Introduction

➢ Geophysical methods and their main application

표 1.1 물리탐사 방법과 적용분야

탐사방법	장	이용되는 물성	적용분야									
			1	2	3	4	5	6	7	8	9	10
중력탐사	2	밀도	P	P	s	s	s	s	!	!	s	!
자력탐사	3	대자율	P	P	P	s	!	m	!	P	P	!
굴절법 탄성파탐사	4, 5	탄성계수, 밀도	P	P	m	P	s	s	!	!	!	!
반사법 탄성파탐사	4, 6	탄성계수, 밀도	P	P	m	s	s	m	!	!	!	!
전기비저항탐사	7	전기비저항	m	m	P	P	P	P	P	s	P	m
자연전위(SP)탐사	8	전위차	!	!	P	m	P	m	m	m	!	!
유도분극(IP)탐사	9	전기비저항, 커패시턴스	m	m	P	m	s	m	m	m	m	m
전자(EM)탐사	10	컨덕턴스, 인덕턴스	s	P	P	P	P	P	P	P	P	m
VLF 전자탐사	11	컨덕턴스, 인덕턴스	m	m	P	m	s	s	s	m	m	!
GPR탐사	12	유전율, 전기전도도	!	!	m	P	P	P	s	P	P	P
MT탐사	11	전기비저항	s	P	P	m	m	!	!	!	!	!

P = 1차로 우선 적용, s = 2차 적용, m = 사용 가능하나 최적의 방법은 될 수 없음. 또는 현 상황에서는 이 분야에 적용되지 못하고 있음, ! = 부적합

적용분야

1 : 탄화수소 탐사(석탄, 석유, 가스)
2 : 광역 지질 연구(수백 km^2 정도의 넓은 지역)
3 : 광물 탐사 및 개발
4 : 토목 · 건설 공사부지 조사
5 : 수리지질학적 조사
6 : 지하공동 탐지
7 : 침출수 및 오염물질의 오염범위 파악
8 : 매몰된 금속물의 위치 및 종류 파악
9 : 고고학적 유물 조사
10 : 법과학 분야(예 : 범죄 수사시 매몰 시신 및 은닉 증거물 등 탐지)

➢ Planning a geophysical survey

- ✓ What is the main objective of the survey?
- ✓ Strategy: The mere acquisition of data does not guarantee the success of the survey. → Interpretation is important, and choosing proper methods is of critical importance. Consider which method yields an anomaly.
- ✓ Survey constraints: The first and most important factor is finance (budget). How much is the survey going to cost and how much money is available? Reconnaissance survey is needed to cut down on expenses. Check accessibility of field area. Besides, there are political, social, and religious constraints.

- Geophysical survey design

  - ✓ Target identification

    - A marked contrast in physical properties gives rise to a geophysical anomaly.
    - The type of target is of great importance.
    - For example, consider the situation where saline water intrudes into a near surface aquifer; saline water has a high conductivity (low resistivity) in comparison with freshwater and so is best detected using electrical resistivity or electromagnetic conductivity methods; gravity methods and seismic methods are not appropriate.

  - ✓ Optimum line configuration: the best orientation of a profile is normally at right-angles to the strike of the target.

  - ✓ Selection of station intervals

    - The smaller the sampling interval, the better the approximation is to the actual anomaly. The loss of high-frequency information is a phenomenon known as spatial aliasing. (Fig. 1.7 and 1.8)

  - ✓ Noise (Figure 1.10)
    - Coherent noise
    - Random noise

  - ✓ Garbage In , Garbage Out

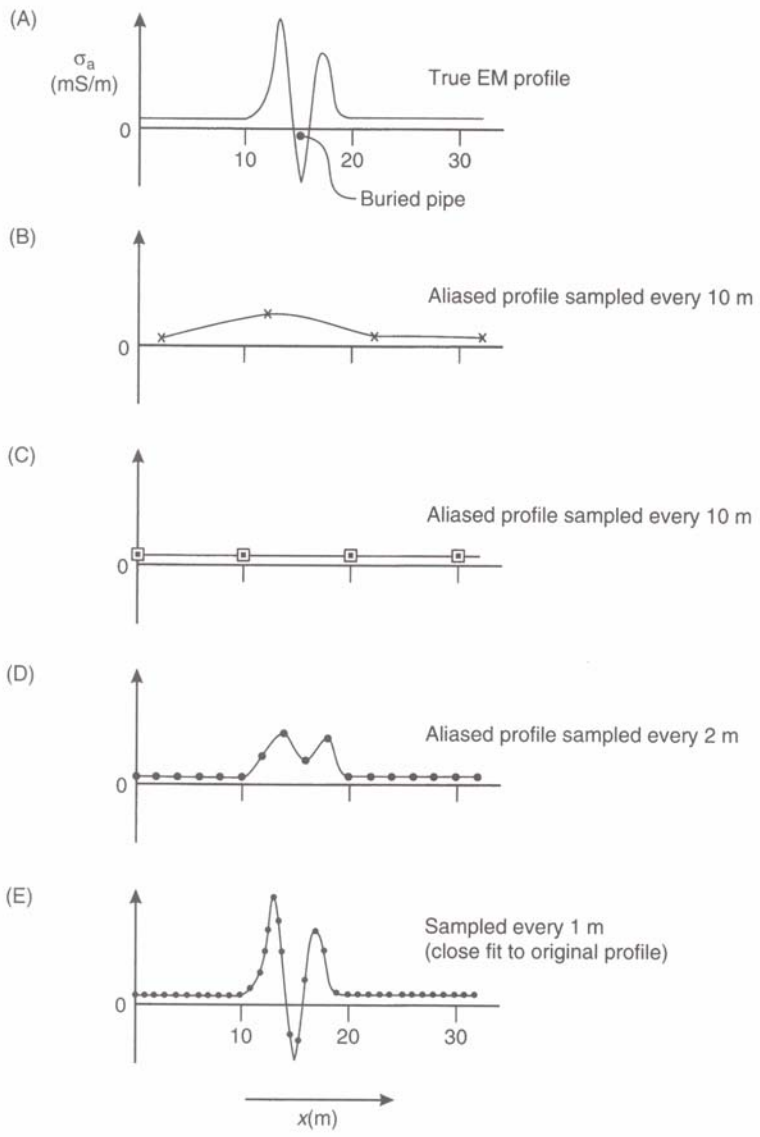


Figure 1.7 Examples of various degrees of spatial aliasing using different sampling intervals. (A) shows a continuously sampled profile. (B) and (C) show sampling every 10 m, but at different points along the profile. (D) shows sampling every 2 m: the profile is still aliased. (E) shows sampling every 1 m: this profile is the closest to that in (A)

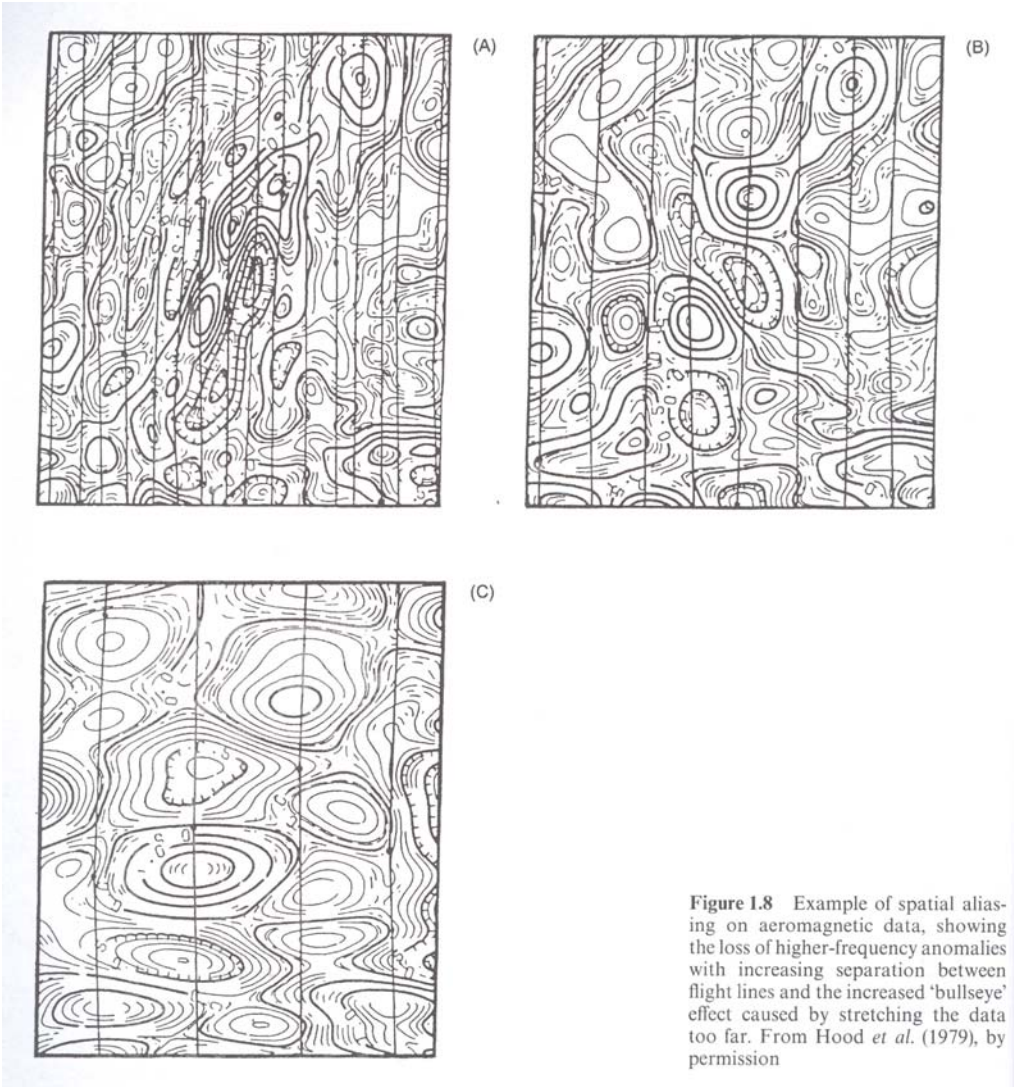


Figure 1.8 Example of spatial aliasing on aeromagnetic data, showing the loss of higher-frequency anomalies with increasing separation between flight lines and the increased 'bullseye' effect caused by stretching the data too far. From Hood *et al.* (1979), by permission

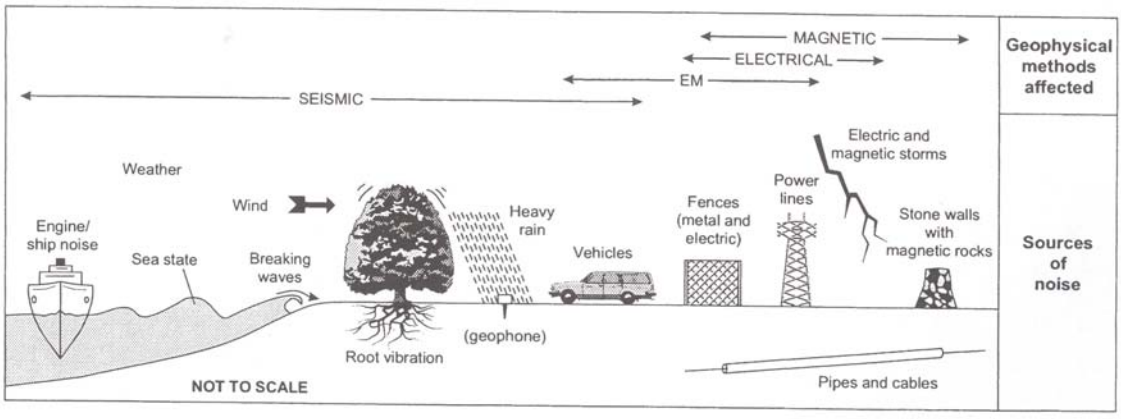


Figure 1.10 Schematic illustrating some common sources of geophysical noise

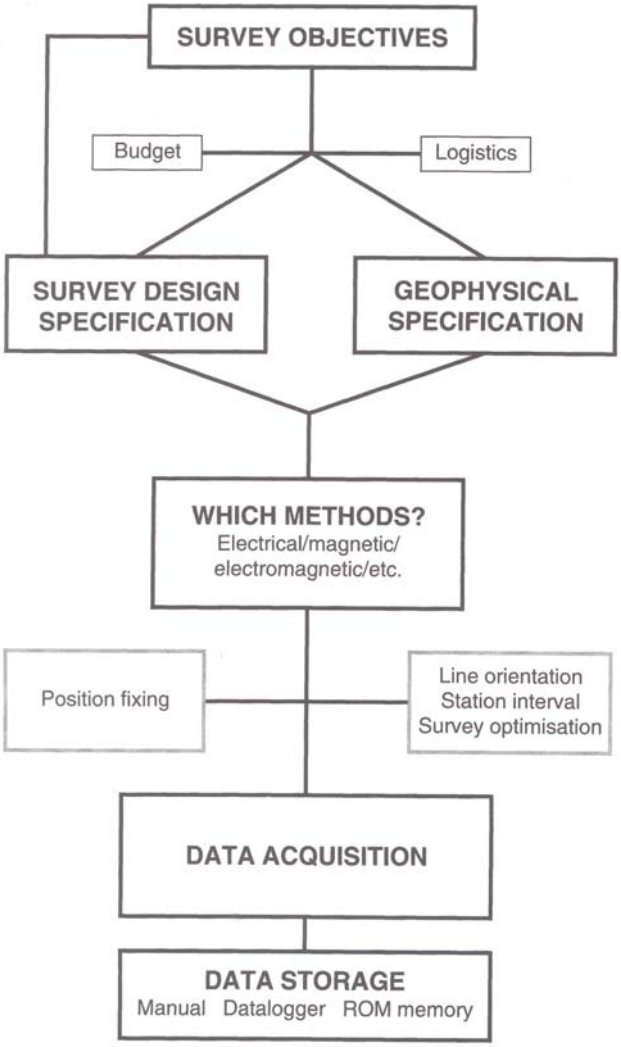


Figure 1.3 Schematic flow diagram to illustrate the decision-making process leading to the selection of geophysical and utility software. From Reynolds (1991a), by permission

Gravity methods

1. Introduction
   - Gravity prospecting involves the measurement of variations in the gravitational field of the earth

   - Observations are normally made within a few feet of the earth's surface or on a ship
   - Airborne tests and underground surveys have occasionally used

   - Uses a natural source method
     ➔ Local variations in density of rocks near the surface cause minute changes in the main gravity field

   - Attempts to measure small differences in a force field which is relatively huge.
   - The main fields vary with position and time
   - Density variations are relatively small and uniform
     ➔ Gravity anomalies are small and smooth
     ➔ Instruments must be more sensitive

   - Gravity prospecting is used as a reconnaissance tool in oil exploration. Although expensive, it is still considerably cheaper than the seismic method. In mineral exploration, it has usually been employed as a secondary method

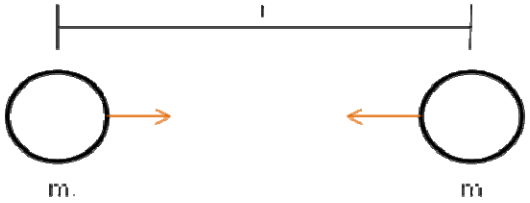
표 2.1 중력탐사의 적용분야

탄화수소 탐사
광역적인 지질 연구
지각평형보정
광물광상의 탐사 및 매장량 평가
지하공동의 탐지(고정밀 중력탐사)
매몰 암석계곡(rock-valley)의 위치 파악(지하수 탐사)
빙하두께 결정
조석진동(tidal oscillation)
고고 지구물리학(고정밀 중력탐사) : 예를 들면 고분위치 탐사
지구의 모양(측지학)
군사용(특히 미사일 탄도)
화산활동 감시

2. Principles and elementary theory

(1) Newton's law of gravitation

- The expression for the force of gravitation is given by Newton's laws
- The law states that the force between two particles of mass m_1 and m_2 is directly proportional to the product of the masses and inversely proportional to the square of the distance between the centers of mass
  (Figure)(Equation)



$$\mathbf{F} = -G\frac{m_1 m_2}{r^2}\mathbf{r}_1$$

$\mathbf{F}$ is the force on $\mathbf{m}_2$

$\mathbf{r}_1$ is a unit vector directed from m_1 towards $\mathbf{m}_2$

$\mathbf{G}$ is the universal gravitational constant is $6.67\times10^{-8}\ dyne\cdot cm^2/g^2$

- Gravitational force is one of the weak forces
- For n interacting particles

(2) Acceleration of gravity

- The acceleration of m_2 due to the presence of m_1 can be found by dividing F by m_2. In particular, if m_1 is the mass of the earth M_e and R_e is the radius of the earth

$$g = \frac{\mathbf{F}}{m_2} = -G\frac{m_1}{r^2}\mathbf{r}_1 = -G\frac{\mathbf{M}_e}{\mathbf{R}_e^{\,2}}\mathbf{r}_1$$

- The numerical value at the earth's surface is 980 cm/sec^2
- The unit of acceleration of gravity 1 cm/sec^2 is called the *gal*

(3) Gravitational potential

- The analysis of certain kinds of force fields, such as gravitational, magnetic, or electric, can often be simplified by using the concept of potential.
- Definition
  - ✓ The potential at a point in a given field is defined as the work done by the force in moving a unit mass from an arbitrary reference point (usually at an infinite distance) to the point

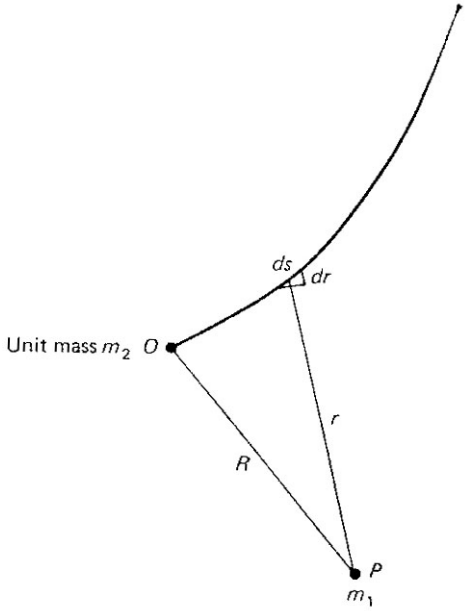


FIGURE 12-1 Gravitational potential is the work done by the attractive force of m_1 on m_2 as it moves to *0* from infinity.

  - ✓ Let us assume that the distance between two masses are initially infinite; the unit mass is moved until it reaches point O which is placed at a distance R from m_1, which has remained at P

  - ✓ The force per unit mass, or acceleration at a distance r from P is

$$-G\frac{m_1}{r^2}$$

  - ✓ The work needed to move the unit mass for a distance *ds* having a component *dr* in the direction of **P** is **(**dW **= F · ds)**

$$W = Gm_1 \int_{\infty}^{R} -\frac{dr}{r^2} = Gm_1 \frac{1}{r}\Big|_{\infty}^{R} = \frac{Gm_1}{R}$$

- ✓ Potential is obtained regardless of the path taken by m_2 in traveling from its starting point at infinite distance from m_1 to its end position at O.
- ✓ The gravitational acceleration is the derivative of the potential with respect to r
- ✓ Equipotential Surface : Any surface along which the potential is constant is referred to as an Equipotential Surface

  -> No work is needed to bring a body from one point on an Equipotential Surface to another. Ex) Sea level

- Potential in 3 dimension

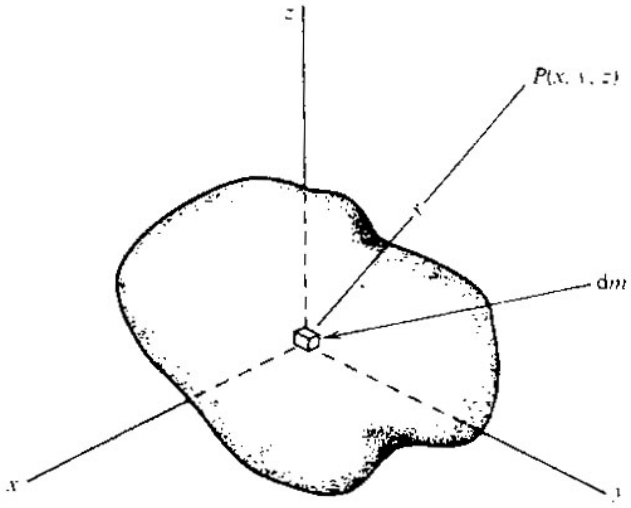


Fig. 2.1 Potential of three-dimensional mass.

- ✓ The potential due to an element of mass dm at a distance r from P
- ✓ In the Cartesian coordinate

$$dU = G\frac{dm}{r} = G\frac{1}{r}\rho\, dx dy dz$$

$$dm = \rho\, dv = \rho\, dx dy dz$$

$$U = G\rho \int_z \int_y \int_x \frac{1}{r} dx dy dz$$

✓ In the cylindrical coordinate

Cylindrical coordinates:

$x = r \cos\theta$

$y = r \sin\theta$

$z = z$

$dV = r\, dr\, d\theta\, dz$

$ds^2 = dr^2 + r^2\, d\theta^2 + dz^2$

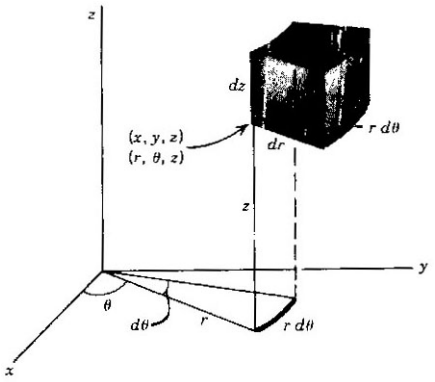
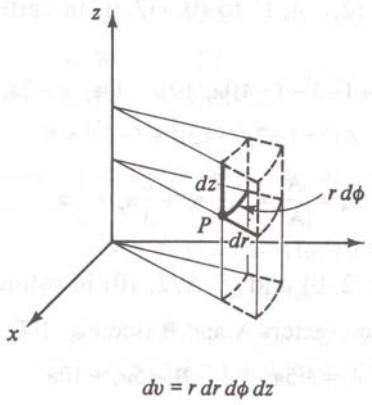


FIGURE 4.4



(b) Cylindrical

$$dm = \rho dv = \rho dr\ r d\phi\ dz$$

$$U = G\rho \int_z \int_\phi \int_r \frac{1}{r} dr\ r d\phi\ dz = G\rho \int_z \int_\phi \int_r dr d\phi dz$$

✓ In the spherical coordinate

Spherical coordinates:

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$ds^2 = dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2$

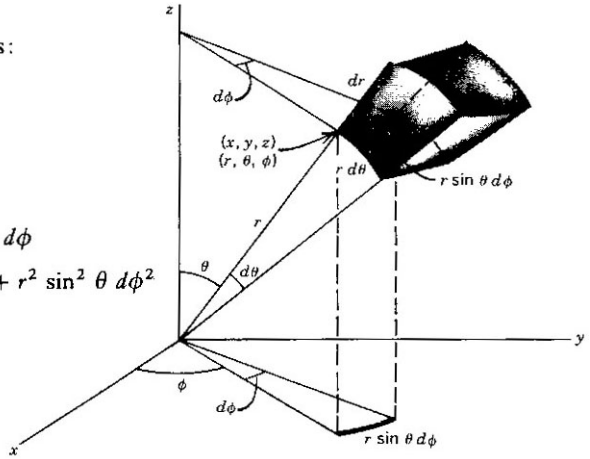
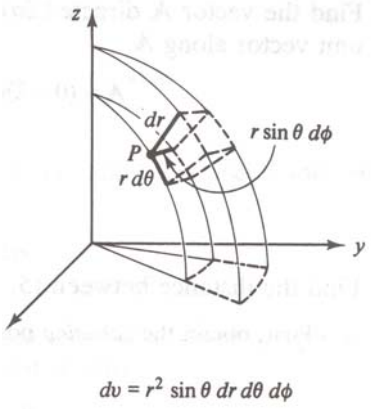


FIGURE 4.5



$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

(c) Spherical

$$dm = \rho dv = \rho dr \; r\sin\theta d\phi \; r d\theta$$
$$= \rho r^2 \sin\theta dr d\phi d\theta$$
$$U = G\rho \int_\theta \int_\phi \int_r r\sin\theta dr d\phi d\theta$$

참고>

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}$$

(4) Governing equation

- Since the gravitational acceleration is the derivative of the potential with respect to r, we can express it by

$$\mathbb{F} = \nabla U$$

- Gravitational field is conservative

$$\nabla \times F = \nabla \times \nabla U = 0$$

- We assume some mass whose volume is V and its enclosed surface is S and

then gravitational flux Φ passing through the surface S is expressed by

$$\Phi = \int_s \mathbb{F} \cdot \vec{\mathrm{n}}\, ds = \int_s F_n ds$$ n : unit vector normal to the surface

- From Gauss' Theorem (divergence theorem)
  → The integral of the divergence of a vector field over a region of space is equivalent to the integral of the outward normal component of the field over the surface enclosing the region:
  $$\int_V \nabla \cdot \mathbb{F}\ dv = \int_s F_n\ ds$$

If there is no attracting matter contained within this volume, the integrals are zero

$$\int_V \nabla \cdot \mathbb{F} dv = \int_V \nabla \cdot \nabla U dv = \int_V \nabla^2 U dv = 0$$

$\nabla^2 U = 0$ (Laplace equation in free space)

- In the Cartesian, cylindrical, Spherical coordinate systems, Laplace's equations is

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$ (in the cartesian coordinate)

$$\nabla^2 U = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2} = 0$$ (in the cylindrical coordinate)

$$\nabla^2 U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2\partial U}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial U}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2 U}{\partial \phi^2}\right) = 0$$

(in the spherical coordinate)

- If there is a particle of mass m within the volume and if we consider it to be at the center of a spherical surface of radius r

$$U = \int_s F_n ds = -G\frac{m}{r^2} 4\pi r^2 = -4\pi G m$$

$$\int_V \nabla \cdot \mathbb{F} dv = \int_s F_n ds = -4\pi G m$$

If the volume is assumed to be very small (infinitesimal)

$\nabla^2 U = -4\pi G\rho$ (Poisson's Equation)

- The gravity potential satisfies Laplace's equation in free space and Poisson's equation in a region containing attracting material.

(5) Figure of the earth

- Gravity prospecting evolved from the study of the earth's gravitational field for determining the shape of the earth

- The earth is not a perfect sphere but a spheroid
  ➔ It bulges at the equator and is flattened at the poles.

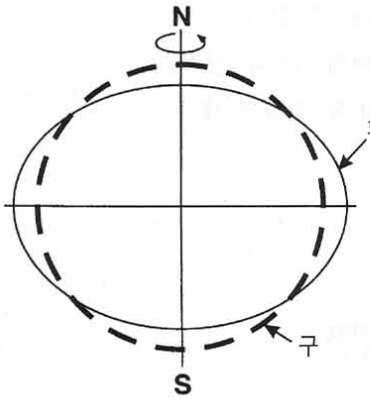


그림 2.1 구와 회전타원체의 차이를 확대한 그림

  ➔ Polar flattening

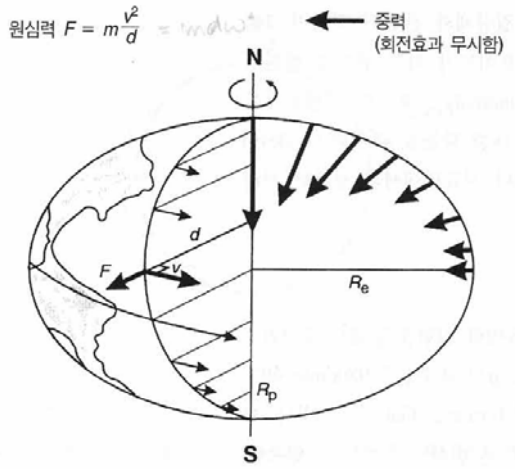


그림 2.3 원심력과 위도 ϕ에 따른 중력의 변화

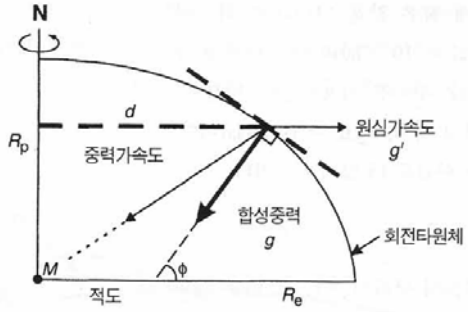


그림 2.4 원심가속도(g')와 중력가속도(g)의 합성력(실제 크기가 아님). 지리위도(geographic latitude)는 ϕ이다(Robinson과 Coruh, 1988).

→ Earth's radius is 6357 km at the poles and 6378 km at the equator.

→ Difference is 21 km.

→ Earth's revolution causes centrifugal forces, which is large at the equator and zero at the poles.

→ Centrifugal force = $md\omega^2=mv^2/d$ ($v = d\omega$)

- The observed gravity is expressed by vector addition of gravitational acceleration and centrifugal acceleration

a. **The reference spheroid**:

✓ The spheroid is defined to describe the gravitational field using mathematical formula. It is a mathematical figure based on the gravity values at all surface points.

✓ Basic assumption: the earth rotates about its polar axis and its density increases with depth, for example, from 3 g/cm^3 to 12 g/cm^3

There is no lateral variation in density

- ✓ The mean sea level with excess land masses removed and ocean deeps filled.

- ✓ Equipotential surface: the force of gravity is everywhere normal to this surface

- ✓ Practically, equipotential surface is a little different from the reference spheroid.

- ✓ International gravity formula based on the reference spheroid

  - ➢ In 1930, the International Union of Geodesy and Geophysics adopted an international gravity formula based on the polar flattening 1/297

    $g_{theo} = g_e\left(1+\alpha\sin^2\phi+\beta\sin^2 2\phi\right)$

    $g_e = 978.049$: gravity at the equator

    ϕ: latitude

    $\alpha = 0.0052884$, $\beta = -0.0000059$

  - ➢ In 1967, the polar flattening was adjusted to be 1/298.247 (eq)

    $g_{theo} = g'_e\left(1+\alpha'\sin^2\phi+\beta'\sin^4\phi\right)$

    $g'_e = 978.03185$: gravity at the equator

    ϕ: latitude

    $\alpha' = 0.005278895$, $\beta' = 0.000023462$

- The international formula adopted by IAG (International Association of Geodesy) in 1971 is

  $g_{theo} = g''_e\left(1+\alpha''\sin^2\phi+\beta''\sin^2 2\phi\right)$

  $g''_e = 978.0318$: gravity at the equator

ϕ: latitude

$\alpha'' = 0.0053024$, $\beta'' = -0.0000059$

- The international formula adopted by IAG at Canberra in 1980 after a critical evaluation of the available absolute g values on the Earth (IAG, 1980)

$$g_{theo} = g_e'''\left(1 + \alpha''' \sin^2\phi + \beta''' \sin^4\phi + \gamma \sin^6\phi\right)$$

$g_e'' = 978.0327$: gravity at the equator

ϕ: latitude

$\alpha''' = 0.0052790414$, $\beta''' = 0.0000232718$, $\gamma = 0.0000001262$

➢ The change in gravity from equatorial to polar regions amounts to about 5 gals, or 0.5% of the average value of g (980 gals)

☞ Additional information

➔ The effect of elevation in some cases might be as large as 0.1 gal or 0.01% of g

➔ Large gravity anomaly in oil exploration would be 10 mgals (0.001% of g), while in mineral areas the value would perhaps be one tenth of this.

➔ Variations in g which are significant in prospecting are not only minute in comparison with the value of g itself, but also in comparison with the effects of large changes in latitude and elevation

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b. The geoid

✓ Practical mean sea level (equipotential surface)

✓ In the ocean: it is defined as average sea level

✓ On land: it is defined as average seal level over the surface of sea water in virtual canals which cut through the land masses.

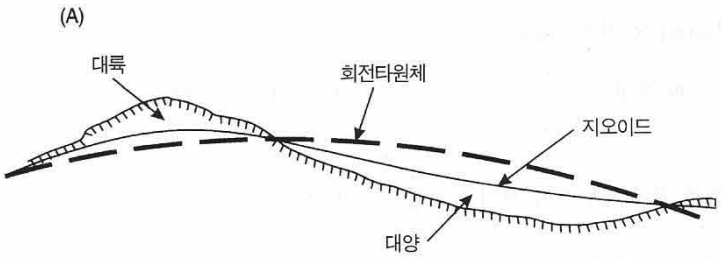


그림 2.2 지오이드의 변화.
(A) 광역적 효과

☞ The geoid and reference spheroid surface do not coincide at all points, since the geoid is warped upward under the continental masses due to attracting material above and downward over the ocean basins due to water (with lower density)

☞ If there is excess mass, its effects is not reflected in the reference spheroid, while it is reflected in the geoid

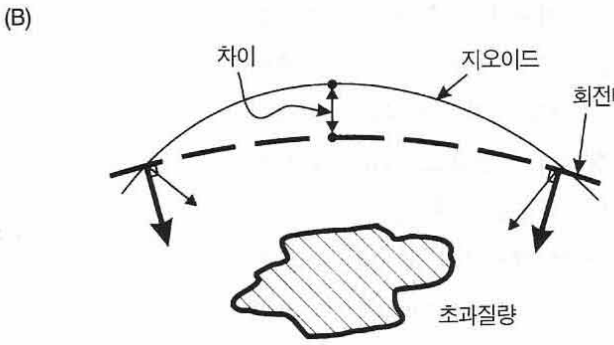


그림 2.2 지오이드의 변화.
(B) 지하의 밀도차에 의한 국지적 효과

The objective of the gravity method is to investigate the cause of the differences between geoid and spheroid.

3. Gravity exploration

(1) Measurements of the gravity

- The detection of an anomaly in gravity prospecting requires that we measure changes in g at least as small as 0.1 mgal. It is not possible to determine gravity variations δg and the absolute gravity g with the same instrument.

- The absolute measurement is carried out at a fixed installation and involves the accurate timing of a swinging pendulum and of a falling body.

- Relative measurements may be made by three types of instruments: the torsion balance, the portable pendulum and the gravimeter.

(a) Absolute measurements

➢ Falling body

The acceleration of gravity can be found from Newton's equation of motion by noting the time interval between two points in a vertical fall.

$$s = \frac{1}{2} g t^2 \qquad g = \frac{2s}{t^2}$$

If the falling body falls distances s_1 and s_2 in time intervals of t_1 and t_2

$$g = \frac{2s_1}{t_1^2}, \qquad g = \frac{2s_2}{t_2^2}$$

Multiply t_1 in both sides of the first equation, and t_2 in both sides of the second equation

$$t_1 g = \frac{2s_1 t_1}{t_1^2}, \quad t_2 g = \frac{2s_2 t_2}{t_2^2}$$

$$\left(t_1 - t_2\right) g = \frac{2s_1}{t_1} - \frac{2s_2}{t_2}$$

$$g = \frac{1}{t_1 - t_2}\left(\frac{2s_1 t_2 - 2s_2 t_1}{t_1 t_2}\right) = \frac{2\left(s_1 t_2 - s_2 t_1\right)}{t_1 t_2 \left(t_1 - t_2\right)}$$

In order to obtain an accuracy of 1 mgal with a fall of one or two metres, it is necessary to measure time to about 10^{-8} seconds and distance to

less than 1/2 micron.

- Pendulum

This is the standard method for measuring g
The value of g is obtained by timing a large number of oscillations

The period of swing pendulum: T
The length of the pendulum: l

$$T = 2\pi\sqrt{\frac{l}{g}} \quad , \qquad g = \frac{4\pi^2 l}{T^2}$$

※ l should be weightless but in practice, we can't ignore the weight

※ Use physical pendulum

$$g = \frac{4\pi^2 I_c}{T^2 mh}$$

m: mass
h: distance between the axis and the center of mass
I_c: the moment of inertia

The limitations of errors: Period: 10^{-8} sec and the length of the pendulum: 0.5 micron

(b) Relative measurement of gravity

Assignment #1

Examine methods for relative measurement of gravity, for example, portable pendulum, torsion balance, and gravimeter (stable type and unstable type for land survey, and instrument for marine survey)

- Portable pendulum

The pendulum has been used for both geodetic and prospecting purposes. Since g varies inversely as the square of the period T, we have

$$T^2 g = \text{constant}$$

Differentiating, we get

$$\Delta g = 2g\Delta T / T$$
$$= -2g\left(T_2 - T_1\right) / T_1$$

If we can measure the periods at two stations to about 1 μs, the gravity difference is accurate to 1 mGal. It took 1/2hr to get the required accuracy.

- Torsion balance

Two equal masses m are separated both horizontally and vertically by rigid bars, the assembly being supported by a torsion fiber with an attached mirror to measure rotation by the deflection of a light beam. Two complete beam assemblies were used to reduce the effects of support sway. Readings were taken at three azimuth positions of the beam assemblies, normally 120° apart. Each station had to be occupied for approximately one hour.

The deflection of the torsion balance beam is due to horizontal and vertical changes in the gravity field resulting from curvature of the equipotential surfaces.

중력의 크기보다는 중력장의 불균형을 측정하기 위한 중력 구배의 측정에 이용. 두개의 질량 m이 양단에 붙어있는 L자형의 막대를 실에 매달아 중력장 내에 위치시킨다. 만약 중력장이 균일하지 않으면 막대는 회전하게 되고, 이로부터 중력장의 불균형을 측정하게 된다. 불균일한 중력장의 수직역선을 곡선이며, 위치가 다른 질량 M_1과 M_2에 작용하는 힘 F_1과 F_2는 서로 평행하지 않다. 이들의 수평분력 H_1과 H_2가 곡선의 곡률의 크기에 따라 야기되고 토오크를 발생시켜, torsion balance를 회전시킨다. Torsion balance에 작용하는 토오크는 질량 M_1과 M_2에 작용하는 힘의 차이에 기인하므로 힘의 변화율, 즉 중력 포텐셜의 2차 편미분에 의하여 구할 수 있다.

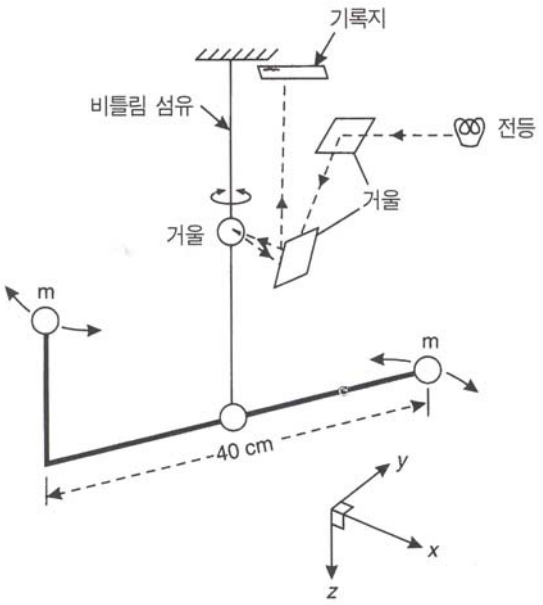


그림 2.6 비틀림 저울(torsion balance)의 모식도

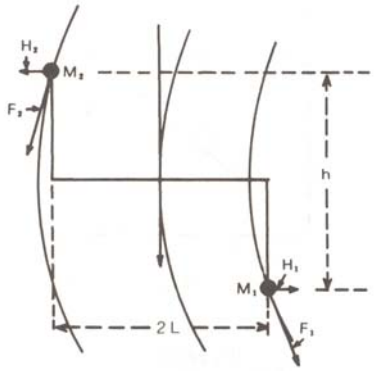


그림 2·23 곡선역장(曲線力場)내의 torsion balance

The unstable gravimeters have been generally used in gravity exploration, which is based on sensitive mechanical balances in which a mass is supported by a spring.

- Stable-type gravimeters

They have been superceded by more sensitive unstable meters. All gravimeters are essentially extremely sensitive mechanical balances in which a mass is supported by a spring. Small changes in gravity move the weight against the restoring force of the spring.

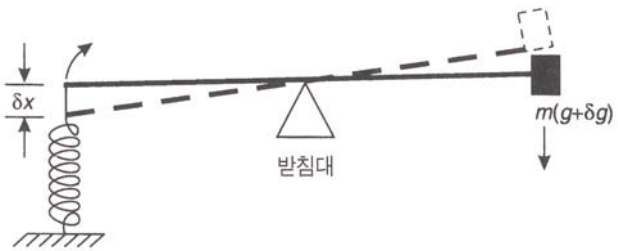


그림 2.8 안정형 중력계의 동작 원리

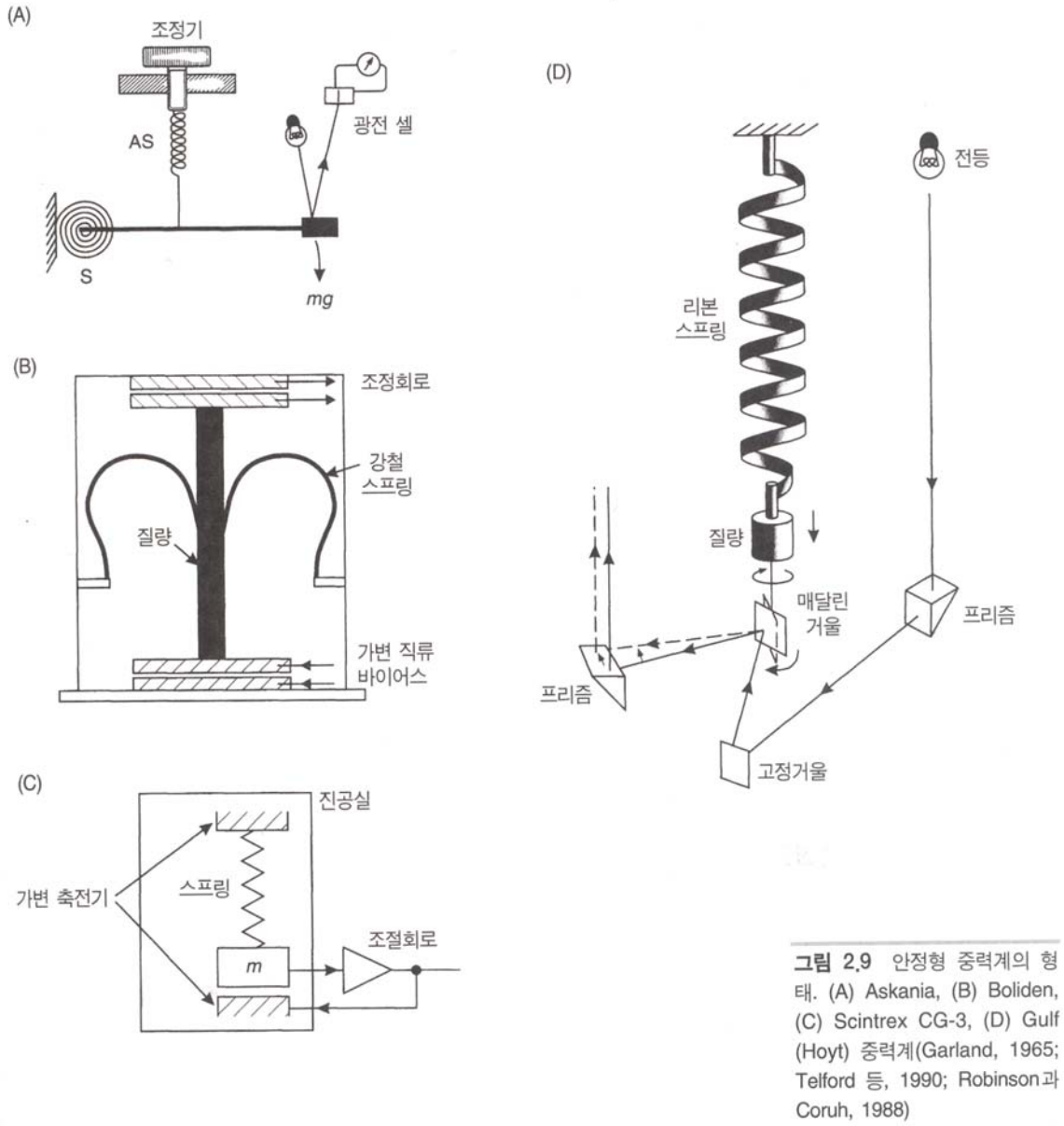


그림 2.9 안정형 중력계의 형태. (A) Askania, (B) Boliden, (C) Scintrex CG-3, (D) Gulf (Hoyt) 중력계(Garland, 1965; Telford 등, 1990; Robinson과 Coruh, 1988)

➢ Unstable-type gravimeters

These instruments have an additional negative restoring force operating against the restoring spring force, that is, in the same sense as gravity. They essentially are in a state of unstable equilibrium and this gives them greater sensitivity than stable gravimeter. These gravimeters magnify the movement of spring.

