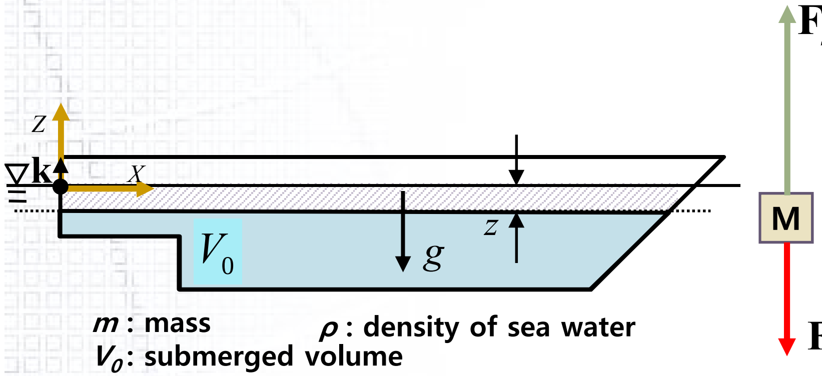
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} \quad + \mathbf{F}_{external,static}$$

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

Oscillation by the restoring force

# Ex) Heave motion of ship – step 4




$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\,\mathbf{n}\,dS = \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z} = \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

***m*** : mass  ***ρ*** : density of sea water

***$V_0$***: submerged volume

***$S_B$*** : submerged surface area

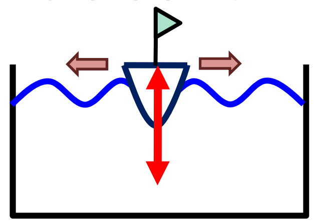
***Awp*** : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F} = \mathbf{F}_{gravity} + \mathbf{F}_{static} = -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} = -\rho g A_{wp}\mathbf{z} = -k\mathbf{z}$$

**Ship will oscillate forever?**

**Energy is dissipated by radiation wave**

**정수 중 선박의 강제 운동에 의해 발생한 힘**



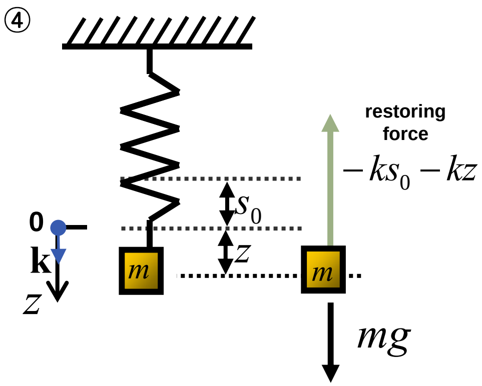
**Radiation Force**

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\,\mathbf{n}\,dS$$

✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**

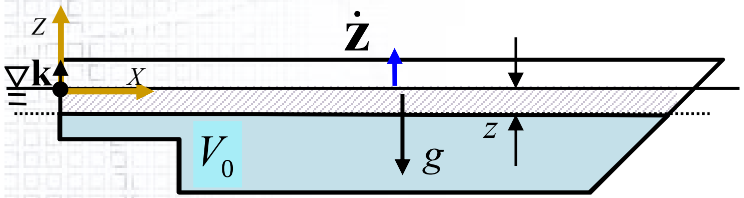
④



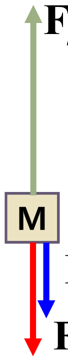

$$m\mathbf{z}'' = \mathbf{F} = mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} = -kz\mathbf{k}$$

$$\boxed{m\mathbf{z}'' + k\mathbf{z} = 0}$$ Oscillation by the restoring force

# Ex) Heave motion of ship – step 5




$m$ : mass  $\rho$ : density of sea water
$V_0$: submerged volume
$S_B$ : submerged surface area
$Awp$ : waterplane area



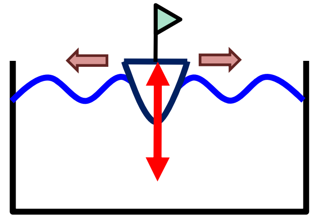

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}}$$
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}}$$
$$= -k\mathbf{z} - c\dot{\mathbf{z}}$$

