Electrical Systems I

Basic Elements

- Active Elements: OP Amp etc.
  (Has transistors/amplifiers that require active source of power to work)

- Passive Elements: Inductor, Resistor, Capacitor etc.
  (Simply respond to an applied voltage or current.)

- Current : the rate of flow of charge

- Charge : (electric charge) the integral of current with respect to time [C]

$$i = \frac{dq}{dt} \qquad [\text{ampere}] = \frac{[\text{coulomb}]}{[\text{sec}]}$$

- Voltage : electromotive force needed to produce a flow of current in a wire [V]
  Change in energy as the charge is passed through a component.
  [V] = [J/C]

- Power: product of voltage and current

  [W] = [J/Sec]

Basic Elements - Resistance

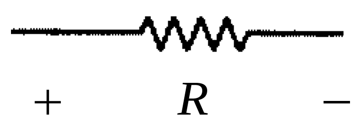
- Resistance : the change in voltage required to make a unit change in current.

Analogous to →

$$R = \frac{Change \ \ in \ \ voltage}{Change \ \ in \ \ current} = \frac{[V]}{[A]} = [Ohm(\Omega)]$$

- Resistor

$$V_R = R \cdot i_R \qquad R = \frac{V_R}{i_R}$$



Basic Elements - Capacitance

- Capacitance: the change in the quantity of electric charge required to make a unit change in voltage.

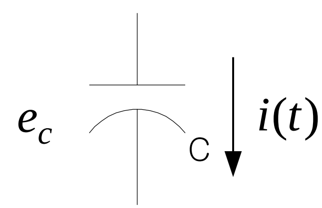
Analogous to →

$$C = \frac{[Coulomb]}{[V]} = [Farad\ (F)]$$

- Capacitor: two conductor separated by non-conducting medium.

$$i = dq/dt\ , \quad e_c = q/C \quad \rightarrow\ i =$$

$$\therefore e_c(t) =$$



$$I(s) = \qquad\qquad V(s) =$$

Basic Elements - Inductance

- Inductance: An electromotive force induced in a circuit, if the circuit lies in a time –varying magnetic field.

Analogous to →

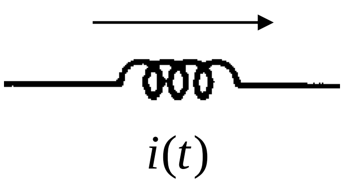
$$L = \frac{[V]}{[A/\sec]} = [Henry\ (H)]$$