Thyssen, LaCoste-Romberg, Worden, Sodin

(2) Field operation

- Gravity exploration is carried out both on land and at sea. Sometimes airborne survey is performed

- Gravity is influenced by latitude, elevation, topography of the surrounding terrain, earth tides and variations in subsurface density

- In gravity exploration, we should
  - ✓ Precisely measure the relative gravity between fundamental gravity station and stations where we want to measure gravity.
  - ✓ Record the time when the measurement is executed
  - ✓ Record the elevation and position
  - ✓ Record changes of gravity with time at fixed stations
    Repeated reading is required at the fixed stations-elastic creep
    (Note that gravimeter is made of mass hanging on to a spring. The elasticity of the spring can changes as we measure gravity)→elastic creep
    The maximum time allowable between repeat readings would not be greater than 2-3 hours.

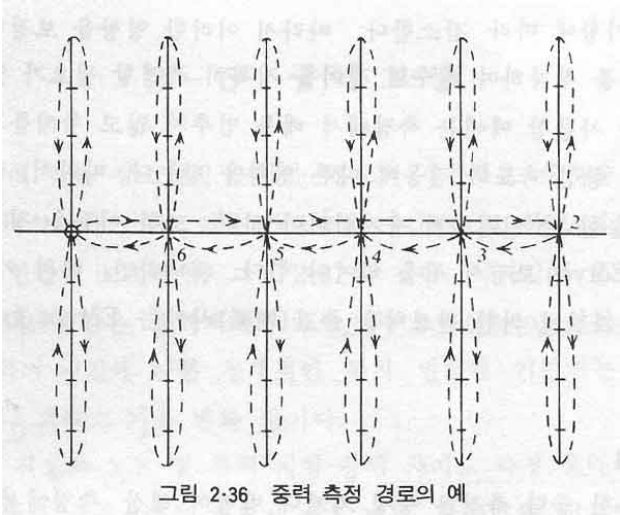


그림 2·36 중력 측정 경로의 예

- Station
  - ✓ Station is placed along the road because of easiness of access.
  - ✓ Station interval depends on the size and depth of the anomalous body
    → in general, the station interval ranges from tens of meters to

kilometers

- ✓ Avoid measuring gravity value at areas with specific terrains, for instance, on the top of hill where gravity value can include errors due to the mass of the hill.

➢ Altitude

- ✓ By topographical map, altimeter or leveling
- ✓ Altimeter: employs the phenomenon that atmospheric pressure changes with height
- ✓ The precision of the altimeter is less than leveling
- ✓ The errors of 30 cm in altitude cause the errors of 0.07 mgal in gravity
- ✓ The errors of altitude should be within ± 3 cm.

➢ Position

- ✓ By GPS (global positioning system)

- ✓ Acquiring field data – data processing (gravity correction) – data interpretation

(3) Gravity correction

- Variation of density is the most important factor in gravity prospecting. So, we need to remove the effects of other factors (such as drift, latitude, elevation, terrain, tide, etc) from the observed data. → this process is called gravity correction (or gravity reduction)

(a) Drift correction

- ✓ When we read gravity values with any kinds of gravimeters even if we measure gravity at a fixed station, the gravity values change with time. This drift mainly results from elastic creep of the spring
- ✓ We need to repeat readings periodically at a fixed station to obtain drift curves. → Differences between two readings at a fixed station are caused by drift and tide force.

- ✓ If we compute tide correction first and then subtract the tide correction from drift curves, we can obtain drift correction.
- ✓ We assume that the gravity changes due to drift linearly with time.
- ✓ Nonlinear changes may be caused by earth tides. In order to remove the effects of tide variations, we need to repeat gravity readings at a fixed station within two or three hours.

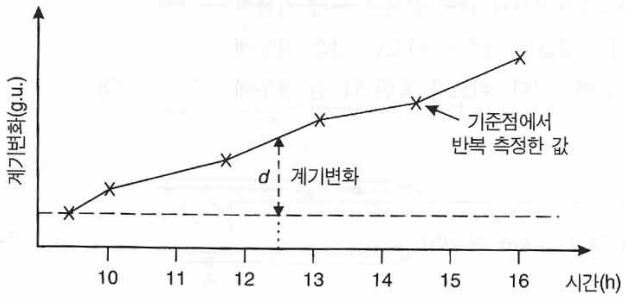


그림 2.14 계기변화 곡선(instrumental drift curve)

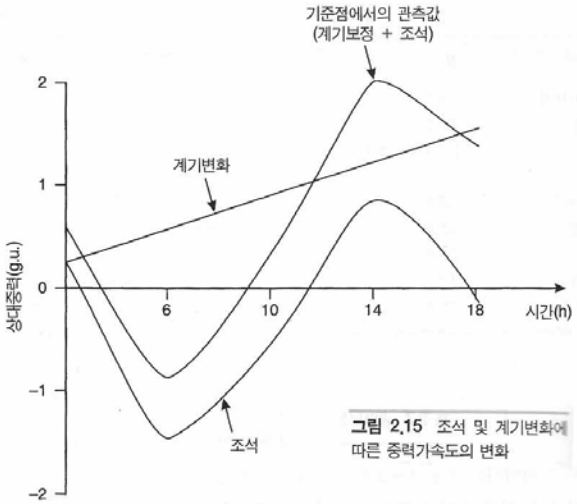


그림 2.15 조석 및 계기변화에 따른 중력가속도의 변화

(b) Earth-tide correction

- ✓ Instruments for measuring gravity are quite sensitive enough to record the changes in g caused by movement of the sun and moon.
- ✓ These variations have amplitudes as large as 0.3 mgal.
- ✓ They (land tides) depend on latitude and time like sea tides, but its amplitude is only a fraction of that of the ocean tides.
- ✓ The changes due to tide can be theoretically calculated for any time and place.

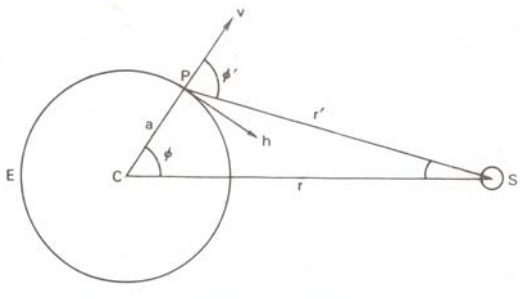
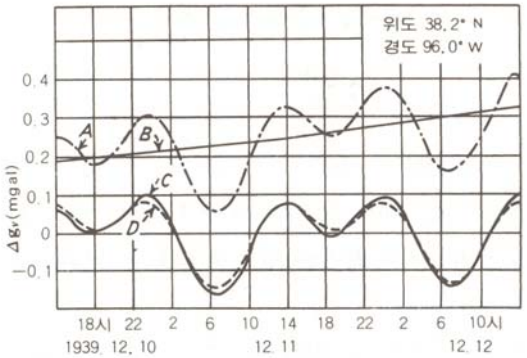


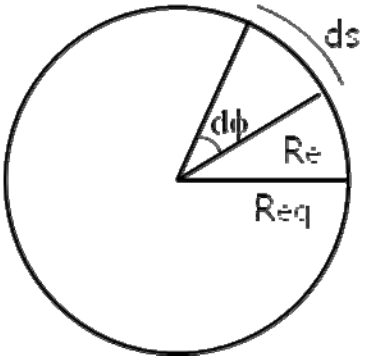
그림 2·39 조석보정식(潮汐補正式)의 유도



(천체가 중심 C와 P에 미치는 힘을 계산하여 차를 구하고, 이 힘을 다시 v방향과, h방향으로 분리시킴. 결국 v방향이 중력에 영향을 미치므로 이에 의한 효과를 구해서 빼줌. 오른쪽 그림은 현장 자료A에서 Drift에 의한 효과B를 빼면 C와 같은 중력값의 변화를 얻게 되는데 이는 위와같이 이론적으로 구한 조석곡선 D와 거의 일치한다고 함. 그런데 실제로는 천체에 의해 땅이 약간씩 올라가므로 이에 의해 중력값이 달라지기도 함. 따라서, 조석에 의한 중력 값은 앞에서 구한 중력값의 1.2배 정도가 된다고 함.)

(c) Latitude correction

- ✓ Gravity changes with latitude, which is due to differences between radii at the equator and at the poles
- ✓ It is necessary to apply a latitude correction where there are any appreciable north-south excursions of the grid stations.
  (If grid stations are placed at different latitudes and its difference is appreciable, we should correct latitude correction)
- ✓ Latitude correction can be obtained by differentiate gravity with respect to distance in a south-north direction between stations
  (eq)



$$g_{theo} = 978.03185(1 + 0.005278895 \times \sin^2\phi + 0.000023462\sin^4\phi)$$

$$ds = R_e d\phi$$

$$\frac{dg}{ds} = \frac{1}{R_e}\frac{dg}{d\phi} \approx \frac{1}{R_{eg}}\frac{dg}{d\phi}$$

$$= \frac{1}{6378\times10^5\,\mathrm{cm}}\left[978.03185\times0.005278895\right]\sin 2\phi$$

$$\cong 0.000812\times10^{-5}\sin 2\phi \text{ gal / 남북거리 1cm}$$

$$\cong 0.00812\sin 2\phi \text{ mgal / 남북거리 10m}$$

- ✓ When the north-south distance is 120 m at latitude 45˚ ($0.0082\cdot\sin 90\times12$) the latitude correction amounts to $0.09744 \approx 0.1$ mgal.
- ✓ Since gravity increases with latitude (either north or south) the correction is (added) as we move towards the equator

(d) Elevation correction (Free-air correction & Bouguer correction)

➢ Free-air correction

- ✓ Gravity varies inversely with the square of the distance.
- ✓ We need to remove the effects of elevation changes between stations so that all the fields are reduced to a datum surface.
- ✓ The datum surface can be sea level or an altitude at which the most stations are placed.
- ✓ This is called free-air correction, because it takes no account of the material between the datum surface and the station.

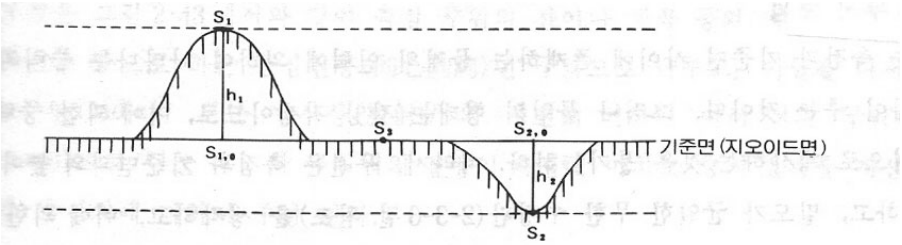


그림 2·41 고도보정

(M_e : the mass of the earth

$$g = \frac{GM_e}{R^2}$$

$$\frac{dg}{dR} = -2G\frac{M_e}{R^3} = -\frac{2g}{R} \qquad g_0 = G\frac{M_e}{R^2}$$

$$R : R + h$$

$$: R - h$$

Gravity at S_1 : g_1

$$g_1 = G\frac{M_e}{(R+h_1)^2} = GM_e(R+h_1)^{-2} = \frac{GM_e}{R^2}(1+\frac{h_1}{R})^{-2}$$

$$= \frac{GM_e}{R^2}\left(1 - 2\frac{h_1}{R} + 3\frac{h_1^2}{R^2} - \cdots\right)$$

Since $h_1 << R$,

$$= g_0\left(1 - 2\frac{h_1}{R}\right)$$

$$\Delta g_{FA} = g_0 - g_1 = 2\frac{g_0}{R}h_1 \cong 0.3086\times10^{-3}h_1\text{gal} = 0.3086h_1\text{mgal}$$

The unit of h_1 = m

※ $(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots$

- ✓ The elevation change of 1 m requires free-air correction of 0.3086 mgal.
- ✓ If the station is above the datum, the free-air correction is added to the field data, while if the station is below the datum, the free-air correction is subtracted.

➢ Bouguer correction

- ✓ The Bouguer correction accounts for attraction of material between the station and the datum plane, which was ignored in the free-air correction.

- ✓ Basic assumption: the slab with uniform density exists between the station and the datum, and the slap's horizontal extent is infinite. Neither is really valid. In order to modify the first, one would need to have knowledge of the local geology as to rock type and actual densities. The second is taken care of in the next correction.

- ✓ The Bouguer correction is given by

$$\Delta g_B = 2\pi G\rho h \cong 41.9\times10^{-8}\,\rho(h\times10^2)cm/s^2$$
$$= 0.04193\rho h\text{mgal}$$

- ✓ For density of rocks, we use the average density of rocks.
- ✓ The Bouguer correction is applied in the opposite sense to free-air correction, that is, it is subtracted when the station is above the datum plane and vice-versa.
- ✓ Elevation correction = free-air correction + Bouguer correction

$$\Delta g_E = \Delta g_{FA} + \Delta g_B = \pm\left(0.3086 - 0.04193\rho\right)h\ mgal$$

Assignment #2 Solve the below problems

Problem 1

If a gravity determination was made at an elevation of 152.7 m, what is the value of the free-air correction (assuming sea level as the datum)? The Bouguer correction (assuming a 2.50 g/cm^3 reduction density)?

Problem 2

A gravity station at a location of 0 m is located in the center of an erosional basin. The floor of the basin has virtually no relief. An escarpment of a plateau is located at a distance of 450 m from the gravity station. The surface of the plateau has a relatively constant elevation of 400 m. Will terrain corrections be necessary?

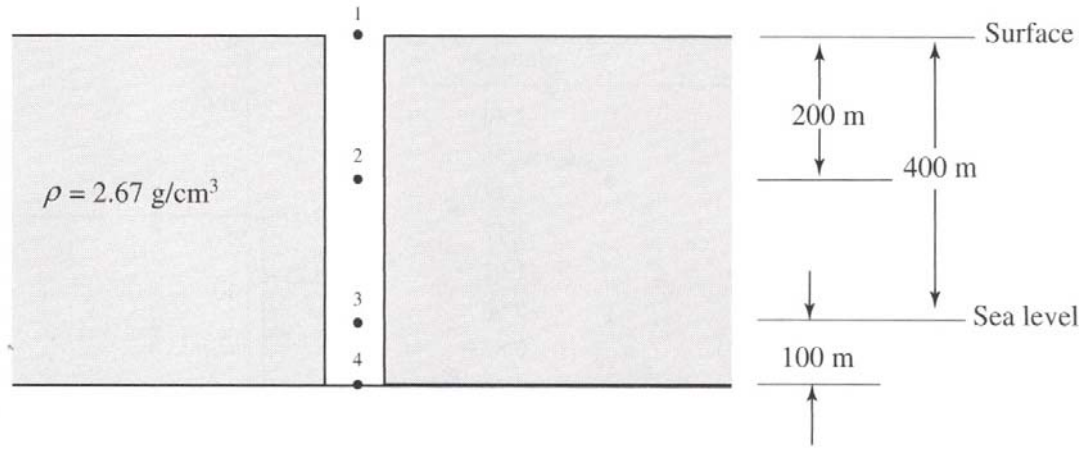
Problem 3

Prepare a drift curve for the following data and make drift corrections. Convert your corrected data to milliGals. The data were collected by a gravimeter with a dial constant of 0.0869 mGal/dial division.

Station	Time	Reading in Dial Divisions
Base	11:20	762.71
GN1	11:42	774.16
GN2	12:14	759.72
GN3	12:37	768.95
GN4	12:59	771.02
Base	13:10	761.18

Problem 4

You are measuring values of g in a drill hole. Refer to the diagram below and calculate the free-air and Bouguer corrections in measured g at positions 1, 2, 3, and 4. Sea level is the reduction datum. The mass loss due to the drill hole is negligible.



➢ Terrain correction

✓ Surface irregularities in the vicinity of the station (hills rising above the gravity station and valleys below it) affect gravity.

✓ Both of these topographic undulations affect the gravity measurement in the same sense, reducing the readings because of upward attraction (hills) or lack of downward attraction (valleys)

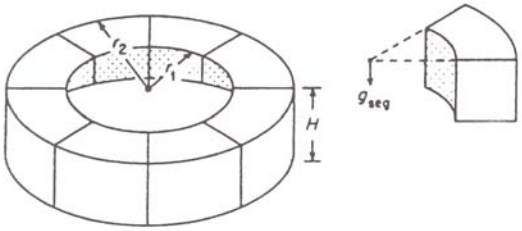
✓ (부게 보정시 산이 있는데 산이 없다고 가정했고, 산이 당기는 힘에 의해 중력값은 산이 없을 때 보다 작게 나타나 있음. 이를 보정하기 위해 산에 의한 효과를 계산하여 더해줌. 또한 주변에 계곡이 있을 때, 무한수평판을 가정하면서 계곡이 밀도가 있는 물질로 채워져 있다고 가정하고 빼줬음. 그러나 실제로는 비어 있음. 따라서 이에 의한 효과를 더해 주어야 함)

✓ Terrain correction is always added to the observed gravity.

✓ There are several methods for calculating terrain corrections. All of them require a good topographical map of the area. Nowadays, DEM (digital elevation models) are used.

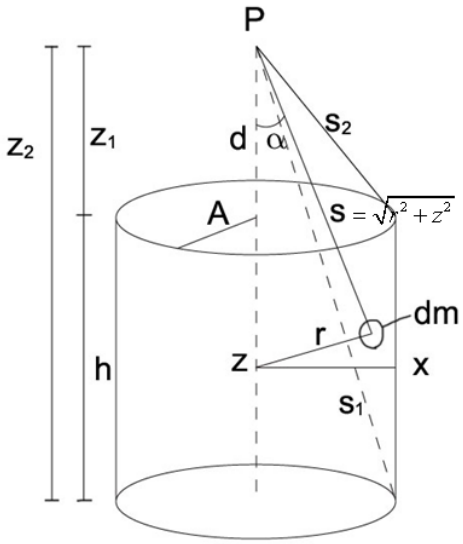
✓ The usual procedure is to divide the area into compartments and compute the average elevation within each compartment. Each compartment is compared to the station elevation.

✓ Conventional method: Sigmund Hammer (1939) devised Hammer chart. Hammer chart is composed of concentric circles, and annuli are divided into several segments.



$$\delta g_{seg} = \frac{2\pi\rho G}{N}\left[r_2 - r_1 + \sqrt{r_1^2 + z^2} - \sqrt{r_2^2 + z^2} \right]$$

Proof: 반경이 각각 r_1, r_2인 원통에 의한 중력 효과의 차를 N으로 나눠준 것과 같음. 후에 부게보정을 위한 유도에서 나오는 식 (응용지구물리 2.69식 이용, $z_1 \rightarrow 0$, $z_2 \rightarrow z$ 로 놓으면 됨)



응용지구물리 2.69식

$g_z = 2\pi G\rho\left[\left(A^2 + z_1^2 \right)^{1/2} - \left(A^2 + z_2^2 \right)^{1/2} + z_2 - z_1 \right]$ 에서

A: 반지름 $z_1=0$ $z_2=z$가 됨.

반지름이 각각 r_2, r_1일 때 구하면

$$g_z(r_1) = 2\pi G\rho\left[r_1 - \left(r_1^2 + z^2 \right)^{1/2} + z \right]$$

$$g_z(r_2) = 2\pi G\rho\left[r_2 - \left(r_2^2 + z^2 \right)^{1/2} + z \right]$$

$$\delta g_{seg} = \frac{1}{N}\left[g_z(r_2) - g_z(r_1) \right]$$

- ✓ When we have digital elevation models, it is convenient to use prism

$$\delta g_{prism} = G\rho D(1 - \cos\alpha) K(i, j)$$

where

D =The length of the side

α =The angle of inclination

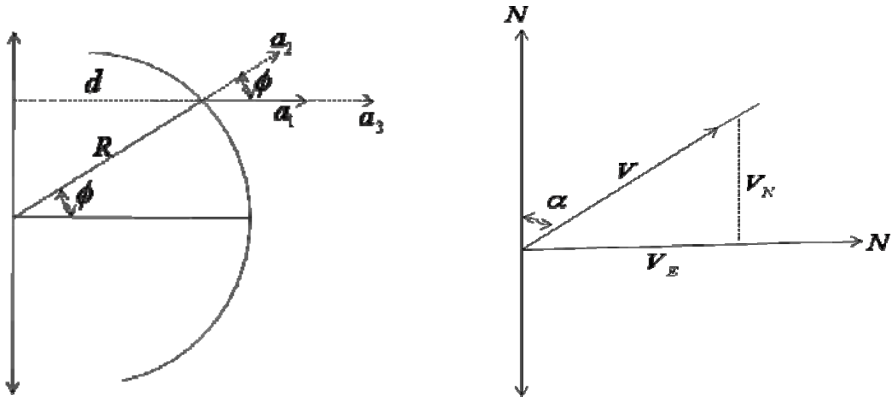
K = Coordinate

- ✓ Terrain corrections are made on a computer by using an appropriate set of grid station elevations.
- ✓ Regardless of the approach, the topography reduction is a slow and very boring task. Furthermore in areas of steep and erratic slopes it is not necessarily accurate, particularly in the vicinity of the station itself.

➢ Eötvös correction

✓ It should be applied to marine or airborne gravity data. In that case, ship or plane move, which causes some centrifugal forces. The centrifugal forces have an effect of reducing gravitational acceleration.

✓ The effects of the centrifugal forces should be removed from the measured data



- Centrifugal acceleration for a static mass $a_1 = \frac{v^2}{d}$ is only due to earth's rotation.
- When the ship moves, the east-west components of the velocity of the ship plays a role of increasing the centrifugal acceleration.

$$a_1 = \frac{\left(v + V_E{}^2\right)}{d}$$

- There exists centrifugal acceleration due to the movement of the ship in the north-south direction

$$a_2 = \frac{V_N{}^2}{R}$$

- Centrifugal acceleration due to the earth's rotation

$$a_3 = \frac{v^2}{d}$$

$$\delta g_E = a_1 \cos\phi + a_2 - a_3 \cos\phi$$

$$d = R\cos\phi, \quad v = \omega R\cos\phi, \quad V = (V_N{}^2 - V_E^2)^{\frac{1}{2}}$$

$$V_E = V\sin\alpha$$

$$\delta g_E = \frac{(v + V_E)^2}{R\cos\phi}\cos\phi + \frac{V_N{}^2}{R} - \frac{v^2\cos\phi}{R\cos\phi}$$

$$\delta g_E = \frac{\left[(v + V_E)^2 + V_N{}^2 - v^2\right]}{R}$$

$$\delta g_E = 2\omega V\cos\phi\sin\alpha + \frac{V^2}{R}$$

$$\omega = 7.2921\times10^{-5}\ radian/s$$

$$R = 6.371\times10^{8}\ cm$$

$$1Knot = 51.479\ cm/s$$

$$1Kn = 1{,}852\ m/h$$

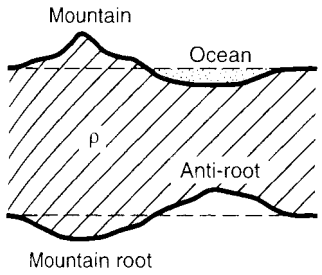
$$1Gal = 10^4\ g\cdot u$$

$$\delta g_E = 75.08V\cos\phi\sin\alpha + 0.0416V^2$$

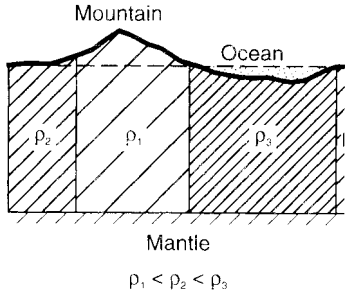
- Isostatic correction
  - ✓ If the earth crust had no lateral variations in density we would find that a set of gravity readings, after suitable corrections for the effects described above, would be identical.

  - ✓ From world-wide gravity measurements, it has been found that the average Bouguer anomaly on land near sea level is approximately zero.
  - ✓ In oceanic areas it is generally positive, while in regions of large elevation it is mainly negative. These large-scale effects are due to density variations in the crust and indicate that the material beneath the oceans is more dense than normal, while in regions of elevated land masses it is less dense than normal.
  - ✓ Two hypotheses were suggested to account this large-scale systematic density variation:

Airy proposed a rigid crust of uniform density floating on a liquid substratum of higher density



Pratt suggested a crust, also floating on a uniform liquid, the density of the crust varying with the topography, being lower in mountain regions and higher where the crust is thin.



Heiskanen has presented a modified form of the hypothesis, in which he combines a lateral variation of crustal density with a variable depth, plus a gradual increase of density with depth.

For isostatic correction, Pratt's assumption is used.
Isostatic correction is to remove the effects of the large-scale density variations.

- Gravity anomaly

  - ✓ Gravity anomaly is defined as the differences between corrected gravity and reference gravity
  - ✓ Once all of the preceding corrections are applied to observed gravity

data, the gravity anomaly reflects the density variation.

- ✓ Free-air anomaly: the differences between reference data and observed data corrected for latitude effects and free-air effects.

- ✓ Simple Bouguer anomaly: the differences between reference data and observed data corrected for latitude, free-air, and bouguer effects

- ✓ Bouguer anomaly: the differences between reference data and observed data that went through latitude, free-air, bouguer, and terrain corrections

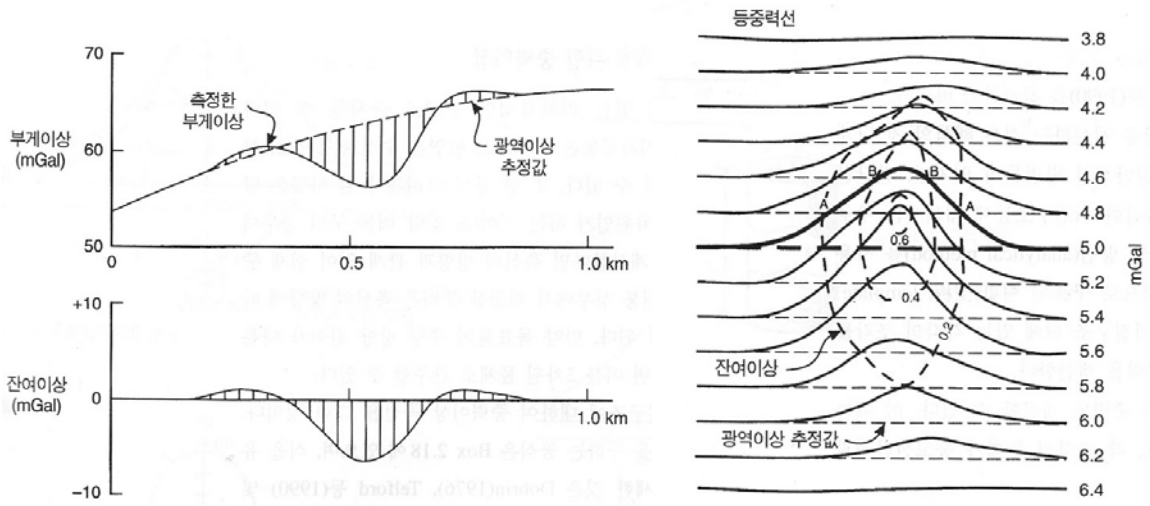
- ✓ Isostatic anomaly: the differences between reference data and observed data that went through latitude, free-air, bouguer, terrain and isostatic correction.

(4) Interpretation

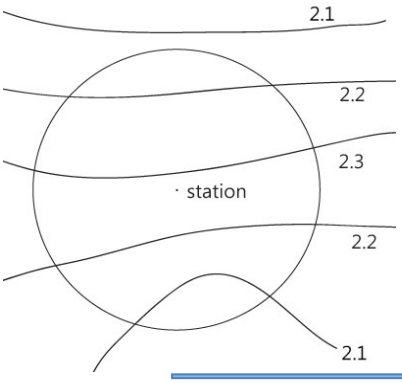
(a) Regionals and residuals

- Regional anomaly: the interesting anomalies on the gravity map frequently are masked by deep structures. The long wavelength gravity anomaly resulting from deep structures is called regional anomaly
- Residual anomaly: the short wavelength gravity anomaly resulting from shallow structures
  (Residual anomaly = observed gravity anomaly – regional anomaly)
- The removal of the regional gravity is a more serious problem in gravity than in other geophysical methods

➢ Graphical and smoothing technique



➢ Empirical gridding method



The regional anomaly is considered to be the average value of gravity in the vicinity of the station, and is obtained by averaging observed values on the circumference of a circle centered at the station.

The regional is dependent on the radius of the circle. If the radius is very small, the residual is zero, and if it is too large, the residual is approximately the observed gravity.

- Second derivative

  The second vertical derivative enhances near-surface effects at the expense of deeper sources

  The second derivatives are a measure of curvature; large curvatures are associated with shallow or residual anomalies.

- Polynomial fitting

  This is a purely analytical method. Regional gravity is matched by a polynomial surface. Regional gravity can be computed by the least squares method from observed gravity data.

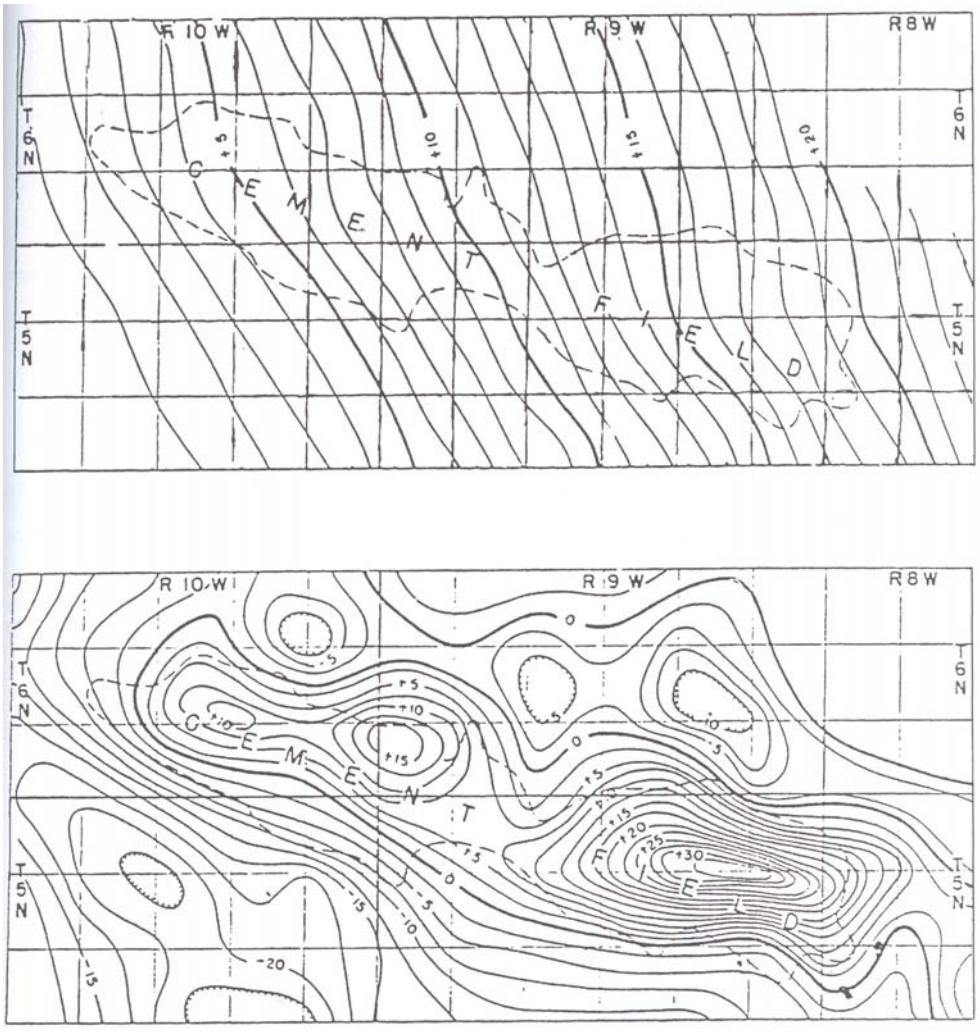


Figure 2.31 Observed Bouguer anomaly (contour interval 1 mGal) and second vertical derivative (contour interval 2.5×10^{-15} c.g.s.u.) maps over a Cement field in Oklahoma. From Elkins (1951), by permission

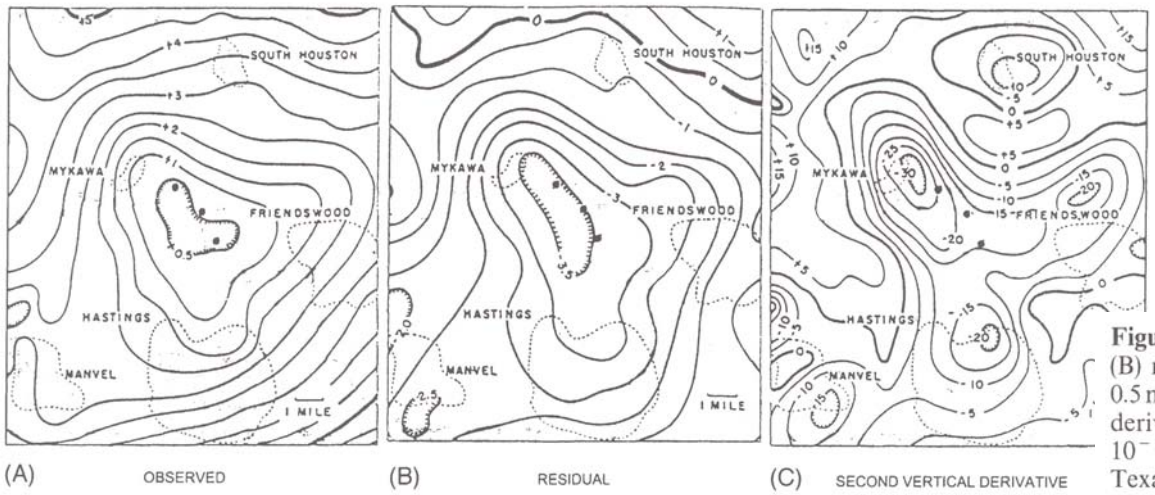


Figure 2.32 (A) Observed gravity, (B) residual gravity (contour interval 0.5 mGal), and (C) second vertical derivative (contour interval 5×10^{-15} c.g.s.u.) maps for Mykawa, Texas Gulf Coast. ● indicate dry boreholes. From Elkins (1951), by permission

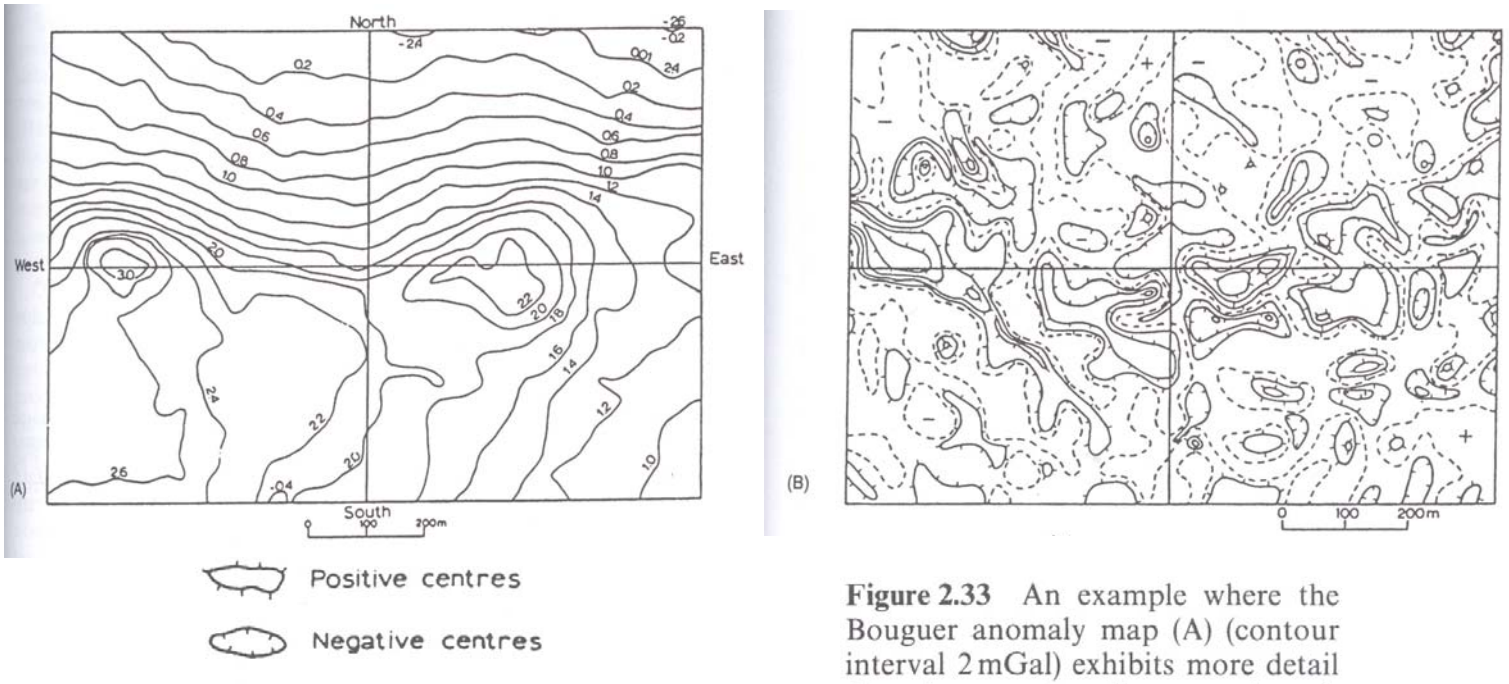


Figure 2.33 An example where the Bouguer anomaly map (A) (contour interval 2 mGal) exhibits more detail than the corresponding second vertical derivative map (B) for the J-jaure titaniferous iron-ore region in Sweden. Contours: 0 (dashed), ± 0.1, ± 0.2, ± 0.4 in units of 0.0025 mGal/m^2). From Parasnis (1966), by permission

➢ Upward and downward continuation

This is the process by which potential field data from one datum surface are mathematically projected upward or downward to level surfaces above or below the original datum.

✓ Upward continuation (Amplify the Noises)

- Straightforward operation, since the surfaces are in field-free space

- Smoothing the anomalies obtained at ground surface: regional structure (Figure 2.34)

✓ Downward continuation

- More dangerous, because of the inherent uncertainty in the location and size of the structures represented by the Bouguer gravity at the datum plane

- Sharpening the anomalies obtained at ground surface: small-scale structure

➢ The downward continuation process can be best suited to interpretation in oil exploration when the principal features of interest are largely controlled by the basement and the overlying relatively uniform sedimentary beds produce smaller gravity effects

☞ For mineral prospecting, the second derivative calculation is probably more suitable (Figure 2.31, 2.32, 2.33)

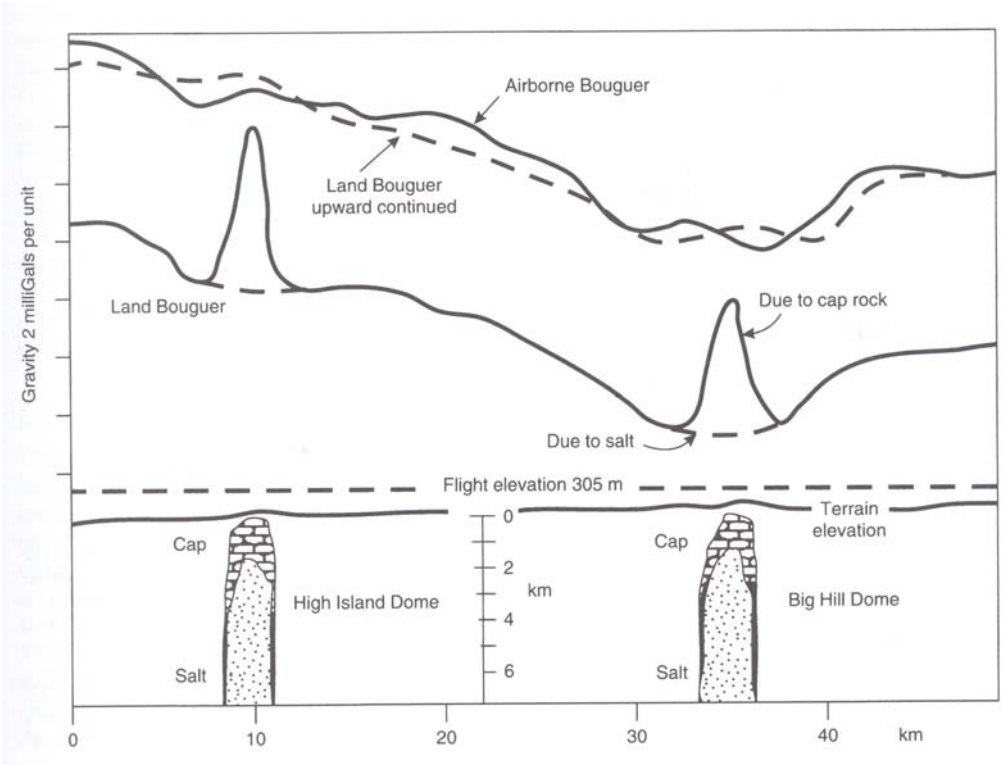
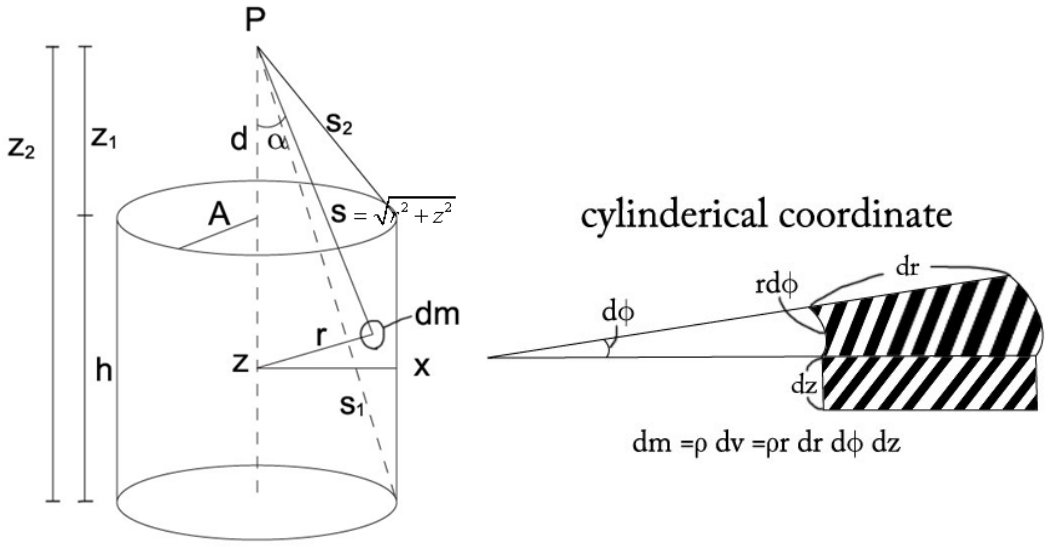


Figure 2.34 Comparison of airborne and upward continued land Bouguer gravity data with those obtained by airborne gravity surveying. From Hammer (1984), by permission

(b) Gravity effects of simple shapes

➢ Gravity effects of a vertical cylinder



$$dg_s = \frac{G \cdot dm}{s^2} = \frac{G\rho r dr d\phi dz}{r^2 + z^2}$$

$$dg_z = dg_s \cdot \cos\alpha = \frac{G\rho r dr d\phi dz}{r^2 + z^2} \cdot \frac{z}{\sqrt{r^2 + z^2}}$$

$$= \frac{G\rho r z dr d\phi dz}{(r^2 + z^2)^{3/2}}$$

$$g_z = G\rho \int_{z=z_1}^{z_2} \int_{r=0}^{A} \int_{\phi=0}^{2\pi} \frac{r z d\phi dr dz}{(r^2 + z^2)^{3/2}} = 2\pi G\rho \int_{z=z_1}^{z_2} \int_{r=0}^{A} \frac{r z dr dz}{(r^2 + z^2)^{3/2}}$$

$$= 2\pi G\rho \int_{z=z_1}^{z_2} \left[\frac{z}{(r^2 + z^2)^{1/2}} \right]_0^A dz = 2\pi G\rho \int_{z=z_1}^{z_2} \left[-\frac{z}{\sqrt{A^2 + z^2}} + 1 \right] dz$$

$$= 2\pi G\rho \left[-\sqrt{A^2 + z^2} + z \right]_{z_1}^{z_2} = 2\pi G\rho \left[\sqrt{A^2 + z_1^2} - \sqrt{A^2 + z_2^2} + (z_2 - z_1) \right]$$

$$= 2\pi G\rho \left[s_2 - s_1 + h \right]$$

① when $z_2 \cong \infty$

$$\sqrt{A^2 + z_2^2} \approx z_2$$

$$g_z = 2\pi G\rho \left[(A^2 + z_1^2)^{1/2} - z_1 \right] = 2\pi G\rho \left[s - z_1 \right]$$

② when $A \cong \infty$

$$\sqrt{A^2 + z_1^2} \approx A, \quad \sqrt{A^2 + z_2^2} \approx A$$

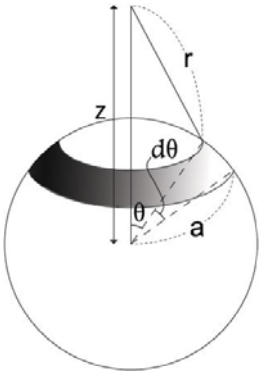
$g_z = 2\pi G\rho h \Rightarrow$ Bouguer slab. Or Bouguer plate

➢ Gravity effects of a spherical shell and a sphere

① spherical shell

$$dA = a^2 \sin\theta d\theta d\phi \qquad dm = \sigma dA$$

※gravitational potential



$$dU = \frac{Gdm}{r} = \frac{G\sigma dA}{r}$$

$$U = \int dU$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{G\sigma a^2 \sin\theta d\theta d\phi}{r}$$

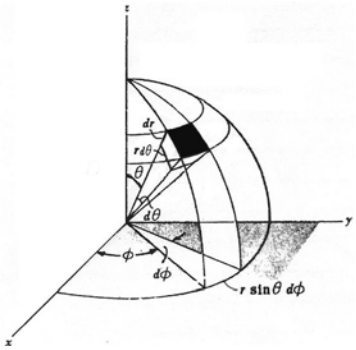
$$= \int_{\theta=0}^{\pi} \frac{G\sigma(2\pi a^2)\sin\theta d\theta}{r}$$

The second cosine law $r^2 = z^2 + a^2 - 2az\cos\theta$

$$U = \int_0^{\pi} \frac{G\sigma(2\pi a^2)\sin\theta d\theta}{(z^2 + a^2 - 2az\cos\theta)^{1/2}} = \frac{G\sigma(2\pi a^2)}{az}\left[(z+a) - |z-a|\right]$$

Using the area for spherical shell $4\pi a^2$

$$U = \frac{G\cdot \mathrm{M}_{\mathrm{shell}}}{2az}\left[(z+a) - |z-a|\right], \quad \mathrm{M}_{\mathrm{shell}} = 4\pi a^2\sigma$$



$z \geq a$ P is located outside the spherical shell

$|z-a| = z-a$

$U_{out} = \frac{G\cdot \mathrm{Mshell}}{z}$: $z \geq a$: function of z

$z < a$

$U_{in} = \frac{G\cdot \mathrm{Mshell}}{z}$: $z < a$: constant

g : the gravity measured at the distance z from the center of the spherical shell when the mass of the shell Mshell are assumed to be gathered at the center

$$g_{out} = \nabla U_{out} = -\frac{G\cdot \mathrm{M}_{\mathrm{shell}}}{z^2}\vec{z} \quad (z \geq a)$$

$$g_{int} = 0 \qquad (z < a)$$

② Sphere

We can think of a sphere as gathering of a number of spherical shells from r=0 to r=a

radius : a, volume density : ρ

$$dV = R^2 \sin\theta dR d\theta d\phi$$

$$dm = \rho R^2 \sin\theta dR d\theta d\phi$$

$$U = \iiint \frac{Gdm}{r} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{R=0}^{a} \frac{G\rho R^2}{r}\sin\theta dR d\theta d\phi$$

$$U_{out} = \frac{G\rho}{z}\left(\frac{4}{3}\pi a^3\right) \quad z \geq a$$

$$= \frac{G\cdot \mathrm{M}_{\mathrm{sphere}}}{z}$$

All the masses are placed at the center

$$g_{out} = \nabla U_{out}$$

$$= -\frac{G\cdot \mathrm{M}_{\mathrm{sphere}}}{z^2}\vec{z} \quad (z > a)$$

※ z<a : imagine the sphere surface whose radius is a

⇒ the surface of the sphere is Gauss surface.

