Mechanical Systems III

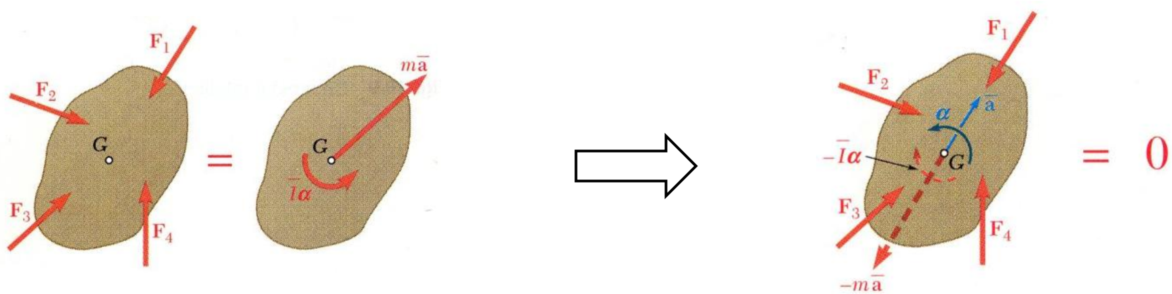
Drawing a F.B.D. using d'Alembert's principle

- ***d'Alembert's Principle*** **:**

  The sum of the differences between the forces acting on a system and the time derivatives of the moments of the system itself along a virtual displacement con sistent with the constraints of the system, is zero.

$$\sum_i (\mathbf{F}_i - m_i \mathbf{a}_i) \cdot \delta \mathbf{r}_i = 0$$

- One can transform an accelerating rigid body into an equivalent **static system** by adding the so-called "**inertial force**" and "**inertial moment**"



Mechanical Impedances

Find TF without writing the differential equation!

Let's define Mechanical Impedance as $Z_M(s) = \dfrac{F(s)}{X(s)}$

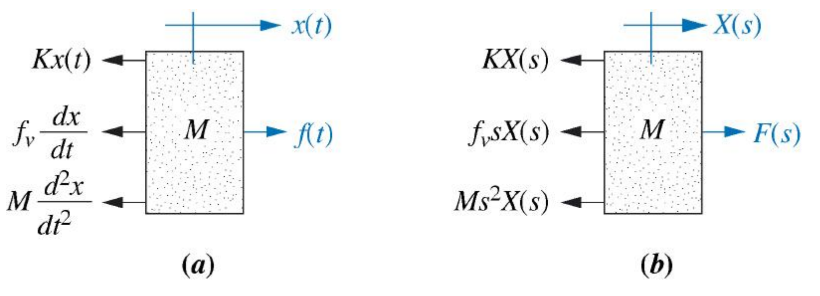
$$F(s) = Ms^2 X(s) \qquad Z_M(s) = Ms^2$$

$$F(s) = CsX(s) \qquad Z_C(s) = Cs$$

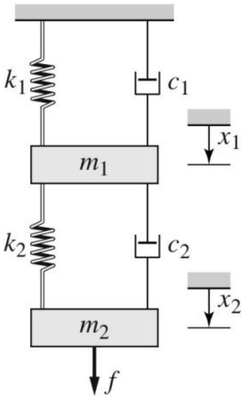
$$F(s) = KX(s) \qquad Z_K(s) = K$$

Problem Solving Technique:

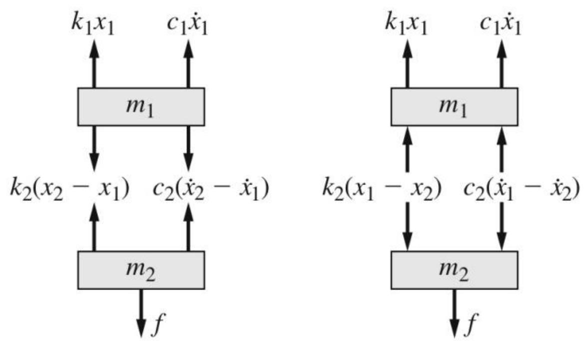
[Sum of impedances] X(s) = [Sum of applied forces]