정수 중 선박의 강제 운동에 의해 발생한 힘



**Radiation Force**

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$
$$= -c\dot{\mathbf{z}}$$

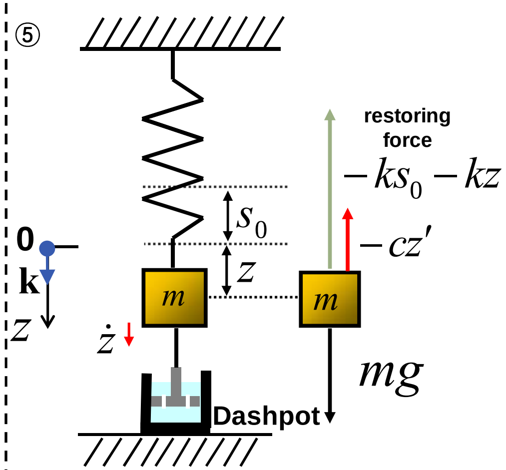
opposite to velocity

$c$ : damping coefficient

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

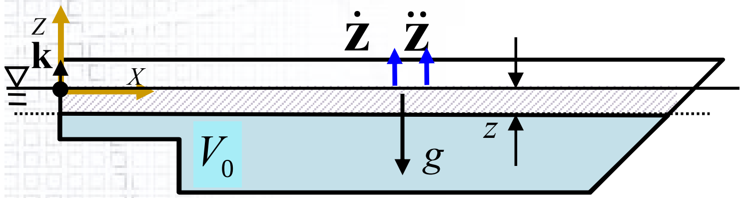
✓ **Mass-Spring-Damper system**

⑤




$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

# Ex) Heave motion of ship – step 5




$m$ : mass  $\rho$ : density of sea water
$V_0$: submerged volume
$S_B$ : submerged surface area
$Awp$ : waterplane area

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z} = \rho g V_0 \mathbf{k} - k\mathbf{z}$$

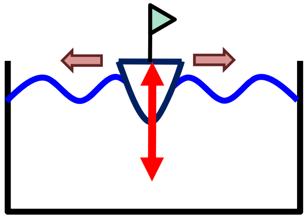
M

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
$$= -k\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

정수 중 선박의 강제
운동에 의해 발생한 힘



Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

opposite to velocity

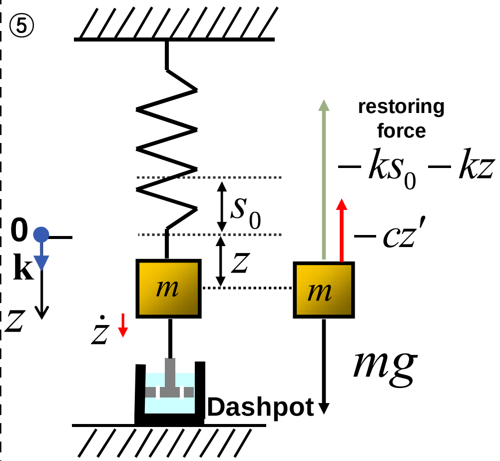
opposite to acceleration

$c$ : damping coefficient
$m_a$ : added mass

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

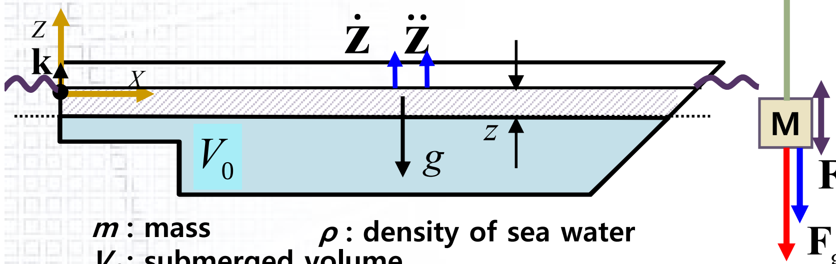
✓ Mass-Spring-Damper system

⑤




$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

# Ex) Heave motion of ship – step 6




$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0\mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0\mathbf{k} - k\mathbf{z}$$