The effect of interior mass + effect of outside mass

$$U_{in} = G\left[\underbrace{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{R=0}^{z}\frac{\rho R^2}{r}\sin\theta dR d\theta d\phi}_{\text{(a)}} + \underbrace{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{R=z}^{a}\frac{\rho R^2}{r}\sin\theta dR d\theta d\phi}_{\text{(b)}}\right]$$

$$\text{(a)} = \frac{4\pi G\rho z^2}{3} \qquad \text{(b)} = 2\pi G\rho\left[a^2 - z^2\right]$$

(a)앞에서와 같은 방법으로 구해지나 반지름이 z가 됨.

(b) $$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{R=z}^{a}\frac{G\rho R^2\sin\theta dR d\theta d\phi}{\sqrt{R^2+z^2-2Rz\cos\theta}} = 2\pi G\rho\int_{R=z}^{a}[2R]dR = 2\pi G\rho\left[a^2 - z^2\right]$$

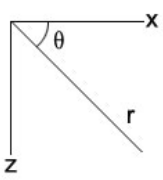
$$\int_{\theta=0}^{\pi}\frac{R^2\sin\theta d\theta}{\sqrt{R^2+z^2-2Rz\cos\theta}} = \left[\frac{R}{z}\sqrt{R^2+z^2-2Rz\cos\theta}\right]_0^{\pi}$$

$$= \frac{R}{z}\left[R+z-\left|R-z\right|\right] = 2R \quad (R > z)$$

$$U_{in} = \frac{2\pi G\rho}{3}(3a^2 - z^2)$$

$$g_{in} = \nabla U_{in} = -\frac{4\pi G\rho z}{3}\vec{z} = -\frac{G\cdot \mathrm{M}_{inside}}{r^2}\vec{z} \quad : z < a$$

Outside mass never affects the gravity



At any point along the horizontal axis

$$\sin\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2+z^2}} \qquad g = G\frac{M_{sphere}}{r^3} = G\frac{4\pi R^3\rho/3}{x^2+z^2}$$

$$g_z = g\sin\theta = \frac{4\pi G\rho R^3}{3}\frac{z}{\left(x^2+z^2\right)^{3/2}}$$

(c) Determination of depth and mass

- Half-width method: using the half-width of the profile at half-maximum

$$g_{\max} \qquad \frac{1}{2}g_{\max}$$

$x=0$

- ✓ For sphere

(Depth)

Gravity at any point along the horizontal axis

$$g_z = \frac{4\pi G\rho R^3}{3}\frac{z}{(x^2+z^2)^{\frac{3}{2}}}$$

$$\Delta g_z = \frac{4\pi G\Delta\rho R^3}{3}\frac{z}{(x^2+z^2)^{\frac{3}{2}}}$$

$$x=0,\ \Delta g_{\max} = \frac{4\pi G\Delta\rho R^3}{3}\frac{1}{z^2}$$

$$x=x_{\frac{1}{2}}\ \frac{1}{2}\Delta g_{\max} = \frac{4\pi G\Delta\rho R^3}{3}\frac{z}{(x_{\frac{1}{2}}^{\ 2}+z^2)^{\frac{3}{2}}}$$

$$\frac{1}{2} = \frac{z^3}{(x_{\frac{1}{2}}^{\ 2}+z^2)^{\frac{3}{2}}}$$

$$2z^3 = (x_{\frac{1}{2}}^{\ 2}+z^2)^{\frac{3}{2}}$$

$$\sqrt[3]{2}z = (x_{\frac{1}{2}}^{\ 2}+z^2)^{\frac{1}{2}}$$

$$\sqrt[3]{4}z^2 = x_{\frac{1}{2}}^{\ 2}+z^2$$

$$\left(\sqrt[3]{4}-1\right)z^2 = x_{\frac{1}{2}}^{\ 2}$$

$$z = \frac{1}{\sqrt{\left(\sqrt[3]{4}-1\right)}}x_{\frac{1}{2}} = 1.305x_{\frac{1}{2}}$$

(Mass)

$$\Delta g_z = \frac{4\pi G\Delta\rho R^3}{3}\frac{z}{(x^2+z^2)^{\frac{3}{2}}}$$

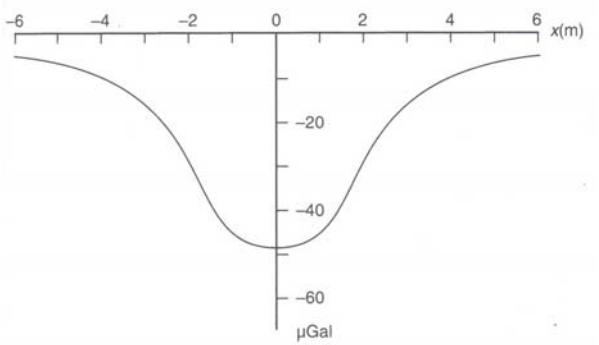
$$x=0,\ \Delta g_{\max} = \frac{4\pi G\Delta\rho R^3}{3}\frac{1}{z^2} = \frac{G\Delta M}{z^2}$$

$$\Delta M = \frac{\Delta g_{\max} z^2}{G}$$

$$G = 6.67\times10^{-8}\,dyne\cdot cm^2/g^2 = 6.67\times10^{-8}\,g\cdot cm^3/g\cdot s^2$$

$$z = 1.305\cdot x_{\frac{1}{2}}\,m = 1.305\times10^2\,x_{\frac{1}{2}}\,cm$$

✓ For examples



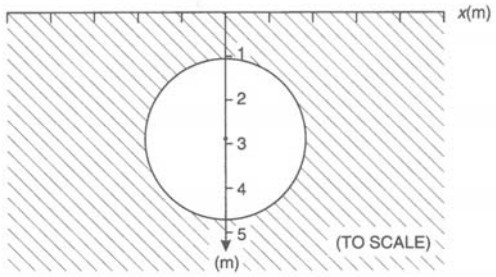
$$\Delta g_{\max} = 0.048 mgal$$

$$x_{\frac{1}{2}} = 2.2m$$

$$\delta\rho = 2.5 Mg/m^3$$

*Depth of the center of sphere

$$z = 1.305 \cdot x_{\frac{1}{2}} = 1.305 \times 2.2 = 2.87m$$



*The radius of the sphere $\Delta g_{\max} : x = 0$

$$R^3 = \frac{0.048 \times 10^{-3} \times 10^{-2} m/s (2.87m)^2}{\frac{4\pi}{3} 6.67 \times 10^{-8} g \cdot cm^3 / g \cdot s^2 \cdot 2.5 g/cm^3} = 5.53 m^3$$

$$R = 1.17m$$

$$d = 2.87 - 1.17 = 1.10m$$

➢ Maximum-depth rules

✓ Smith (1959) has given several formulae for maximum depths of gravity distributions whose shapes are not known.

✓ The only restriction is that the anomalous bodies have a density contrast with the host rock which is either entirely positive or entirely negative.

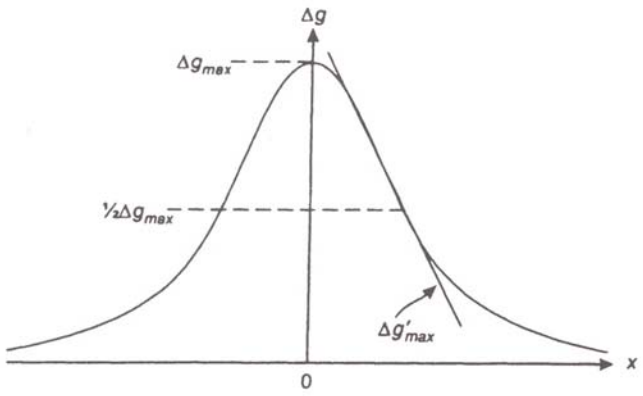


Figure 2.30 Limiting depth calculations: half-width method and gradient–amplitude ratio method (see text for details)

(1) Where the entire anomaly has been isolated:

$$d \le c \Delta g_{\max} / \Delta g'_{\max}$$

$c = 0.65$ for a 2D body

$= 0.86$ for a 3D body

(2) When only part of an anomaly is isolated, for any point x:

$$d \le K \Delta g_x / \Delta g'_x$$

$K = 1.0$ for a 2D body

$= 1.5$ for a 3D body

(c) Density of rocks

표 2.2 지질매질의 밀도(Telford 등, 1990)

매질의 종류	밀도범위(Mg/m^3)	평균밀도(Mg/m^3)
퇴적암		
충적층	1.96 ~ 2.00	1.98
점토	1.63 ~ 2.60	2.21
자갈	1.70 ~ 2.40	2.00
황토	1.40 ~ 1.93	1.64
실트	1.80 ~ 2.20	1.93
토양	1.20 ~ 2.40	1.92
모래	1.70 ~ 2.30	2.00
사암	1.61 ~ 2.76	2.35
셰일	1.77 ~ 3.20	2.40
석회암	1.93 ~ 2.90	2.55
백운암	2.28 ~ 2.90	2.70
백악(chalk)	1.53 ~ 2.60	2.01
암염(halite)	2.10 ~ 2.60	2.22
빙하얼음	0.88 ~ 0.92	0.90
화성암		
유문암(rhyolite)	2.35 ~ 2.70	2.52
화강암(granite)	2.50 ~ 2.81	2.64
안산암(andesite)	2.40 ~ 2.80	2.61
섬장암(syenite)	2.60 ~ 2.95	2.77
현무암(basalt)	2.70 ~ 3.30	2.99
반려암(gabbro)	2.70 ~ 3.50	3.03
변성암		
편암(schist)	2.39 ~ 2.90	2.64
편마암(gneiss)	2.59 ~ 3.00	2.80
천매암(phyllite)	2.68 ~ 2.80	2.74
점판암(slate)	2.70 ~ 2.90	2.79
그래뉼라이트(granulite)	2.52 ~ 2.73	2.65
각섬암(amphibolite)	2.90 ~ 3.04	2.96
에클로자이트(eclogite)	3.20 ~ 3.54	3.37

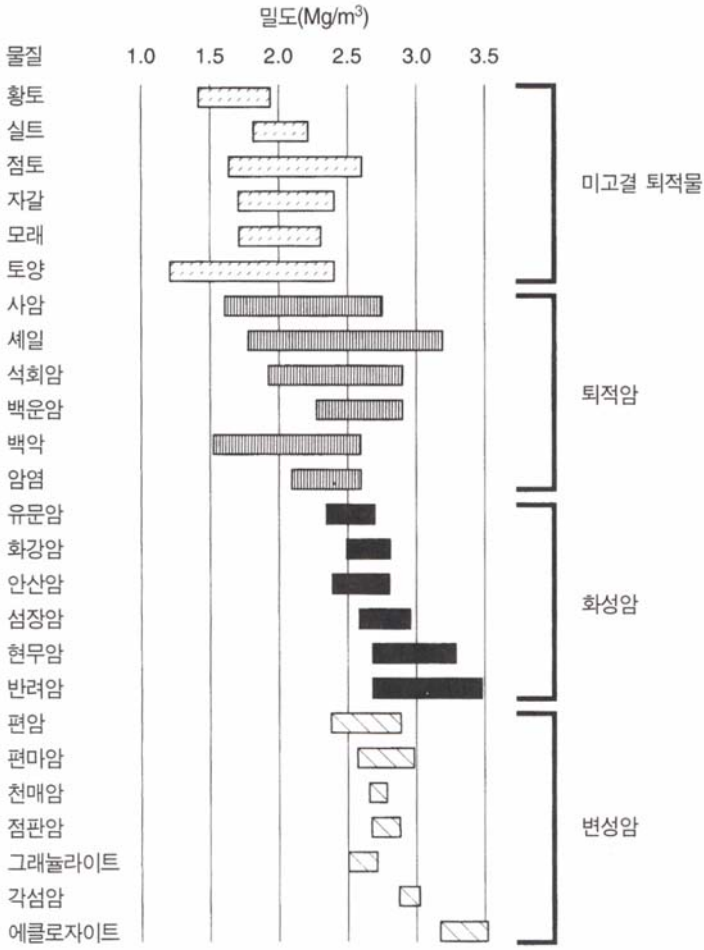


그림 2.5 여러 가지 암석의 밀도변화(Telford 등, 1990)

표 2.3 여러 요인이 밀도에 미치는 영향

요 인	밀도에 미치는 영향(%)
조 성	35
교 결	10
연령 및 깊이	25
조구조 운동	10
공극률 및 공극유체	10

(d) Examples

✓ Sedimentary Basin or Deep granite?
(Figure 2.36 and Box 2.23)

1) Sedimentary Basin $\left|g''_{\max}\right| / \left|g''_{\min}\right| > 1.0$

2) Deep granite $\left|g''_{\max}\right| / \left|g''_{\min}\right| \leq 1.0$

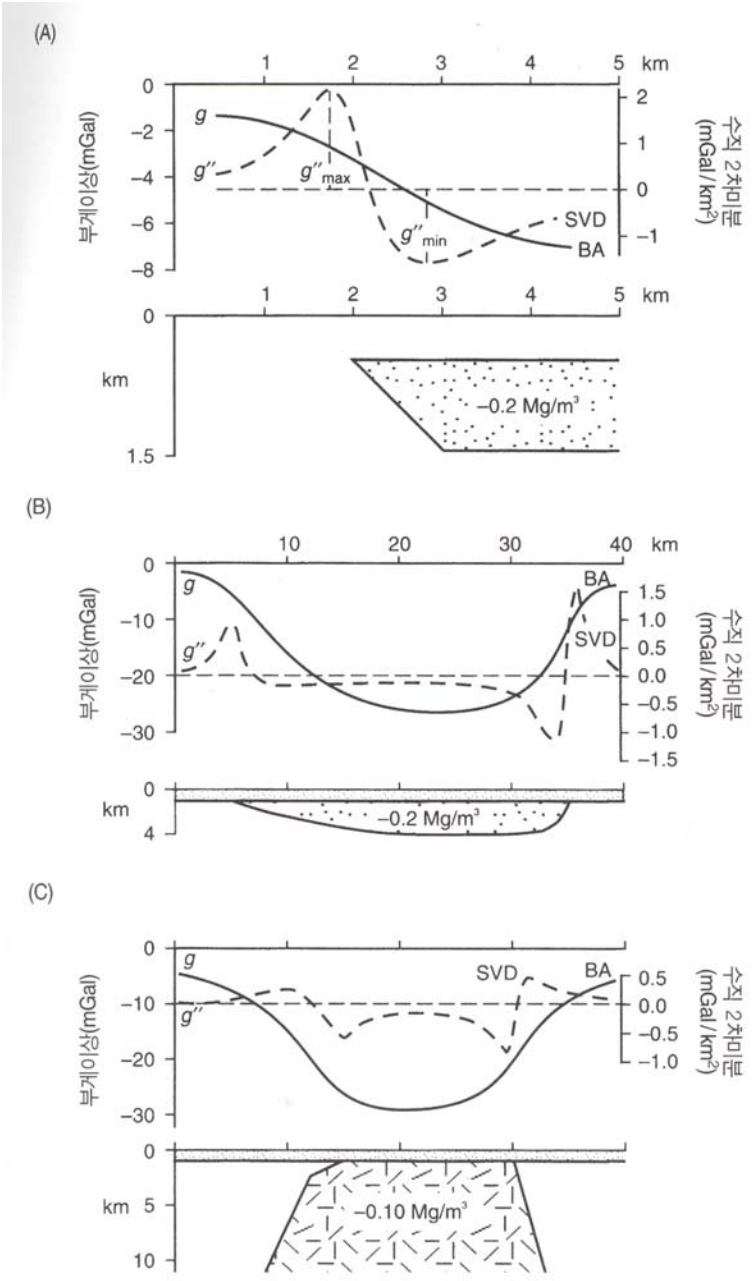


그림 2.36 (A) 수평 각주, (B) 퇴적분지, (C) 심부 화강암에 대한 부게이상 곡선을 서로 구분하는 Bott의 기준

(5) Application

a. Exploration of salt domes (Mors salt dome, Denmark, waste disposal)

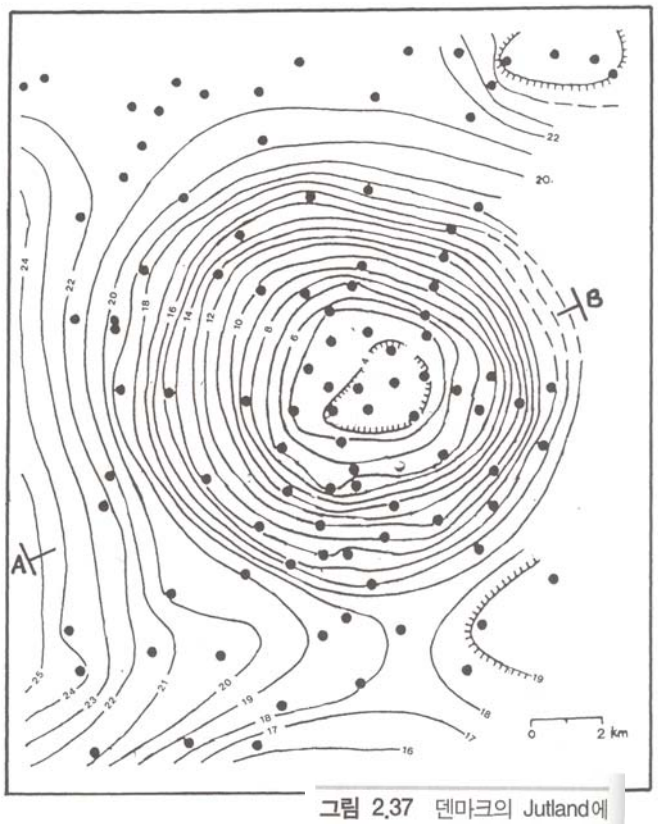


그림 2.37 덴마크의 Jutland에 있는 Mors 암염돔의 부게이상도. 까만 점(●)은 측정점이며, 등고선간격은 1mGal이다(Saxov, 1956; Sharma, 1986).

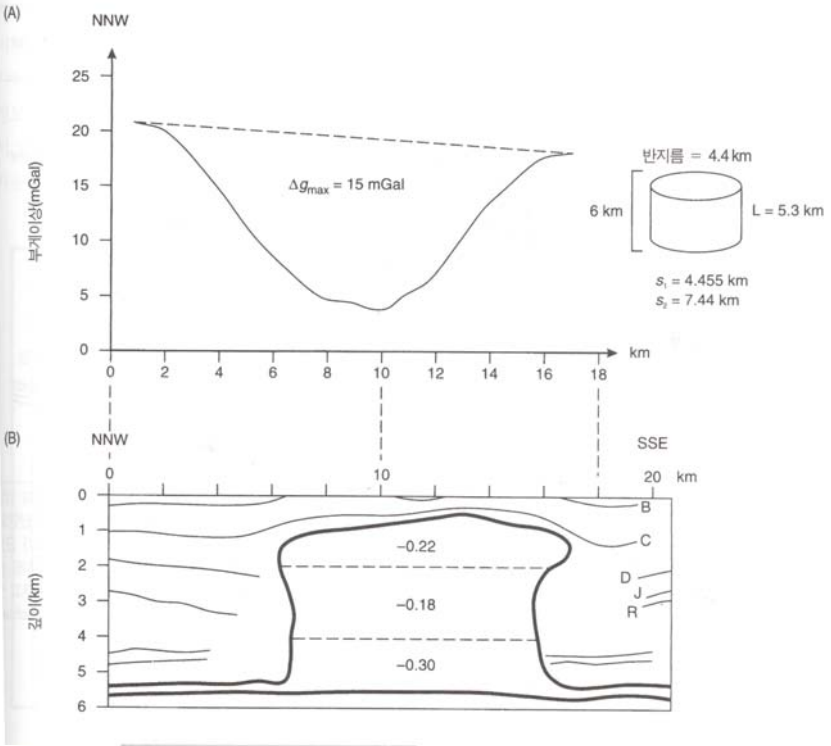


그림 2.38 그림 2.37의 A–B 측선에 대한 (A) 부게이상, (B) 탄성파 단면(Kreitz, 1982; LaFehr, 1982; Sharma, 1986)

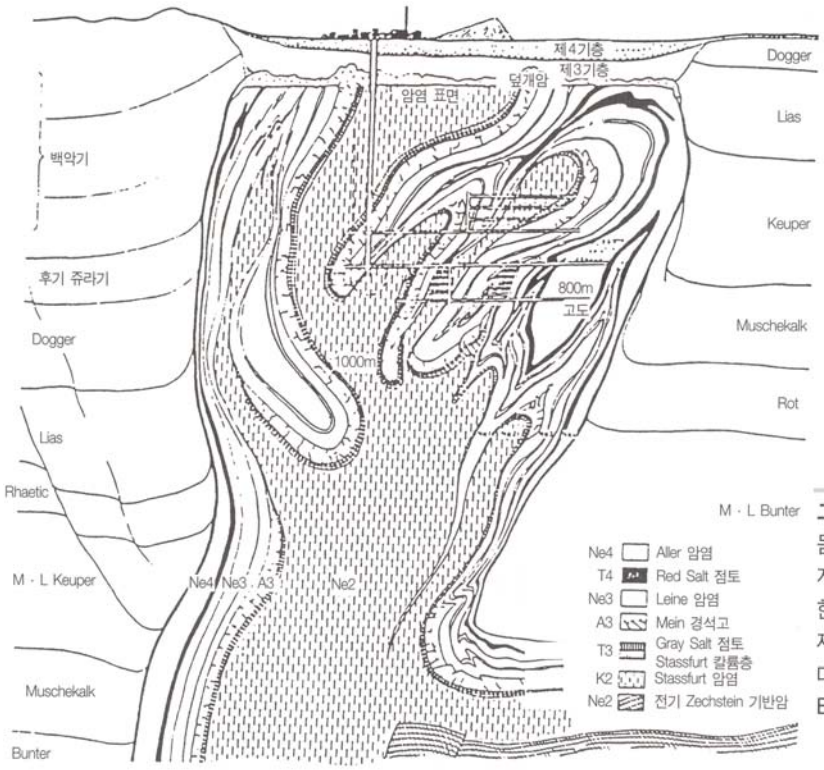


그림 2.39 독일 북서부의 암염돔 구조에 대한 모식적인 단면도. 지구물리학적으로는 밀도가 균일한 수직 원통으로 근사하지만 실제로는 복잡한 모양을 이루고 있다(Sorgenfrei, 1971; Richter-Bernburg, 1982).

b. Mineral exploration

➢ Discovery of the Faro lead-zinc deposit, Yukon

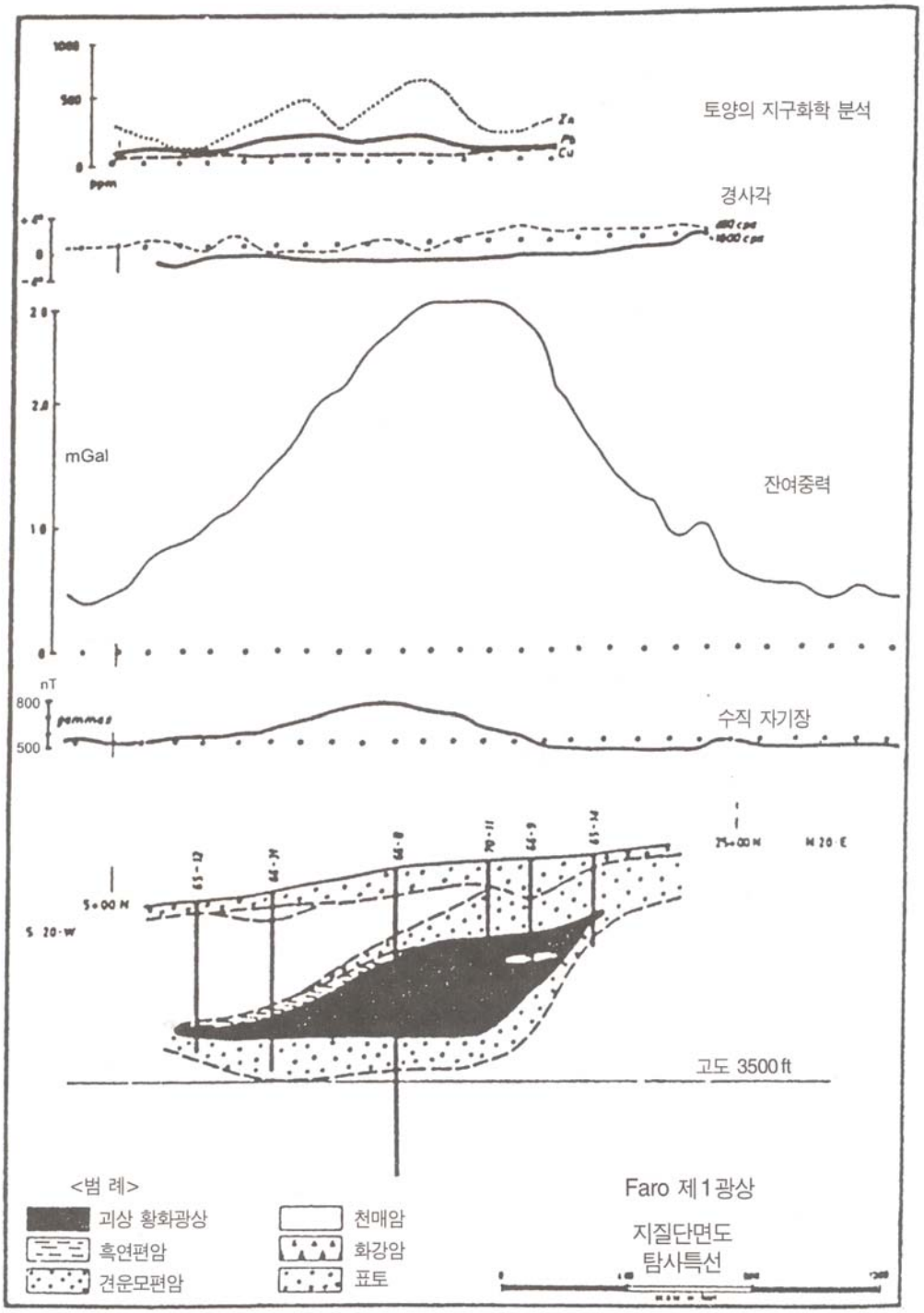


그림 2.41 캐나다 Yukon 주의 Faro 납-아연 광상 상부에서 중력의 잔여이상을 포함한 여러 가지 물리탐사 결과(Brock, 1973; Tanner와 Gibb, 1979)

➢ Sourton Tors, Dartmoor, SW England

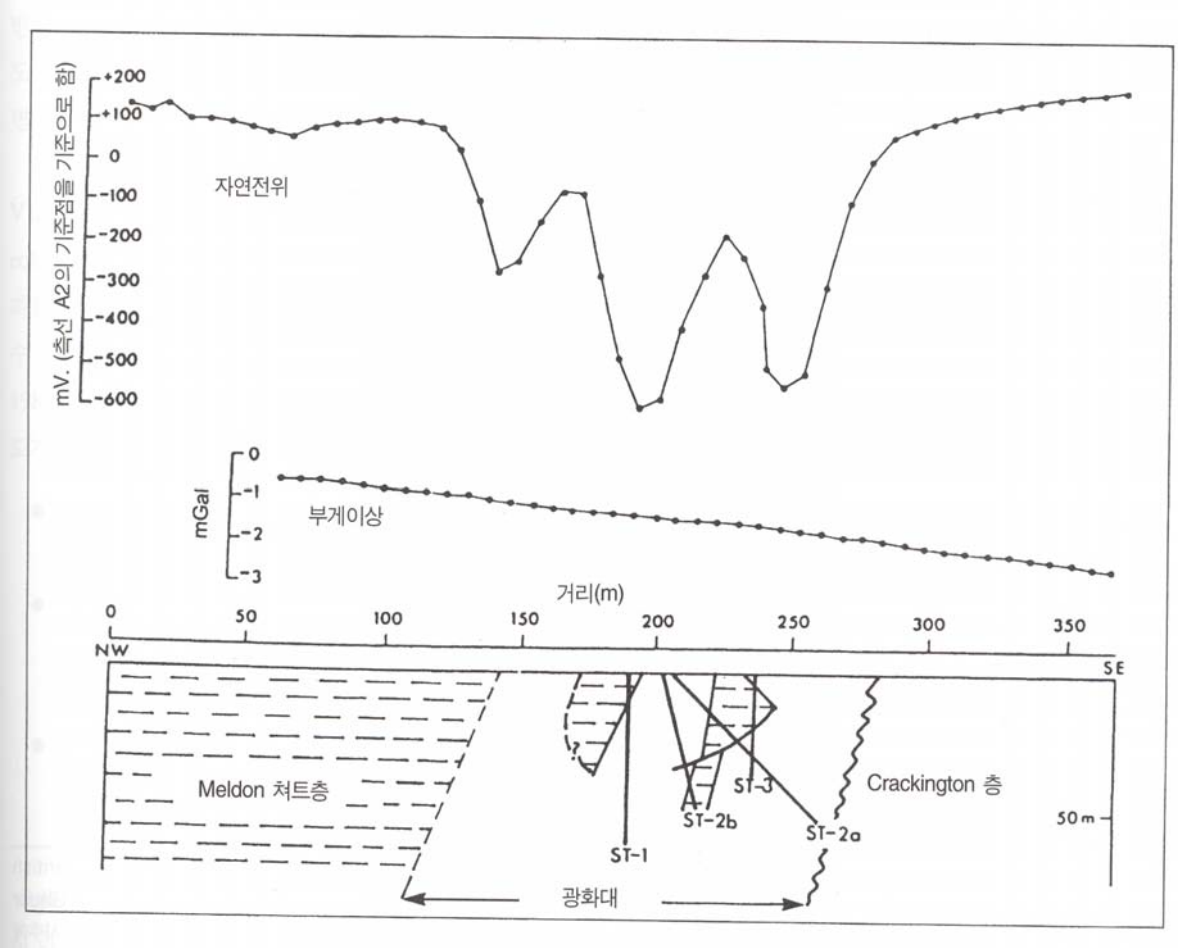


그림 2.43 영국 Dartmoor 북서부의 Sourton Tors에 있는 쳐트(chert) 광상 내에서의 광화대에 대한 부게이상 곡선. 식별할 수 있는 이상이 나타나지 않는다 (Beer와 Fenning, 1976).

➢ Glacier thickness determination

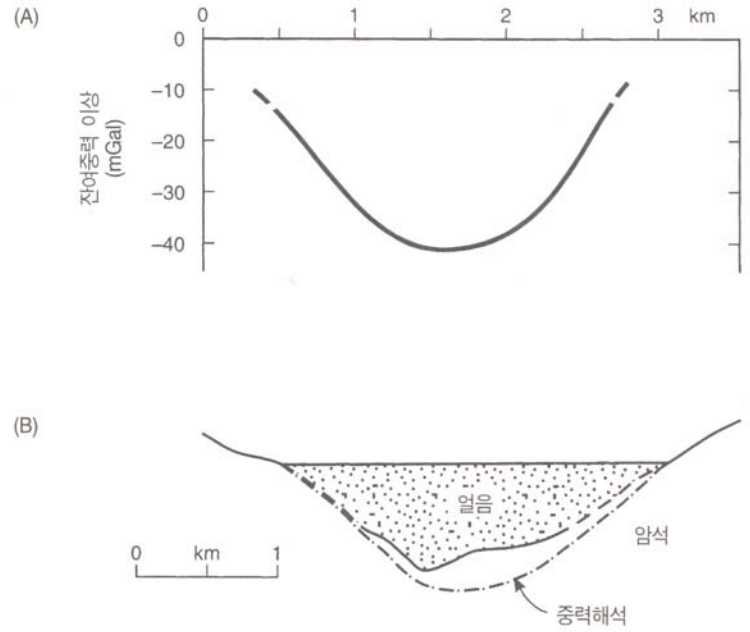


그림 2.44 캐나다 British Columbia 주의 Salmon Glacier에 대한 잔여중력 곡선. 시추에 의해 확인한 빙하의 두께와 중력탐사에 의한 두께를 비교하였다 (Grant와 West, 1965).

➢ Detection of underground cavities

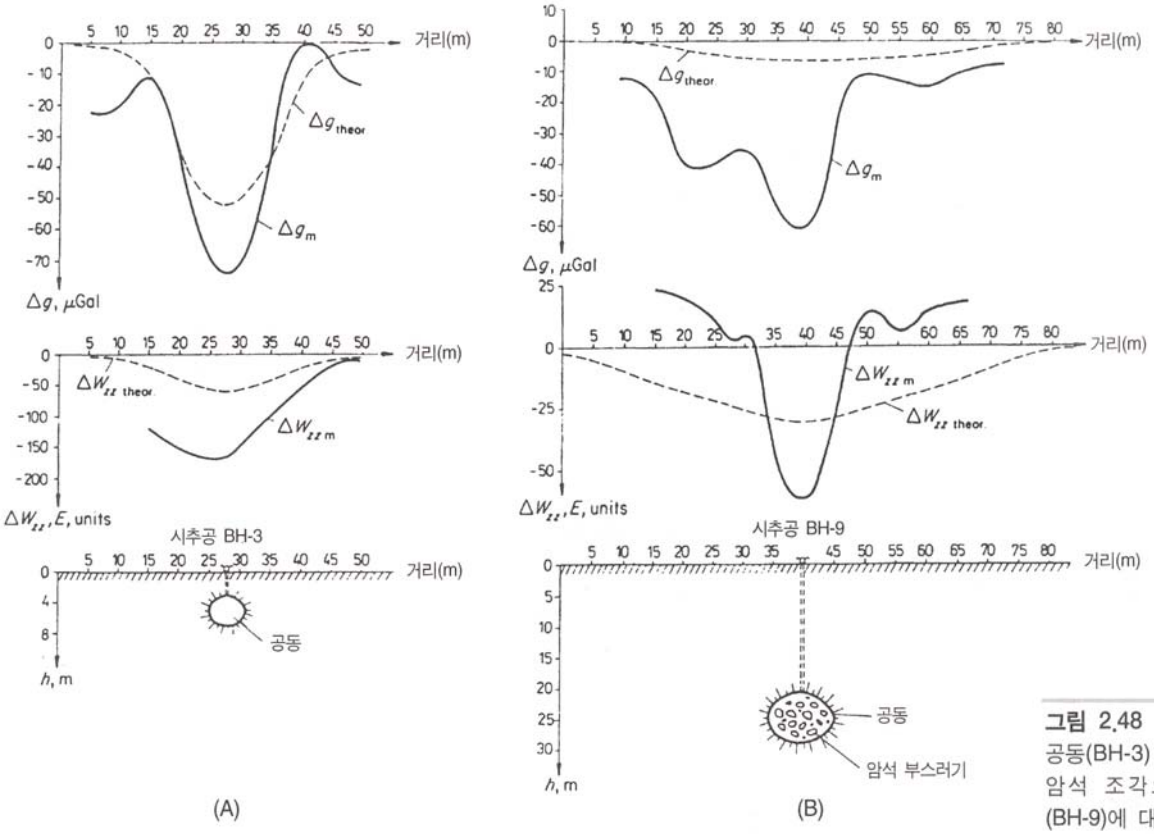


그림 2.48 (A) 공기로 채워진 공동(BH-3) 및 (B) 부분적으로 암석 조각으로 채워진 공동(BH-9)에 대한 고정밀 중력이상 Δg_m과 중력기울기이상 $\Delta W_{zz,m}$. 'theor' 라는 첨자가 붙은 곡선은 공동(空洞)에 대하여 계산한 이론적인 이상이다.

➢ A micro-gravity survey within a deep coal mine

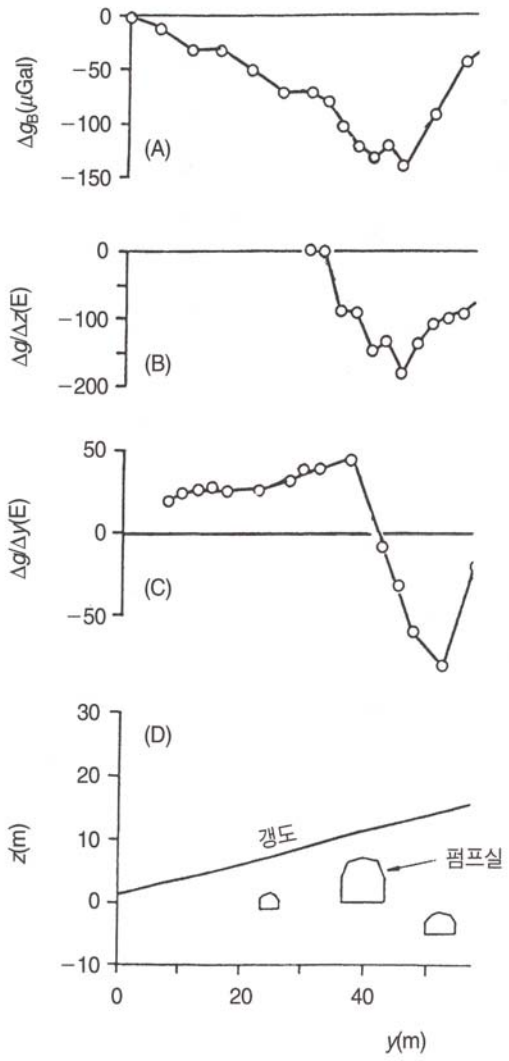


그림 2.49 심부 석탄광 내에서의 고정밀 중력탐사. 알고 있는 공동들과 펌프실 위를 지나는 갱도에서 측정하였다. (A) 측정한 잔여중력 곡선, (B) 측정 · 계산한 수직 중력기울기, (C) 측정한 수평 중력기울기, (D) 이미 알고 있는 공동의 지하위치

➢ Volcanic hazards

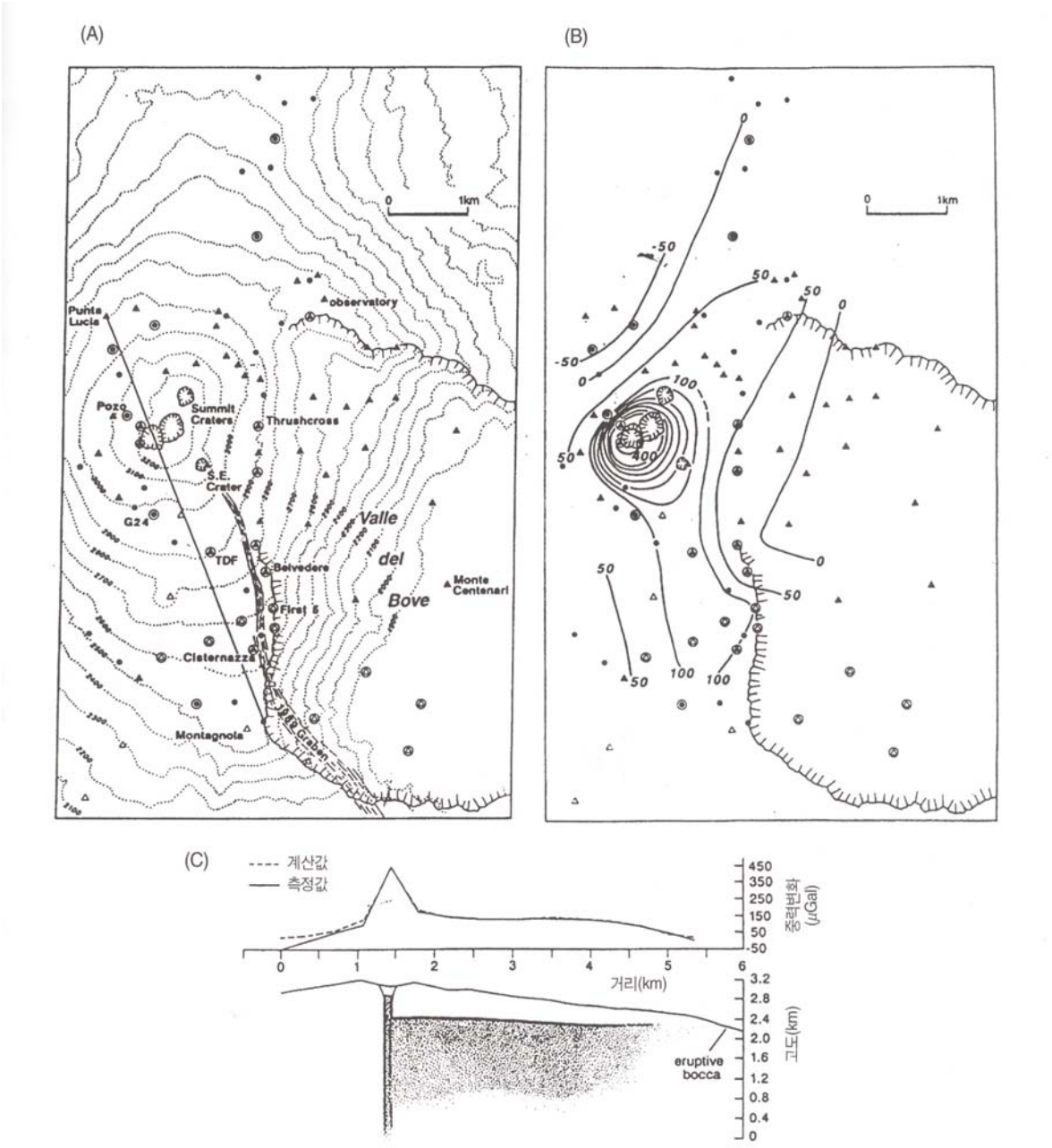


그림 2.55 Sicily의 Etna 화산에서 (A) 고정밀 중력과 지표 변형을 관측하는 측점의 위치도 및 (B) 산 정상지역의 고정밀 중력 등고선. 등고선간격은 50μGal이다. (C)는 (A)에 표시한 측선을 따라 Etna 화산 정상을 지나는 단면도. 측정한 중력변화와 가장 잘 맞는 모형은 폭이 4m인 암맥과 1990년 6월 및 1991년 6월의 관측 사이에 마그마로 채워진 직경 50m의 암맥이다(Rymer, 1993; Rymer 등 1993).