- Inductor: $e_L =$

$V(s) =$

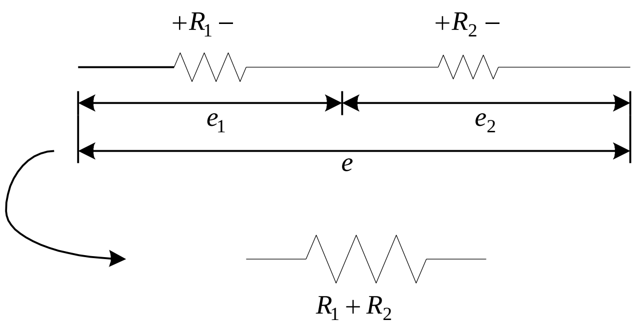
$\therefore\ i_L(t) =$

$I(s) =$



Series & Parallel Resistance

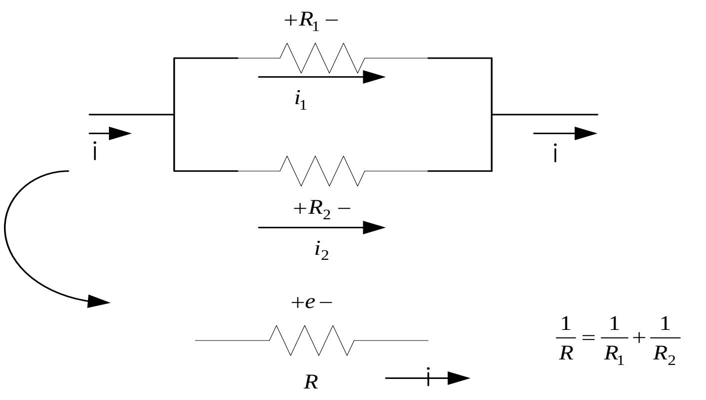
- Series Resistance



$$e_1 = iR_1, \quad e_2 = iR_2$$

$$e = \qquad\qquad =$$

- Parallel Resistance



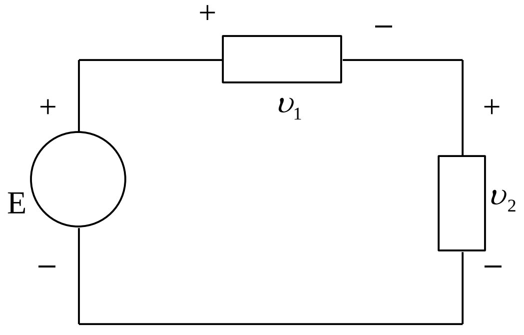
$$-e_1 + e_2 = 0 \quad \Rightarrow$$

$$i = \qquad\qquad = \epsilon$$

Series/Parallel Capacitance?

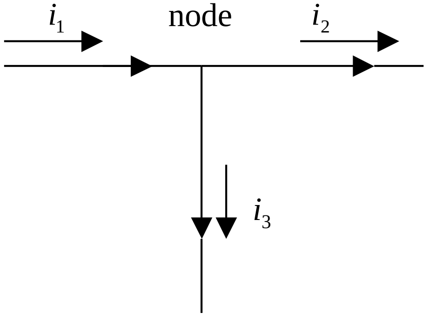
Kirchhoff's laws

1. The algebraic sum of the potential difference around a closed path equals zero.



$$-\upsilon_1 - \upsilon_2 + E = 0$$

2. The algebraic sum of the currents entering (or leaving) a nod is equal to zero.

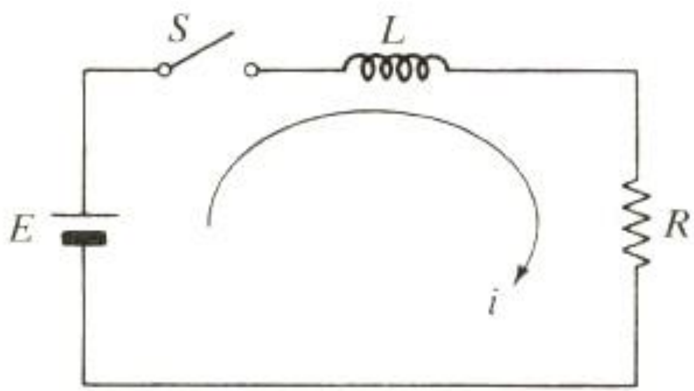


$$i_1 - i_2 - i_3 = 0$$

Current in = Current out

$$\rightarrow \quad i_1 = i_2 + i_3$$

Mathematical Modeling of Electrical Systems



The switch S is closed at t=0

or

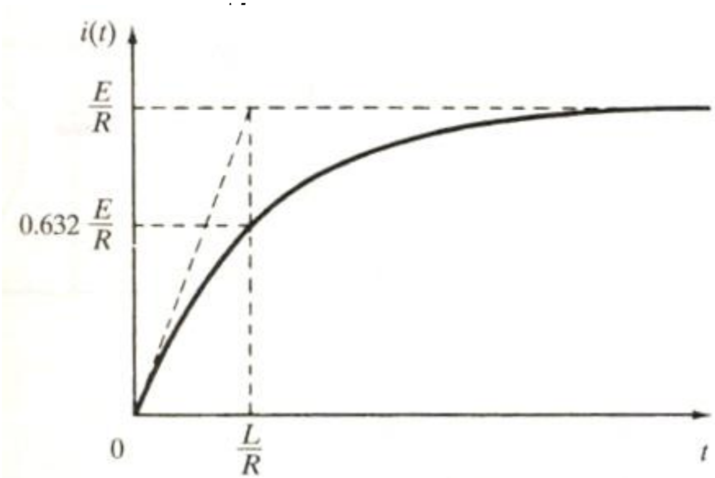
At the instant that switch S is closed, the current $i(0) = 0$