$\mathbf{F}_{exciting}$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

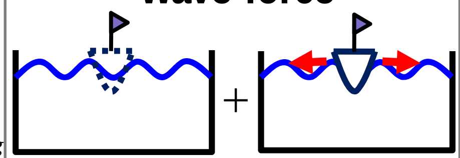
$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

***m*** : mass
***ρ*** : density of sea water
***$V_0$***: submerged volume
***$S_B$*** : submerged surface area
***Awp*** : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation} + \mathbf{F}_{exciting}$$
$$= -mg\mathbf{k} + \rho g V_0\mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$
$$= -k\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$

***c*** : damping coefficient
***$m_a$*** : added mass

## Wave force



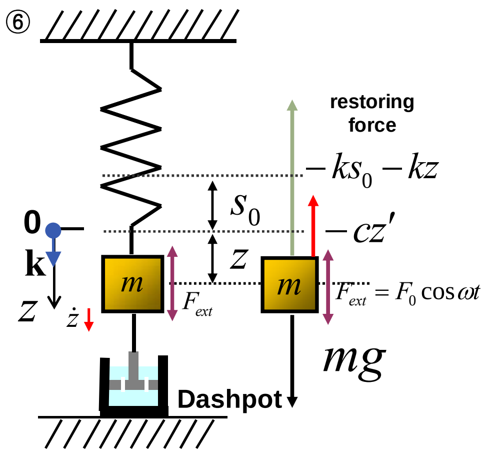

Froude-Kriloff Force    Diffraction Force

$$\mathbf{F}_{wave\ exciting} = \iint_{S_B} P_{wave\ exciting}\mathbf{n}dS$$
$$\left(= \mathbf{F}_{exciting}\right)$$

✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

✓ **Mass-Spring-Damper system**

⑥

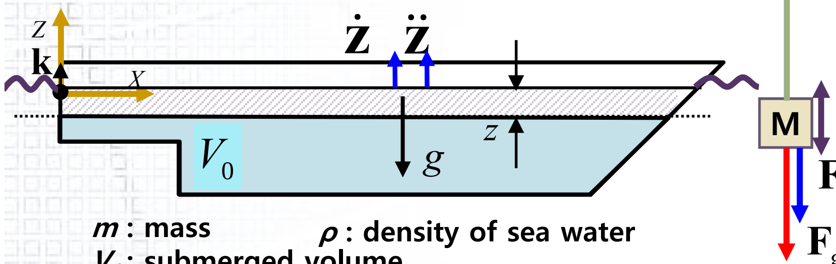



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0\cos\omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0\cos\omega t$$

NAOE/SNU

@SDAL Advanced Ship Design Automation Lab. http://asdal.snu.ac.kr

# Ex) Heave motion of ship – step 6




$$\mathbf{F}_{static} = \iint_{S_B} P_{static} \mathbf{n} dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP} \mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$\mathbf{F}_{exciting}$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

*m* : mass
*ρ* : density of sea water
$V_0$: submerged volume
$S_B$ : submerged surface area
*Awp* : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation} + \mathbf{F}_{exciting}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp} \mathbf{z} - c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$
$$= -\rho g A_{wp} \mathbf{z} - c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$
$$= -k\mathbf{z} - c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$

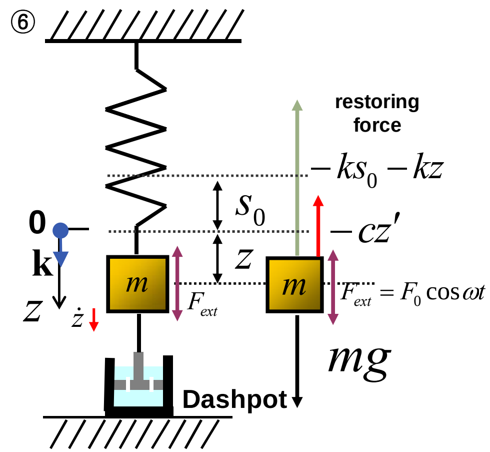
$$\boxed{(m + m_a)\ddot{\mathbf{z}} + c\dot{\mathbf{z}} + k\mathbf{z} = \mathbf{F}_{exciting}}$$