Determination of density

- Core samples
- Nettleton
- Parasnis
- Subsurface gravity measurement
- Density (gamma-gamma) logger in boreholes
- P-wave velocities

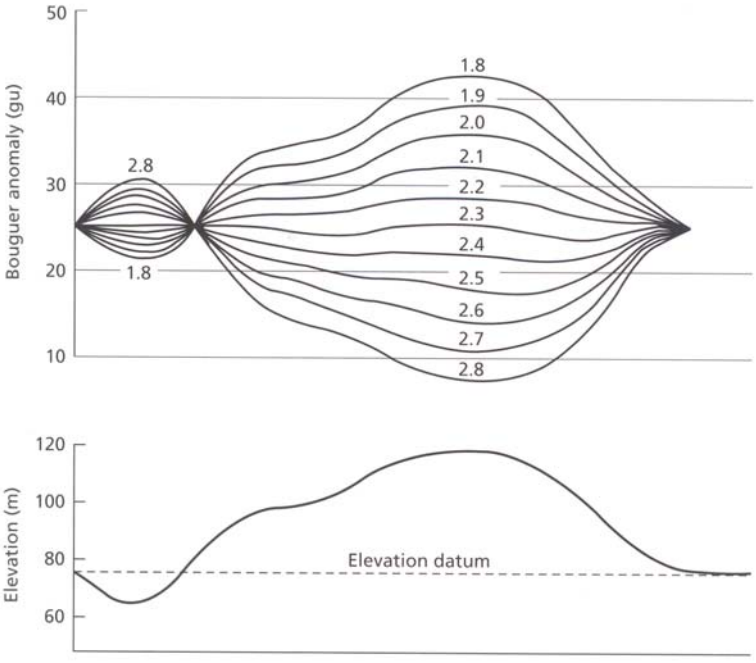


Fig. 6.15 Nettleton's method of density determination over an isolated topographic feature. Gravity reductions have been performed using densities ranging from 1.8 to 2.8 Mg m^{-3} for both Bouguer and terrain corrections. The profile corresponding to a value of 2.3 Mg m^{-3} shows least correlation with topography so this density is taken to represent the density of the feature. (After Dobrin & Savit 1988.)

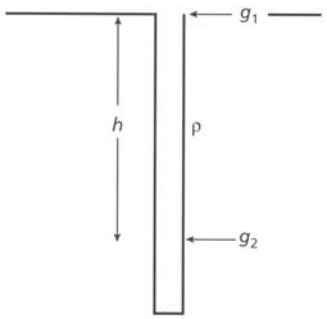


Fig. 6.14 Density determination by subsurface gravity measurements. The measured gravity difference $g_1 - g_2$ over a height difference h can be used to determine the mean density ρ of the rock separating the measurements.

Magnetic methods

1. Introduction

- 2nd century BC: Chinese first used *lodestone* (magnetite-rich rock) or *leading stone* in primitive direction-finding

- 12th century: in Europe, reference was made to the use of a magnetic compass for navigation

- In 1600, the first scientific analysis of the Earth's magnetic field and associated phenomena was published by the English physicist William Gibert in his book *De Magnete*

- In 1640, measurements of variations in the Earth's magnetic field were made in Sweden to locate iron ore deposits

- In 1870, Thalen and Tiberg developed instruments to measure various components of the Earth's magnetic field accurately and quickly for routine prospecting

- In 1915, Adolf Schmidt made a balance magnetometer which enabled more widespread magnetic surveys to be undertaken.

- During the Second World War, advances in technology were made that enabled more efficient, reliable and accurate measurements to be made

- In the 1960s, optical absorption magnetometers were developed which provided the means for extremely rapid magnetic measurements with very high sensitivity, ideally suited to airborne magnetic surveys.

- Since the early 1970s, magnetic gradiometers have been used that measure not only the total Earth's magnetic field intensity but also the magnetic gradient between sensors.

- Geomagnetic methods can be used in a wide variety of applications and range from small-scale investigations to locate pipes and cables in the very near surface, and engineering site investigations, through to large-scale regional geological mapping to determine gross structure, such as in hydrocarbon exploration.

- In the larger exploration investigations, both magnetic and gravity methods are used to complement each other.

- Gravity and magnetic methods can provide more information about the sub-surface, particularly the basement rocks.

- The range of magnetic measurements is extremely large, and it includes Palaeomagnetism.

(Table 3.1)

Applications of geomagnetic surveys

- Locating
  - Pipes, Cable, and Metallic objects
  - Buried military ordnance (Shell, Bombs, etc)
  - Buried metal drums of contaminated or toxic waste
  - Concealed mineshafts and adits (horizontal entrance to amine)

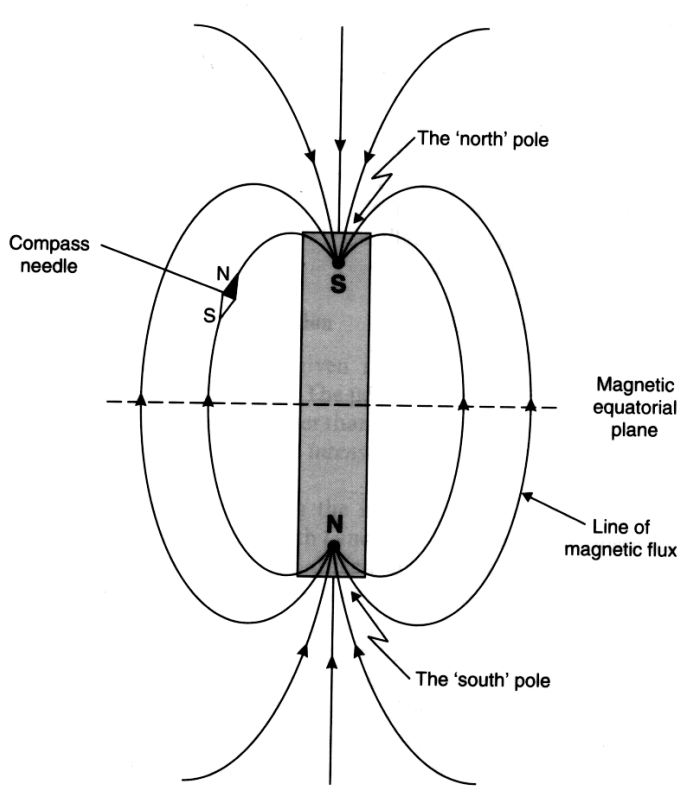
- Mapping
  - Archaeological remains
  - Concealed basic igneous dykes
  - Metalliferous mineral lodes
  - Geological boundaries between magnetically contrasting lithologies including faults
  - Large-scale geological structures

2. Principles and elementary theory

(6) Definitions

(a) Flux line:

- ✓ A magnetic field strength gives rise to a magnetic flux, just as an electric field strength can give rise to an electric charge flux (current).
- ✓ Around a bar magnet, a magnetic flux exists, as indicated by the flux lines, and converges near the ends of the magnet, which are known as the magnetic poles.



Wrong flux line

- ✓ Magnetic pole always exists in pairs of opposite sense to form a *dipole*
- ✓ When one pole is sufficiently far removed from the other so that it no longer affects the other one: a *monopole*

(b) Magnetic flux density (**B**)

- ✓ Magnetic flux per area
- ✓ Unit: Tesla (=Wb/m^2) in MKS unit, and gauss in cgs unit.

$$B = \frac{F}{qv}$$

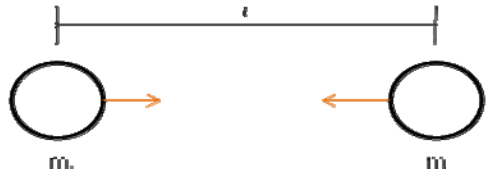
1N force is acting on an electric charge 1C that is moving at the speed 1m/s → 1T

$$1\mathrm{T} = 1N / C / 1\,m / s = 1N / Am$$
$$A = C / s$$

- ✓ Unit: Tesla (=Wb/m^2) in MKS unit, and gauss in cgs unit.
- ✓ 1 gauss = 10^{-4} Tesla
- ✓ 1 nT (10^{-9} T) = 1 gamma (γ) = 10^{-5} gauss

(c) Magnetic force (**F**):

- ✓ When two magnetic poles whose strengths are m_1 and m_2 are separated by a distance r, a force exists between them:



$$\vec{\mathbf{F}} = \frac{m_1 m_2}{4\pi\mu r^2}\vec{\mathbf{r}}_1 \quad \text{in mks system}$$

$$\vec{\mathbf{F}} = \frac{m_1 m_2}{\mu r^2}\vec{\mathbf{r}}_1 \quad \text{in cgs system}$$

F is the force on m_2

μ is magnetic permeability of the medium separating the poles

m_1, m_2 is pole strengths

r_1 is a unit vector directed from m_1 towards m_2

r is distance between two magnetic poles

(d) Magnetic field strength (**H**):

- ✓ A more practical quantity that the force is the strength of magnetic field existing at a point in space, as a result of a pole of strength m located at a distance r from it. The magnetic field strength H is defined as the force

on a unit pole:

$$\mathbf{H} = \frac{\mathbf{F}}{m'} = \frac{m}{4\pi\mu r^2}\mathbf{r}_1 \quad \text{in mks system}$$

$$\mathbf{H} = \frac{\mathbf{F}}{m'} = \frac{m}{\mu r^2}\mathbf{r}_1 \quad \text{in cgs system}$$

m' is fictitious pole – in effect the instrument of measurement

It is assumed that m' is not large enough to disturb the field H at the point of measurement, i.e., $m' << m$

(e) Permeability

In vacuum, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Wb}/(\mathrm{Am})$ in MKS unit

$\mu_0 = 1\mathrm{gauss}/\mathrm{oersted}$ in cgs unit

$\mu_r = \mu / \mu_0$

$\mathbf{B} = \mu\mathbf{H}$

(f) Intensity of magnetization (**I**) and Magnetic moment (**M**):

- ✓ A magnetic body placed in an external magnetic field becomes magnetized by induction.
- ✓ Practically this magnetization by induction amounts to lining up the dipoles of the magnetic material. For this reason, **I** is often referred to as the *magnetic polarization*.
- ✓ If **I** is constant and has the same direction throughout, the body is said to be *uniformly magnetized*.

  The magnetic intensities due to all the individual north and south poles will cancel out except at the end faces of the whole magnet
- ✓ Uniformed magnetized bar can be regarded as a collection of aligned dipoles producing a pole strength of $\pm m$

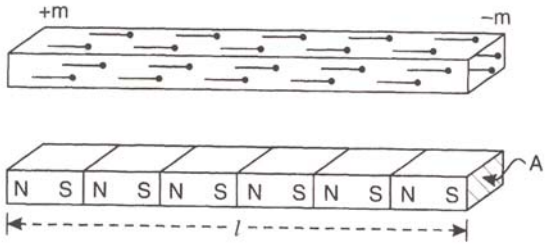


그림 3.2 균일하게 자화된 막대자석을 자극의 세기가 $\pm m$인 쌍극자들의 정렬로 생각하는 모식도와 일련의 작은 막대자석들의 모임으로 보는 모식도

- ✓ Since magnetic poles always exist in pairs, the fundamental magnetic

entity is the magnetic dipole: two poles of strength $+m$ and $-m$ separated by a distance l:

✓ Magnetic moment

$$\mathbb{M} = ml\mathfrak{r}_1$$

$\mathfrak{r}_1$: Extends from the negative pole towards the positive pole

✓ The intensity of magnetization is defined as the magnetic moment per unit volume, that is,

$$\mathrm{I} = \frac{\mathbb{M}}{V} = \frac{ml}{V}\mathfrak{r}_1 = \frac{m}{A}\mathfrak{r}_1$$

V : Volume of the magnetized body= Al

(g) Magnetic susceptibility (k):

✓ The degree to which the body is magnetized is determined by its magnetic susceptibility, defined as

$\mathrm{I} = k\mathrm{H}$ for a vacuum k =0

✓ Susceptibility is the fundamental parameter in magnetic prospecting, since the magnetic response of rocks and minerals is determined by the amount of magnetic materials in them

(h) Magnetic Induction (**B**)

✓ When the magnetic body is placed in the external field **H**, the magnetic body which we are considering has its internal poles more or less lined up by the field **H** to produce a field of its own, **H'**, which increases the total field within the body. This extra field is related to the intensity of magnetization.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{I})$$
$$= \mu_0(\mathbf{H} + k\mathbf{H}) \quad \text{in mks system}$$
$$= \mu_0(1+k)\mathbf{H}$$
$$\mu = \mu_0(1+k)$$

$$\mathbf{B} = \mathbf{H} + \mathbf{H}' = \mathbf{H} + 4\pi\mathbf{I}$$
$$= \mathbf{H} + 4\pi k\mathbf{H}$$
$$= (1 + 4\pi k)\mathbf{H} = \mu\mathbf{H} \quad \text{in cgs system}$$

$$\mu = 1 + 4\pi k$$
$$\mathbf{B} = \mu\mathbf{H}$$

표 3·3 여러 자기량과 이들의 단위

자 기 량	SI(mks 단위계) 단위*	기 호	cgs 단위계 단 위*	기 호
자기장의 세기, 자기강도, 자기유도, 자속밀도	1 tesla =1 weber/meter² =1 newton/ampere-meter	$B=\mu_0(H+M)$ $=\frac{\Phi}{\text{area}}$	$=10^4$ gauss $=10^9\ \gamma$	$B=H+4\pi I$
쌍극자모우먼트	1 ampere-meter²	md	$=10^{10}$ pole centimeters	md
자 극	1 ampere-meter	m	$=10^8$ unit poles	m
자속, 자력선	1 weber	Φ	$=10^8$ maxwells	Φ
기자력, 자기포텐셜	1 ampere-turn	mmf	$=0.4\pi$ gilbert $=1.26$ gilberts	mmf
자 화 력	1 ampere-turn/meter	$H=\frac{B}{\mu\mu_0}$	$=4\pi 10^{-3}$ oersted $=0.0126$ oersted $=4\pi 10^{-3}$ gilbert/centimeter	$H=\frac{B}{\mu'}$
자화, 단위체적당자기쌍극자모우먼트, 자기분극, 자화강도	1 ampere-meter²/meter³ =ampere/meter	$M=kH$ $=\frac{md}{V}$		$I=k'H$
투 자 율†			1 gauss/oersted	$\mu'=\frac{\lvert B\rvert}{\lvert H\rvert}$
상대투자율	Dimensionless	$\mu=\frac{\lvert B\rvert}{\mu_0\lvert H\rvert}$	Dimensionless	$\mu=\lvert\mu'\rvert$
대 자 율	Dimensionless k_{SI}	$k=\frac{\lvert M\rvert}{\lvert H\rvert}$ $=\mu-1$	Dimensionless $=4\pi k_{cgs}$	$k'=\frac{\lvert I\rvert}{\lvert H\rvert}$
릴럭턴스 (Reluctance)	1 ampere-turn/weber	$R=\frac{\text{mmf}}{\Phi}$	$=4\pi 10^{-7}$ gilbert/maxwell	$R=\frac{\text{mmf}}{\varphi}$
인덕턴스 (Inductance)	1 henry =1 weber/ampere	L		

* SI와 cgs 단위계에서의 크기는 같음.
† μ_0=진공에서의 투자율$=4\pi 10^{-7}$ weber/ampere-meter
$=12.57\times10^{-7}$ weber/ampere-meter
$=12.57\times10^{-7}$ henry/meter
=1 gauss/oersted

(7) Basic Theory

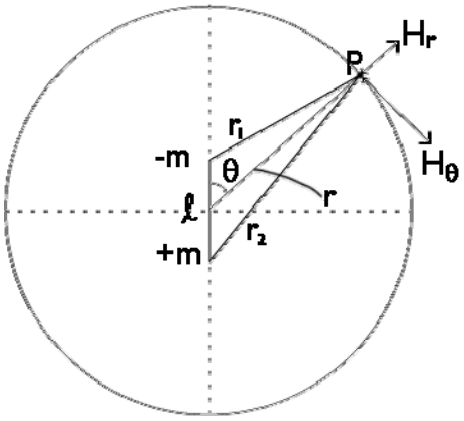
(a) Magnetic potential (dipole field)

Force between two magnetic poles

$\mathbf{F} = \frac{1}{\mu}\frac{m_1 m_2}{r^2}\mathbf{r}$ (cgs system)

$\mathbf{H} = \frac{1}{\mu}\frac{m_1}{r^2}\mathbf{r}$ dyne/unit pole = oersted

$\mathbf{M} = ml\vec{l}$



dipole field : magnetic field generated by the dipole when two poles of strength + m and –m are separated by a distance l

vector magnetic field may be derived from a scalar potential function

$\mathbf{F(r)} = -\nabla V(\mathbf{r})$ '-' means the repulsive force when positive pole is located at the point P.

Magnetostatic field is defined as a work

$V(\mathbf{r}) = -\int_{\infty}^{r} \mathbf{H}\bullet d\mathbf{r} = -\int_{\infty}^{r} \frac{m}{\mu r^2} dr = \frac{m}{\mu r}$ for vacuum, $V(\mathbf{r}) = \frac{m}{r}$

※ for a dipole,

$$V(\mathbf{r}) = \frac{m}{r_2} - \frac{m}{r_1} = \left[\frac{1}{\sqrt{r^2 + \frac{l^2}{4} + rl\cos\theta}} - \frac{1}{\sqrt{r^2 + \frac{l^2}{4} - rl\cos\theta}} \right]$$

$$r_1 = r^2 + \left(\frac{l}{2}\right)^2 - 2r\left(\frac{l}{2}\right)\cos\theta = r^2 + \left(\frac{l}{2}\right)^2 - rl\cos\theta$$

$$r_2 = r^2 + \left(\frac{l}{2}\right)^2 - 2r\left(\frac{l}{2}\right)\cos(\pi - \theta) = r^2 + \left(\frac{l}{2}\right)^2 + rl\cos\theta$$

$$r \gg l: \ \frac{m}{r}\left\{\left(1 - \frac{l}{2r}\cos\theta\right) - \left(1 + \frac{l}{2r}\cos\theta\right)\right\} = -\frac{ml}{r^2}\cos\theta$$

$M = ml$ (magnitude of magnetic moment)

$$V(\mathbf{r}) = -\frac{M}{r^2}\cos\theta$$

In the spherical coordinate (r,θ,ϕ),

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

$$\mathbf{H} = -\nabla V(\mathbf{r}) = -\hat{\mathbf{r}}\frac{\partial V}{\partial r} - \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

$$H_r = -\frac{\partial V}{\partial r} = -\frac{2M}{r^3}\cos\theta$$

$$H_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\frac{M}{r^3}\sin\theta$$

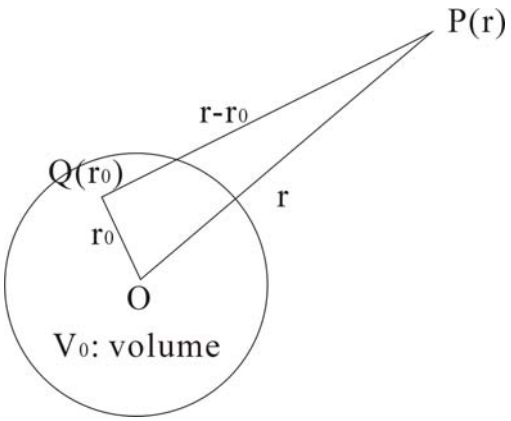
$$H_\phi = 0$$

When the dipole is placed along the z axis, the magnetic field due to dipole at the surface of the sphere is expressed

$$\mathbf{H} = -\hat{\mathbf{r}}\frac{2M}{r^3}\cos\theta - \hat{\boldsymbol{\theta}}\frac{M}{r^3}\sin\theta$$

$$V(\mathbf{r}) = -\frac{M}{r^2}\cos\theta = -\mathbf{M}\cdot\nabla\left(\frac{1}{r}\right)$$

(b) Poisson's relation



Magnetic intensity
=Magnetic moment per unit volume

$$\mathbf{I} = \mathbf{M}/V_0$$

Potential at P(r) due to I(r_0) at Q(r_0)

$$dV = -\mathbf{I}(\mathbf{r}_0)\cdot\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}_0|}\right)$$

$$V(\mathbf{r}) = \int dV = -\int \mathbf{I}(\mathbf{r}_0)\cdot\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}_0|}\right)dV_0$$

Magnetization direction is constant(α) all over the magnetic body

$\mathbf{I}(\mathbf{r}_0)\cdot\nabla = I\frac{\partial}{\partial\alpha}$ α is independent of the integral

$$V(\mathbf{r}) = -\frac{\partial}{\partial\alpha}\int_{V_0} I\frac{1}{|\mathbf{r}-\mathbf{r}_0|}dV_0$$

If we assume that the density of the magnetic body is ρ

$$V(\mathbf{r}) = -\frac{I}{G\rho}\frac{\partial}{\partial\alpha}\int_{V_0}\frac{G\rho}{|\mathbf{r}-\mathbf{r}_0|}dV_0 = -\frac{I}{G\rho}\frac{\partial}{\partial\alpha}U(\mathbf{r}),\ U(\mathbf{r})\text{: gravitational potential}$$

⇒ Poisson's relationship

(c) Field equations

In the homogeneous region outside the volume V, magnetic potential satisfies Laplace equation $\nabla^2 V = 0$ similarly the magnetic potential everywhere within a region containing magnetic material within a mass anomaly, satisfies Poisson's equation $\nabla^2 V = 4\pi\mu m = 4\pi\mu\ \nabla\cdot\mathbf{M}(\mathbf{r}_0)$

3. Magnetism of the earth

(1) Nature of the geomagnetic field

- Main field: It is not constant in time and varies relatively slowly and is of internal origin
- External field: It is a small fraction of the main field. It varies rather rapidly, partly cyclically and partly randomly and which originates outside the earth

- Minor variations in the field from place to place: it usually but not always much smaller than the main field, relatively constant in time and place and caused by local magnetic anomalies in the near-surface crust of the earth; these are the targets in magnetic prospecting

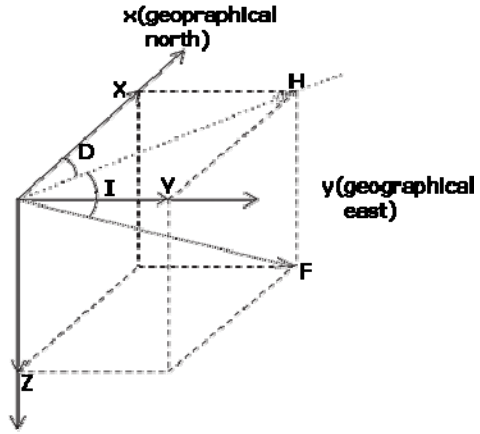
(2) The main field

(a) Elements of the earth's magnetic field

➢ If a steel needle, not previously magnetized, could be hung at its centre by a thread so that it were free to orient itself in any direction in space, it points in the direction of the earth's total magnetic field at this point.

➢ Requirements to completely define the magnetic field
  ✓ The magnitude of this field F

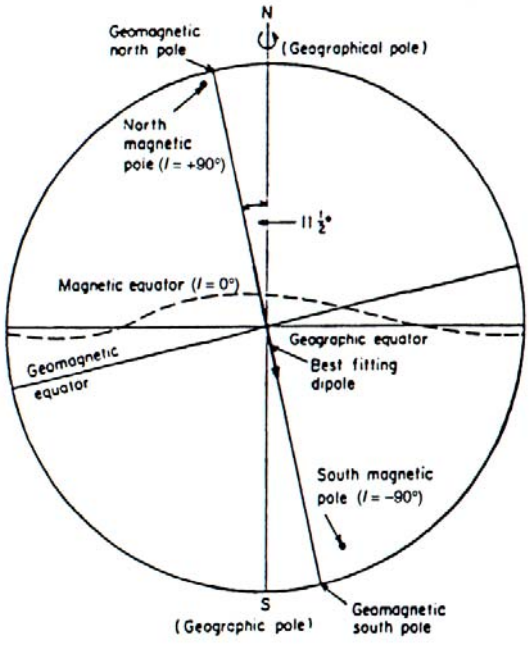
- ✓ The inclination of the needle I: the angle between the total field and its horizontal component
- ✓ The declination of the needle D: the angle between the horizontal component of the total field and geographic north



$$F^2 = H^2 + Z^2 = X^2 + Y^2 + Z^2$$
$$H = F\cos I, \quad Z = F\sin I$$
$$\tan I = \frac{Z}{H}$$

$$X = H\cos D, \quad Y = H\sin D$$
$$\tan D = \frac{Y}{X}$$

- ➢ Isomagnetic charts: Lines of equal declination, inclination, horizontal intensity, etc., when plotted on maps
- ➢ The geomagnetic field reflects little or nothing of the variation in surface geology and geography such as mountain ranges, submarine ridges, earthquake belts, which indicates that the source of the field lies deep within the earth or far outside it.



- ➢ The geomagnetic poles:
  - ✓ The positions on the earth's surface through which the axis of the

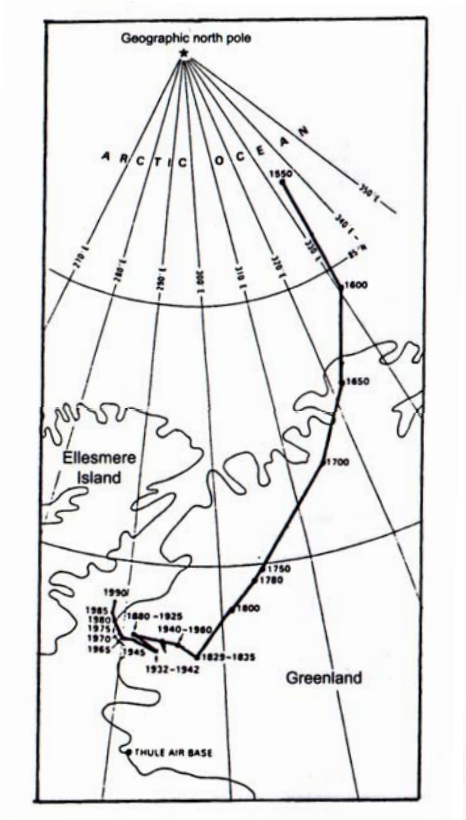
best-fitting dipole passes

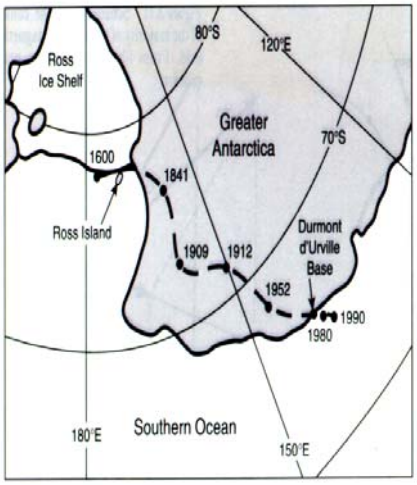
- ✓ They are located in Hayes Peninsula in northern Greenland and near the Russian Vostok research station in Greater Antarctica (left figure)

- ➢ Magnetic poles:
  - ✓ They are located where the magnetic field is directed vertically ($I = \pm 90°$).

  - ✓ The north magnetic pole is currently just north of Bathurst Island in the Canadian Arctic Archipelago.
  - ✓ The north magnetic pole is slowly drifting across the Canadian Arctic.

  - ✓ The south magnetic pole is currently about 150 km offshore from the French Research Station Durmont d'Urville on the Adelie Coast of Greater Antarctica. (right figure)
  - ✓ The south magnetic pole is just off the coast of Wikes land, Antarctica. That point lies outside the Antarctic Circle. The pole is moving northwest by about 10 to 15 kilometers per year.

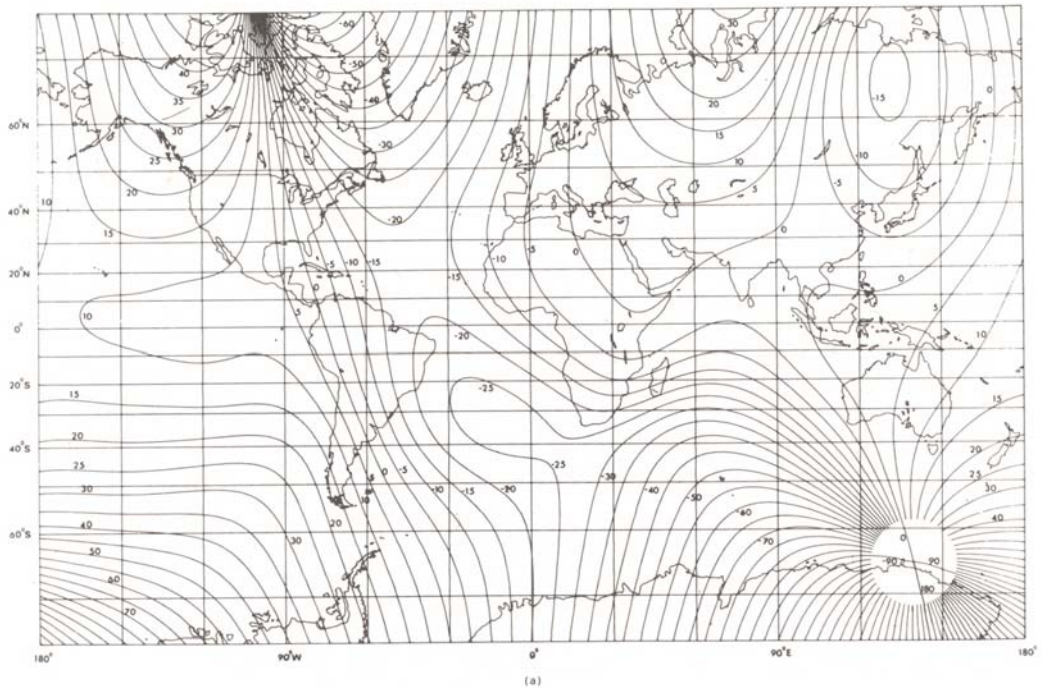




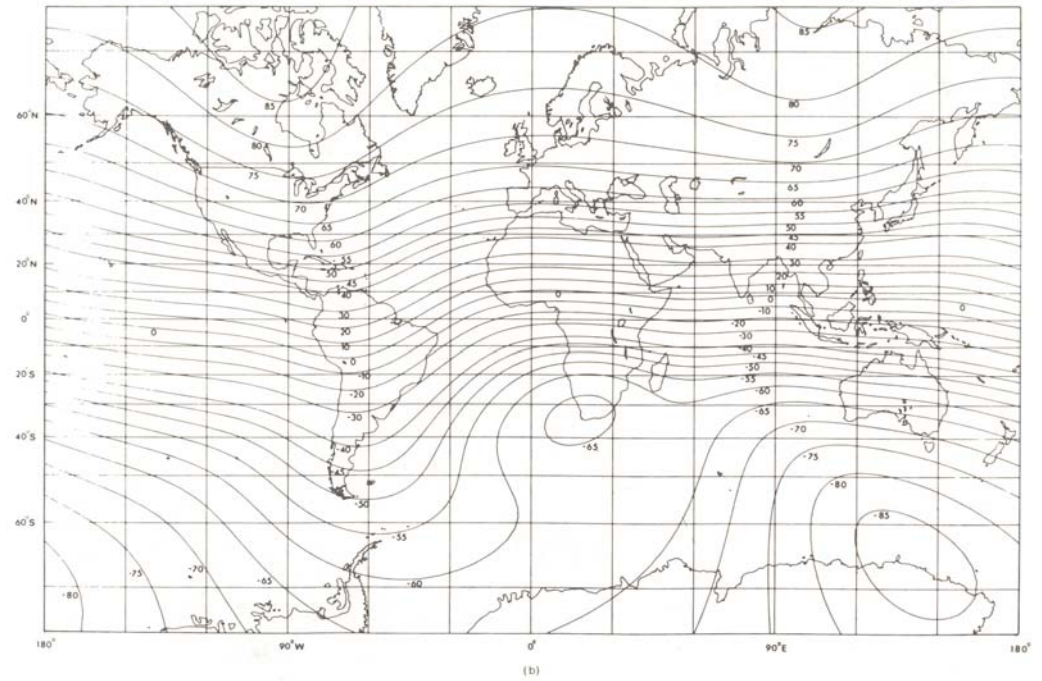
Year	North magnetic pole	
	Latitude (°N)	Longitude (°W)
2001	81.3	110.8
2002	81.6	111.6
2003	82.0	112.4
2004	82.3	113.4
2005	82.7	114.4
	South magnetic pole	
	Latitude (°S)	Longitude (°E)
1998	64.6	138.5
2004	63.5	138.0
2005	64.53	137.86

- The line of zero inclination is never more than 15˚ from the equator, the largest deviation being to the south, in South America and in the eastern Pacific; In Africa and Asia, it is slightly north of the equator

- Isogonic, Isoclinic, and isodynamic maps show lines of equal declination, dip and equal values of F, H, and Z, respectively.


60°N
40°N
20°N
0°
20°S
40°S
60°S
180°
90°W
0°
90°E
180°

(a)


60°N
40°N
20°N
0°
20°S
40°S
60°S
180°
90°W
0°
90°E
180°

(b)

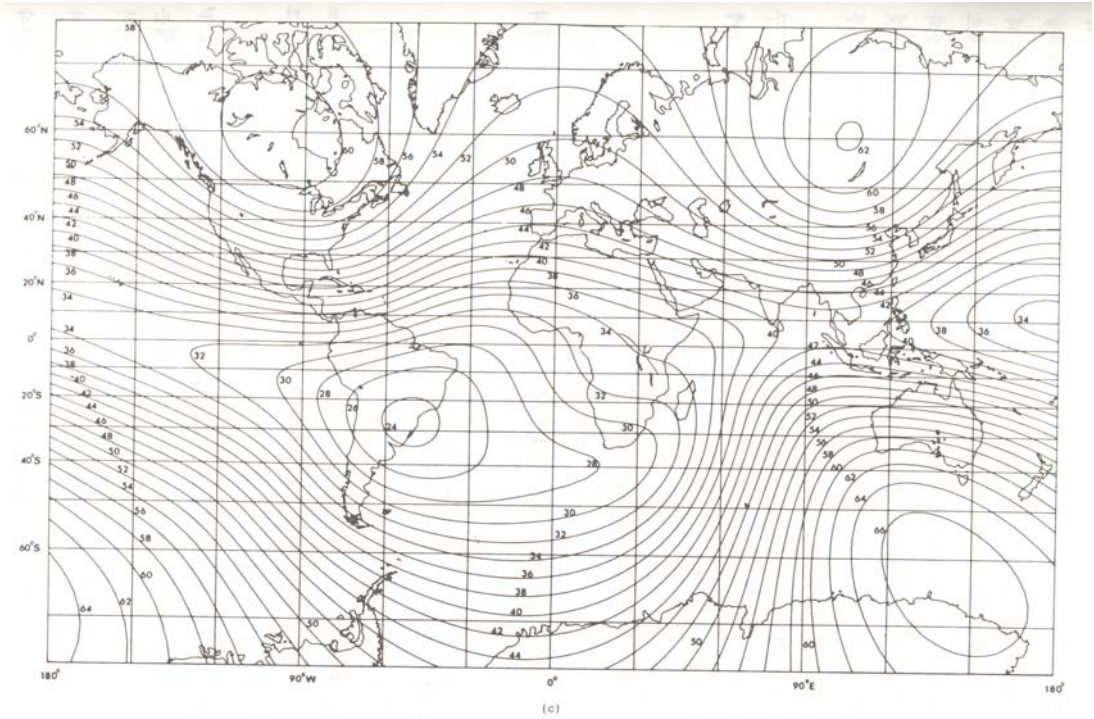


그림 3·9 1980년도의 편각, 복각 및 총 자기장의 세계적 분포. 그림(a), (b)의 단위는 도(度, degree)이고, 그림(c)의 단위는 1000 nT임. (Merrill과 McElhinny, 1983)
(a) 등편각 분포도, (b) 등복각 분포도, (c) 등총자기장 분포도

(b) Origin of the main field

- The main geomagnetic field theoretically could be caused by an internal or external source, either of permanent magnetism or of unidirectional current flow, or it could be the result of current flowing in and out of the earth's surface. The latter possibility can be ruled out because the observed air-to-earth currents are much too small to account for the existing magnetic field of the earth.

- Spherical harmonic analysis of the observed surface magnetic fields shows that at least 99% is due to sources inside, and the remaining 1% is due to sources outside the earth.

- Several hypotheses

  - ✓ The field is a fundamental property associated with rotating bodies → Magnetization arising from the angular velocity of the earth is much too small.
  - ✓ The field is caused by the rotation of electric charges → potential gradients in the earth would be impossibly large.

  - ✓ The earth is uniformly magnetized requires a value of induction far too large for known surface rocks.
  - ✓ The inevitably high temperatures in the earth's interior (~2000˚C) probably rule out the possibility of permanent magnetism at depth, since all known magnetic materials lose their magnetic properties at much lower temperatures (Fe: 750˚C, Ni: 310˚C, Magnetite: 515˚C).

  - ✓ The temperature of demagnetization is known as the Curie point.

- The present theory

  - ✓ The main field is caused by electric currents circulating in the outer core – known to be liquid from seismic evidence – which extends from a radius of 1300 km to 3500 km (from the center of the earth).

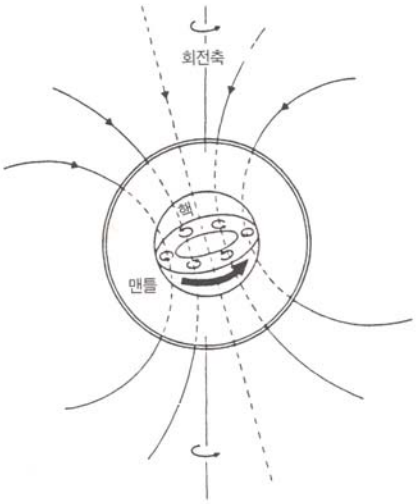


그림 3.11 지자기장의 주된 원인을 설명하는 모식도

- ✓ For several reasons the earth's core is assumed to be a combination of iron and nickel, both good electrical conductors.

- ✓ Even if the internal material were a nonconductor, however, the enormous pressures in the core (~10^6 bars) may conceivably squeeze out electrons to form some sort of free-electron gas of suitable conductivity.

- ✓ Whatever the constituents of the core is, the magnetic source is thought to be a type of self-excited dynamo, in which a highly conductive fluid moves about in complex mechanical motion, while electric currents, possibly caused by chemical or thermal variations, flow through it.

Assignment: Investigate Dynamo theory

- ✓ The combination of motion and current creates a magnetic field.

- ✓ Laboratory tests have shown that the dynamo source may be a valid explanation and also that it may account for certain slow or secular variations which are known to have occurred in the earth's magnetic field.

(c) Secular variations

- The geomagnetic field is far from permanent
- Inclination and Declination change as time goes by
- Although their source is not understood, it is thought to be internal, possibly connected with changes in convection currents in the core, in the core-mantle coupling and in the rotational speed of the earth.
- The intensity of the main magnetic field is decreasing at about 5% per century.

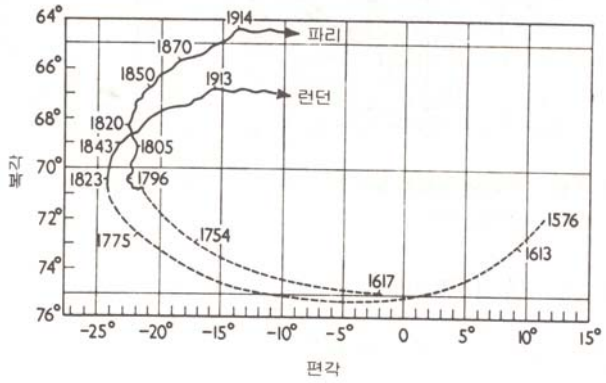


그림 3·13 수세기에 걸쳐 런던과 파리에서 측정한 지자기의 영년 변화 (Runcorn, 1962)

(3) The external magnetic field

- Most of the remaining 1% of the geomagnetic field, which originates outside the earth, appears to be associated with electric currents in the ionized layers of the outer atmosphere.

- The variation in time is much more rapid than for the so-called permanent field.

- Several well-documented effects are listed below

  - ✓ A cycle of 11 years duration, correlated with sunspot activity, has a latitude distribution which indicates **an external origin**.

  - ✓ **Solar diurnal variations**, with a period of 24 hours and range of 30 γ vary with latitude and season and are probably controlled by action of the sun on ionospheric currents.

Upper atmosphere can be ionized by plasma such as UV and Xray.

Ionization leads to current, which induces electromagnetic field,

Variation can be affected by the amount and velocity of plasma.

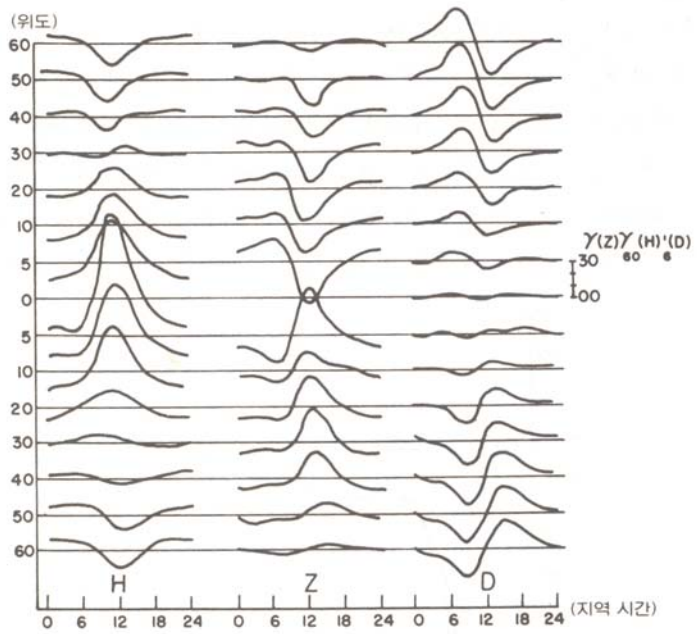


그림 3·11 위도에 따른 지자기장의 일변화 (Matsushita 외, 1967)
H : 수평선분, Z : 수직선분, D : 편각

✓ **Lunar diurnal variations** of 25-hour period and amplitude about 2γ vary cyclically through the month and seem to be associated with a moon-ionosphere interaction

Tidal force also affects the ionized layer of the upper atmosphere.

✓ **Internal source for diurnal variation** is due to electromagnetic field induced by solar and lunar diurnal variation in the outer core.

✓ **Magnetic storms** are transient disturbances with amplitudes as great as 1000γ in most latitudes and even larger in polar regions, where they are usually associated with the aurora (Fig. 3.12 in the handout) Magnetic storms last several hours or 1-2 days.