Laplace Transformation :

$$i(0) = 0 \quad \rightarrow \quad (Ls + R)I(s) = \frac{E}{s}$$

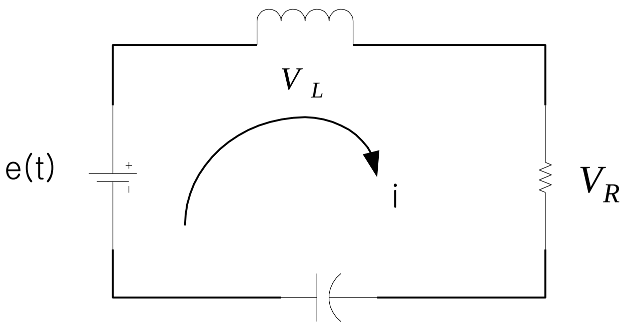
$$I(s) = \qquad\qquad =$$

$$\therefore \; i(t) =$$



Examples of Circuit Analysis

ex) R-L-C Circuit



$$-V_L - V_R - V_C + e(t) = 0$$

$$V_L = L\frac{di}{dt}, \quad V_R = iR, \quad V_c = \frac{1}{C}\int i\,dt + V_C(t)$$

$$\frac{dV_c}{dt} = \frac{1}{C}i$$

$$L\frac{di}{dt} + Ri = e(t) - V_C(t)$$

Laplace Transform, $$(Ls + R + \frac{1}{Cs})I(s) = E(s)$$

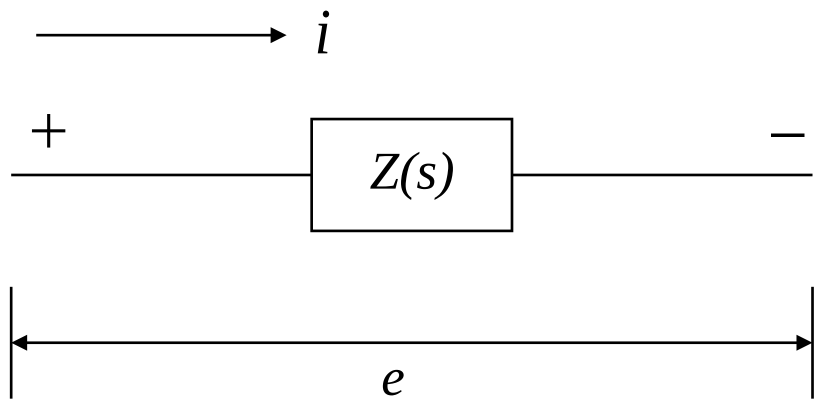
$$\frac{I(s)}{E(s)} = \frac{1}{(Ls + R + \frac{1}{Cs})}$$

$V_o(s) =$

$V_o(s) =$ $\qquad\qquad =$

Step Response?

Complex Impedance

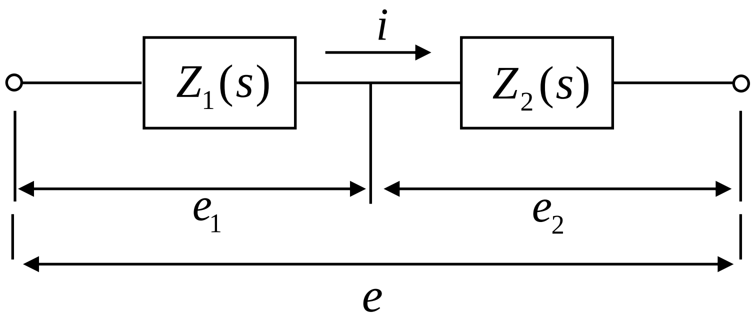


$$I(s) = \frac{E(s)}{Z(s)} \qquad E(s) = Z(s)I(s)$$

Z(s) : complex impedance

Complex Impedance

The complex impedance Z(s) of a two-terminal circuit is :