*c* : damping coefficient
$m_a$ : added mass

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**

⑥




$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0 \cos \omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0 \cos \omega t$$

$$\boxed{m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = \mathbf{F}_0 \cos \omega t}$$

# 선박 유체정역학(Ship Hydrostatics) vs 선박 유체동역학(Ship Hydrodynamics)

선박에 작용하는 **유체동역학적**인 힘/모멘트와

**중력**에 의한 힘/모멘트가 평형을 이루는 **자세와 이때의 힘/모멘트**를

구하는 것이 선박 유체동역학의 목표이다.
(선박 유체동역학은 6자유도 운동방정식에서 가속도가 있는 경우)

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \underbrace{(\mathbf{F}_{Gravity} + \mathbf{F}_{static})}_{\mathbf{F}_{restoring}} + \mathbf{F}_{wave\ exciting} - \underset{\text{added mass}}{\mathbf{M}_A}\ddot{\mathbf{x}} - \underset{\text{Damping Coefficient}}{\mathbf{b}}\dot{\mathbf{x}} + \mathbf{F}_{external,dynamic} + \mathbf{F}_{external,static}$$

---

선박에 작용하는 **유체정역학적**인 힘/모멘트와
**중력**에 의한 힘/모멘트가 평형을 이루는 **자세와 이때의 힘/모멘트**를
구하는 것이 선박 유체정역학의 목표이다.
(선박 유체정역학은 6자유도 운동방정식에서 속도, 가속도가 0인 경우)

$$\cancel{\mathbf{M}\ddot{\mathbf{x}}} = \sum \mathbf{F} = (\mathbf{F}_{Gravity} + \mathbf{F}_{static}) + \cancel{\mathbf{F}_{external,dynamic}} - \cancel{\mathbf{M}_A\ddot{\mathbf{x}}} - \cancel{\mathbf{b}\dot{\mathbf{x}}} + \mathbf{F}_{external,static}$$

$$0 = (\mathbf{F}_{Gravity} + \mathbf{F}_{static}) + \mathbf{F}_{external,static}$$

$$0 = \mathbf{F}_{restoring} + \mathbf{F}_{external,static}$$

# – Shear Force & Bending Moment acting On the ship

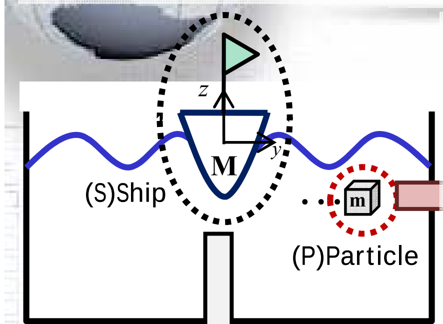
$\mathbf{x}$: 유체입자의 시간에 따른 변위

$\mathbf{V} = \frac{d\mathbf{x}}{dt}, \mathbf{a} = \frac{d^2\mathbf{x}}{dt^2}$

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bendidng Moment

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

✓ Assumption
① 뉴턴 유체 (Newtonian fluid)
② Stokes Assumption
③ 비점성 유동 (Invicid flow)
④ 비회전 유동 (Irrotational flow)




✓유체입자의 운동방정식

Lagrangian & Eulerian Description

Newton's 2nd Law

$m\ddot{\mathbf{x}} = \sum \mathbf{F}_P = \mathbf{F}_{body,P} + \mathbf{F}_{surface,P}$

유체입자 6자유도

Cauchy equation ①, ②

전단 응력

Navier-Stokes equation ③

유체입자 3자유도 (회전 3자유도 X)

Euler equation ④

Curl과 회전

Bernoulli equation

미시적 유도/ 거시적 유도(RTT[1])