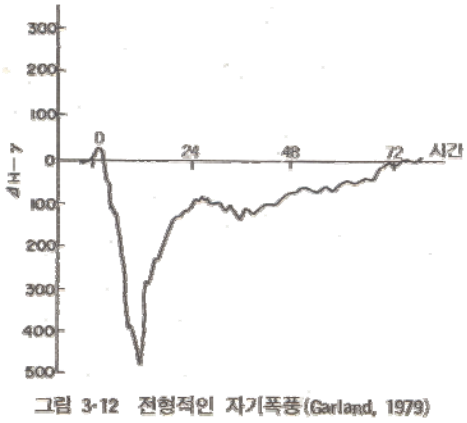


그림 3-12 전형적인 자기폭풍(Garland, 1979)

Although rather erratic, they often occur at intervals of about 27 days, the period correlating with sunspot activity.

Magnetic prospecting may be impractical.

Magnetic storm breaks out more often in polar regions than in the equator.

- ✓ When we interpret magnetic survey data, we should remove these effects.

(4) Local magnetic anomalies

- ➢ Important changes occur in the main field as the result of variations in magnetic mineral content of near-surface rocks.

- ➢ These anomalies are occasionally large enough to double the main field locally. They do not persist over very great distances; magnetic maps do not exhibit large-scale regional features, such as isostatic anomalies in gravity.
- ➢ The sources of these local magnetic anomalies can hardly be very deep, since temperatures at the bottom of the crust should be well above the Curie point. They are associated with near-surface features.

(5) Magnetism of rocks and minerals

(a) Basic types of magnetism

- Magnetic anomalies are entirely caused by the amount of magnetic minerals contained in the rocks.
- All minerals can be classified in three groups according to their magnetic properties:
  diamagnetism(반자성), paramagnetism(상자성) and ferromagnetism(강자성)

- **Diamagnetism**
  - ✓ All atoms have a magnetic moment as a result of the orbital motion of electrons around the nucleus and the spin of the electrons.
  - ✓ According to quantum theory, two electrons can exist in the same electron shell (or state) as long as they spin in opposite directions.
  - ✓ The magnetic moments of two such electrons (paired electrons) will cancel out.
  - ✓ In the majority of substances, when there is no external magnetic field, the spin magnetic moments of adjacent atoms are distributed randomly so there is no overall magnetization.
  - ✓ In a diamagnetic material, all the electron shells are complete, so there are no unpaired electrons.
  - ✓ A diamagnetic substance has a negative magnetic susceptibility.
  - ✓ The intensity of magnetization induced in the substance by the field **H** is in the opposite direction to **H**.
  - ✓ The diamagnetism will prevail only if the net atomic magnetic moment of all the atoms is zero when H is zero.

- ✓ The most common diamagnetic earth materials are graphite (흑연), gypsum (석고), marble (대리석), quartz (석영) and salt (암염).

➢ Paramagnetism

- ✓ Unpaired electrons in incomplete electron shells produce unbalanced spin magnetic moments and weak magnetic interactions.
- ✓ In a paramagnetic substance each atom or molecule has a net magnetic moment in zero external fields.
- ✓ When an external magnetic field is applied, the magnetic moments align themselves into the same direction, although this process is retarded by thermal agitation (열적교반).
- ✓ The result is a weak positive susceptibility but one which decreases inversely with the absolute temperature according to the Curie-Weiss Law.
- ✓ Paramagnetism is generally at least 10 times stronger than diamagnetism.
- ✓ Positive susceptibility
- ✓ Amphiboles(각섬석), pyroxenes(휘석), olivines(감람석), garnets(석류석) and biotite(흑운모).

➢ Ferromagnetism

- ✓ The susceptibility is large but is dependent upon temperature and the strength of the applied magnetic field.
- ✓ The susceptibility of the ferromagnetism is about 10^6 times those of the diamagnetism and paramagnetism.
- ✓ The spin moments of unpaired electrons are coupled magnetically

due to the very strong interaction between adjacent atoms and overlap of electron orbits.

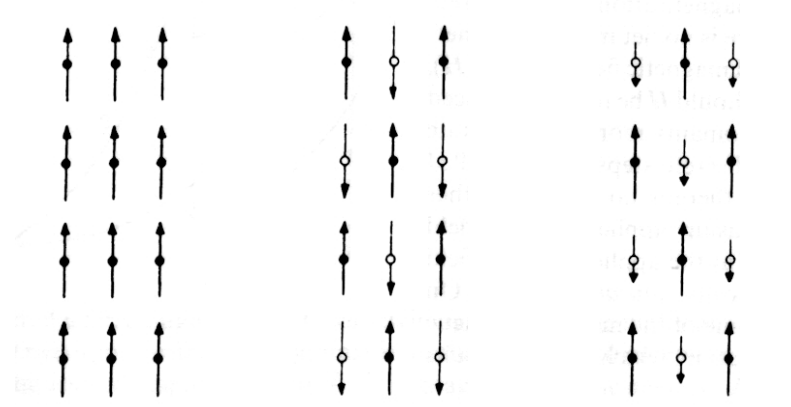
- ✓ The moments align within relatively large regions called magnetic domains, whose dimension is micron

- ✓ This gives rise to a strong 'spontaneous magnetisation' which can exist even when there is no external applied field.

- ✓ The magnetic coupling can be such that the magnetic moments are aligned either parallel or antiparallel.

- ✓ Ferromagnetism disappears when the temperature of the material is raised above the Curie temperature as inter-atomic magnetic coupling is severely restricted and the material thereafter exhibits paramagnetic behavior.

- ✓ Ferromagnetic materials occur only rarely in nature but include substances such as cobalt, nickel and iron, all of which have parallel alignment of moments.



Ferromagnetism Antiferromagnetism Ferrimagnetism

➢ Antiferromagnetism

- ✓ The magnetic domains are subdivided into subdomains
- ✓ The magnetic moments of subdomains are aligned in an antiparallel manner.

- ✓ Although the magnetic fields of the oppositely orientated dipoles cancel each other out, crystal lattice defects result in a net residual moment or parasitic (anti)-ferromagnetism.
- ✓ Hematite(적철석) is the most common example

➢ Ferrimagnetism

- ✓ Materials in which the magnetic domains are subdivided into regions which may be aligned in opposition to one another, but whose net moment is not zero when **H**=0, are called ferrimagnetic.
- ✓ One set of sub-domains has a stronger magnetic alignment than the other.

- ✓ The first type of ferrimagnetic minerals are magnetite(자철석), titanomagnetite(티탄자철석), ilmenite(티탄철석), and pyrrhotite(자류철석).

- ✓ Although the temperature dependence of ferromagnetic behavior is complex, ferrimagnetism disappears at temperatures above the Curie point.
- ✓ The majority of naturally occurring magnetic minerals exhibit either ferrimagnetic or imperfectly antiferromagnetic characteristics.

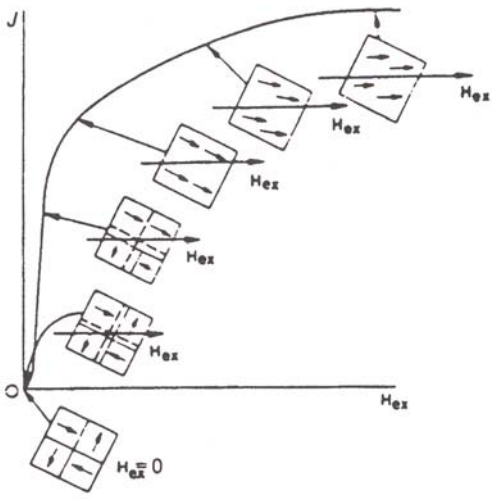


그림 3.5 자구이론에 따라 강자성체가 자화되는 과정(Sharma, 1986)

- **The hysteresis loop:** The relation between B and H can be quite complex in ferromagnetic materials. This is illustrated in the cycle of magnetization, a familiar elementary physics experiment performed with a steel rod in a solenoid.

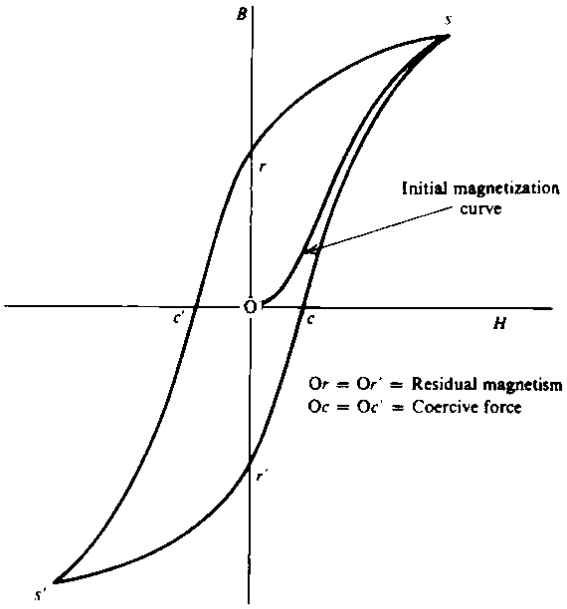


Figure 3.2. Hysteresis loop. s, s' = saturation, r and r' = remanent magnetism, c and c' = coercive force.

  - ✓ If a ferromagnetic or ferrimagnetic material, such as magnetite or pyrrhotite, with grains larger than 10 mcrons if placed in an increasing applied magnetic field, its magnetic intensity I increases to the point of saturation following a hysteresis loop.
  - ✓ After reaching saturation, the applied field H is reduced to zero at which point the intensity of magnetization is that attributed to the remanent magnetization.
  - ✓ To eliminate this magnetization, a negative field, -Hc, the coercive force, has to be applied.
  - ✓ The coercivity, Hc is an indication as to the 'hardness' or permanence of the magnetization.

(6) Magnetic susceptibilities of rocks and minerals

- Rocks that have a significant concentration of ferro- and ferri-magnetic

materials tend to have the highest susceptibility

- Rock susceptibilities are also dependent upon a number of factors in addition to mineralogical composition: the alignment and shape of the magnetic grains dispersed throughout the rock
- Magnetic fabric: the variation of magnetic properties as a function of orientation and shape of mineral grains
- Basic and ultrabasic (mafic and ultramafic) igneous rocks: highest susceptibility
- Acid (felsic) igneous and metamorphic rocks: intermediate to low values

표 3.2 암석과 광물의 대자율(SI 단위)

광물 또는 암석 종류	대자율*
퇴적암	
백운암(순수)	−12.5 ~ +44
백운암(불순물)	20000
석회암	10 ~ 25000
사암	0 ~ 21000
세일	60 ~ 18600
평균	**0 ~ 360**
변성암	
편암(schist)	315 ~ 3000
점판암(slate)	0 ~ 38000
편마암(gneiss)	125 ~ 25000
사문암(serpentenite)	3100 ~ 75000
평균	**0 ~ 73000**
화성암	
화강암	10 ~ 65
화강암(m)	20 ~ 50000
유문암(rhyolite)	250 ~ 37700
페그마타이트(pegmatite)	3000 ~ 75000
반려암(gabbro)	800 ~ 76000
현무암(basalt)	500 ~ 182000
해양 현무암	300 ~ 36000
감람암(peridotite)	95500 ~ 196000
산성(acid) 화성암 평균	**40 ~ 82000**
염기성(basic) 화성암 평균	**550 ~ 122000**
광물	
얼음(d)	−9
암염(d)	−10
석고(d)	−13
석영(d)	−15
흑연(d)	−80 ~ −200
황동광(chalcopyrite)	400
황철광(pyrite)(o)	50 ~ 5000
적철광(hematite)(o)	420 ~ 38000
자류철광(pyrrhotite)(o)	1250 ~ 6300000
티탄철광(ilmenite)(o)	314000 ~ 3800000
자철광(magnetite)(o)	70000 ~ 20000000

(d) = 반자성 물질, (o) = 광석, (m) = 자성 물질 포함
*$k \times 10^6$ SI 단위. cgs 단위로 변환하려면 4π로 나누면 됨.

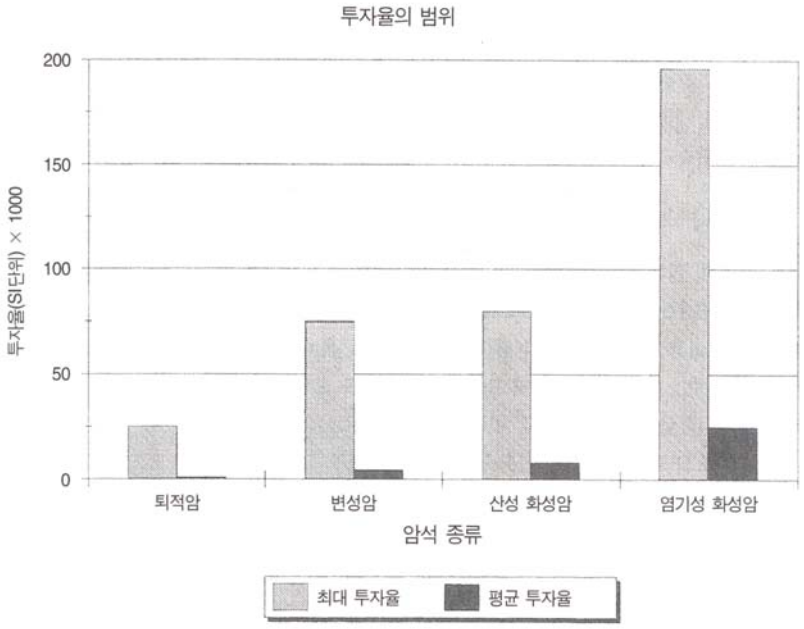


그림 3.7 주요 암석 종류에 따른 대자율

(7) Remanent magnetization

- Remanent magnetization indicates the magnetization remains after the external magnetic field (the earth's magnetic field) is eliminated.
- The magnetization of rocks depends mainly upon the strength of the ambient geomagnetic field and the magnetic mineral content.
- In practice, residual magnetism often contributes to the total magnetization in rocks, both in amplitude and direction.
- The effect is very complicated because of the dependence upon the magnetic history of the rocks.
- Natural remanent magnetization (NRM), which is acquired by a rock and mineral under natural condition, is classified in several groups according to their causes.

  - ✓ **Thermal remanent magnetization (TRM)** is acquired when magnetic material is cooled from the Curie point in the presence of an external field. TRM is particularly stable. In some cases it may be opposite in direction to the present earth's magnetic field (which is due to the phase reversal of geomagnetic field). This is the main mechanism in the magnetization of igneous rocks.

  - ✓ **Isothermal remanent magnetization (IRM)** is acquired over a short time (of the order of seconds) in the present of a strong magnetic field at a constant temperature. Lightning strikes produce IRM over very small irregular areas.

  - ✓ **Chemical or crystallization remanent magnetization (CRM)** takes place when magnetic grains increase in size or are changed from one form to another as a result of chemical reaction at moderate temperatures below the Curie point. This process is particularly significant in sedimentary and metamorphic rocks

  - ✓ **Thermal-chemical remanent magnetization (TCRM)** is acquired by both chemical alteration and cooling

- ✓ **Detrital or depositional remanent magnetization (DRM)** is produced by the settling out of previously magnetized particles to form consolidated sediments. This is a weak net magnetization, but prior to any chemical alteration through diagenetic processes. Varied clays (a layer or series of layers of sediment deposited in a body of still water) exhibit this type of remanence.

- ✓ **Post-depositional remanent magnetization (PDRM)** is acquired by physical processes acting on sediments after deposition (e.g., bioturbation and compaction)

- ✓ **Viscous remanent magnetization (VRM)** is produced by a long exposure to an ambient external field when all other factors are constant. VRM is probably more characteristic of fine-grained rocks than coarse-grained rocks. The remanence is quite stable.

- ✓ **Anhysteretic remanent magnetization (ARM)** is acquired when the peak amplitude of alternating magnetic field decreases from a large value to zero in the presence of a weak but constant magnetic field.

- ✓ Studies of the magnetic history of the earth (paleomagnetism) indicate that the earth's field has varied in magnitude and has actually reversed its polarity several times in the past. On the other hand, there is no specific evidence that it has disappeared entirely for any appreciable periods.

  Many rocks have remanent magnetism that is oriented neither in the direction of, nor opposite to, the present Earth's field. Such results support the plate tectonics theory.

  Paleomagnetic laboratory methods separate residual from induced magnetization.

(8) Königsberger ratio Q

- The ratio of the remanent magnetic intensity to the induced magnetic intensity: $\frac{\mathbf{I}_r}{\mathbf{I}_i}$
- For slowly crystallized igneous and thermally methamorphosed rocks in the continental areas: $Q \approx 1$
- For volcanic rocks: $Q \approx 10$
- For many rapidly quenched basaltic rocks: $Q \approx 30 \sim 50$
- In the sedimentary and metamorphic rocks, except when iron ore is involved $Q < 1$

Assignment: Read through the textbook and some reference books, and then summarize several **magnetic instruments (magnetometers)**.

4. Magnetic exploration

(1) Field operations

- Magnetic exploration is carried out on land, at sea, and in the air

- Among them, airborne survey is the most convenient and the fastest method.

- Diurnal variation curves should be obtained for later correction.
- In airborne or marine surveys, it is not easy to return to the base station repeatedly. In this case, the survey is designed so that track lines intersect.

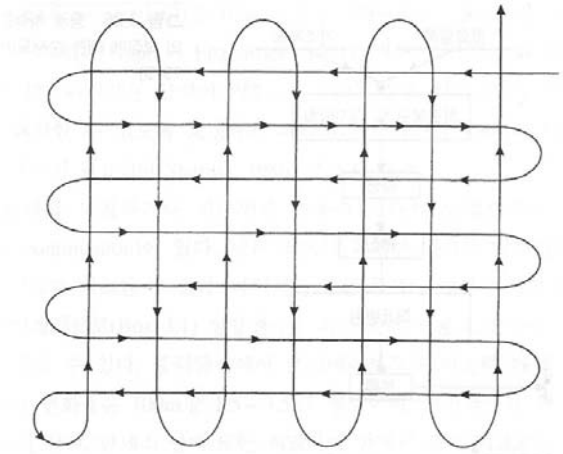


그림 3.25 선박 또는 항공기에 탑재된 자력계의 궤도. 때로는 모든 측선의 간격을 일정하게 하지 않고 측선간격의 10배마다 한 번씩 교차측선을 만들 수 있다.

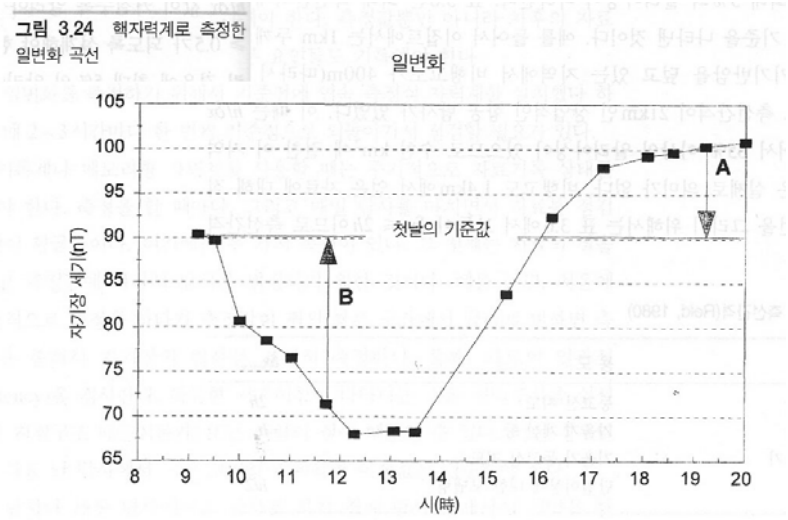
⇒ Tracklines for airborne survey

- We may obtain diurnal variation curves by going back to a base station and measure magnetic field repeatedly every 30 -45 minutes.

- Sometimes, if we have an extra magnetometer, then we can obtain diurnal variation curves by measuring magnetic field every 10-15 minutes. In this case, we still need to go back to the base station, and check magnetic field every 2-3 hours.

- For the diurnal variation curves, we can use the data provided by an institute. In Korea, Korean Astronomy & Space Institute provides the diurnal variation curves observed at several base stations. We only apply

the data at a base station that is located near the survey area.



⇒ Diurnal variation curves: the magnetic fields increases by 10nT
we subtract 10nT from the measured field data

- When the magnetic targets have a substantial (long) strike length, survey profile should be conducted across strike with tie-lines along strike.
- When the targets are more equidimensional, north-south and east-west orientations are commonly used.

- Detailed and accurate field notes are important. Practical details of how magnetic surveys should be undertaken are recorded in the field notes.
- Check your data as they are collected and at the end of each survey day. → It provides a data quality check and allows the operator to alter the survey in response to the magnetic values measured. We can change station interval.
- If the magnetic field changes abruptly between stations, we should reduce the station intervals.

(a) Airborne magnetic surveys

- The sensitivity of airborne magnetometers is generally greater than those used in ground exploration.
- There are certain problems in mounting the sensitive magnetic detector in an airplane, because the airplane has a complicated magnetic field of its own.

- One obvious method to eliminate these effects is to tow the sensing element at some distance behind the aircraft. The detector is housed in a streamlined cylindrical container, known as a bird, connected by a cable 30 to 150 m long.
- An alternative scheme is to mount the detector on a wing tip or slightly behind the tail. The magnetic effects of묻힌 파이프 the plane are minimized by permanent magnets and soft iron or permalloy shielding (영구자석용 합금)strips, by currents in compensating coils, and by metallic sheets for electric shielding of the eddy currents.

- Determine an adequate line separation or flying height and orientation of survey line so that spatial aliasing can be avoided.

- For spatial aliasing, see Figures 1.7 and 1.8 in the textbook.

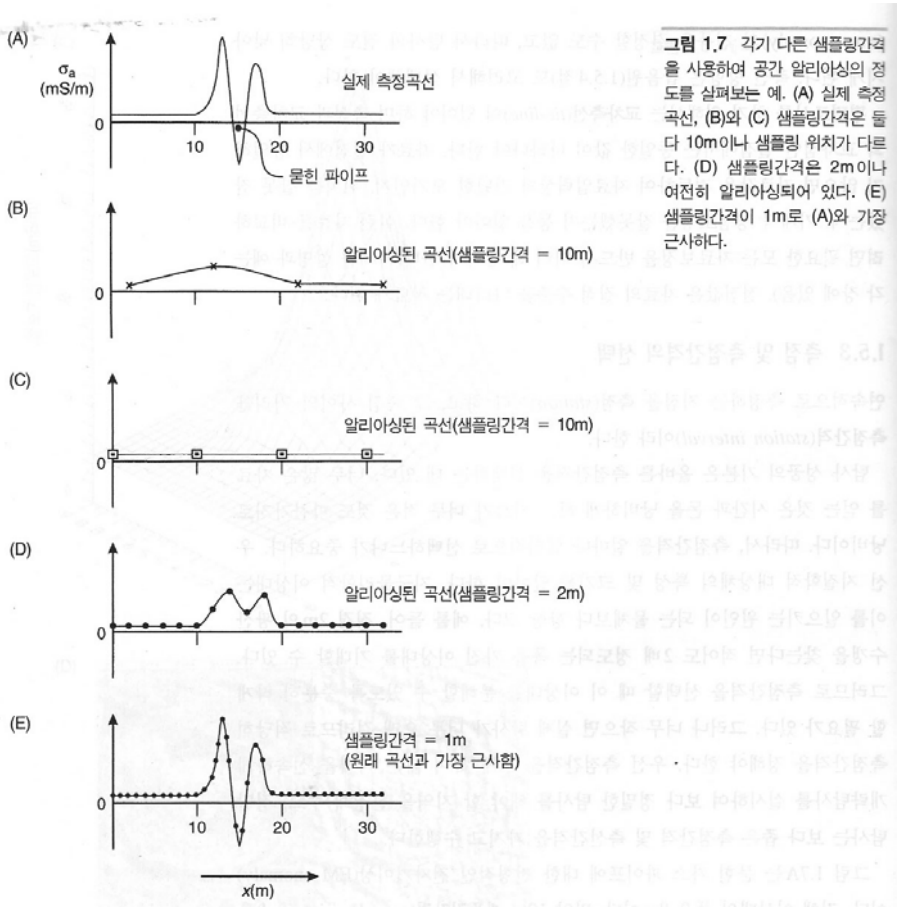


그림 1.7 각기 다른 샘플링간격을 사용하여 공간 알리아싱의 정도를 살펴보는 예. (A) 실제 측정곡선, (B)와 (C) 샘플링간격은 둘 다 10m이나 샘플링 위치가 다르다. (D) 샘플링간격은 2m이나 여전히 알리아싱되어 있다. (E) 샘플링간격이 1m로 (A)와 가장 근사하다.

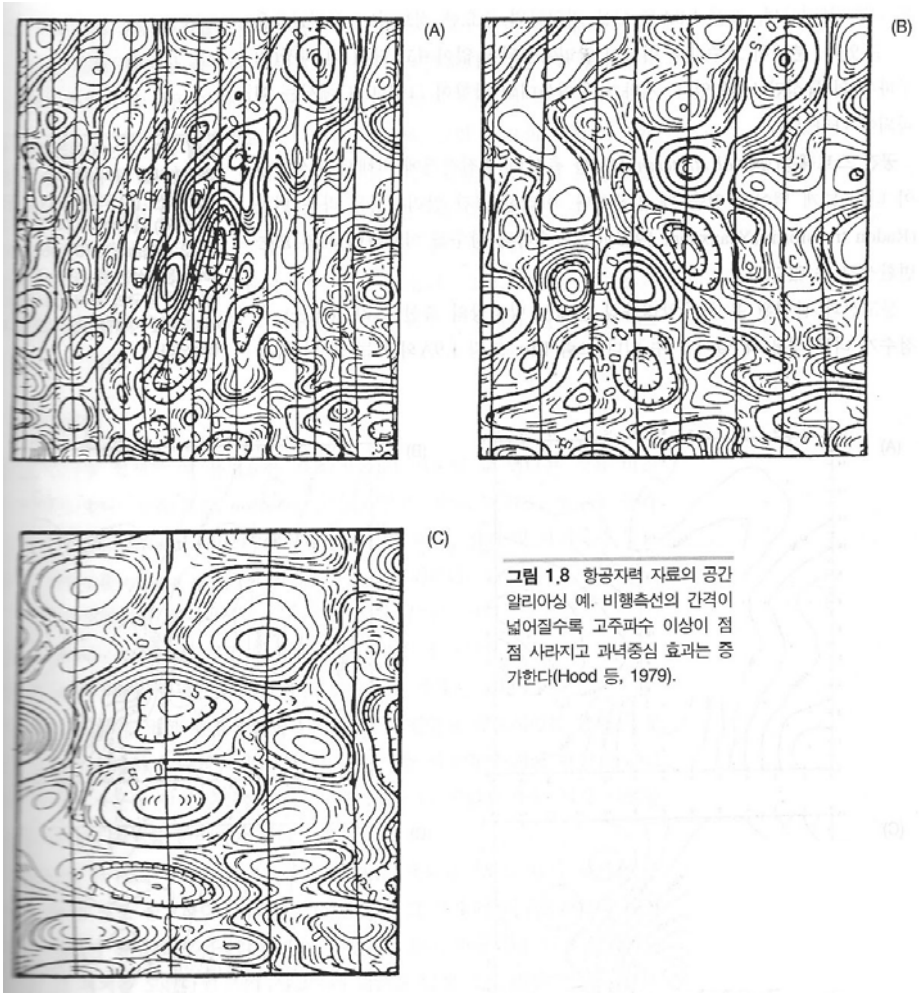


그림 1.8 항공자력 자료의 공간 알리아싱 예. 비행측선의 간격이 넓어질수록 고주파수 이상이 점점 사라지고 과녁중심 효과는 증가한다(Hood 등, 1979).

- Tables 3.5 and 3.6 show degree of aliasing and guideline for maximum line spacing.

표 3.5 알리아싱 정도(Reid, 1980)

$h/\delta x$	F_T(%)	F_G(%)
0.25	21	79
0.5	4.3	39
1	0.19	5
2	0.0003	0.03
4	0	0

F_T : 총 자기장, F_G : 수직 자기장의 기울기

표 3.6 최대 측선간격(Reid, 1980)

탐사 종류	용 도	δx_{max}
총 자기장	등고선 지도	$2h$
총 자기장	기울기 계산 등	h
수직 기울기	기울기 등고선 지도	h
총 자기장	단일이상에 대한 모델링	$h/2$

- The larger the value of h/dx is, the less aliasing will occur
- If contouring is to be undertaken on total field data, the ratio h/dx is more than 0.5. In that case, maximum spatial aliasing will be 5%.

- For instance, there have been examples of commercially flown surveys, where the flying height was 400 m above ground, and the thickness of sedimentary cover over magnetic basement is 1 km. The line spacing was 21 km. → Evaluate the survey design: When the target is the basement, the overall height is 1.4 km. h/dx = 1.4/21 = 0.07, which results in 65% aliasing. → The contour maps of these surveys are useless.

(b) Marine surveys

- Both the fluxgate and proton-precession magnetometers have been used.
- The magnetic sensor is towed at some distance (150 to 300 m) behind the ship to reduce magnetic effects of the vessel.
- The sensor is in a watertight housing called a fish, which usually rides about 15 m below the surface.
- Use of a ship rather than an aircraft provides no advantage and incurs considerable cost increase unless the survey is carried out in conjunction with other surveys, such as gravity or seismic survey.
- The main application has been in large-scale oceanographic surveying related to earth physics and petroleum search.
- Much of the evidence supporting plate tectonincs has come from marine magnetics.

(c) Ground-based magnetic surveys

- The portable proton-precession magnetometer is used to measure the total field
- The main application is in detailed surveys for minerals.
- The ground magnetic surveys are employed after geochemical reconnaissance in base-metal search.

- Station spacing is usually 15 to 60 m; occasionally it is as small as 1 m.

- Since most ground magnetometers have a sensitivity of 1 nT, stations should not be located near any sizeable objects containing iron, such as railroad tracks, drill-hole casings, and culverts(배수구)
- It is important to establish a local base station in an area away from suspected magnetic targets or magnetic noise and where the local field gradient is relatively flat. A base station is needed to obtain the diurnal or erratic variation curves of the magnetic field. A base station should be quick and easy to relocate and re-occupy

(d) Gradiometer surveys

- Measures the vertical gradient directly in the field.
- The effect of diurnal variations is also minimized, which is especially beneficial in high magnetic latitudes. $\frac{F(z_2)-F(z_1)}{z_2-z_1}$

(2) Corrections

(a) Drift correction

- ✓ Instrument drift is generally not a major problem.

(b) Diurnal variation correction

- ✓ In precise work, either repeat readings should be made every few hours at a previously occupied station or a base-station recording magnetometer should be employed.
- ✓ This provides corrections for diurnal and erratic variations of the magnetic field. However, such precautions are unnecessary in most mineral prospecting because anomalies are large (>500 nT).

(c) IGRF correction (normal correction)
(International geomagnetic reference field)

- ✓ The vertical gradient varies from approximately 0.03 nT/m at the

poles to 0.01 nT/m at the magnetic equator.

- ✓ The latitude variation is rarely > 6 nT/km → elevation and latitude corrections are generally unnecessary.
- ✓ For regional surveys, latitude corrections should be applied using International Geomegnetic Reference Field (IGRF) model.

  ⇒roughly shows latitude variations

  By subtracting IGRF from the measured data, normal correction is applied

(d) Terrain corrections

- ✓ Terrain corrections are not needed in most cases. However, if there exist abruptly changing terrains and they consist of igneous rock with high susceptibility, terrain corrections should be applied.

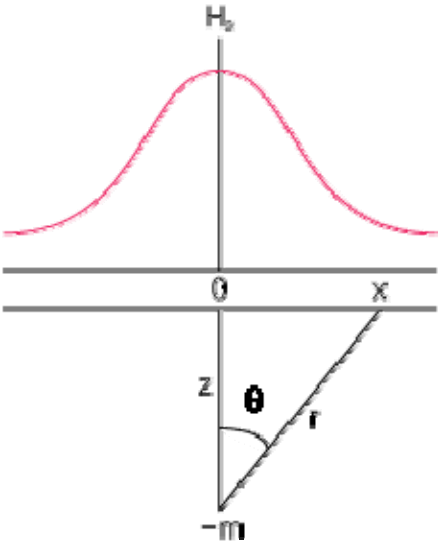
(e) Reduction to the pole (RTP)

- ✓ This operation changes the actual inclination to the vertical.
- ✓ It can be performed by convolving the magnetic field with a filter.
- ✓ convert the data as we measure magnetic field at the pole

5. Magnetic effects of simple shapes

(1) The Isolated Pole (Monopole)

- Although an isolated pole is a fiction, in practice it may be used to represent a steeply dipping dipole whose lower pole is so far away that it has a negligible effect
- The induced magnetization in a long, slender, near-vertical body tends to be along the axis of the body except near the magnetic equator.



$$\mathbf{H} = \frac{m}{r^2}\mathbf{r}$$

$$H_z = \frac{zm}{(x^2+z^2)^{3/2}} = H\cos\theta \qquad \left(\cos\theta = \frac{z}{r}\right)$$

(2) The dipole

$$m = IA = kH_eA$$

A: 쌍극자의 단면, k : susceptibility, He : earth's magnetic field

Hz=(-m에 의한 vertical magnetic field)=-(m에 의한 vertical component)

$$H_z = kH_eA\left\{\left(\frac{1}{r_1^2}\right)\left(\frac{z}{r_1}\right)-\left(\frac{1}{r_2^2}\right)\left(\frac{z+L\sin\alpha}{r_2}\right)\right\}$$

$$= kH_eA\left\{\frac{z}{r_1^3}-\frac{z+L\sin\alpha}{r_2^3}\right\}$$

In a similar manner,

$$H_h = kH_eA\left\{\frac{x}{r_1^3}-\frac{x+L\cos\alpha}{r_2^3}\right\}$$

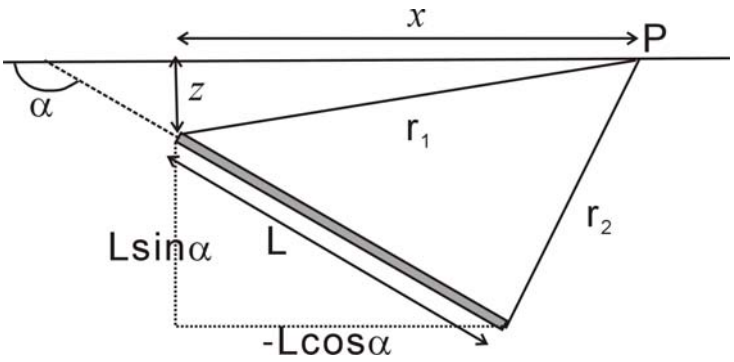
magnetic fields due to the dipole are greater than the earth's magnetic fields and their axis is the same as that of the earth's magnetic fields.

① For a vertical dipole,

$$\alpha = \frac{\pi}{2},\ H_e = (H_z)_0$$

$$H_z = k(H_z)_0 A\left(\frac{z}{r_1^3} - \frac{z+L}{r_2^3}\right)$$

$$H_h = k(H_z)_0 Ax\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)$$



② for a horizon dipole,

$$\alpha = \pi,\ H_e = (H_h)_0$$

$$H_x = k(H_h)_0 A\left(\frac{x}{r_1^3} - \frac{x-L}{r_2^3}\right)$$

$$H_z = k(H_h)_0 Az\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)$$

(3) Sphere

for a vertically magnetic sphere,

$$\mathbf{H} = -\nabla V(\mathbf{r})$$

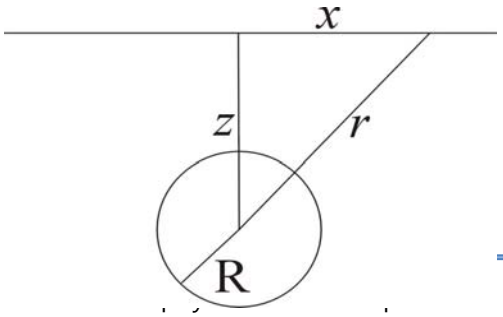
$$V(\mathbf{r}) = -\frac{I}{G\rho}\frac{\partial}{\partial\alpha}U(\mathbf{r})$$ (Poisson's relationship)

$$H_z = \frac{I}{G\rho}\frac{\partial^2}{\partial z^2}U(\mathbf{r})$$ (U:gravitational potential)

$$H_z = \frac{\frac{4}{3}\pi R^3 I}{z^3}\cdot\frac{2-(\frac{x}{z})^2}{\left[1+(\frac{x}{z})^2\right]^{\frac{5}{2}}}$$

$$U = \frac{GM}{r} = \frac{\frac{4}{3}\pi R^3\rho G}{r} = \frac{\frac{4}{3}\pi R^3\rho G}{\sqrt{x^2+z^2}}$$

$$\frac{\partial U}{\partial z} = -\frac{2}{3}\frac{\pi R^3\rho G\cdot 2z}{(x^2+z^2)^{3/2}}$$



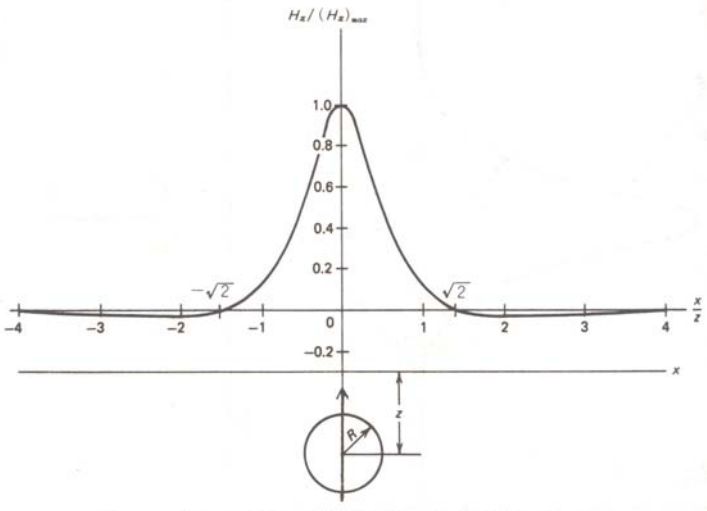


그림 3-24 수직으로 분극된 구형 자성체에 의한 수직 자기성분 이상

$$\frac{\partial^2 U}{\partial z^2} = -\frac{4}{3}\pi R^3 \rho G\left[\frac{1}{(x^2+z^2)^{3/2}} - \frac{3}{2}\frac{z\cdot 2z}{(x^2+z^2)^{5/2}}\right]$$
$$= -\frac{4}{3}\pi R^3 \rho G\left[\frac{x^2+z^2-3z^2}{(x^2+z^2)^{5/2}}\right]$$
$$= -\frac{4}{3}\pi R^3 \rho G\left[\frac{2z^2-x^2}{(x^2+z^2)^{5/2}}\right]$$

$$H_z = \frac{4}{3}\pi R^3 I \frac{2z^2-x^2}{\left(x^2+z^2\right)^{5/2}}$$

for a spherical body whose radius is 100m, and which is composed of magnitude by 30%. let's compute the maximum value of Hz.

the spherical body is located at te depth of 200m.

the earth's magnetic intensity (gauss/Oe) is He=0.5Oe

$$I = kH_e = 0.3\times 0.5\times 0.5\times O_e = 0.075 cgs$$

$$\left(H_z\right)_{\max} = \frac{\frac{4}{3}\pi(100)^3\times 0.075}{(200)^3}\times 2$$
$$= 0.07875 gauss$$
$$= 7875 gamma$$

Assignment

1. Compute magnetic effect (vertical component, horizontal component, and total magnetic intensity) of a monopole and draw graphs of vertical component, horizontal component and total magnetic intensity of the magnetic effect.

   Depth to negative pole: 5 m
   Pole cross-sectional area: 0.5 m^2
   Susceptibility of pole: 0.003 cgs emu
   Earth's field (nT): 55000 nT
   Earth's field inclination: 70
   Horizontal axis ranges from -15 to 15 with a horizontal increment of 1 m.

2. Compute magnetic effect (vertical component, horizontal component, and total magnetic intensity of a dipole and draw graphs of vertical component, horizontal component and total magnetic intensity of the magnetic effect.

   Depth to negative pole: 5 m
   Pole cross-sectional area: 1 m^2
   Susceptibility of pole: 0.003 cgs emu
   Earth's field (nT): 55000 nT
   Earth's field inclination: 70
   Length of the dipole: 15 m
   Horizontal axis ranges from -30 to 30 with a horizontal increment of 2 m.

3. Compute magnetic effect of a sphere and draw graphs

   Depth to sphere center : 5 m
   Sphere radius: 1
   Intensity of magnetization: 750 nT
   Horizontal axis ranges from -30 to 30 with a horizontal increment of 2 m.

TABLE 7.1 MAGNETIC EFFECT OF A MONOPOLE

Depth to negative pole (m)	**5**	Pole cross-sectional area (m^2)	**0.5**
Horizontal increment (m)	**1**	Susceptibility of pole (cgs emu)	**0.003**
		Earth's field (nT)	**55000**
(All anomaly values are in nT.)		Earth's field inclination (°)	**70**

x position (m)	y position (m)	Z_A	H_{A_x}	H_{A_y}	F_{A_T}
−15	**0**	0.10	0.31	0.00	0.21
−14	0	0.13	0.35	0.00	0.24
−13	0	0.15	0.40	0.00	0.28
−12	0	0.19	0.45	0.00	0.33
−11	0	0.23	0.51	0.00	0.40
−10	0	0.30	0.59	0.00	0.48
−9	0	0.38	0.68	0.00	0.59
−8	0	0.49	0.79	0.00	0.73
−7	0	0.65	0.91	0.00	0.92
−6	0	0.87	1.04	0.00	1.17
−5	0	1.17	1.17	0.00	1.50
−4	0	1.57	1.26	0.00	1.91
−3	0	2.08	1.25	0.00	2.38
−2	0	2.64	1.06	0.00	2.84
−1	0	3.11	0.62	0.00	3.14
0	0	3.30	0.00	0.00	3.10
1	0	3.11	−0.62	0.00	2.71
2	0	2.64	−1.06	0.00	2.12
3	0	2.08	−1.25	0.00	1.53
4	0	1.57	−1.26	0.00	1.05
5	0	1.17	−1.17	0.00	0.70
6	0	0.87	−1.04	0.00	0.46
7	0	0.65	−0.91	0.00	0.30
8	0	0.49	−0.79	0.00	0.19
9	0	0.38	−0.68	0.00	0.12
10	0	0.30	−0.59	0.00	0.08
11	0	0.23	−0.51	0.00	0.04
12	0	0.19	−0.45	0.00	0.02
13	0	0.15	−0.40	0.00	0.01
14	0	0.13	−0.35	0.00	0.00
15	0	0.10	−0.31	0.00	−0.01

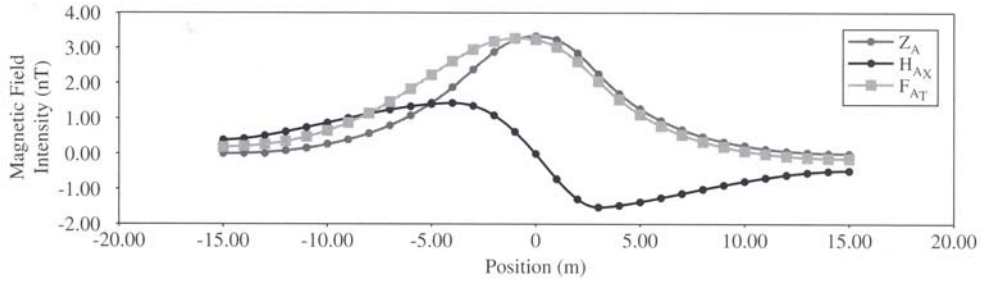


TABLE 7.2 MAGNETIC EFFECT OF A DIPOLE

Dipole inclination (°)	120
Depth to negative pole (m)	5
Length of dipole (m)	15
Horizontal increment (m)	2

(All anomaly values are in nT.)