$$Z(s) = \qquad\qquad E(s) =$$

$$E_1(s) = Z_1(s)I(s), \qquad E_2(s) = Z_2(s)I(s)$$

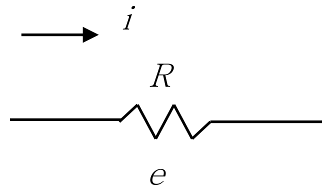
$$\begin{aligned} E(s) &= E_1(s) + E_2(s) \\ &= Z_1(s)I(s) + Z_2(s)I(s) \\ &= \left(Z_1(s) + Z_2(s)\right)I(s) \end{aligned}$$

Direct derivation of transfer function,

without writing differential equations first.

Complex Impedance

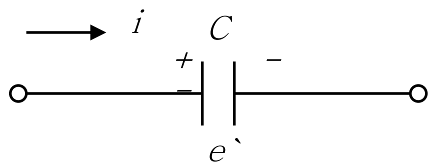
Resistance :



$$e = Ri, \quad E(s) = R\,I(s)$$

$$Z(s) =$$

Capacitance :

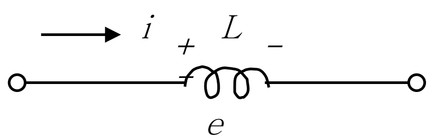


$$\frac{de}{dt} = \frac{1}{C} i$$

$$sE(s) = \frac{1}{C} I(s) \rightarrow E(s) = \frac{1}{Cs} I(s)$$

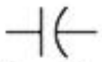
$$\therefore Z(s) =$$

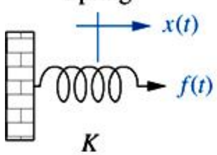
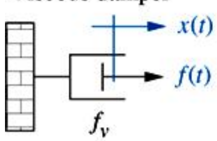
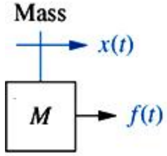
Inductance :



$$e = L\frac{di}{dt}, \qquad E(s) = L\,s\,I(s)$$

$$\therefore Z(s) =$$

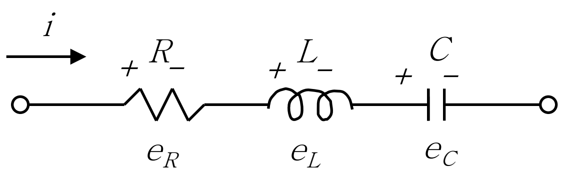
Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Examples of Complex Impedance

Series Impedances

ex1)



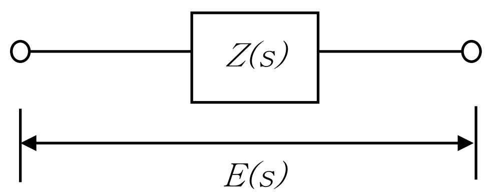
$$e_R = iR, \quad e_L = L\frac{di}{dt}, \quad \frac{de_C}{dt} = \frac{1}{C}i$$

$$e = e_R + e_L + e_C$$

$$E(s) = E_R(s) + E_L(s) + E_C(s)$$

$$=$$

$$=$$

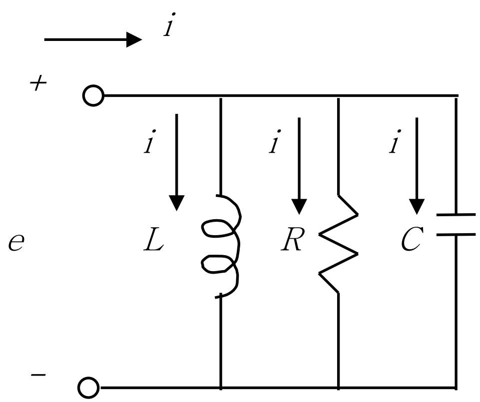


$$\therefore Z(s) = R + Ls + \frac{1}{Cs} = Z_R(s) + Z_L(s) + Z_C(s)$$

Examples of Complex Impedance

Parallel Impedances

ex2)