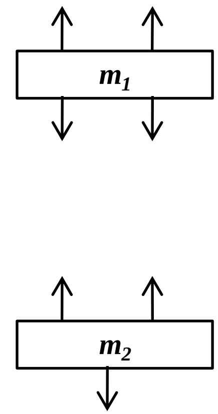
Example: Two mass system



Using differential equation



Using Mechanical Impedances



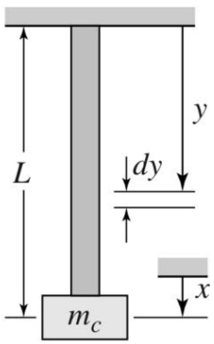
$$m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1) - c_1\dot{x}_1 + c_2(\dot{x}_2 - \dot{x}_1)$$

$$m_1\ddot{x}_2 = f - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1)$$

$$(m_1s^2 + (c_1 + c_2)s + (k_1 + k_2))X_1(s) = (c_2s + k_2)X_2(s)$$

$$(m_1s^2 + c_2s + k_2)X_2(s) = F(s) + (c_2s + k_2)X_1(s)$$

Energy Method for Deriving Equivalent Mass of Elastic Elements



Mass of an infinitesimal element of thickness dy

$\Rightarrow$ $dm_r = \rho dy$ $\quad$ ρ : mass density per unit length of the material

Kinetic Energy of infinitesimal element $\Rightarrow$ $\frac{1}{2}\dot{y}^2 dm_r$

Kinetic Energy of the rod $\Rightarrow$ $KE = \frac{1}{2}\int_0^L \dot{y}^2 dm_r = \frac{1}{2}\int_0^L \dot{y}^2 \rho dy$

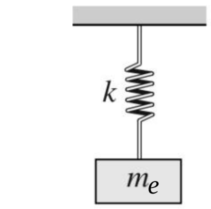
Rewrite the KE using $\dot{y} = \dot{x}\frac{y}{L}$

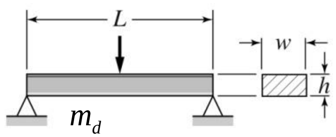
$$\Rightarrow \quad KE = \frac{1}{2}\int_0^L \left(\dot{x}\frac{y}{L}\right)^2 \rho dy = \frac{1}{2}\frac{\rho \dot{x}^2}{L^2}\int_0^L y^2 dy = \frac{1}{2}\frac{\rho \dot{x}^2}{L^2}\frac{y^3}{3}\bigg|_0^L$$

Rewrite the KE in the form of the total mass of the rod

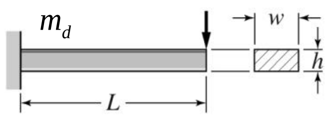
$$KE = \frac{1}{2}\left(\frac{\rho L}{3}\right)\dot{x}^2 = \frac{1}{2}\left(\frac{m_r}{3}\right)\dot{x}^2 \quad \Rightarrow \quad m_e = \left(\frac{m_r}{3}\right)$$

Equivalent masses of common elements

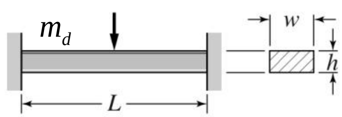




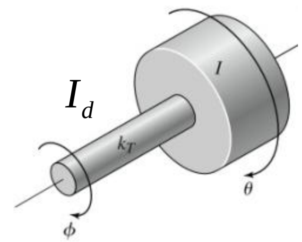
$k = \frac{4Ewh^3}{L^3}$ $\quad$ $m_e = 0.38 m_d$



$k = \frac{Ewh^3}{4L^3}$ $\quad$ $m_e = 0.23 m_d$



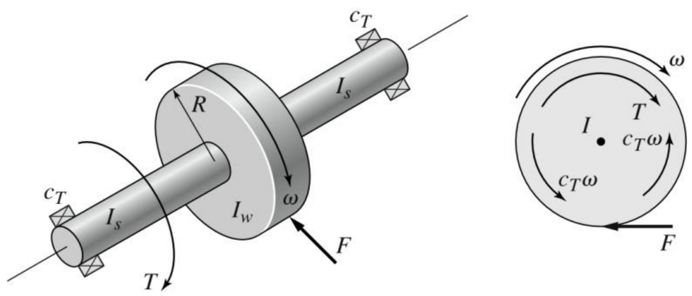
$k = \frac{16Ewh^3}{L^3}$ $\quad$ $m_e = 0.5 m_d$