Position (m)	R_{A_n}	R_{A_p}	Z_{A_n}	Z_{A_p}	H_{A_n}	H_{A_p}	Z_A	H_A	F_{A_T}
−30	0.18	−0.10	0.03	−0.04	0.18	−0.09	−0.01	0.09	0.03
−28	0.20	−0.10	0.04	−0.05	0.20	−0.09	−0.01	0.11	0.04
−26	0.24	−0.11	0.04	−0.05	0.23	−0.10	−0.01	0.13	0.06
−24	0.27	−0.13	0.06	−0.06	0.27	−0.11	−0.01	0.16	0.07
−22	0.32	−0.14	0.07	−0.07	0.32	−0.12	0.00	0.20	0.10
−20	0.39	−0.15	0.09	−0.08	0.38	−0.13	0.01	0.25	0.13
−18	0.47	−0.17	0.13	−0.10	0.46	−0.14	0.03	0.32	0.18
−16	0.59	−0.19	0.18	−0.11	0.56	−0.15	0.06	0.41	0.26
−14	0.75	−0.21	0.25	−0.13	0.70	−0.16	0.12	0.54	0.37
−12	0.98	−0.23	0.38	−0.16	0.90	−0.17	0.22	0.73	0.55
−10	1.32	−0.26	0.59	−0.19	1.18	−0.18	0.40	1.00	0.85
−8	1.85	−0.29	0.98	−0.22	1.57	−0.19	0.76	1.38	1.35
−6	2.70	−0.33	1.73	−0.26	2.08	−0.20	1.47	1.88	2.21
−4	4.02	−0.36	3.14	−0.30	2.51	−0.19	2.84	2.32	3.62
−2	5.69	−0.40	5.28	−0.35	2.11	−0.19	4.93	1.93	5.23
0	6.60	−0.43	6.60	−0.40	0.00	−0.17	6.20	−0.17	5.29
2	5.69	−0.47	5.28	−0.45	−2.11	−0.14	4.84	−2.25	3.06
4	4.02	−0.49	3.14	−0.48	−2.51	−0.09	2.66	−2.61	1.00
6	2.70	−0.51	1.73	−0.50	−2.08	−0.04	1.23	−2.12	0.00
8	1.85	−0.51	0.98	−0.51	−1.57	0.01	0.47	−1.56	−0.37
10	1.32	−0.50	0.59	−0.50	−1.18	0.07	0.09	−1.11	−0.47
12	0.98	−0.48	0.38	−0.47	−0.90	0.12	−0.09	−0.78	−0.47
14	0.75	−0.45	0.25	−0.42	−0.70	0.15	−0.17	−0.55	−0.42
16	0.59	−0.42	0.18	−0.38	−0.56	0.18	−0.20	−0.38	−0.37
18	0.47	−0.38	0.13	−0.33	−0.46	0.19	−0.20	−0.26	−0.31
20	0.39	−0.34	0.09	−0.28	−0.38	0.20	−0.19	−0.18	−0.25
22	0.32	−0.31	0.07	−0.24	−0.32	0.19	−0.17	−0.12	−0.21
24	0.27	−0.28	0.06	−0.20	−0.27	0.19	−0.15	−0.08	−0.17
26	0.24	−0.25	0.04	−0.17	−0.23	0.18	−0.13	−0.05	−0.14
28	0.20	−0.22	0.04	−0.15	−0.20	0.17	−0.11	−0.03	−0.11
30	0.18	−0.20	0.03	−0.12	−0.18	0.16	−0.09	−0.02	−0.09

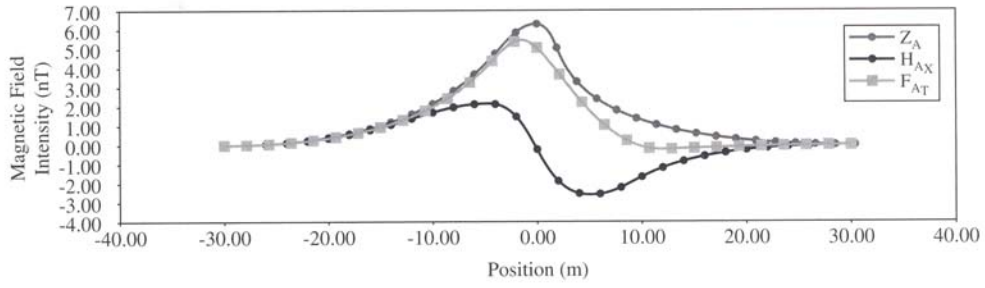


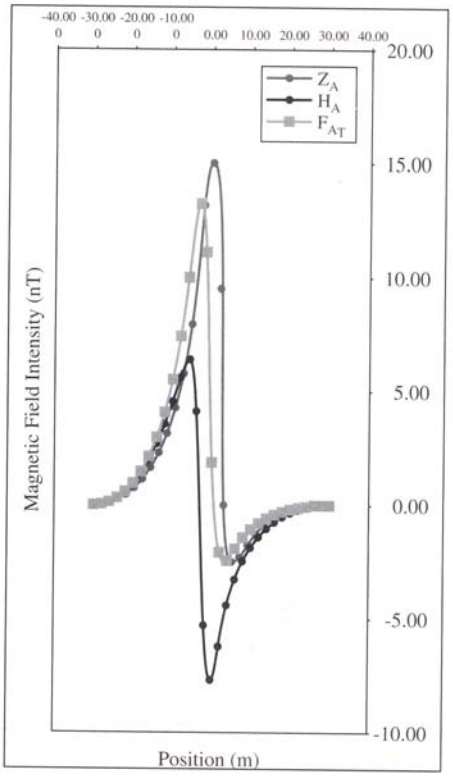
TABLE 7.3 MAGNETIC EFFECT OF A SPHERE

Position (m)	Z_A	H_A	F_{A_T}
−30	−0.02	0.05	0.01
−28	−0.02	0.06	0.01
−26	−0.03	0.08	0.02
−24	−0.03	0.10	0.02
−22	−0.04	0.14	0.04
−20	−0.04	0.18	0.05
−18	−0.05	0.25	0.08
−16	−0.05	0.36	0.14
−14	−0.03	0.53	0.24
−12	0.02	0.81	0.43
−10	0.19	1.30	0.82
−8	0.67	2.18	1.67
−6	2.06	3.66	3.61
−4	5.78	5.40	7.71
−2	12.68	4.04	13.00
0	14.51	−4.19	10.47
2	5.74	−7.97	0.99
4	−0.05	−4.71	−2.40
6	−1.18	−1.96	−2.01
8	−1.01	−0.73	−1.24
10	−0.71	−0.25	−0.74
12	−0.48	−0.07	−0.45
14	−0.33	0.00	−0.29
16	−0.24	0.03	−0.19
18	−0.17	0.04	−0.13
20	−0.13	0.04	−0.09
22	−0.10	0.03	−0.07
24	−0.08	0.03	−0.05
26	−0.06	0.03	−0.04
28	−0.05	0.02	−0.03
30	−0.04	0.02	−0.02

Depth to sphere center (m)	**5**
Sphere radius (m)	**1**
Susceptibility (cgs emu)	**0.005**
Horizontal increment (m)	**2**

Earth's field (F_E) (nT)	**50000**
Earth's field inclination (°)	**60**

(All anomaly values are in nT.)



6. Interpretation of Magnetic Data
   - We may face a difficult not present in gravity surveys: that of remanent magnetization.
   - If sufficient remanence is present, the anomaly we view is a combination of this remanence and the induced magnetization and may be very different in form than if no remanence existed.
   - Many data processing techniques such as the separation of regional triend, the half-maximum technique, slope method, the second derivative method and upward and downward continuations used in gravity work can be adapted for magnetic work.
   - Half -maximum technique

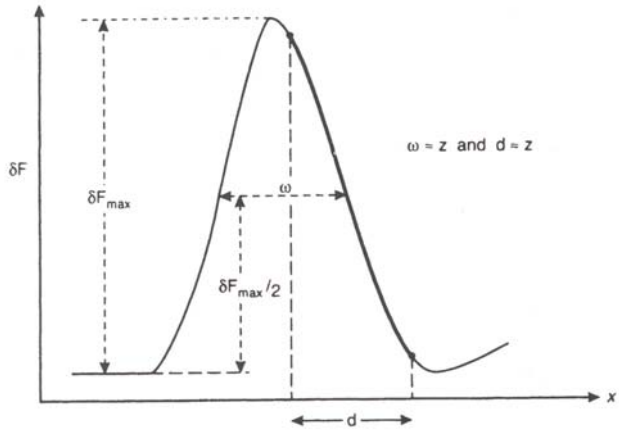


그림 3.44 자성체 상부까지의 깊이를 추정하는 간단한 방법

For monopole,

$$H_z = \frac{zkH_eA}{\left(x^2+z^2\right)^{3/2}}$$

$$H_{z(\max)} = \frac{kH_eA}{2z^2} \quad , \quad H_{z(\max/2)} = \frac{kH_eA}{2z^2} = \frac{zkH_eA}{\left(x_{1/2}^2+z^2\right)^{3/2}}$$

$$z = x_{1/2} / 0.766$$

For sphere and horizontal cylinder, the width for Half maximum approximates the depth to the object.

For declined plane?, gradient method is good

   - Slope methods

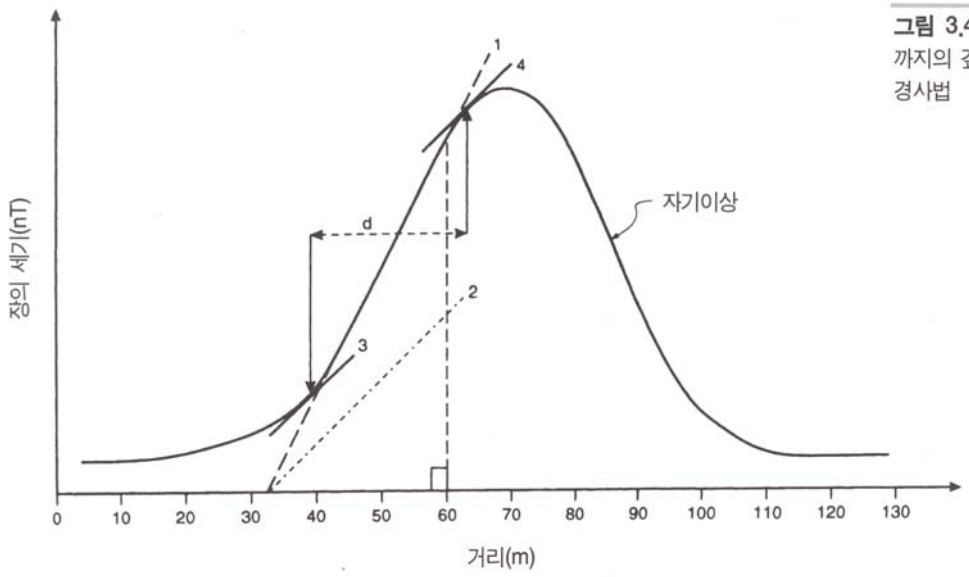


그림 3.45 자화된 암맥의 상부까지의 깊이를 구하는 Peters 반경사법

7. Applications

➢ Mineral Exploration

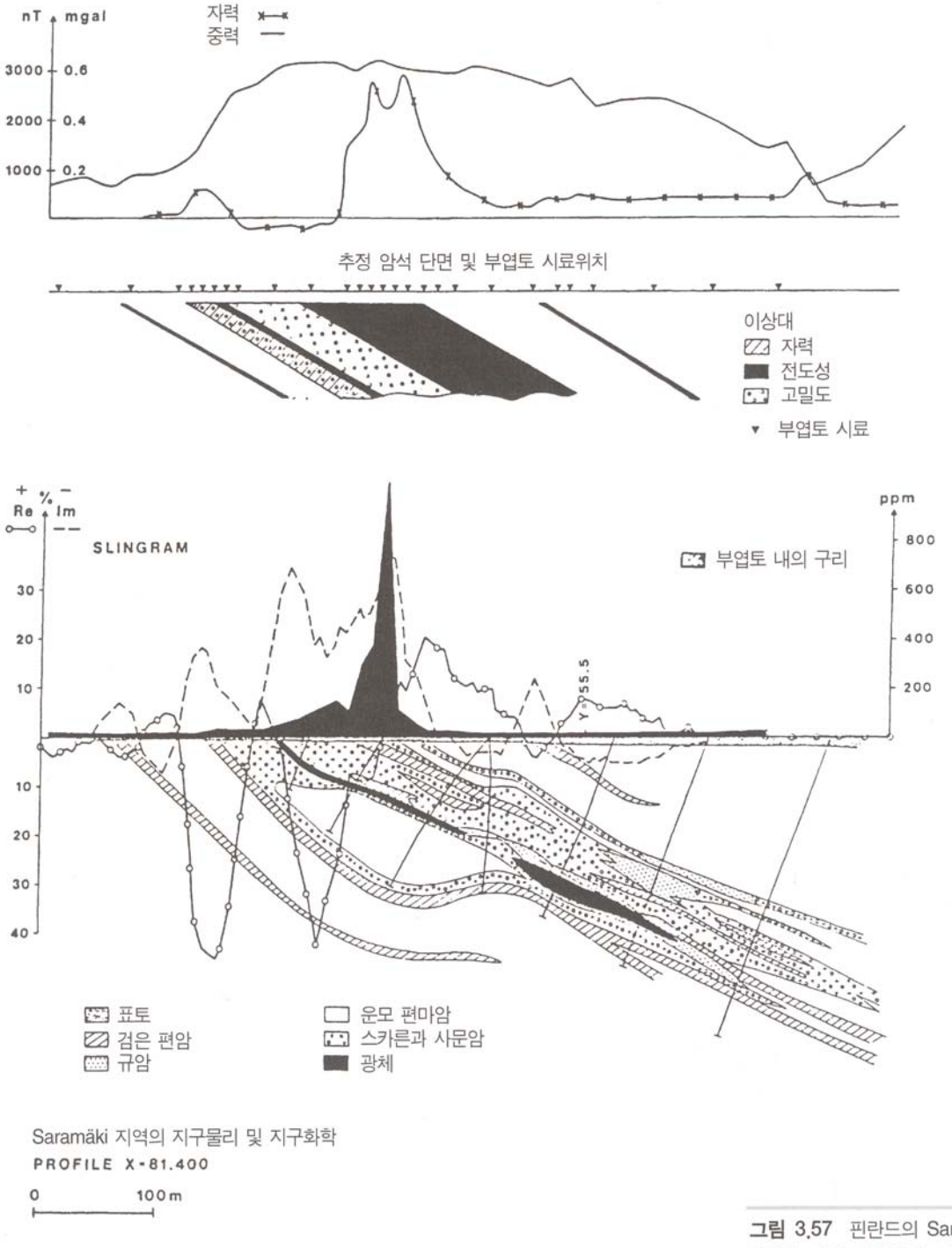


그림 3.57 핀란드의 Saramäki 광체의 지질 단면도와 자력, 중력 및 SLINGRAM 전기비저항 자료(Ketola, 1979)

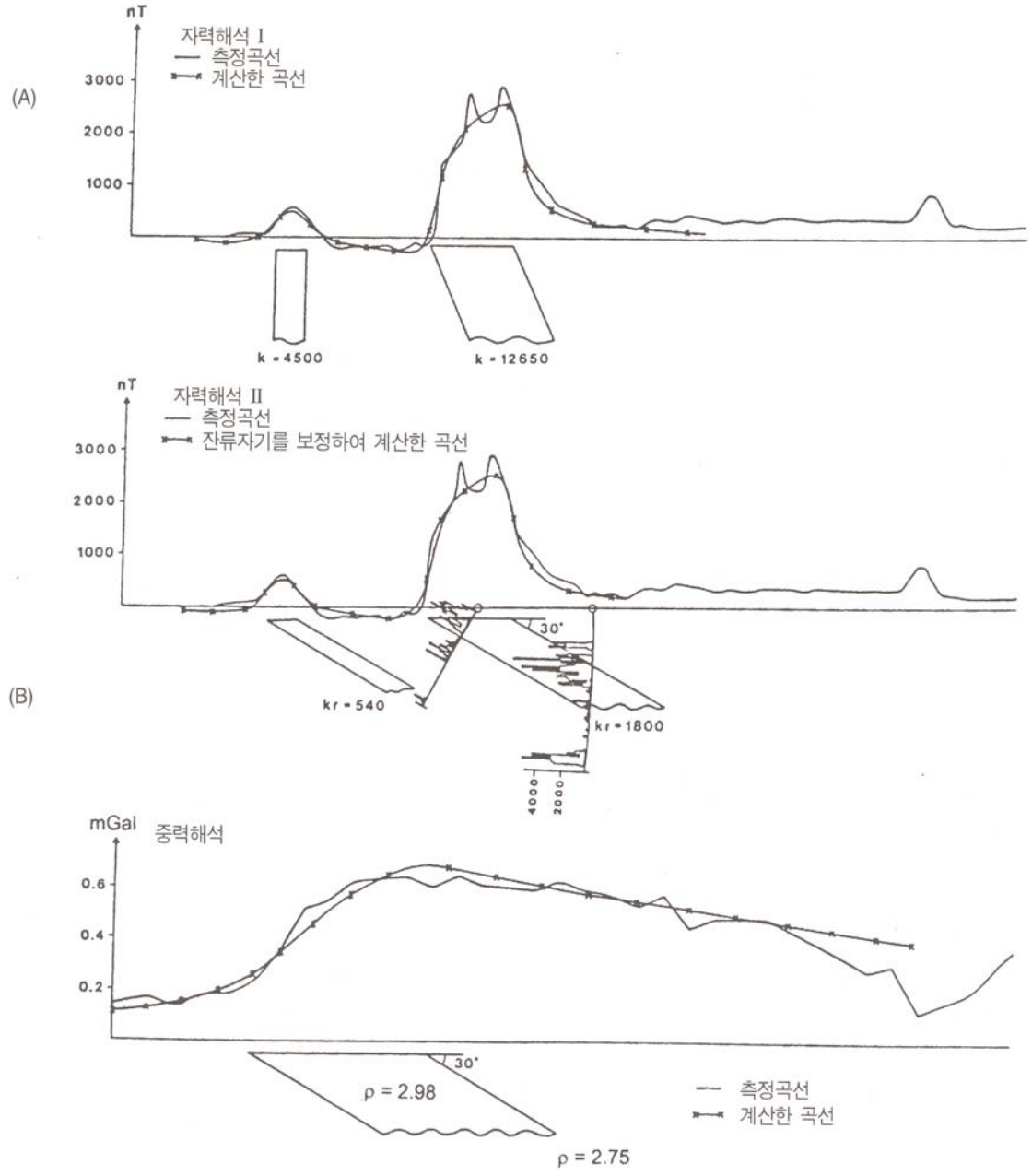


그림 3.58 Saramäki 광체의 모델링. (A) 유도자화만 고려한 모형, (B) 유도자화와 잔류자기를 고려한 모형과 중력모형의 비교 (Ketola, 1979)

➢ Engineering application: Detection of underground pipes

(A)

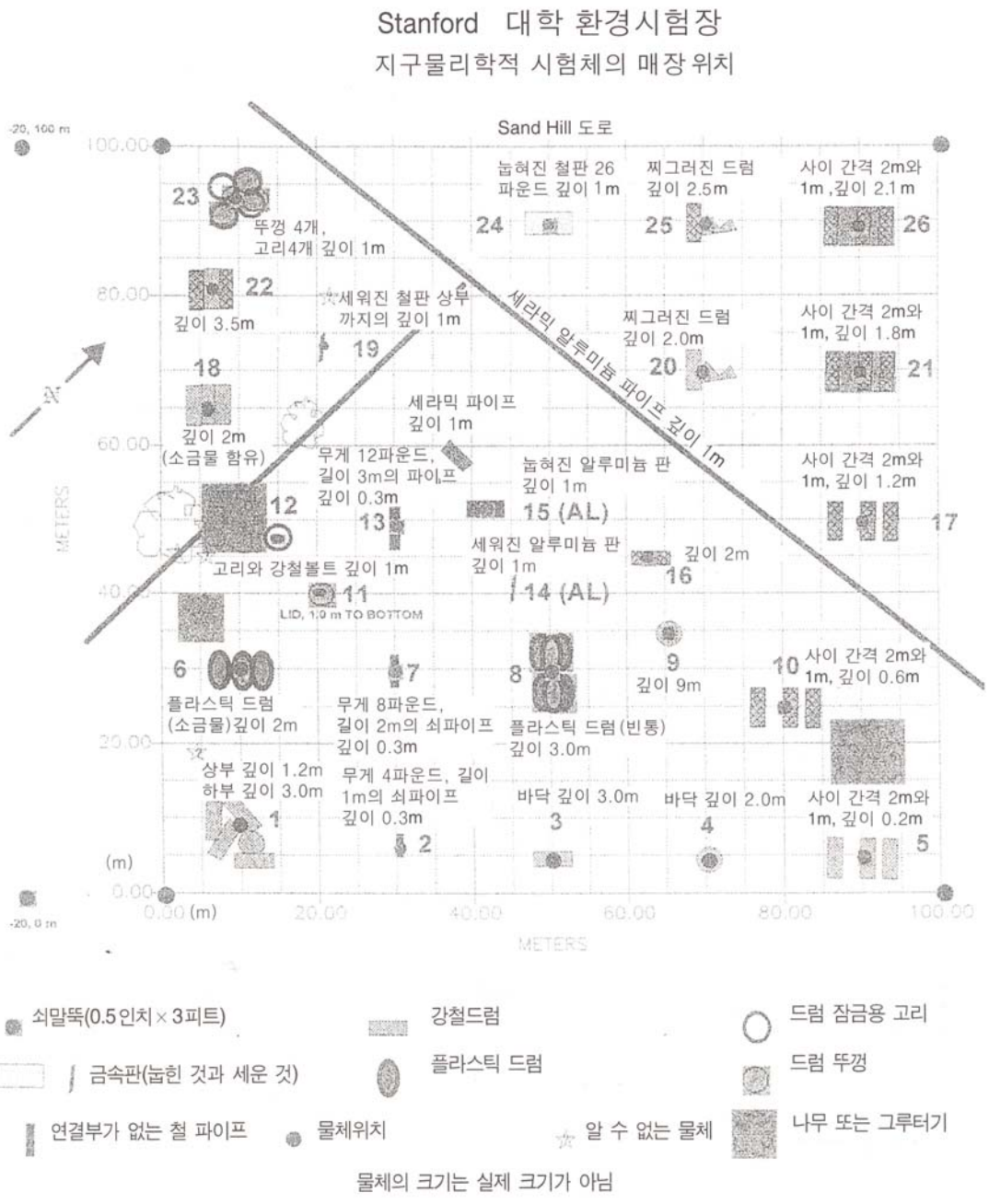


그림 3.60 (A) Stanford 대학 환경시험장의 상세한 지하 매설물 지도

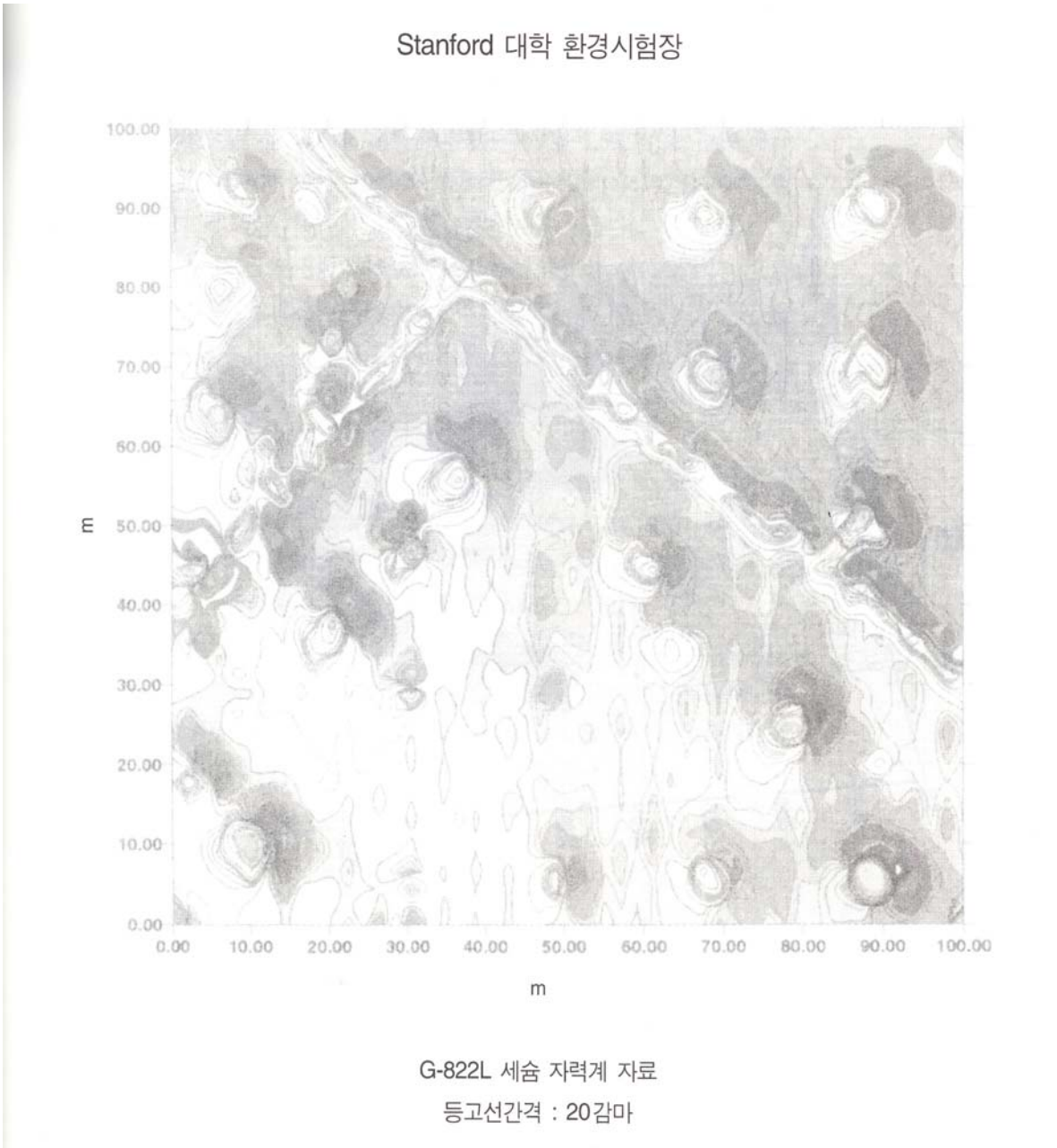


그림 3.60 (B) 세슘 자력계를 이용하여 2m 간격의 측선에서 1m당 10회 측정한 자기이상도

Seismic methods

1. Introduction

- The seismic method is the most important geophysical technique. Its predominance is due to high accuracy, high resolution, and great penetration.
- The widespread use of seismic methods is principally in exploring for petroleum.

- Seismic methods are also important in groundwater searches and in civil engineering, especially to measure the depth to bedrock in connection with the construction of large buildings, dams, highways, and harbor surveys.

- Exploration seismology is an offspring of earthquake seismology. When an earthquake occurs, the earth is fractured and the rocks on opposite sides of the fracture move relative to one another. Such a rupture generates seismic waves that travel outward from the fracture

- Seismologists use the data to deduce information about the nature of the rocks through which the earthquake waves traveled.

- Exploration seismic methods are different from the earthquake seismology in that the energy sources are controlled and movable, and the distances between the source and the recording points are relatively small.

- Source: explosives and other energy sources
- Receiver: arrays of seismometers or geophones or hydrophones

- The basic technique of seismic exploration consists of generating seismic waves and measuring the time required for the waves to travel from the sources to a series of geophones, usually disposed along a straight line directed toward the source.

- Structural information is derived principally from paths that fall into two main categories:

- *headwave or refracted paths*, in which the principal portion of the path is along the interface between two rock layers.
- *Reflected paths*, in which the wave travels downward initially and at some point is reflected back to the surface, and the overall path is essentially vertical.

- For both types of path, the traveltimes depend on the physical properties of the rocks and the attitudes of the beds

- The objective of seismic exploration is to deduce information about the rocks, especially about the attitudes of the beds, from the observed arrival times and from variations in amplitude, frequency, phase, and wave shape.

표 4.1 탄성파탐사에서 얻는 정보와 그 적용분야

전반적인 지질구조
기반암의 깊이
단층 및 파쇄대의 위치
단층 변위
매몰된 계곡의 위치와 특성
암상 결정
층서
염기성 화성암맥의 위치
암석학적인 정보
탄성계수
밀도
감쇠
공극률
탄성파 속도
이방성
굴착난이도
적용분야
공학 부지 조사
암석강도
모래/자갈 자원 조사
공동탐지
해저 조사(시추 리그의 위치 선정)
해저 퇴적층의 탈가스 및 탈수
예정부지 평가
새로운 쓰레기 매립지
주요 건물
항구 및 부두
하수 배출관
터널 건설
수리지질 및 지하수탐사
지반 입자속도
법과학적인 적용
항공기 추락지점
항공기 설계
핵실험 금지조약의 감시
대규모 지하 군사무기고 위치

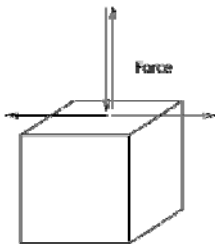
2. Seismic Theory

(1) Stress and strain

a. Stress

- When an external force F is applied across an area A of a surface of a body, forces inside the body are established in proportion to the external force.
- The ratio of the force to area (F/A) is defined as *stress* $P = F/A$
- Stress can be resolved into two components, one at right-angles to the surface (normal or dilatational stress, pressure) and one in the plane of the surface (shear stress).
- Stress at a point

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$



b. Strain

- The stressed body undergoes *strain*, which is the amount of deformation expressed as the ratio of the change in length (or volume) to the original length (or volume).

c. Elasticity

- The size and shape of a solid body can be changed by applying forces to the external surface of the body. These external forces are opposed by internal forces that resist the changes in size and shape
- As a result, the body tends to return to its original condition when the external forces are removed.

- This property of resisting changes in size and shape and of returning to the original condition when the external forces are removed is called *elasticity*.

- Stress and strain are linearly dependent and the body behaves elastically until the yield point (elastic limit or proportional limit) is reached. → *Hooke's law*

- Below the yield point, on relaxation of stress, the body reverts to its pre-stressed shape and size.

- At stresses beyond the yield point, the body behaves in a plastic or ductile

manner and permanent damage results.

- If further stress is applied, the body is strained until it fractures.

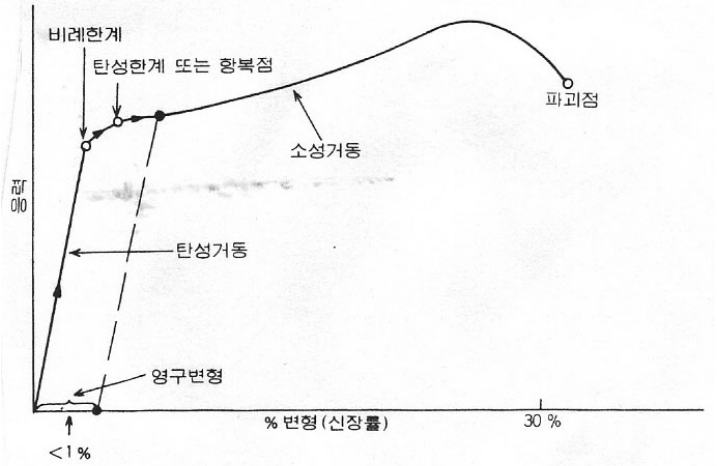


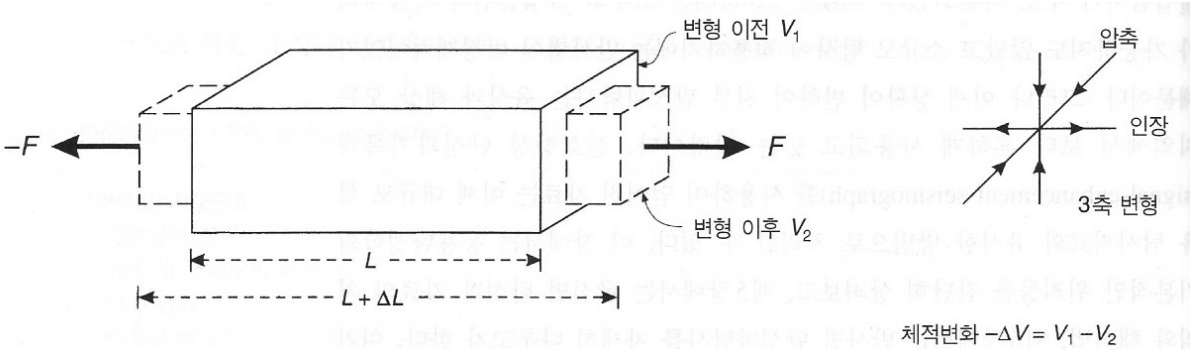
그림 6·1 고체의 응력 – 변형 관계

- Earthquakes occur when rocks are strained until fracture, when stress is then released.

d. Elastic moduli

- The relationship between stress and strain for any material is defined by various *elastic moduli*.

  - ✓ Young's modulus (영률)

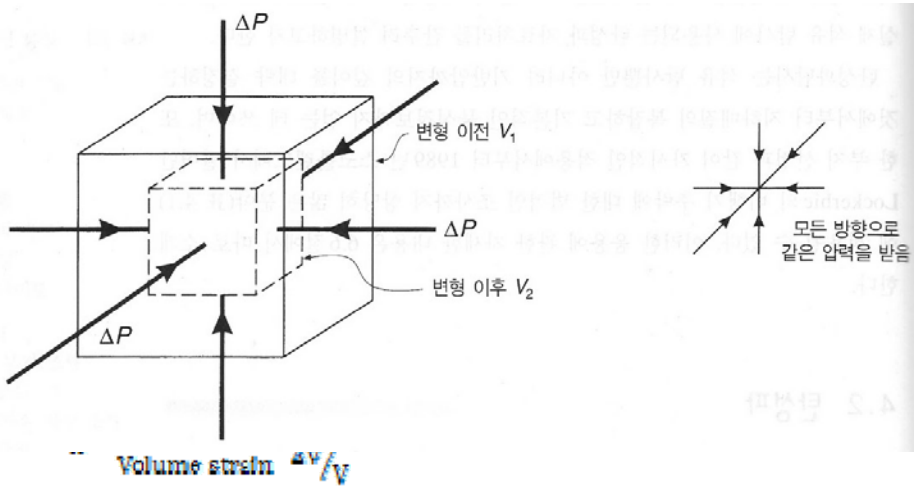


$$E = \frac{\text{Longitudinal stress } \sigma = F/A}{\text{Longitudinal strain } \Delta L/L} = \frac{P}{\varepsilon}$$

※ Poisson's ratio

$$\sigma = -\frac{\text{contractional strain}}{\text{extensional strain}}$$

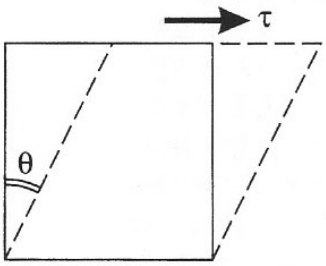
✓ Bulk modulus (체적탄성률)



Volume strain $\Delta V/V$

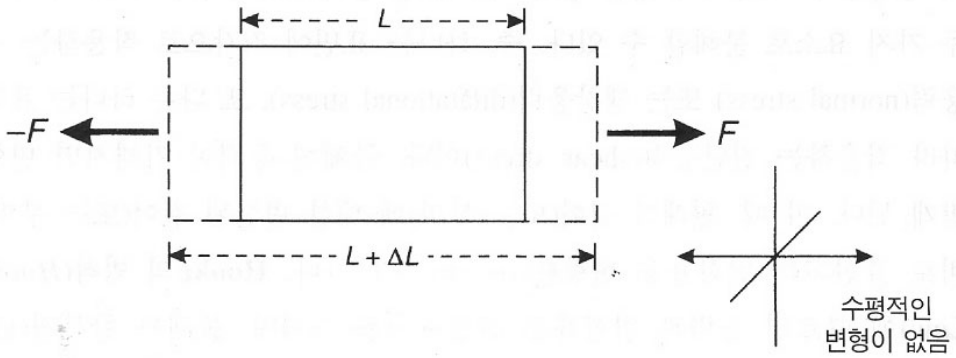
(in the case of excess hydrostatic pressure)

✓ Shear (rigidity) modulus (a Lamé constant) (전단계수, 강성률)



($\mu = 1 - 7 \times 10^4$Mpa ; $\mu = 0$ for fluid)

✓ Axial modulus



$$U = \frac{\text{Longitudinal stress } \Delta F/A}{\text{Longitudinal strain } \Delta L/L} = \frac{\sigma}{\varepsilon}$$

(in the case with no lateral strain)

➢ Relationship between Young's modulus (E), Poisson's ratio (σ) and the two

Lame constants (λ and μ)

$$E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)} \quad \sigma = \frac{\lambda}{2(\lambda + \mu)} \quad k = \lambda + \frac{2}{3}\mu$$

$$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}$$

$$\frac{v_s}{v_p} = \sqrt{\frac{1-2\sigma}{2(1-\sigma)}} \quad \Rightarrow \quad \sigma = 0.5 \quad v_s = 0 : \text{for fluid}$$

$$\lambda \to \infty \quad k \to \infty \text{ incompressibility}$$

- Poisson's ratio ranges from 0.05 (very hard rocks) to 0.45 (for loose sediments). For fluid, it is 0.5. for general rocks, $\sigma \approx 0.25$

Box 4.1 탄성계수

영률(Young's modulus)

$$E = \frac{\text{단축응력 } \Delta F/A}{\text{단축변형률 } \Delta L/L} = \frac{\sigma}{\varepsilon} \quad (3\text{축 변형의 경우})$$

체적탄성률(bulk modulus)

$$k = \frac{\text{체적응력 } \Delta P}{\text{체적변형률 } \Delta v/v} \quad (\text{초과 정수압의 경우})$$

전단계수(shear modulus) 또는 강성률(rigidity modulus)

$$G = \frac{\text{전단응력 } \tau}{\text{전단변형률 } \varepsilon} \quad (G \approx 1 \sim 7 \times 10^4 \text{MPa}, \text{ 유체에서는 } G = 0)$$

축방향계수(axial modulus)

$$U = \frac{\text{단축응력 } \Delta F/A}{\text{단축변형률 } \Delta L/L} = \frac{\sigma}{\varepsilon} \quad (\text{수평 변형이 없는 경우})$$

영률 E, 포아송비 μ, 2개의 Lamé 상수(G, λ)와의 관계

$$E = \frac{G(3\lambda + 2G)}{(\lambda + G)} \qquad \mu = \frac{\lambda}{2(\lambda + G)}$$

$$k = \frac{3\lambda + 2G}{3} \qquad \lambda = \frac{E\mu}{(1 + \mu)(1 - 2\mu)}$$

암석의 탄성계수는 물리상수를 수록한 핸드북에서 찾을 수 있다.

(2) Types of seismic waves

- Seismic waves travel away from any seismic source at speeds determined by elastic moduli and the densities of the media through which they pass

- There are two main types of seismic waves: body waves and surface waves

  ✓ Body waves: waves that pass through the bulk of a medium are known as body waves

  ✓ Surface waves: waves confined to the interfaces between media with contrasting elastic properties, particularly the ground surface, are called surface waves

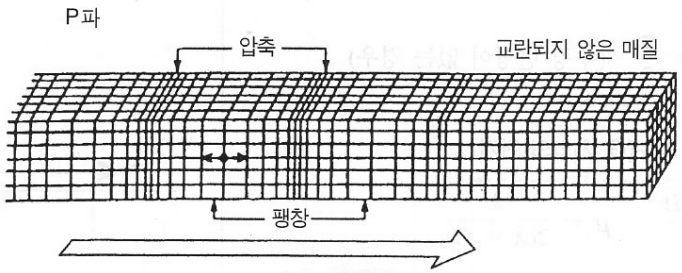
  ✓ Guided waves: are encountered in some applications, which are confined to particular thin bands sandwiched between layers with higher seismic velocities by total internal reflection.

  (For example)
  Channel or seam waves, which propagate along coal seams
  Tube waves, which travel up and down fluid-filled boreholes

- Body waves

  ➢ In unbounded homogeneous isotropic media, body waves only exist.
  ➢ Two types of body waves can travel through an elastic medium.

  ➢ P-wave:
    ✓ Primary, longitudinal, dilatational, irrotational, or compressional waves (for example, sound waves)
    ✓ Material particles oscillate about fixed points in the direction of wave propagation by compressional and dilatational strain



    ✓ Velocity

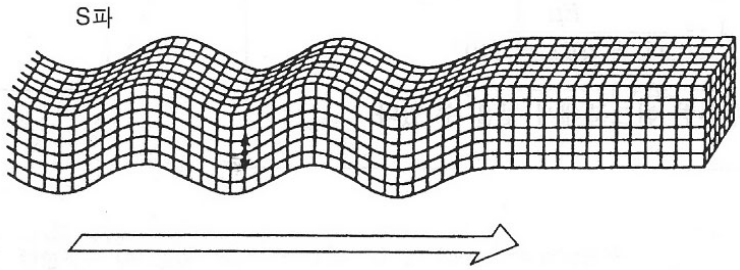
$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}$$

In fluid,

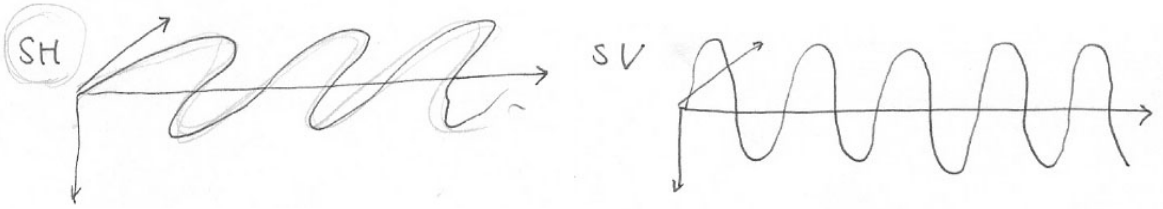
$$\alpha = \sqrt{\frac{\lambda}{\rho}} = \sqrt{\frac{k}{\rho}} \quad k = \lambda$$

- S-wave:

  - ✓ Secondary, transverse, rotational, or shear waves.
  - ✓ Particle motion is at right-angles to the direction of wave propagation and occurs by pure shear strain



  - ✓ When particle motion is confined to one plane only, the S-wave is said to plane-polarized
  - ✓ SH and SV



  - ✓ Velocity

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

In fluid, $\mu = 0, \quad \beta = 0$

Poisson's ratio

$$\frac{\alpha}{\beta} = \sqrt{\frac{2(1-\alpha)}{1-2\alpha}}$$

- All the frequencies contained within body waves travel through a given material at the same velocity. → not dispersive

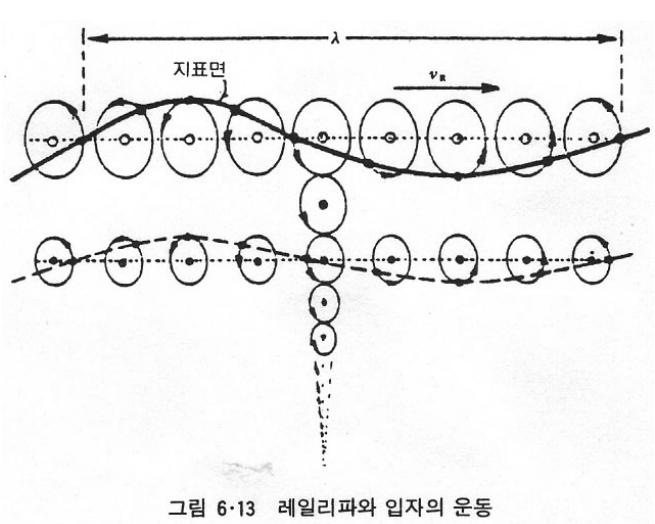
- Surface waves

  - In an infinite homogeneous isotropic medium, only P and S wave exist. However, when the medium does not extend to infinity in all directions, other types of waves can be generated. They are called surface waves.
  - The waves do not penetrate deep into subsurface media. They are confined to the interfaces.