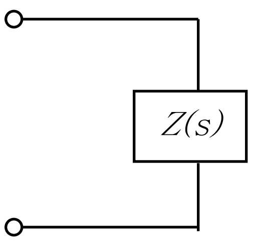
$$i = i_L + i_R + i_C \quad E(s) = Z(s)I(s)$$

$$I(s) =$$

$$=$$

$$= \Bigg(\qquad \Bigg) E(s)$$

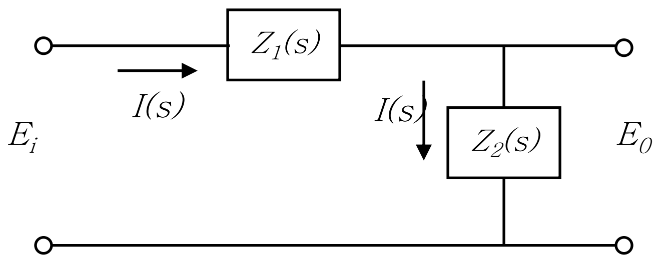
$$= \quad E(s)$$



$$\therefore Z(s) = \frac{1}{\dfrac{1}{Z_R(s)} + \dfrac{1}{Z_L(s)} + \dfrac{1}{Z_C(s)}} = \frac{1}{\dfrac{1}{Ls} + \dfrac{1}{R} + Cs}$$

Examples of Complex Impedance

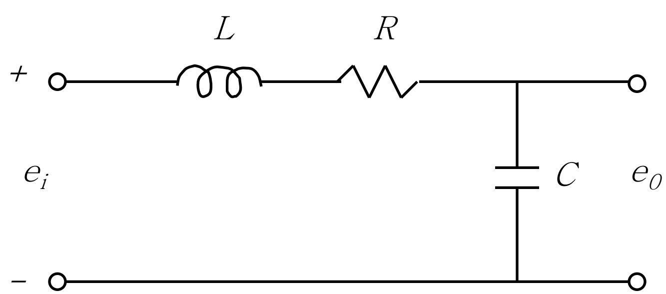
Deriving transfer functions of Electrical circuits by the use of complex impedances.



$$E_i(s) = Z_1(s)I(s) + Z_2(s)I(s) , \qquad E_o(s) = Z_2(s)I(s)$$

$$\frac{E_o(s)}{E_i(s)} =$$

ex)