Mass Conservation Law ② ④

Laplace Equation

$\nabla^2\Phi = 0$ $(\mathbf{V} = \nabla\Phi)$

$\frac{\partial\Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = F(t)$

Velocity potential

✓선박의 6자유도 운동 방정식

① Coordinate system 정의
(Global & Body-fixed coordinate)
② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F}_S = \mathbf{F}_{body,S} + \mathbf{F}_{surface,S}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{Fluid}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$
$$= \mathbf{F}_{restoring} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

$$\boxed{(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} = \mathbf{F}_{restoring} + \mathbf{F}_{exciting}}$$

cf) 선형화 된 복원력 ($\mathbf{F}_{restoring} = -\mathbf{C}\mathbf{x}$)

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

z방향 성분

✓유체력 계산

$\Phi_T = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction velocity potential)
$+\Phi_R$ (Radiation velocity potential)

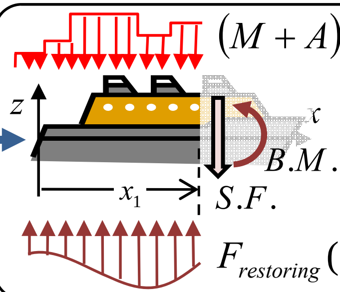
Linearization

$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho\frac{\partial\Phi_T}{\partial t}$$

( $S_B$: wetted surface)

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$




$(M+A)a_z(x)$

($a_z$ : z방향 가속도)
($v_z$ : z방향 속도)

$F_{restoring}(x) + F_{exciting}(x) - Bv_z(x)$

✓Shear force(S.F.) 및 bending moment(B.M.)

Shear force($S.F.$) 계산

🡻(적분)

Bending moment($B.M.$)

## – Shear Force & Bending Moment acting On the ship

✓ Newton 제2법칙 : $\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Body} + \mathbf{F}_{Surface}$

$$\mathbf{M}\ddot{\mathbf{x}} = \underbrace{\mathbf{F}_{Gravity}}_{\mathbf{F}_{Body}} + \underbrace{\mathbf{F}_{Static} + \mathbf{F}_{F.K.} + \mathbf{F}_{D} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}}_{\mathbf{F}_{Surface}}$$

(동적 평형 상태 : d'Alembert principle)

$$0 = -\mathbf{M}\ddot{\mathbf{x}} + \mathbf{F}_{Gravity} + \mathbf{F}_{Static} + \mathbf{F}_{F.K.} + \mathbf{F}_{D} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

Heave motion equation 예 (z방향 성분만 고려함)

$z$ : heave displacement

$\theta$ : pitch angle

✓ 운동 방정식에서 나온 속도,가속도 성분 대입

| |
|---|
| $x$위치에서의 수직 방향 가속도: $a_z = \ddot{z} - x\ddot{\theta}$ |
| $x$위치에서의 수직 방향 속도: $v_z = \dot{z} - x\dot{\theta}$ |

$$0 = -M\boxed{a_z} + F_{Gravity} + F_{Static} + F_{F.K.} + F_D - A_{33}\boxed{a_z} - B_{33}\boxed{v_z}$$

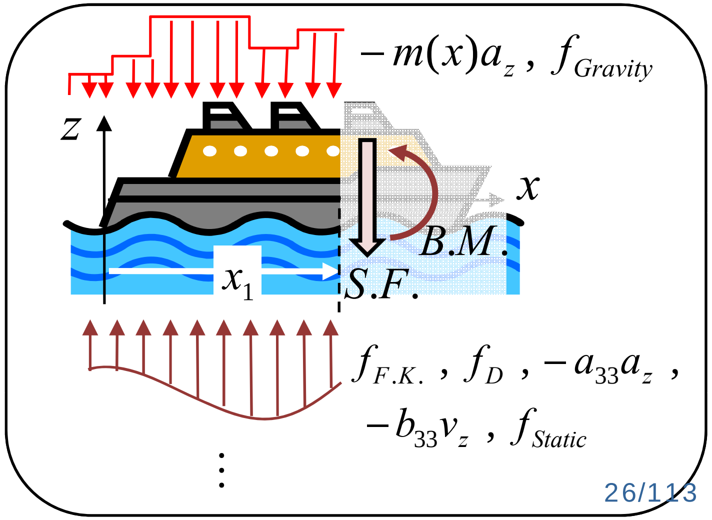
(단위 길이당 작용하는 힘)

$$q(x) = \underbrace{-m(x)a_z}_{\text{Mass inertia}} + \underbrace{f_{Gravity}}_{\text{Structural weight}} + \underbrace{f_{Static}}_{\text{Hydrostatic force}} + \underbrace{f_{F.K.}}_{\text{Froude-Krylov}} + \underbrace{f_D}_{\text{Diffraction}} \underbrace{-a_{33}a_z}_{\text{Added mass force}} \underbrace{-b_{33}v_z}_{\text{Potential damping}}$$