  - There are two types of surface waves: Rayleigh and Love waves
  - Large amplitude and low frequency waves

  - Rayleigh waves

    - ✓ Are generated by the combination of P and SV waves.
    - ✓ Travel along the free surface of the Earth with amplitudes that decrease exponentially with depth
    - ✓ Particle motion is in a retrograde elliptical sense in a vertical plane with respect to the surface



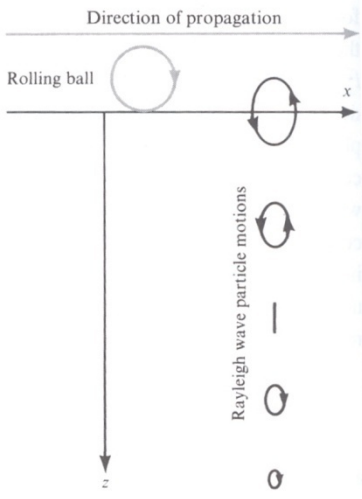
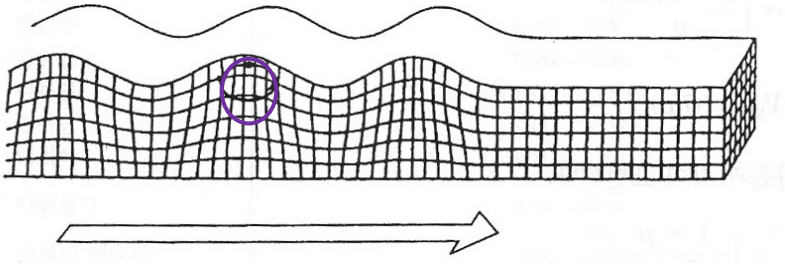


그림 6·13 레일리파와 입자의 운동



    - ✓ The velocity of Rayleigh waves depends upon the elastic constants near the surface and is always less than the S wave velocity β. When the Poisson's ratio is 0.25, the Rayleigh wave velocity is 0.92β

- ✓ Because the elastic constants change with depth, the velocity of Rayleigh waves varies with wavelength. → A variation of velocity with wavelength (or frequency) is called dispersion.

- ✓ Rayleigh waves are dispersive in layered media, whereas they are not dispersive in semi-infinite homogeneous media.

- ✓ Rayleigh waves manifest themselves normally as large-amplitude, low-frequency waves called ground roll which can mask reflections on a seismic record

- ✓ Rayleigh waves are used in research on assessment of stability of structure such as dam and road (Spectral analysis of the surface waves: SASW). The SASW employs the dispersive features of Rayleigh waves.

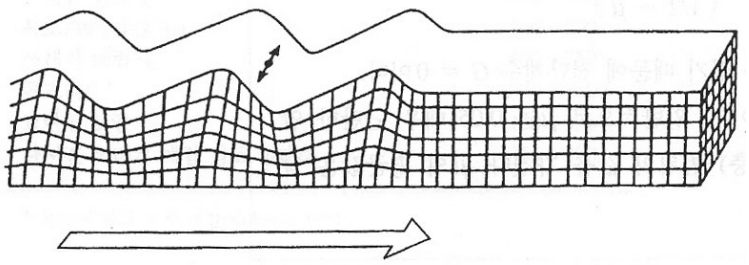
➢ Love waves

- ✓ Love waves occur only where a medium with a low S-wave velocity overlies a half space with a higher S-wave velocity

- ✓ Velocities are intermediate between the S-wave velocity at the surface and that in deeper layers.

- ✓ Particle motion is at right-angles to the direction of wave propagation but parallel to the surface

- ✓ These are polarized shear waves



- ✓ Energy sources used in seismic work do not generate Love waves to a significant degree and they are not important in ordinary seismic exploration

- Stoneley waves

  - ✓ They propagate along the interfaces.

- Surface waves have the characteristic that their waveform changes as they travel because different frequency components propagate at different rates, a phenomenon known as wave dispersion.
- The dispersion patterns are indicative of the velocity structure through which the waves travel, and thus surface waves generated by earthquakes can be used in the study of the lithosphere and asthenosphere.

(3) Seismic wave velocities

- Seismic wave velocities are expressed by the elastic moduli and densities of the rocks through which they pass.

- As a broad generalization, velocities increase with increasing density.
  Table 4.2 (textbook) P wave velocity

  - P wave velocity
    $$v_P = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}$$

  - S wave velocity
    $$v_s = \sqrt{\frac{\mu}{\rho}}$$

  - Find the ratios of P wave velocity to S wave velocity, when Poisson's ratio is 0.25 and 0.33.
    $$\frac{v_p}{v_s} = \sqrt{\frac{2(1-\sigma)}{1-2\sigma}}$$

  - Rayleigh wave velocity
    $v_R = 0.9v_s$: $0.92v_s$ when $\sigma = 0.25$

  - Love wave velocity
    $v_1 < v_L < v_2$

- The seismic wave velocities in sedimentary rocks in particular increase both with depth of burial and age.
  Elastic wave velocity as a function of geological age and depth

Box 4.3 지질 연령과 깊이에 따른 탄성파 속도(Faust, 1951)

> 셰일과 사암에 대한 탄성파 속도 V는 다음과 같다.
>
> $$V = 1.47(ZT)^{1/6} \text{ km/s}$$
>
> 여기서 Z는 깊이(km), T는 지질 연령(백만 년)이다.

for shale and sands, the elastic wave velocity V is given by $V = 1.47(ZT)^{1/6} km/s$
Z is the depth and T is the geological age in millions of years

- Velocities can be determined from seismic data and from laboratory analysis.

  - Refraction method
    - ✓ The in situ velocities measured by using the refraction method are different from those obtained from laboratory measurements, especially when the rock is heavily jointed or fractured.
      The refraction velocities are related to the rock and the discontinuity.

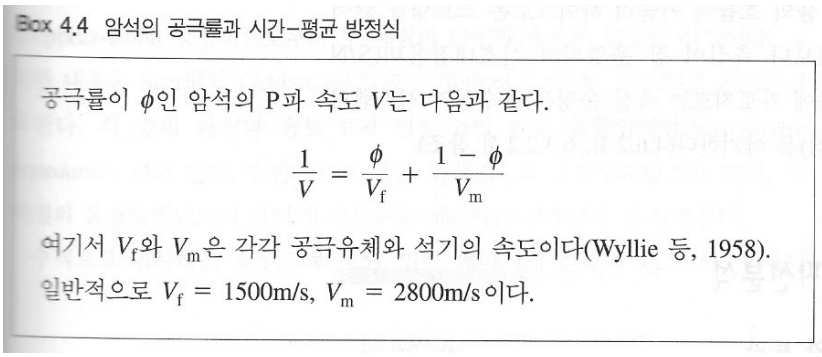
  - Schmidt hammer system
    - ✓ Two geophones are attached to the exposed material at a small distance apart.
    - ✓ P-wave is directly generated by a hammer at a known distance from the receivers.
    - ✓ The velocity is obtained from the difference in traveltime between the two receivers relative to their separation.

  - Laboratory measurements using ultrasonic transducers

    - ✓ Ultrasonic transducers transmit a ultrasonic pulse through a sample of the material.
    - ✓ From the measured traveltime of this pulse through the material whose length is known, a velocity can be calculated.
    - ✓ Ultrasonic frequencies (0.5-1.5 MHz) are used
    - ✓ In addition to the frequency of the transducers, we need to check whether the samples are dry or fully saturated. If they are fully

saturated, the salinity of the fluid and temperature are measured.

- ✓ Mechanical relaxation of retrieved core pieces is more significant. If a sample of rock is obtained from significant depth below ground where it is normally at substantial pressure, the rock expands on its return to the surface, resulting in the formation of microcracks due to the relaxation of confining pressure.
- ✓ In porous rocks, the nature of the material within the pores strongly influences the elastic wave velocity; water-saturated rocks have different elastic wave velocities compared with gas-saturated rocks
- ✓ (Box 4.4 in the textbook) Time-average equation to estimate rock porosity



Box 4.4 암석의 공극률과 시간–평균 방정식

공극률이 ϕ인 암석의 P파 속도 V는 다음과 같다.

$$\frac{1}{V} = \frac{\phi}{V_f} + \frac{1-\phi}{V_m}$$

여기서 V_f와 V_m은 각각 공극유체와 석기의 속도이다(Wyllie 등, 1958).
일반적으로 V_f = 1500m/s, V_m = 2800m/s이다.

The empirical time-average equation is often used to relate velocity v and porosity ϕ

It assumes that the traveltime per unit path length in a fluid-filled porous rock is the average of the traveltimes per unit path length in the matrix material $1/v_m$ and in the fluid $1/v_f$, where the traveltimes are weighted in proportion to their respective volumes $\frac{1}{v} = \frac{\phi}{v_f} + \frac{(1-\phi)}{v_m}$

The phase velocity v for a rock with fractional porosity (ϕ)

$$\rho_b = \rho_m(1-\phi) + \rho_f\phi \qquad \frac{1}{v} = \frac{\phi}{v_f} + \frac{(1-\phi)}{v_m}$$

v_f and v_m are the acoustic velocities in the pore fluid and the rock matrix respectively.

typical values : $v_f = 1500m/s$, $v_m = 2800m/s$

- ✓ (Box 4.5 in the textbook) P-wave velocity as a function of temperature and salinity in water

Box 4.5 온도와 염도에 따른 물의 P파 속도

$$V = 1449.2 + 4.6T - 0.055T^2 + 0.0003T^3 + (1.34 - 0.01T)(S - 35) + 0.016d$$

여기서 S와 T는 염도(‰)와 온도(°C)이고, d는 깊이(m)이다(Ewing 등, 1948; Fofonoff와 Millard, 1983).

$$V = 1449.2 + 4.6T - 0.055T^2 + 0.0003T^3 + (1.34 - 0.01T)(S - 35) + 0.016d$$

S : solinity (parts per thousand)

T: Temperature(^{o}C)

d: depth

✓ In stratified media, seismic velocities exhibit anisotropy. Velocities may be up to 10-15% higher for waves propagating parallel to strata than at right-angles to them.

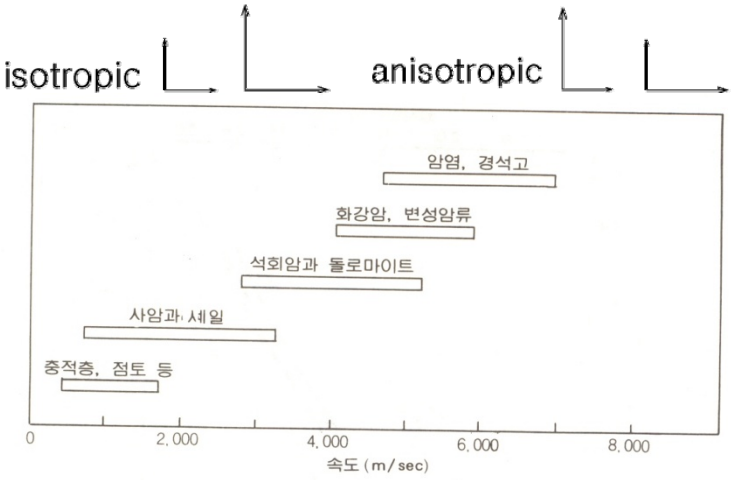


그림 6·28 각종 암석의 탄성파속도 분포

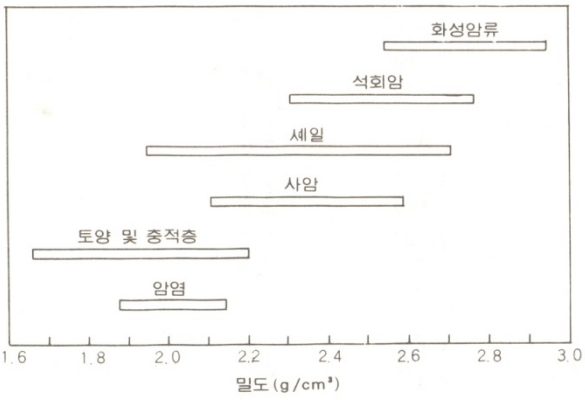


그림 6·27 각종 암석의 체적밀도 분포

표 4.2 P파 속도

물 질	V_P(m/s)
공기	330
물	1450 ~ 1530
석유	1300 ~ 1400
황토	300 ~ 600
토양	100 ~ 500
눈	350 ~ 3000
단단한 빙하(*)	3000 ~ 4000
무른 모래	200 ~ 2000
마르고 무른 모래	200 ~ 1000
물로 포화된 무른 모래	1500 ~ 2000
빙퇴석	1500 ~ 2700
지표 부근의 모래와 자갈	400 ~ 2300
깊이 2km의 모래와 자갈	3000 ~ 3500
점토	1000 ~ 2500
하구의 진흙/점토	300 ~ 1800
범람원 충적층	1800 ~ 2200
영구 동토대(제4기 퇴적층)	1500 ~ 4900
사암	1400 ~ 4500
연한 석회암	1700 ~ 4200
단단한 석회암	2800 ~ 7000
돌로마이트	2500 ~ 6500
경석고	3500 ~ 5500
암염	4000 ~ 5500
석고	2000 ~ 3500
셰일	2000 ~ 4100
화강암	4600 ~ 6200
현무암	5500 ~ 6500
반려암	6400 ~ 7000
감람암	7800 ~ 8400
사문암	5500 ~ 6500
편마암	3500 ~ 7600
대리석	3780 ~ 7000
황화광석	3950 ~ 6700
분쇄된 연료재	600 ~ 1000
복토지반(자갈 등)	160 ~ 600
쓰레기 매립지	400 ~ 750
콘크리트	3000 ~ 3500
교란된 토양	180 ~ 335
다져진 쓰레기 매립 점토 덮개층	355 ~ 380

* 온도에 따라 크게 변함(Kohnen, 1974).

- Phase and group velocities:

  - Phase velocity: is defined as the distance traveled per unit time by a point of constant phase, such as a peak and trough. (α and β are the phase

velocity)

- Group velocity: is defined as the speed with which a pulse of energy travels. Group velocity is obtained by measuring the distance that the envelope travels in unit time.

- Phase velocity is not necessarily the same as group velocity.

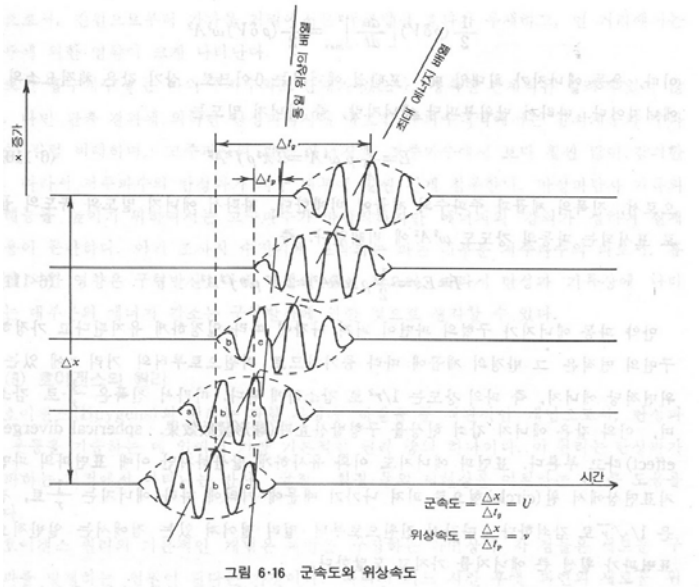


그림 6·16 군속도와 위상속도

- If we decompose a pulse into its component frequencies by Fourier analysis, we find a spectrum of frequencies. If the velocity is the same for all frequencies, the pulse shape will remain the same and the group velocity will be the same as the phase velocity → Medium is not dispersive

- If the velocity varies with frequency, the pulse changes shape as it travels and the group velocity is different from the phase velocity. → Medium is dispersive.

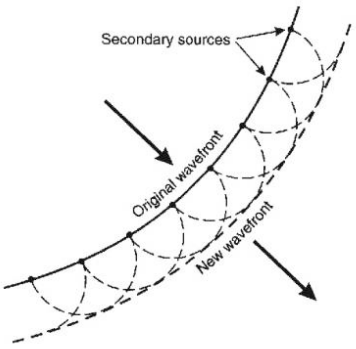
- When the phase velocity decreases with frequency, the phase velocity is larger than the group velocity. → normal dispersion

- When the phase velocity increases with frequency, the opposite is true → Inverse dispersion

- Dispersion is not a dominant feature of exploration seismology because most rocks exhibit little variation of velocity with frequency in the seismic frequency range. Dispersion is important in connection with surface waves and certain other phenomena.

(4) Wave motion

(a) Huygen's principle



- Huygen's principle states that every point on a wavefront can be regarded as a new source of waves.
- The new wavefront is the envelope of these waves after a given time interval.

(b) Reflection and refraction

- Ray: It is easier to consider a line at right-angles (normal) to the wavefront as a ray along which energy travels.

- The propagation of seismic waves is frequently discussed in terms of rays and raypaths.

- Whenever a wave impinges upon an interface across which there is a contrast in elastic properties, some of the energy is reflected back off the interface, and the remainder passes through the boundary and is refracted on entering the second medium.

- The relative amplitudes of the partitioned energy at such an interface into reflected and transmitted components are described by the seismic velocities and densities of the two layers.

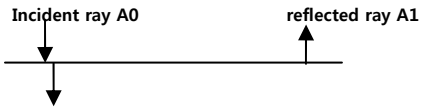
- The product of the density (ρ) and the seismic velocity (v) for each respective layer is known as the acoustic impedance (Z)

  $z = \rho v$

- The more consolidated a rock is, the greater will be its acoustic impedance.

- In the case of normal incidence, if the amplitude of incident energy is A_0 and those of the reflected and transmitted energy are respectively A_1 and A_2.

$z_1 = \rho_1 v_1$



$$z_2 = \rho_2 v_2$$ **Transmitted ray A2**

- Assuming no loss of energy along any raypath – the energy within the incident wave must equal the sum of the energy contained within the reflected and transmitted waves (i.e., $A_0 = A_1 + A_2$)

- Reflection coefficient (R) is the ratio of the amplitudes of the reflected wave (A_1) to the incident wave (A_0)

$$R = \frac{A_1}{A_0}$$

- Transmittivity of an interface for normal and low angles of incidence is described by the transmission coefficient $T = \frac{A_2}{A_0}$

  Reflection and transmission coefficients can be obtained by using acoustic impedances (from the boundary conditions at the interfaces):

$$v_2(A_0 - A_1) = v_1 A_2$$
$$\rho_1(A_0 + A_1) = \rho_2 A_2$$
$$\frac{A_1}{A_0} = R = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1} = \frac{z_2 - z_1}{z_2 + z_1}$$
$$\frac{A_2}{A_0} = T = \frac{2\rho_1 v_2}{\rho_2 v_2 + \rho_1 v_1} = \frac{\rho_1}{\rho_2}\frac{2z_2}{z_2 + z_1}$$

for potential amplitude

$$A_1 + A_2 = A_0$$
$$z_1 A_1 - z_2 A_2 = -z_1 A_0$$
$$\frac{A_1}{A_0} = R = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1} = \frac{z_2 - z_1}{z_2 + z_1}$$
$$\frac{A_2}{A_0} = T = \frac{2\rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1} = \frac{2z_1}{z_2 + z_1}$$

for displacement amplitude

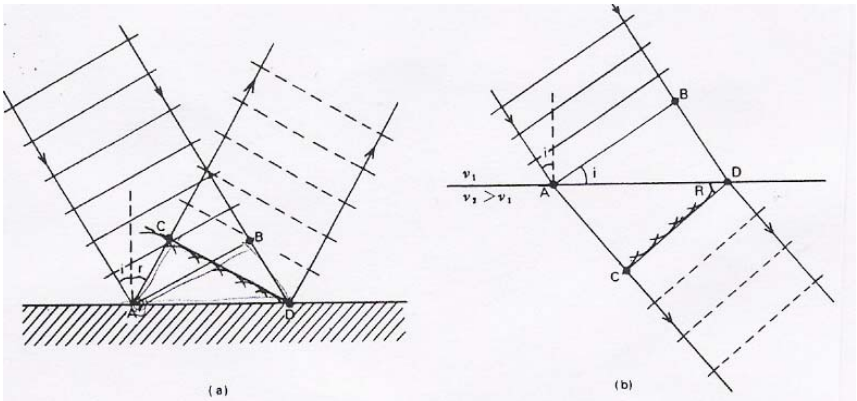


그림 6·20 평면파의 반사(a) 및 굴절(b)

- Snell's law

  (Fig. 6.20)

  Traveltime required for waves to propagate from B to D is the same as traveltime required for waves to propagate from A to C

$$\Delta t = \frac{\overline{BD}}{v_1} = \frac{\overline{AC}}{v_2} \Rightarrow \frac{\overline{AD}\sin i}{v_1} = \frac{\overline{AD}\sin R}{v_2}$$

$$\Rightarrow \frac{\sin i}{v_1} = \frac{\sin R}{v_2}$$

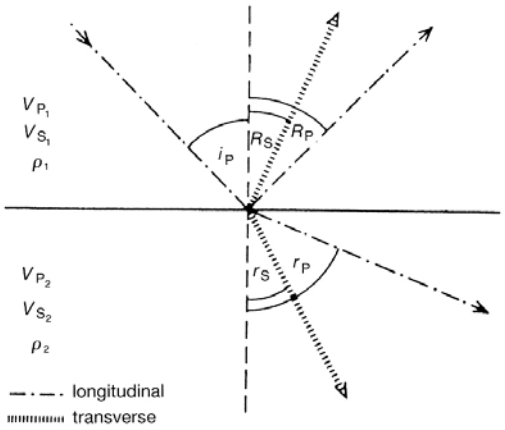
$$R = 90^o, \rightarrow \quad \sin i = \frac{v_1}{v_2} \ (i \text{ is critical angle})$$

Over the critical angle, total reflection occurs

(c) Mode conversion

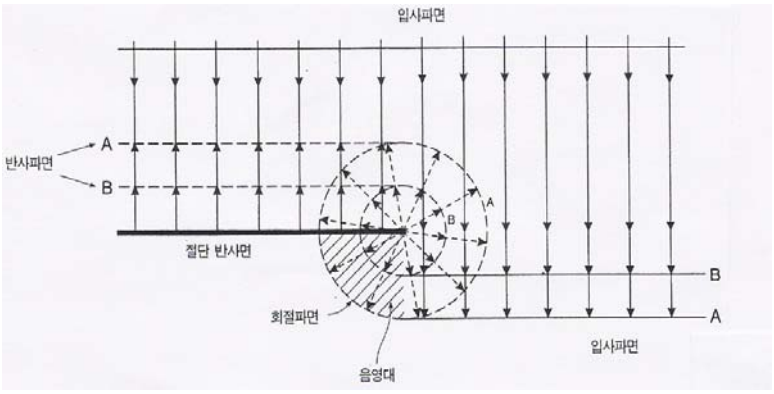
- Consider the case of an incident wave impinging obliquely on an interface. At intermediate angles of incidence, reflected S-waves can be generated by conversion from the incident P-waves, or reflected P-waves can be generated by conversion from the incident S-waves.

  (figure 4.7 in the text book, and Box 4.8)



(d) Diffraction

- Seismic energy travels along other paths besides those given by Snell's law
- Whenever a wave encounters a feature whose radius of curvature is comparable to or smaller than the wavelength, the ordinary laws of reflection and reflection no longer apply.
- In such cases, the energy is diffracted rather than reflected or refracted.



Traveltime for reflected waves:

$$t_r = \frac{\sqrt{x^2+4z^2}}{v} = \frac{2z}{v}\left[1+\left(\frac{x}{2z}\right)^2\right]^{1/2}$$

$$\approx \frac{2z}{v}\left[1+\frac{1}{2}\frac{x^2}{4z^2}+\cdots\right] \approx \frac{2z}{v}+\frac{x^2}{4vz}$$

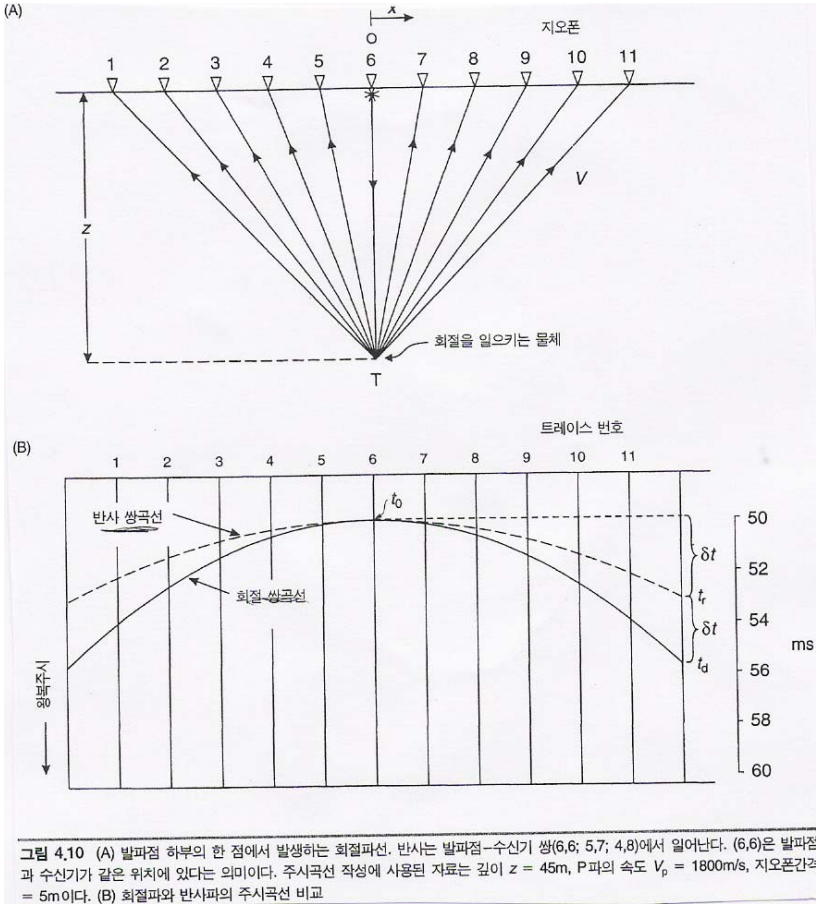


그림 4.10 (A) 발파점 하부의 한 점에서 발생하는 회절파선. 반사는 발파점–수신기 쌍(6,6; 5,7; 4,8)에서 일어난다. (6,6)은 발파점과 수신기가 같은 위치에 있다는 의미이다. 주시곡선 작성에 사용된 자료는 깊이 z = 45m, P파의 속도 V_p = 1800m/s, 지오폰간격 = 5m이다. (B) 회절파와 반사파의 주시곡선 비교

Traveltime for diffracted waves

$$t_d = \frac{z}{v}+\frac{\sqrt{x^2+z^2}}{v} = \frac{z}{v}+\frac{z}{v}\left[1+\left(\frac{x}{z}\right)^2\right]^{1/2}$$

$$\approx \frac{z}{v}+\frac{z}{v}\left[1+\frac{1}{2}\frac{x^2}{z^2}+\cdots\right] \approx \frac{2z}{v}+\frac{x^2}{2vz}$$

(e) Energy of waves

- We are concerned with the energy in the vicinity of the point where we observe it.
- The energy density is the energy per unit volume in the neighborhood of a point.

- Energy density E

  - ✓ Consider a spherical harmonic P wave for which the radial displacement for a fixed value of r is given by

    $u = A\cos(\omega t + \phi)$,

    where ω is the angular frequency, ϕ is a phase angle, and t is the time. The displacement u ranges from –A to +A.

  - ✓ Since the displacement varies with time, each element of the medium has a velocity du/dt and an associated kinetic energy.
    The kinetic energy is expressed by

    $$\frac{1}{2}(mass)(velocity)^2 = \frac{1}{2}(\rho\delta V)\left(\frac{du}{dt}\right)^2$$

    and, the kinetic energy per unit volume is

    $$\frac{1}{2}\rho\left(\frac{du}{dt}\right)^2 = \frac{1}{2}\rho\left[-A\omega\sin(\omega t+\phi)\right]^2 = \frac{1}{2}\rho\omega^2 A^2\sin^2(\omega t+\phi)$$

  - ✓ The wave also involves potential energy resulting from the elastic strains created during the passage of the wave. As the medium oscillates back and forth, the energy is converted back and forth from kinetic to potential form and the total energy remains fixed.

    Because the total energy equals the maximum value of the kinetic energy, the energy density D for a harmonic wave is

    $$D = \frac{1}{2}\rho\omega^2 A^2$$

    The energy density is proportional to the first power of the density of the medium and to the second powers of the frequency and amplitude of the wave.

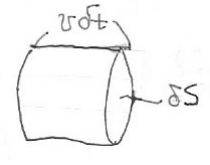
- Intensity
  - ✓ Intensity is defined as the quantity of energy that flows through a unit area

normal to the direction of wave propagation in unit time.

Take a cylinder of infinitesimal cross section area δS, whose axis is parallel to the direction of propagation and whose length is equal to the distance traveled in the time δt.

$$\delta l = v\delta t$$

The total energy inside the cylinder at any instant *t* is $Dv\delta t\delta S$. Dividing it by the area and time interval, we get the intensity I, the amount of energy passing through unit area in unit time:



$$I = Dv$$

For a harmonic wave, this becomes

$$I = Dv = \frac{1}{2}\rho\omega^2 A^2 v$$

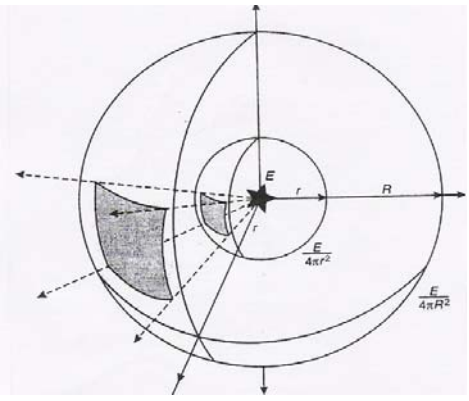
Reflection energy $I_R = \frac{\frac{1}{2}v_1\rho_1\omega^2 A_1^2}{\frac{1}{2}v_1\rho_1\omega^2 A_0^2} = \left(\frac{z_2 - z_1}{z_2 + z_1}\right)^2$

Transmission energy $I_T = \frac{\frac{1}{2}v_2\rho_2\omega^2 A_2^2}{\frac{1}{2}v_1\rho_1\omega^2 A_0^2} = \frac{4z_1 z_2}{(z_2 + z_1)^2}$

$$I_R + I_T = 1$$

- Spherical divergence

  - ✓ Consider a spherical wavefront diverging from a center O. (Figure) Wavefront S_1 and S_2 at the radii r_1 and r_2. The flow of energy per second is the product of the intensity and the area



Since the area S_1 and S_2 are proportional to the square of their radii, we get

$$\frac{I_2}{I_1} = \frac{S_1}{S_2} = \left(\frac{r_1}{r_2}\right)^2$$

Geometrical spreading causes the intensity and the energy density of spherical waves to decrease inversely as the square of the distance from the source.

This is called spherical divergence.

- Absorption

  - ✓ In addition to spherical divergence, elastic energy is absorbed by the medium by being transferred into heat by the friction of individual particles moving against each other as the seismic wave passes through the medium.

  - ✓ This process is called energy absorption or intrinsic attenuation and is responsible for the eventual complete disappearance of the wave motion.

  - ✓ Absorption varies with frequency
  - ✓ The loss of energy by absorption appears to be exponential with distance for elastic waves in rocks.

$$\frac{A}{A_0} = \frac{r_0}{r}\exp\left[-\alpha\left(r - r_0\right)\right]$$

$$\alpha = \frac{\pi f}{Qv} \qquad Q = 2\alpha\lambda$$

where Q is the quality factor, f is frequency, and v is velocity.

  - ✓ Energy losses by geometrical spreading are more important than losses by absorption for low frequencies and short distances.

  - ✓ As the frequency and distance increase, absorption losses increase more rapidly than spreading losses, and eventually become dominant. The more rapid loss of higher frequencies results in change of wave shape with distance.

- Scattering

  - ✓ There are three levels of scattering that can be described in terms of the product of the wavenumber ($k = 2\pi f / v$) and scale of the heterogeneity (a).
  - ✓ For ka<< 0.01: it is called quasi-homogeneous, and the scatterers are too small to be seen by the seismic waves except as an apparent anisotropy if enough cracks within the rock are aligned.
  - ✓ For ka < 0.1: Where the wavelength of the seismic waves is large compared to the heterogeneities (ka < 0.1), Rayleigh scattering occurs and produces an apparent attenuation. This situation is the most common.
  - ✓ For 0.1 < ka < 10: In areas where there is very rapid variation in seismic wave velocity and density laterally and vertically, such as over old landfill sites, scattering can be very significant and will be evident on the seismic record as a chaotic jumble of signals that appears as noise. The energy dissipation is

known as Mie scattering.

- ✓ In high-resolution shallow seismic surveys, sediments such as boulder clay with boulders of the order of 0.5-1 m across are sufficiently large relative to the wavelength of the incident wave for scattering to be a significant cause of amplitude loss.

- ✓ For a survey with seismic waves of frequency 500 Hz, in a medium with P-wave velocity 2000 m/s, boulders of the size given above would yield a value of ka between 0.8 and 1.6, in which case Mie scattering would occur.

Assignment: Read through the textbook and some references, and summarize the seismic energy sources.

3. Refraction method

- Seismic refraction experiments can be undertaken at three distinct scales: global (using earthquake waves), crustal (using explosion seismology), and near-surface (engineering applications).

- At global scale using earthquake waves, the existence of shadow zone discovered by Gutenberg, and Moho discontinuity is revealed by the refraction method.

- In 1920s, the refraction method was also used to find salt dome that is important in oil exploration.

- Recently, the refraction method is popularly used in conventional engineering applications of foundation studies for dams and major buildings etc.

  Seismic refraction is increasingly being used in hydrogeological investigations to determine saturated aquifer thickness, weathered fault zones etc.

  The location of faults, joints, and other such disturbed zones using seismic refraction is of major importance in the consideration of the suitability of potential sites for the safe disposal of particularly toxic hazardous wastes.

(a) General principles

- Critical refraction

  - ✓ The seismic refraction method is based on the principle that when a seismic wave (P- and/or S-wave) impinges upon a boundary across which there is a contrast in velocity, then the direction of wave propagation changes on entry into the new medium

  - ✓ The amount of change of direction is governed by the contrast in seismic velocity across the boundary according to Snell's law.

  - ✓ The refraction method can be applied only when critical refraction occurs.

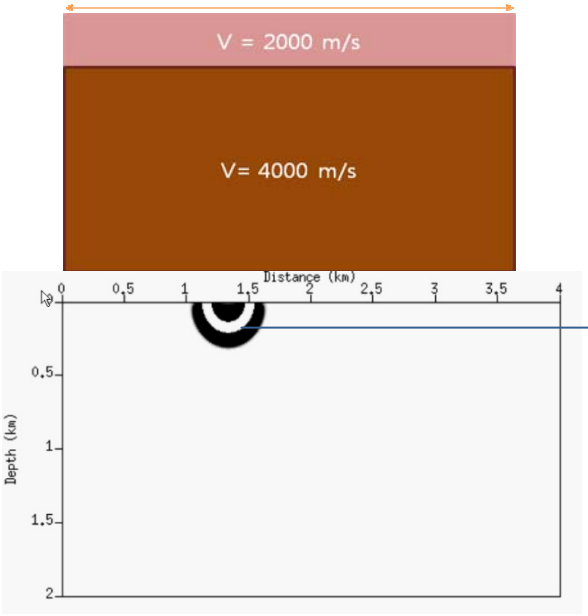
  - ✓ The refraction method can be applied when velocity increases with depth.
  - ✓ If the lower medium has a velocity lower than that from which the wave is emerging (i.e., V_2 is less than V_1, a velocity inversion) then the refracted wave

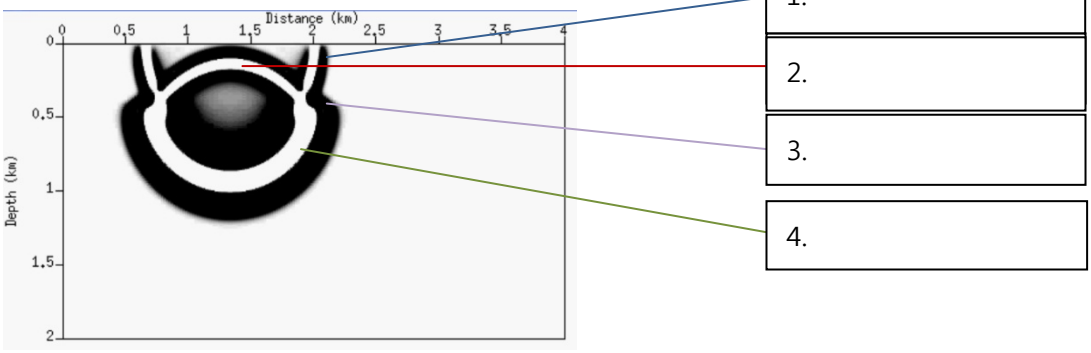
will bend towards the normal. → This gives rise to a situation known as a hidden layer.

✓ The thickness of each layer present must be greater than the wavelength of the incident energy, and that each successive layer is as thick as, or is thicker than, the one lying above it.

➢ Traveltime and refractor depth for a two-layer case

✓ Find the traveltime of critical refraction wave and calculate refractor depth.





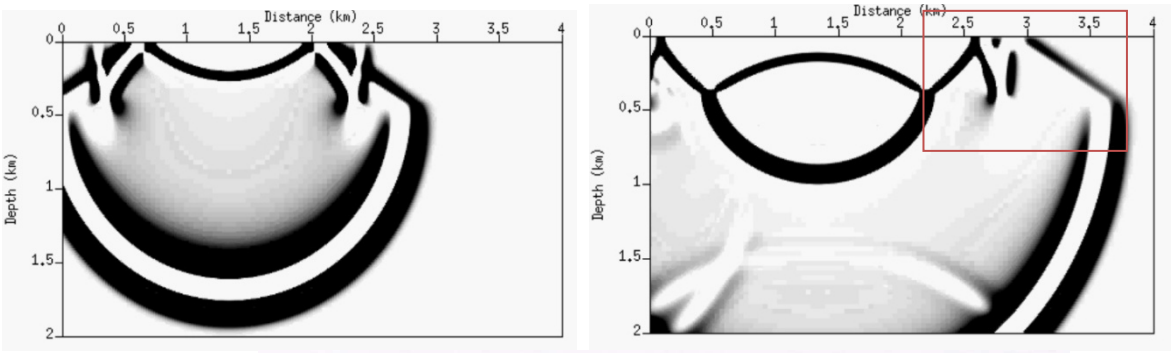
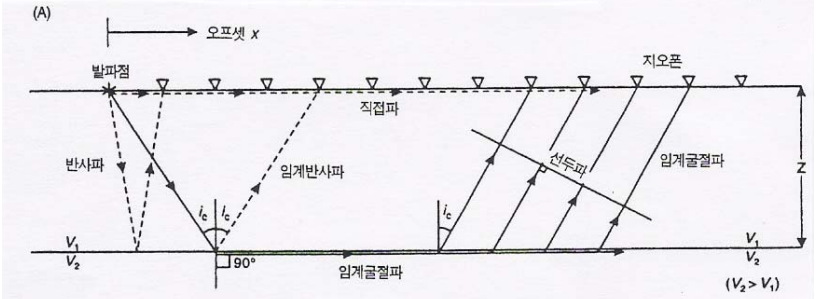
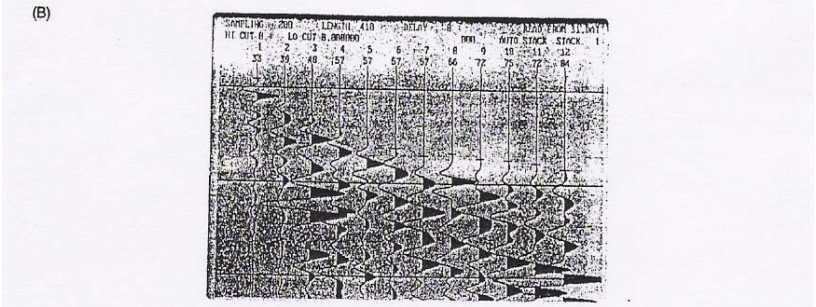
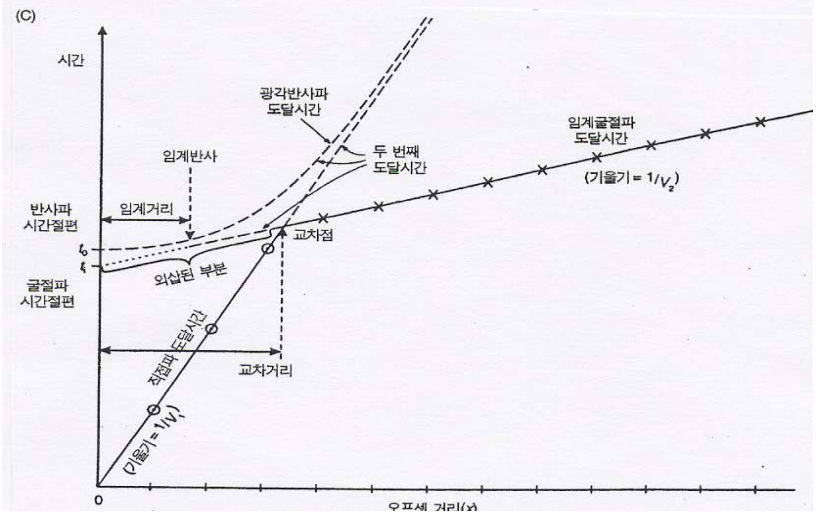


그림 5.1 (A) 직접파, 반사파, 굴절파의 파선경로, (B) 굴절법 탄성파 기록자료. 초동은 각 트레이스에서 짧은 선(cross line)으로 표시되어 있다. (C) 주시곡선에 표시된 초동







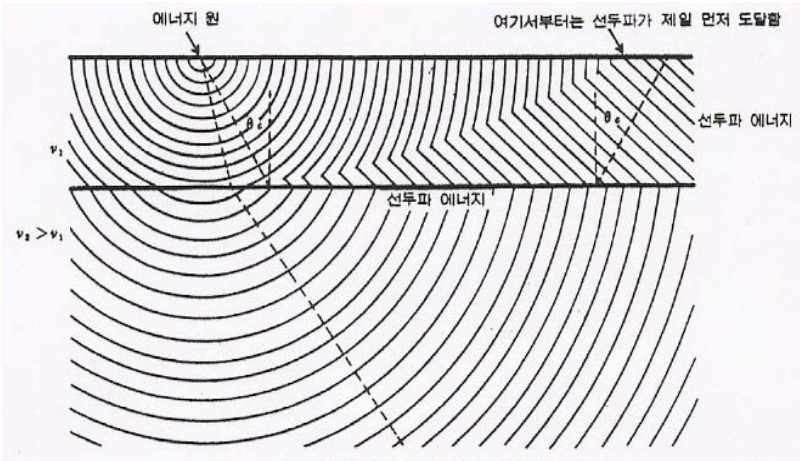
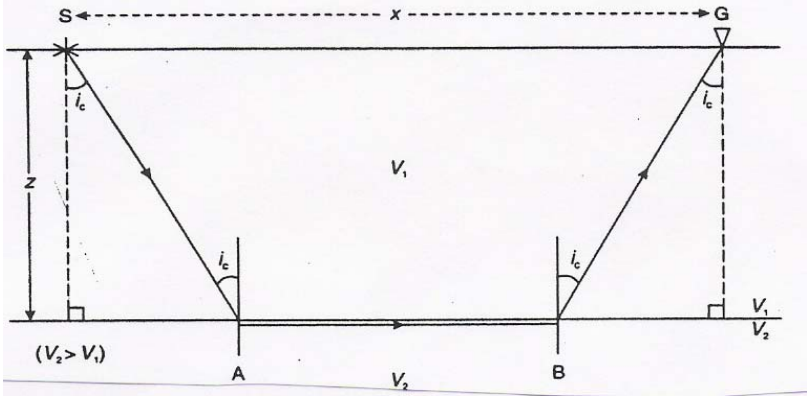


그림 6·64 초동의 에너지와 선두파를 나타내는 파면



$T = \frac{\overline{SA}}{v_1} + \frac{\overline{AB}}{v_2} + \frac{\overline{BG}}{v_1}$ 에서,

$$\left(\begin{array}{l} \frac{z}{\overline{BG}} = \frac{z}{\overline{SA}} = \cos i_c \;\rightarrow\; \overline{SA} = \overline{BG} = \frac{z}{\cos i_c} \\ \frac{x_{BG}}{z} = \frac{x_{SA}}{z} = \tan i_c \;\rightarrow\; x_{BG} = x_{SA} = z \tan i_c \\ \Rightarrow\; \overline{AB} = x - x_{SA} - x_{BG} = x - 2z \tan i_c \end{array} \right)$$ 이므로,

$$T = \frac{z}{v_1 \cos i_c} + \frac{x - 2z \tan i_c}{v_2} + \frac{z}{v_1 \cos i_c} = \frac{x}{v_2} + \left(\frac{2z}{v_1 \cos i_c} - \frac{2z \sin i_c}{v_2 \cos i_c} \right)$$