$I_e = I + I_d / 3$

$k_T = \frac{\pi G D^4}{32L}$

Example: Wheel-axle system with bearing damping

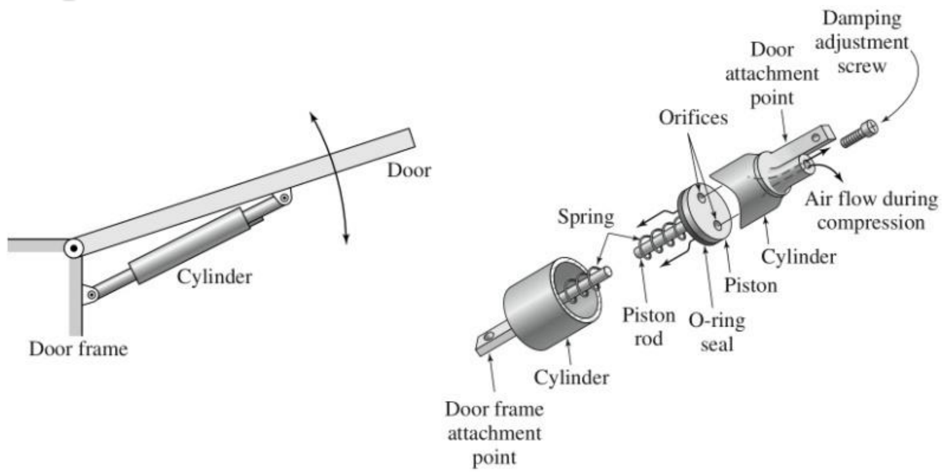


(a) (b)

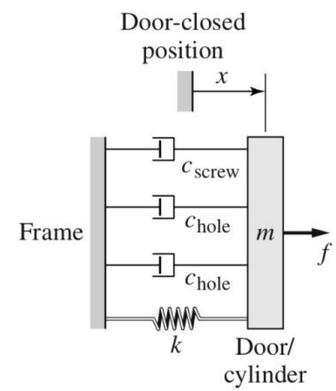
$$I = I_w + 2I_s$$

$$I\dot{\omega} = T - RF - 2c_T\omega$$

Example: Pneumatic Door Closer

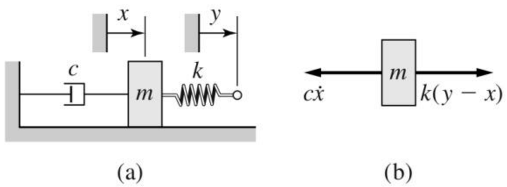


(a) (b)

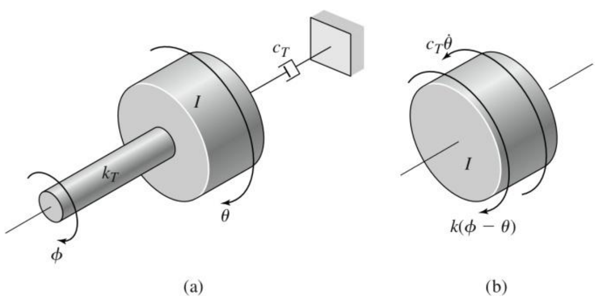


$$m\ddot{x} = f - c_{screw}\dot{x} - c_{hole}\dot{x} - c_{hole}\dot{x} - kx$$

Example: System with displacement input



$$m\ddot{x} = k(y-x) - c\dot{x}$$



$$I\ddot{\theta} = k_T(\phi-\theta) - c_T\dot{\theta}$$

Note that ultimately motion is generated by a force and that this force must be great enough to generate the specified motion.