457.562 Special Issue on River Mechanics (Sediment Transport)

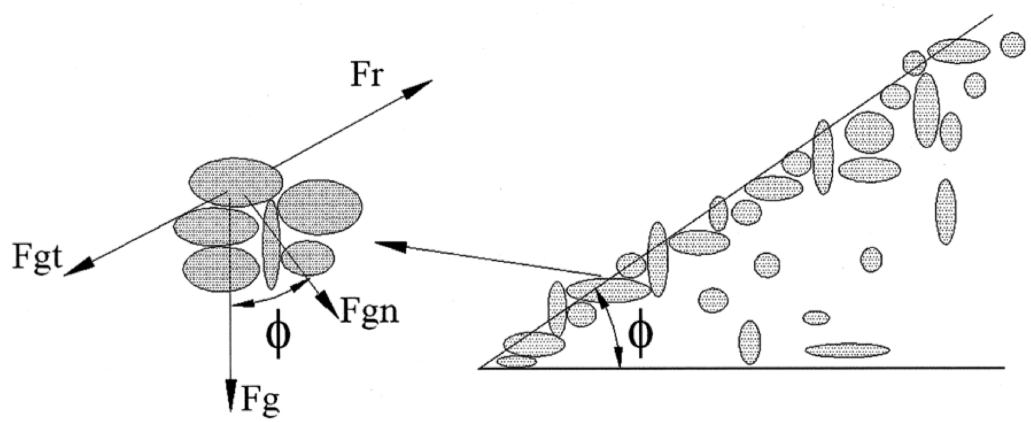
.05 Macroscopic view of sediment transport

Prepared by Jin Hwan Hwang

1. Submerged Angle of Repose

- If granular particles are allowed to pile up while submerged in a fluid, there is a specific slope angle beyond which spontaneous failure of the slope occurs.
- This angle is the angle of repose (or friction angle).
- The coefficient of Coulomb friction

$$\mu = \frac{\text{tangential resistive force}}{\text{downward normal force}}$$



1. Submerged Angle of Repose

- The forces acting on the particle along the slope are:
  - The submerged force of gravity, which has a downslope component F_{gt} and normal component F_{gn}
  - A tangential resistive force F_r due to Coulomb friction.

$$F_{gt} = (\rho_s - \rho) g V_p \sin\phi$$

$$F_{gn} = (\rho_s - \rho) g V_p \cos\phi$$

$$F_r = \mu F_{gn}$$

- The condition for incipient motion is

$$F_{gt} = F_r$$

  - Down slope impelling force of gravity should just barely balance with the Coulomb resistive force.

1. Submerged Angle of Repose

- From the previous four relations, it is found

$$\mu = \tan\phi$$

- The angle of repose is an empirical quantity.
- Test with well-sorted material indicate that this angle is near 30 degree for sand, gradually increasing to 40 for gravel.
- Poorly-sorted angular material tends to interlock, giving greater resistance to failure, and as a result, a higher friction angle of repose.

2. Critical Stress for Flow over a Granular Bed

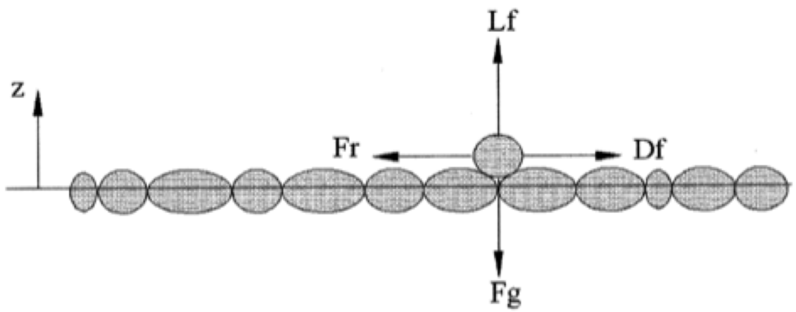
- When a granular bed is subjected to a turbulent flow, it is found that virtually no motion of the grains is observed at some flows, but that the bed is noticeably mobilized at ot her flows.
- Factor that affect the mobility of grains subjected to a flo w are
  - Randomness: including grain placement, and turbulent
  - Forces on grains: Gravity and fluid
    - Fluid : lift and drag (mean & turbulent)
- In the presence of turbulent flow,
  - random fluctuations typically prevent the clear definition of a critical or threshold condition for motion