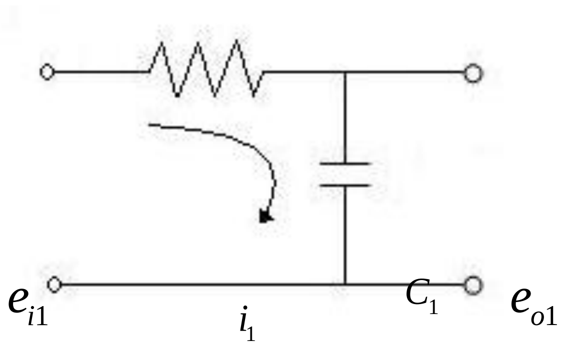
$$\frac{E_o(s)}{E_i(s)} =$$

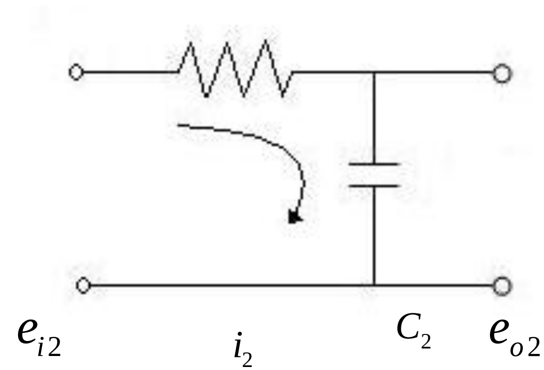
$$Z_1(s) = \qquad\qquad Z_2(s) =$$

$$\frac{E_o(s)}{E_i(s)} =$$

Transfer Functions of Cascaded Elements

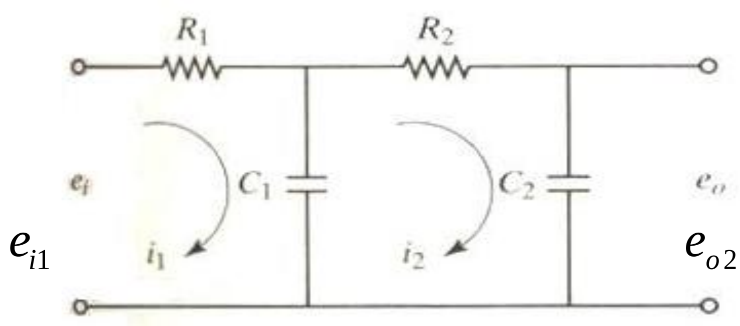
Consider two RC circuits





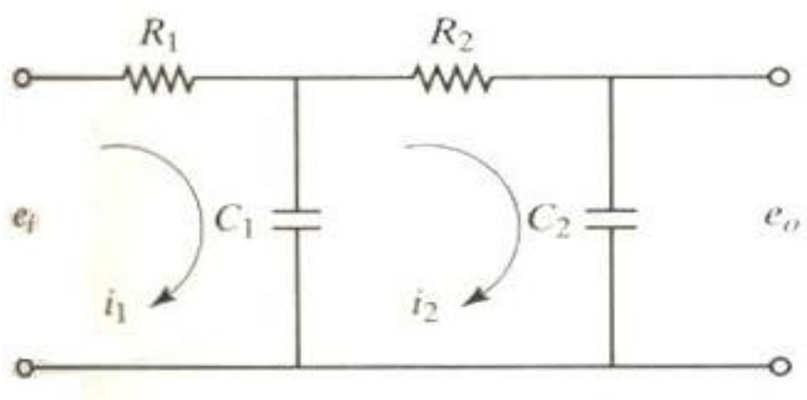
$$\frac{E_{o1}(s)}{E_{i1}(s)} = \frac{1}{R_1 C_1 s + 1} = G_1(s)$$

$$\frac{E_{o2}(s)}{E_{i2}(s)} = \frac{1}{R_2 C_2 s + 1} = G_2(s)$$



$$\frac{E_{o2}(s)}{E_{i1}(s)} = ?$$

Transfer Functions of Cascaded Elements
Loading Effect



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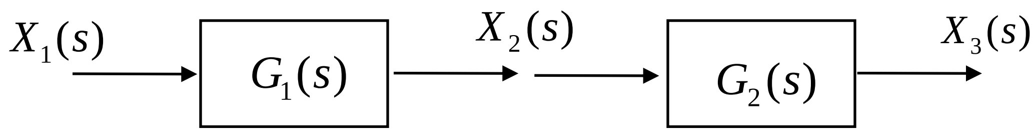
,

$$T.F\ :\ \frac{E_o(s)}{E_i(s)} = \frac{1}{(R_1C_1s+1)(R_2C_2s+1)+R_1C_2s} \neq \frac{1}{(R_1C_1s+1)(R_2C_2s+1)}$$

$$\therefore\quad \frac{E_o(s)}{E_i(s)} \neq \frac{E_{o1}(s)}{E_{i1}(s)}\cdot\frac{E_{o2}(s)}{E_{i2}(s)} \longrightarrow$$ **Loading effect**