From Snell's law, $\sin i_c = \frac{v_1}{v_2}$,

$$= \frac{x}{v_2} + \frac{2z}{v_1 \cos i_c} \left[1 - \frac{v_1^2}{v_2^2} \right]$$

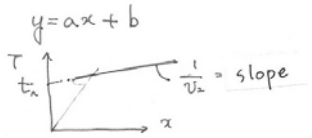
$$= \frac{x}{v_2} + \frac{2z}{v_1}\left[1 - \left(\frac{v_1^2}{v_2^2}\right)\right]^{\frac{1}{2}} \quad \left(\because \cos^2 i_c = 1 - \left(\frac{v_1}{v_2}\right)^2\right)$$

$$= \frac{x}{v_2} + \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

$$= \frac{x}{v_2} + t_i$$

$1/v_2$ is the slope

t_i is intercept time, $t_i = \dfrac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$



From t_i, we can determine $z = \dfrac{t_i v_1 v_2}{2\sqrt{v_2^2 - v_1^2}}$

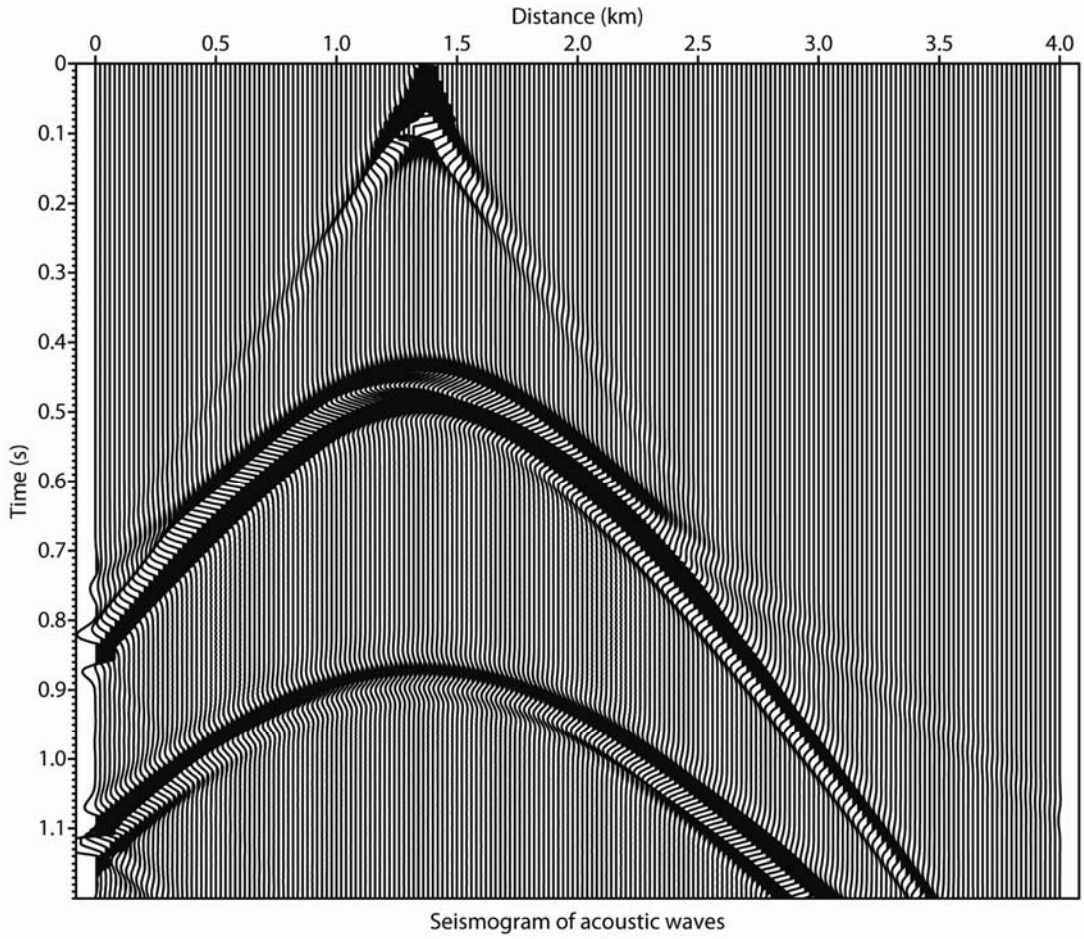
✓ The use of crossover distance to calculate refractor depth

· The traveltime of direct wave at the crossover distance is the same as the traveltime of critically refracted ray at the crossover distance:

$$\frac{x_{cross}}{v_1} = \frac{x_{cross}}{v_2} + \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

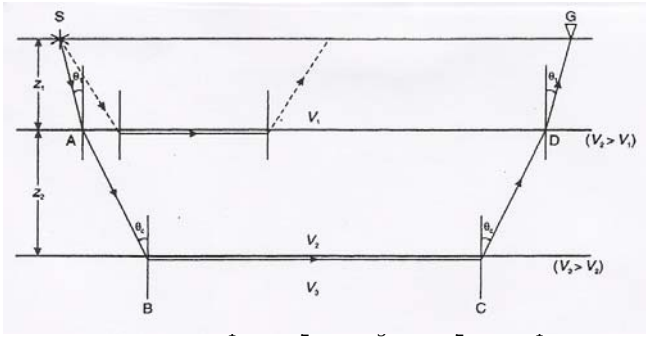
$$\left(\frac{1}{v_1} - \frac{1}{v_2}\right) x_{cross} = \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

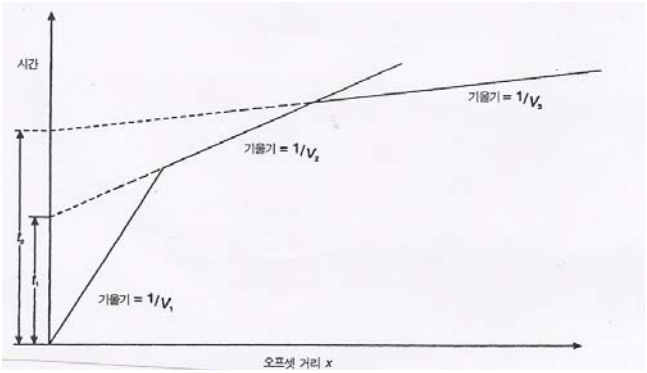
$$z = \frac{x_{cross}}{2}\frac{v_2 - v_1}{\sqrt{v_2^2 - v_1^2}} = \frac{x_{cross}}{2}\sqrt{\frac{v_2 - v_1}{v_2 + v_1}}$$



Seismogram of acoustic waves

➢ Traveltime and refractor depth for a three-layer case





$$\overline{SA} = \overline{DG} = \frac{z_1}{\cos\theta_i},\ \overline{AB} = \overline{CD} = \frac{z_2}{\cos\theta_c},$$

$$\overline{BC} = x - 2z_1\tan\theta_i - 2z_2\tan\theta_c$$

$$T = \frac{2}{v_1}\frac{z_1}{\cos\theta_i} + \frac{2}{v_2}\frac{z_2}{\cos\theta_c} + \frac{1}{v_3}\left(x - \frac{2z_1\sin\theta_i}{\cos\theta_i} - \frac{2z_2\sin\theta_c}{\cos\theta_c}\right)$$

$$= \frac{x}{v_3} + 2z_1\left(\frac{1}{v_1}\frac{1}{\cos\theta_i} - \frac{1}{v_3}\frac{\sin\theta_i}{\cos\theta_i}\right) + 2z_2\left(\frac{1}{v_2}\frac{1}{\cos\theta_c} - \frac{1}{v_3}\frac{\sin\theta_c}{\cos\theta_c}\right)$$

From Snell's law, $\dfrac{\sin\theta_i}{v_1} = \dfrac{\sin\theta_c}{v_2} = \dfrac{1}{v_3}$

$$= \frac{x}{v_3} + 2z_1\frac{\cos\theta_i}{v_1} + 2z_2\frac{\cos\theta_c}{v_2}$$

$$= \frac{x}{v_3} + 2z_1\frac{\sqrt{v_3^2 - v_1^2}}{v_1 v_3} + 2z_2\frac{\sqrt{v_3^2 - v_2^2}}{v_2 v_3}$$

$$t_2 = 2z_1\frac{\sqrt{v_3^2 - v_1^2}}{v_1 v_3} + 2z_2\frac{\sqrt{v_3^2 - v_2^2}}{v_2 v_3},\quad z_2 = \frac{t_2 v_2 v_3}{2\sqrt{v_3^2 - v_2^2}} - \frac{v_2\sqrt{v_3^2 - v_1^2}}{v_1\sqrt{v_3^2 - v_2^2}}z_1$$

➢ Multilayer case

The general expressions governing the traveltime – velocity relationship

$$T = \frac{x}{v_m} + \sum_{i=1}^{n-1}\left[\left(\frac{2z_i\cos\theta_i}{v_i}\right)\right]$$

$\sin\theta_i = \dfrac{v_i}{v_n}$, θ_i are not critical angle except for θ_{n-1}

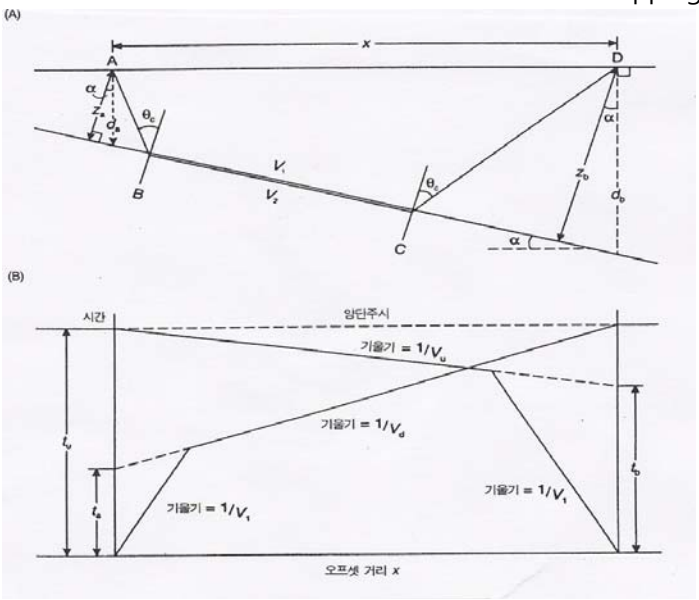
- Dipping-layer case

  - ✓ When a refractor lies at an angle to the horizontal, the simple geometries so far described are complicated by the angle of dip.

  - ✓ It is necessary to carry out both forward and reverse shooting in order to determine all the parameters required to solve for the refractor geometry.

  - ✓ Furthermore, the refractor velocities determined in the case of dip are referred to as apparent velocities, as the values of velocity determined from the inverse gradients of the straight-line segments on the travel time-distance graphs are higher in the upslope direction (V_u) and lower in the downslope direction (V_d).

  - ✓ Travel time calculations for a dipping refractor



  - · For Down dip,

$$T = \frac{\overline{AB}}{v_1} + \frac{\overline{BC}}{v_2} + \frac{\overline{CD}}{v_1}$$

$$\overline{AB} = \frac{z_a}{\cos\theta_c},\ \overline{CD} = \frac{z_b}{\cos\theta_c},\ \overline{BC} = x\cos\alpha - z_a\tan\theta_c - z_b\tan\theta_c$$

$$= \frac{1}{v_1}\frac{z_a}{\cos\theta_c} + \frac{1}{v_2}\left(x\cos\alpha - z_a\tan\theta_c - z_b\tan\theta_c\right) + \frac{z_b}{v_1\cos\theta_c}$$

$$= \frac{x}{v_2}\cos\alpha + \left(\frac{1}{v_1}\frac{\left(z_a+z_b\right)}{\cos\theta_c} - \frac{z_a}{v_2}\frac{\sin\theta_c}{\cos\theta_c} - \frac{z_b}{v_2}\frac{\sin\theta_c}{\cos\theta_c}\right)$$

$$= \frac{x}{v_1}\cos\alpha\sin\theta_c + \left(\frac{1}{v_1}\frac{\left(z_a+z_b\right)}{\cos\theta_c} - \frac{z_a}{v_1}\frac{\sin^2\theta_c}{\cos\theta_c} - \frac{z_b}{v_1}\frac{\sin^2\theta_c}{\cos\theta_c}\right)$$

$$= \frac{x}{v_1}\cos\alpha\sin\theta_c + \frac{z_a}{v_1}\cos\theta_c + \frac{z_b}{v_1}\cos\theta_c$$

$$= \frac{x}{v_1}\cos\alpha\sin\theta_c + \frac{z_a}{v_1}\cos\theta_c + \frac{\left(z_a + x\sin\alpha\right)}{v_1}\cos\theta_c$$

$$= \frac{x}{v_1}\left[\cos\alpha\sin\theta_c + \sin\alpha\cos\theta_c\right] + \frac{2z_a}{v_1}\cos\theta_c$$

$$= \frac{x}{v_1}\sin(\theta_c+\alpha) + \frac{2z_a}{v_1}\cos\theta_c$$

$$t_d = \frac{x}{v_d} + t_a \qquad v_d = \frac{v_1}{\sin(\theta_c + \alpha)}$$

$$t_u = \frac{x}{v_u} + t_b \qquad v_u = \frac{v_1}{\sin(\theta_c - \alpha)},$$

$$v_d = \frac{v_1}{\sin(\theta_c + \alpha)} = \frac{v_2 \sin\theta_c}{\sin(\theta_c + \alpha)}$$

$$v_u = \frac{v_1}{\sin(\theta_c - \alpha)} = \frac{v_2 \sin\theta_c}{\sin(\theta_c - \alpha)}$$

$$\theta_c = \frac{1}{2}\left\{\sin^{-1}\left(\frac{v_1}{v_d}\right) + \sin^{-1}\left(\frac{v_1}{v_u}\right)\right\}$$

$$\alpha = \frac{1}{2}\left\{\sin^{-1}\left(\frac{v_1}{v_d}\right) - \sin^{-1}\left(\frac{v_1}{v_u}\right)\right\}$$

$$z_a = \frac{v_1 t_d}{2\cos\theta_c}$$

$$z_b = \frac{v_1 t_u}{2\cos\theta_c}$$

$$d_a = \frac{z_a}{\cos\alpha}$$

$$d_b = \frac{z_b}{\cos\alpha}$$

$$\frac{1}{v_d} = \frac{\sin(\theta_c + \alpha)}{v_2 \sin\theta_c}$$

$$\frac{1}{v_u} = \frac{\sin(\theta_c - \alpha)}{v_2 \sin\theta_c}$$

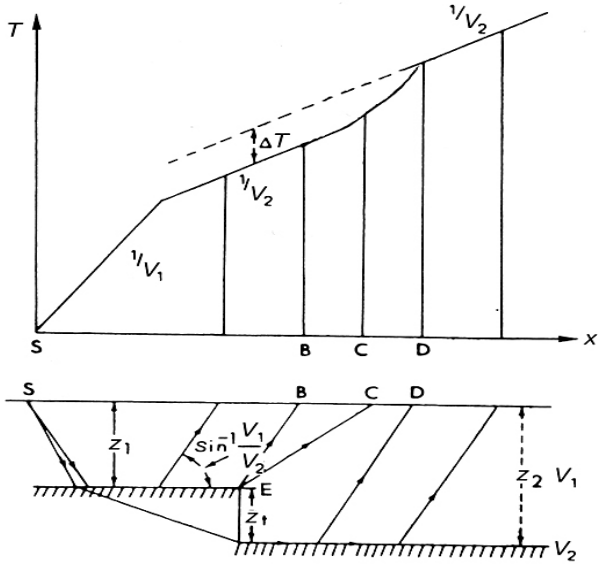
$$\frac{1}{v_d} + \frac{1}{v_u} = \frac{2\sin\theta_c \cos\alpha}{v_2 \sin\theta_c} = \frac{2\cos\alpha}{v_2}$$

$$v_2 = \frac{2 v_d v_u}{v_d + v_u}\cos\alpha$$

$$v_2 \simeq \frac{1}{2}(v_d + v_u)$$

❖ Reverse shooting is necessary

- The case of a step discontinuity



- Fault

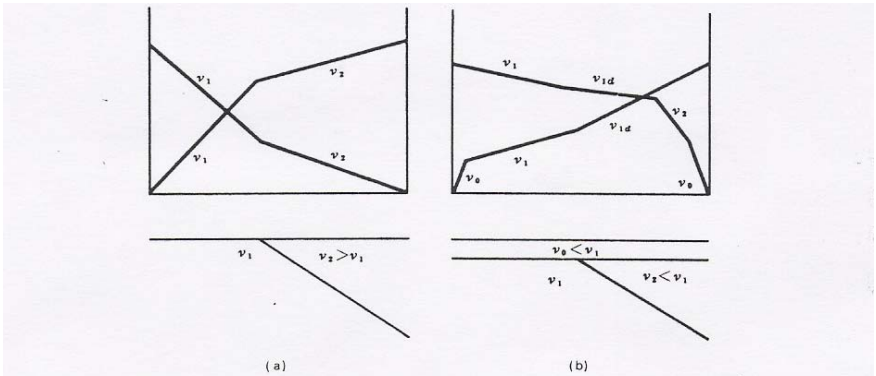


그림 6·73 수평적 속도 불연속의 구조와 주시곡선간의 관계
(a) 속도가 높은 층이 상부에 가로놓여 있는 경우에는 경계면의 기울기가 변하더라도 주시곡선의 양상은 변하지 않는다. (b) $v_2 < v_1$인 경우에는 겉보기속도가 v_1d, v_2d로 주어지는 주시곡선이 부분적으로 나타나게 된다. 저속도의 표층의 영향도 함께 도시되어 있다.

(b) Field exploration

- Refraction field work mostly result from the long source-to-geophone distances → 5 times longer than the depth
- The energy input to the ground must be larger

- The longer travelpaths result in the higher frequencies being mostly absorbed so that refraction data are generally of low frequency.

- Consequently refraction geophones have lower natural frequencies than reflection geophones.
- Refraction shooting is usually slower than reflection shooting because the large offset distances involve more moving time and create problems of communications and logistics.
- Although refraction profiles are often not as closely spaced as reflection lines, the cost of mapping an area is not necessarily greater.
- Reverse shooting is required.

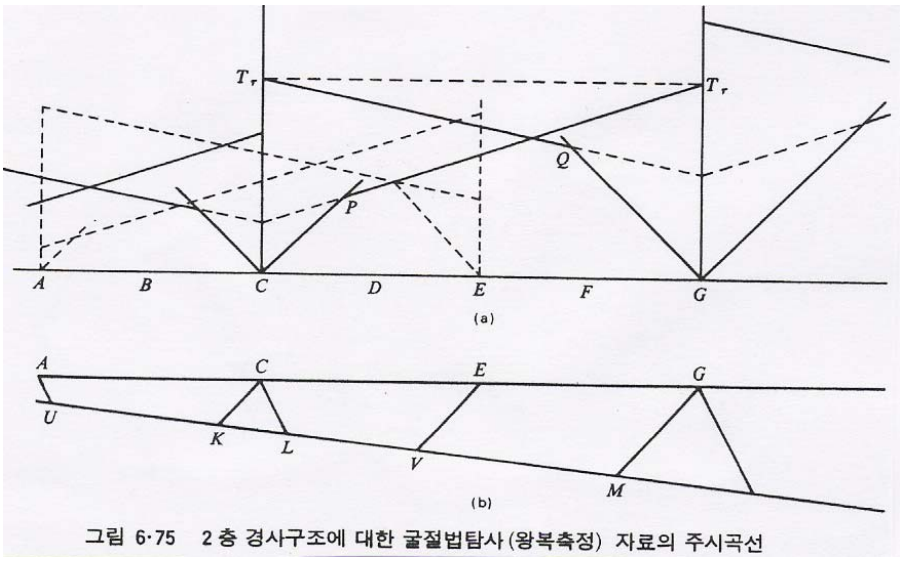


그림 6·75 2 층 경사구조에 대한 굴절법탐사(왕복측정) 자료의 주시곡선

➢ In-line survey

➢ fan shooting

· The first extensive use of refraction was in searching for salt domes by the fan-shooting technique.

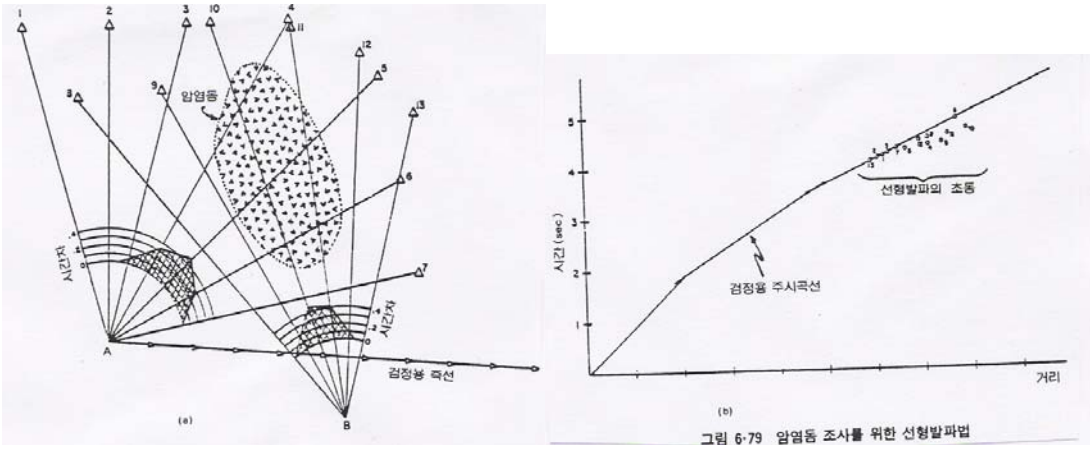


그림 6·79 암염돔 조사를 위한 선형발파법

· Salt dome is a high-velocity mass, so that horizontally traveling energy arrives earlier than that obtained when the salt dome were not present; The difference in traveltime between that actually observed and that expected without salt dome is called a time lead.

➢ Marine refraction work

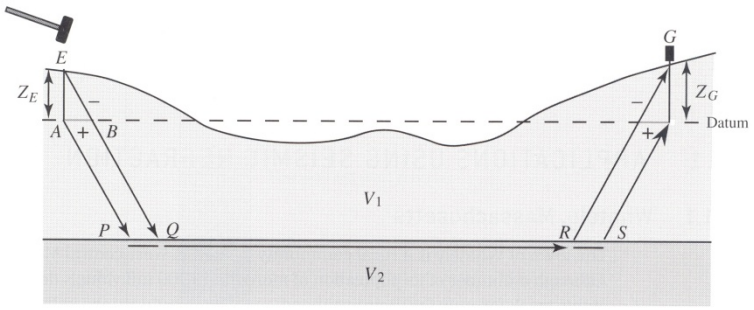
· Because refraction recording requires that there be appreciable distance between the source and the recording locations, two ships have usually been required for marine refraction recording.

· To shoot a reversed refraction profiles in one traverse requires three ships – a shooting ship at each end and a recording ship that travels between them

· The sonobuoy permits recording a refraction profile with only one ship.
· The sonobuoy is an expendable listening station that radios the information it receives back to the shooting ship.
· The salt water activates batteries in the sonobuoy as well as other devices that cause a radio antenna to be extended upward and one or two hydrophones to be suspended beneath the buoy.
· As the ship travels away from the buoy, shots are fired and the signals received by the hydrophones are radioed back to the ship where they are recorded.

(c) Refraction data reduction and interpretation

➢ Elevation correction: The depth of source and surface undulations cause elevation differences between a source and receivers. → A reference datum is selected and corrections are calculated so that, in effect, the shot points and geophones are located on the datum surface.



$V_2 > V_1$

$$AP = BQ, RC = SD$$

$$\Delta x_l = PQ - EB = AB - EB$$

$$\Delta x_r = RS - CG = CD - CG$$

$$\Delta t_l = \frac{z_E \tan\theta_{ic}}{v_2} - \frac{z_E}{v_1 \cos\theta_{ic}} = \frac{z_E\left[\sin^2\theta_{ic} - 1\right]}{v_1 \cos\theta_{ic}}$$

$$= -\frac{z_E \cos\theta_{ic}}{v_1} = -\frac{z_E\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

$$\left(\frac{1}{v_2} = \frac{\sin\theta_{ic}}{v_1}, \cos\theta_{ic} = \frac{\sqrt{v_2^2 - v_1^2}}{v_2}\right)$$

$$\Delta t_r = -\frac{z_G\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

$$\Delta T = -\frac{\left(z_E + z_G\right)\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

- Weathering correction: is to remove weathering layer effects. In this case, we should know the depth and velocity of the weathering layer.

- Several different interpretational methods have been published, falling into two approaches: delay-time and wavefront construction methods. The most commonly used methods for the former method are Hagedoorn's plus-minus method and the generalized reciprocal method (GRM).

- Hidden-layer problem

  - ✓ A hidden layer or blind zone occurs when a layer that is present is not detected by seismic refraction

  - ✓ Four causes of this problem:

    - Velocity inversion: no critical refraction can occur
    - Lack of velocity contrast: it may be extremely difficult to identify the arrivals of headwaves from the top of this zone.
    - The presence of a thin bed: velocities increase with depth, but the thickness of a layer is less than one wavelength of the incident wave.
    - Inappropriate spacing of geophones: the portion of traveltime graph arising from this thin bed may not be sampled if the geophone interval is too large

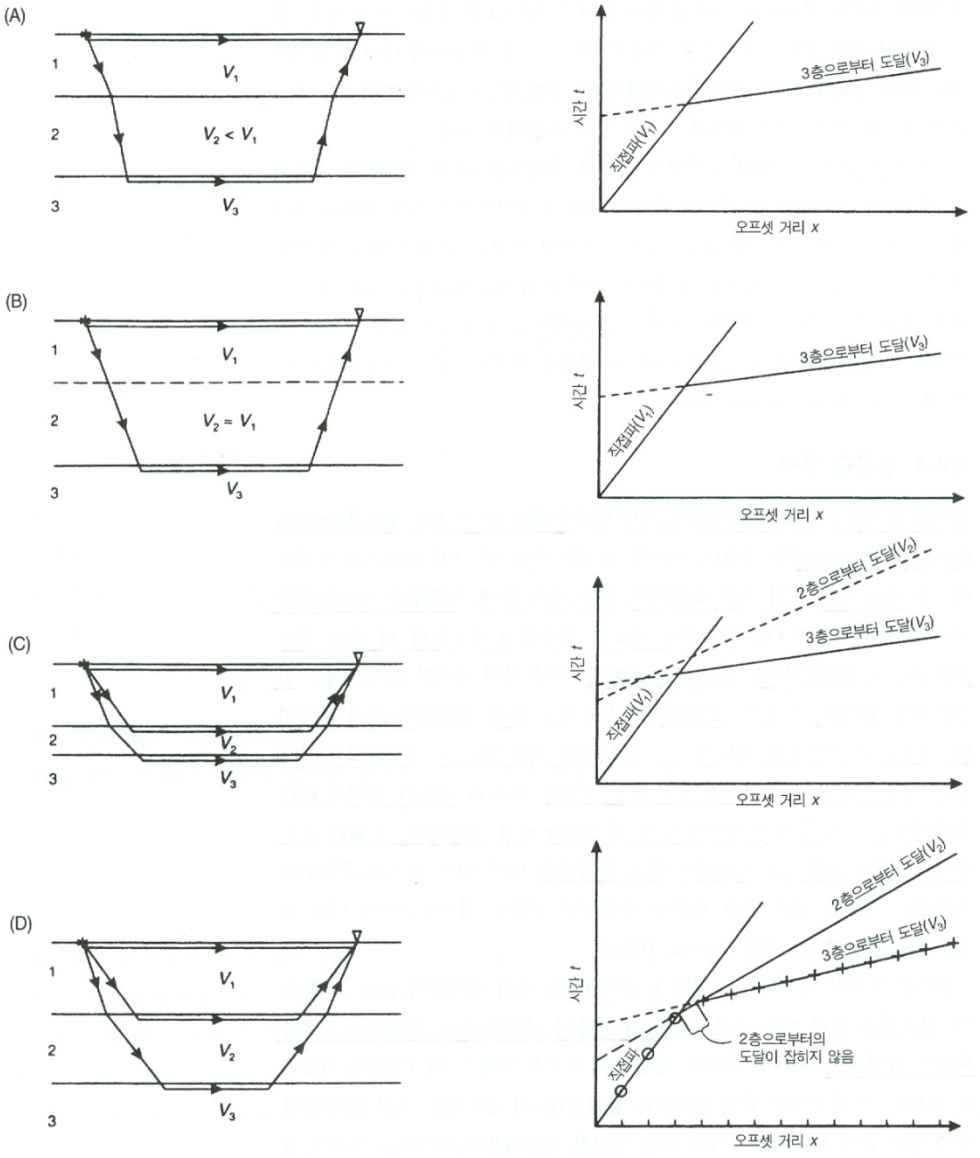


그림 5.14 숨은층 문제. (A) 속도역전($V_2 < V_1$), (B) 속도대비가 거의 없음($V_2 \approx V_1$), (C) 1층과 3층 사이에 끼여 있는 얇은 2층, (D) 지오폰간격이 너무 커서 2층이 샘플링되지 않음.

(d) Examples and Applications

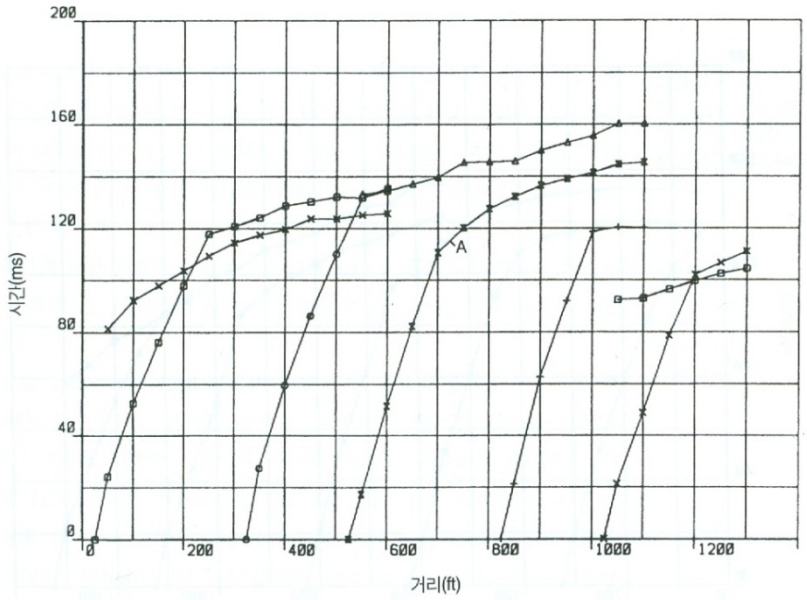


그림 5.15 정방향 발파의 주시곡선. 왼쪽의 원거리 오프셋 발파에 대한 초동(x)은 근거리 오프셋에 의한 초동(□)보다 일찍 도착한다. 이것은 발파점과 굴절면 사이의 거리가 근거리 오프셋과 원거리 오프셋 사이에서 심하게 감소할 때 일어난다(Lankston, 1990).

- Indicate the presence of an intermediate layer
- Geophone interval is too large

- The first breaks for the far-offset shots for the leftmost spread arrive earlier than those from the near-offset shot.
- This occurs when the distance between the shot point and the refractor decreases significantly between the near and far offset shots.

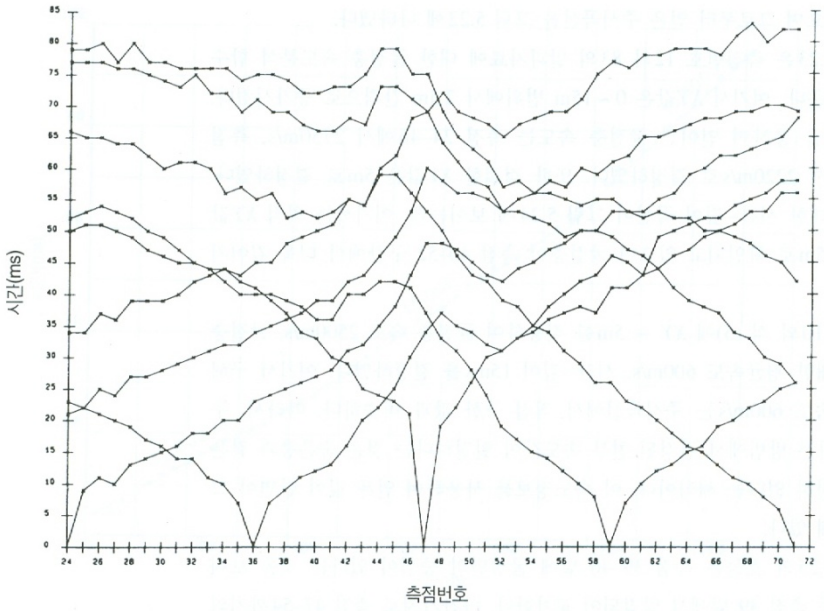


그림 5.22 호주 동부에 있는 붕괴된 돌리네에서 기록된 주시곡선. 지오폰 사이의 간격은 2.5m이다(Palmer, 1991).

collapsed doline

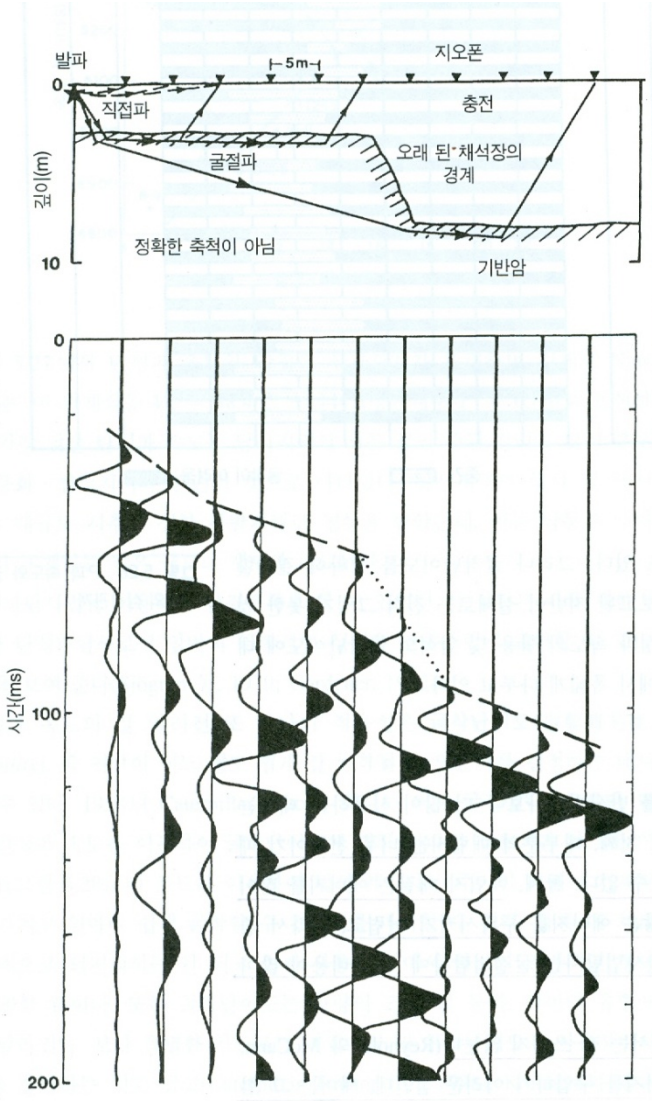
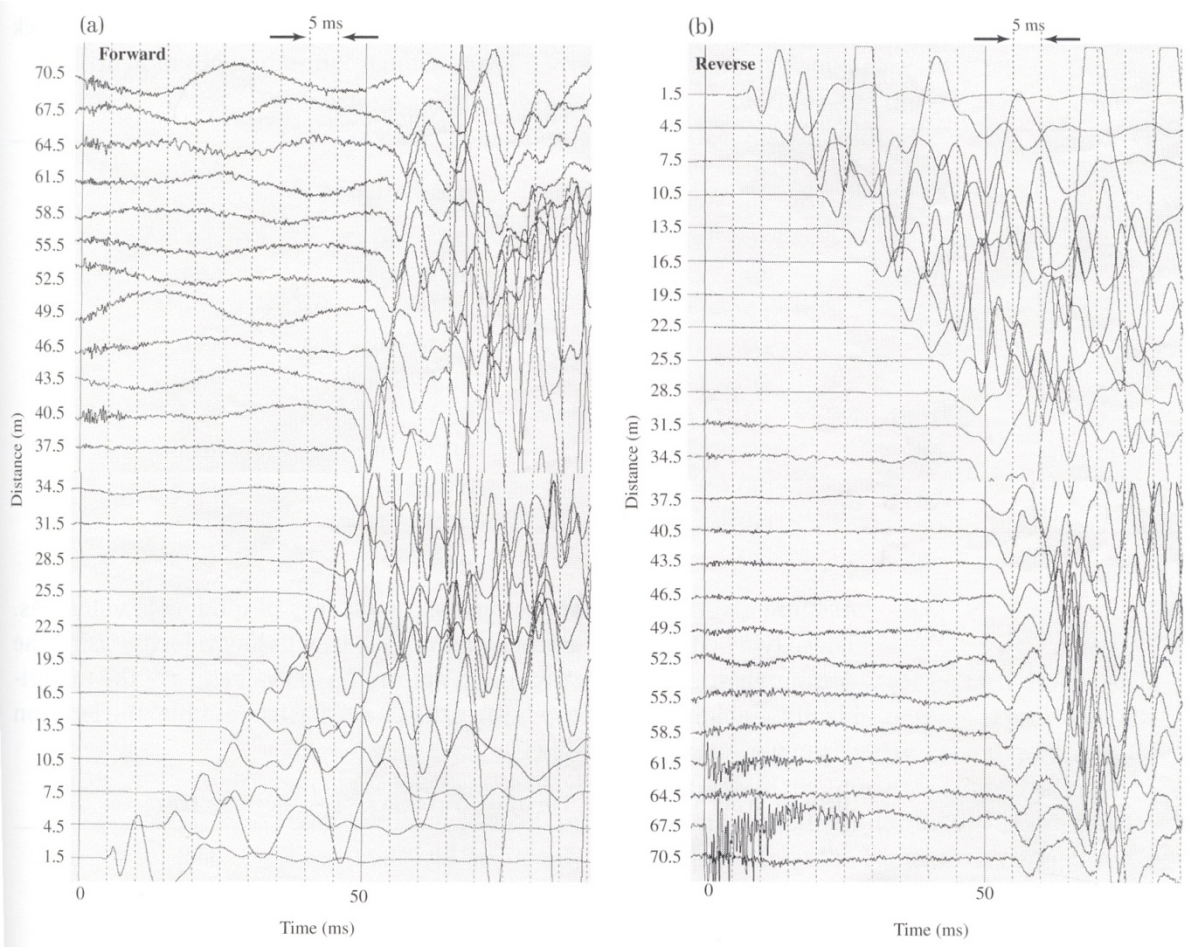
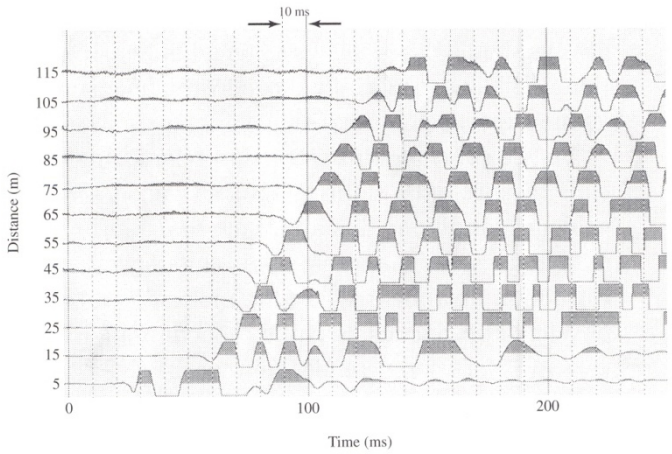
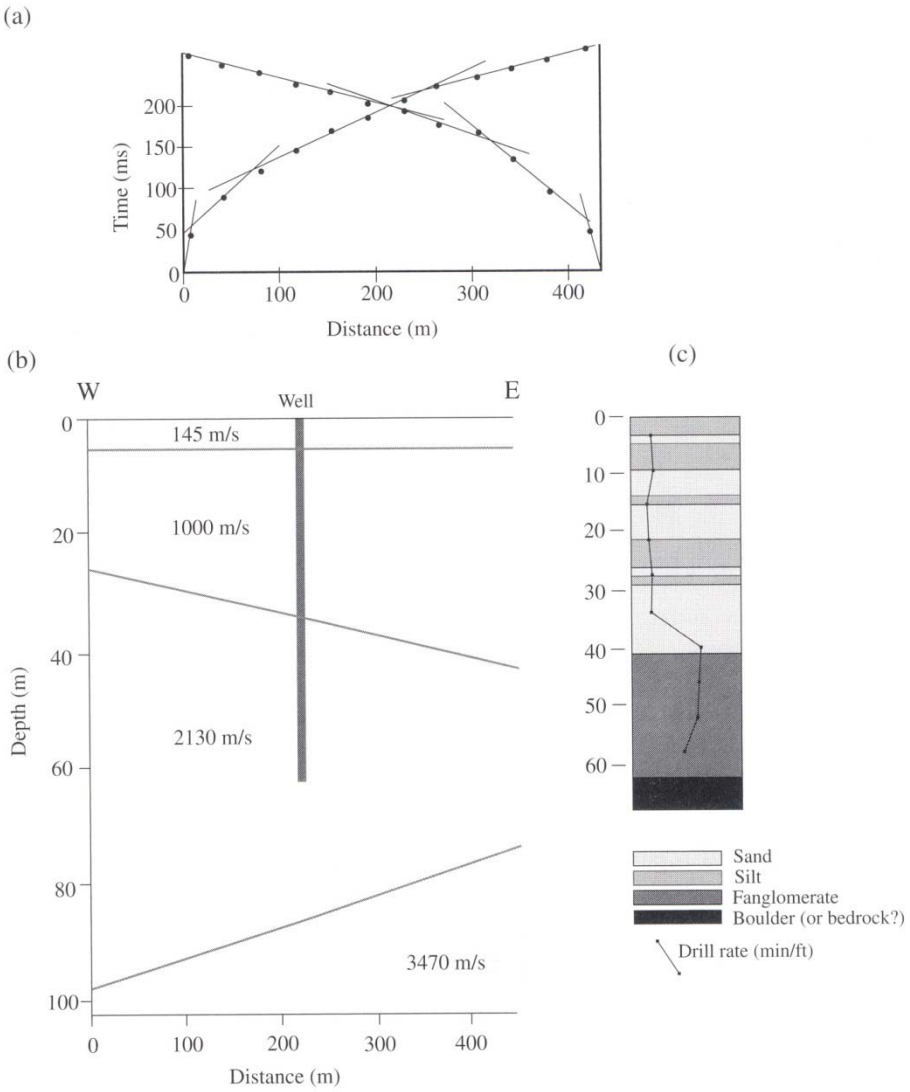


그림 5.29 이회토(marl)로 메꾸어진 쓰레기 매립지 안의 매몰 채석장에 대한 탄성파 기록. 초동선들은 점선과 쇄선으로 표시되어 있다(Reynolds와 McCann, 1992).

An example of a seismic refraction record with displaced first arrivals due to a vertical step associated with a step within an old backfilled quarry.


(a)
5 ms
Forward
70.5
67.5
64.5
61.5
58.5
55.5
52.5
49.5
46.5
43.5
40.5
37.5
34.5
31.5
28.5
25.5
22.5
19.5
16.5
13.5
10.5
7.5
4.5
1.5
Distance (m)
0
50
Time (ms)
(b)
5 ms
Reverse
1.5
4.5
7.5
10.5
13.5
16.5
19.5
22.5
25.5
28.5
31.5
34.5
37.5
40.5
43.5
46.5
49.5
52.5
55.5
58.5
61.5
64.5
67.5
70.5
Distance (m)
0
50
Time (ms)


10 ms
115
105
95
85
75
65
55
45
35
25
15
5
Distance (m)
0
100
200
Time (ms)


(a)
Time (ms)
0
50
100
150
200
0
100
200
300
400
Distance (m)
(b)
W
Well
E
145 m/s
1000 m/s
2130 m/s
3470 m/s
Depth (m)
0
20
40
60
80
100
0
100
200
300
400
Distance (m)
(c)
0
10
20
30
40
50
60
Sand
Silt
Fanglomerate
Boulder (or bedrock?)
Drill rate (min/ft)