$m(x)$ : 선박의 단위길이당 질량

$x = x_1$에 작용하는 S.F.(Shear force)와 B.M.(Bending Moment)

$$S.F.(x_1) = \int_{A.P.}^{x_1} q(x)dx \Rightarrow B.M.(x_1) = \int_{A.P.}^{x_1} S.F.(x)dx$$



# – Shear Force & Bending Moment acting On the ship

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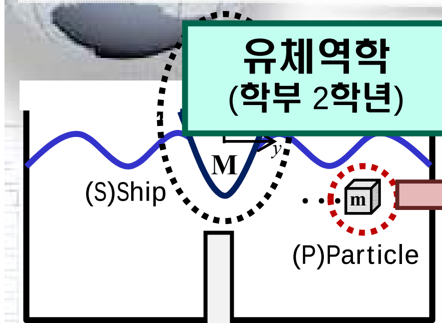



**유체역학 (학부 2학년)**

**✓유체입자의 운동방정식**

Lagrangian & Eulerian Description

Newton's 2nd Law

$$m\ddot{\mathbf{x}} = \sum \mathbf{F}_P = \mathbf{F}_{body,P} + \mathbf{F}_{surface,P}$$

유체입자 6자유도

Cauchy equation → ①, ② **전단 응력** Navier-Stokes equation → ③ Euler equation → ④ Bernoulli equation

유체입자 3자유도 (회전 3자유도 X)

**Curl과 회전**

미시적 유도/ 거시적 유도(RTT[1])

Mass Conservation Law ②④

**공학수학 (학부2학년)**

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\frac{\partial\Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla\Phi|^2 + gz = F(t)$$

Velocity potential

**✓선박의 6자유도 운동 방정식**

**운항제어론 (학부 3학년)**

① Coordinate system **정의** (Global & Body-fixed coordinate)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F}_S = \mathbf{F}_{body,S} + \mathbf{F}_{surface,S}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Fluid}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

$$= \mathbf{F}_{restoring} + \ldots\dot{\mathbf{x}}$$

**조선해양공학계획 (학부 2학년)**

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} = \mathbf{F}_{restoring} + \mathbf{F}_{exciting}$$

cf) 선형화 된 복원력 ($\mathbf{F}_{restoring} = -\mathbf{C}\mathbf{x}$)

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

z방향 성분

**공학수학 (학부2학년)**

**✓유체력 계산**

**해양파역학 (학부 3학년)**

$$\Phi_T = \Phi_I \text{ (Incident wave potential)}$$
$$+\Phi_D \text{ (Diffraction velocity potential)}$$
$$+\Phi_R \text{ (Radiation velocity potential)}$$

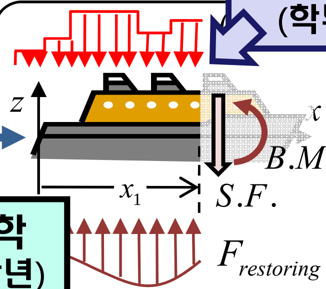
$$P = -\rho g z - \rho\frac{\partial\Phi_T}{\partial t}$$

( $S_B$: wetted surface)

$$\mathbf{F}_{Fluid} = \iint_S P\mathbf{n}dS = \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

**선체구조설계시스템 (학부 3학년)**

**재료역학 (학부2학년)**




($a_z$ : z방향 가속도)
($v_z$ : z방향 속도)

$$F_{restoring}(x) + F_{exciting}(x) - Bv_z(x)$$

Shear force(S.F.) 및 bending moment(B.M.)

Shear force(S.F.) **계산**

**↓(적분)**

Bending moment(B.M.)