Transfer Functions of Cascade Elements

Input Impedance, Output Impedance



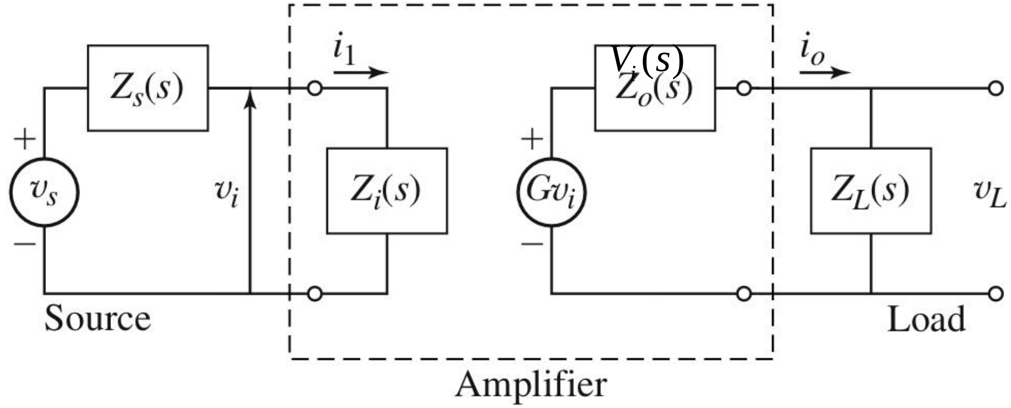
$$G(s) = \frac{X_3(s)}{X_1(s)} = \frac{X_2(s)}{X_1(s)} \cdot \frac{X_3(s)}{X_2(s)} = G_1(s)G_2(s)$$

If the "input Impedance" of the second element is infinite, the output of the first element is not affected by connecting it to the second element.

Then, $G(s) = G_1(s)G_2(s)$

Transfer Functions of Cascade Elements

Isolating Amplifier



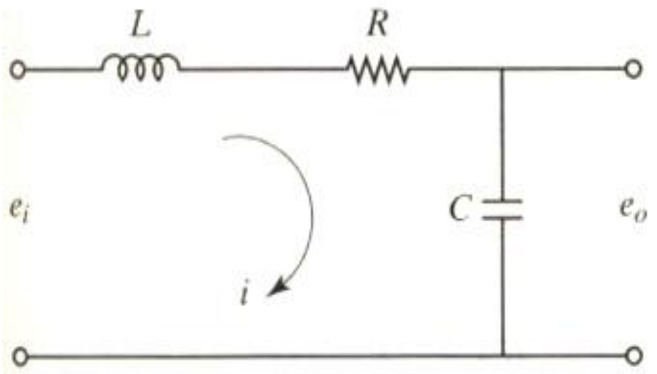
This amplifier circuit has to

1. Not affect the behavior of the source circuit.
2. Not be affected by the loading circuit.

This isolating amplifier circuit has to have

1. a very high input impedance,
2. very low output impedance

State-Space Mathematical Modeling of Electrical Systems



By Kirchhoff's voltage law

$$L\frac{di}{dt}+Ri+v_c=e_i, \quad \frac{dv_c}{dt}=\frac{1}{C}i, \quad e_o=v_c$$

Assume, initial condition is 0,

$$LsI(s)+RI(s)+V_c(s)=E_i(s), \quad V_c(s)=\frac{1}{Cs}I(s)$$

$$T.F \ : \frac{E_o(s)}{E_i(s)}=\frac{1}{LCs^2+RCs+1}$$

State-Space Mathematical Modeling of Electrical Systems

Differential equation : $\ddot{e}_o + \frac{R}{L}\dot{e}_o + \frac{1}{LC}e_o = \frac{1}{LC}e_i$

State variable : $x_1 = e_o, \quad x_2 = \dot{e}_o$

Input and output : $u = e_i, \quad y = e_o = x_1$

State-space equation :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$