2. Critical Stress for Flow over a Granular Bed

- But, still we need some conditions for describing it.
- Ikeda-Coleman-Iwagaki model for threshold conditions.
- Assumptions:
  - The forces acting on a particle taken to “dangerously place d.
  - The flow is taken to follow the turbulent rough logarithmic l aw near the boundary.
  - Turbulent forces on the particle are neglected.
  - Drag and lift forces act through the particle center
  - On drag force, boundary effects can be neglected
  - Very low streamwise slopes
  - Roughness height is equated to diameter of sediment

2. Critical Stress for Flow over a Granular Bed

- Particle is centered at z=D/2
- To solve this problem, it is ne cessary to use some informati on about turbulent boundary l ayers to to define effective flui d velocity u_f acting on the part icle.



- In the viscous sublayer, a linear velocity profile

$$\frac{u}{u_*} = \frac{u_* z}{\nu} \quad \Rightarrow \quad \frac{u_f}{u_*} = \frac{1}{2}\frac{u_* D}{\nu} \quad \left(\text{when } \frac{D}{\delta_v} = \frac{u_* D}{\nu} < 0.5 \right)$$

$D / \delta_v > 2$ then no viscous sublayer exist.

2. Critical Stress for Flow over a Granular Bed

- If no viscous sublayer exists, apply the rough pipe log v elocity profile

$$\frac{u_f}{u_*} = 2.5\ln\left(30\frac{z}{D}\right)\Bigg|_{z=\frac{1}{2}D} = 6.77$$

- Then the general form for the both cases,

$$\frac{u_f}{u_*} = F\left(\frac{u_* D}{\nu}\right)$$

- Where

$$\cancel{F = \frac{1}{2}\frac{u_* D}{\nu} \quad for \quad \frac{u_* D}{\nu} < 13.5} \longrightarrow \text{Since rough flow}$$

$$F = 6.77 \quad for \quad \frac{u_* D}{\nu} > 13.5$$

2. Critical Stress for Flow over a Granular Bed

- However, it would be more convenient to have a contin uous function

$$F=\left\{\left(\frac{2\nu}{u_*D}\right)^{10/3}+\left[\kappa^{-1}\ln\left(1+\frac{\frac{9}{2}\frac{u_*D}{\nu}}{1+0.3\frac{u_*D}{\nu}}\right)\right]^{-10/3}\right\}^{-0.3}$$

- Forces acting on the particle

$$D_f=\rho\frac{1}{2}\pi\left(\frac{D}{2}\right)^2 c_D u_f^2,\qquad L_f=\rho\frac{1}{2}\pi\left(\frac{D}{2}\right)^2 c_L u_f^2,\quad F_g=\rho Rg\frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

- Coulomb resistive force

$$F_r=\mu\left(F_g-L_f\right)$$

2. Critical Stress for Flow over a Granular Bed

- For a channel with a downstream slope angle, the downslope effect of gravity has to be included in the force balance presented earlier, resulting in

$$\boxed{\tau_c^* = \frac{4}{3}\frac{\mu}{C_D + \mu C_L}\frac{1}{F^2(u_{*c}D/\nu)}}$$

- This is dimensionless critical shear stress and called as "the Shields pa

$$\tau_c^* = \frac{\tau_{bc}}{\rho g R D}$$

2. Critical Stress for Flow over a Granular Bed

- The critical condition for incipient motion of the particle is that the impelling drag force is just balanced by the resisting Coulomb frictional force:

$$D_f = F_r$$

- With all forces above,

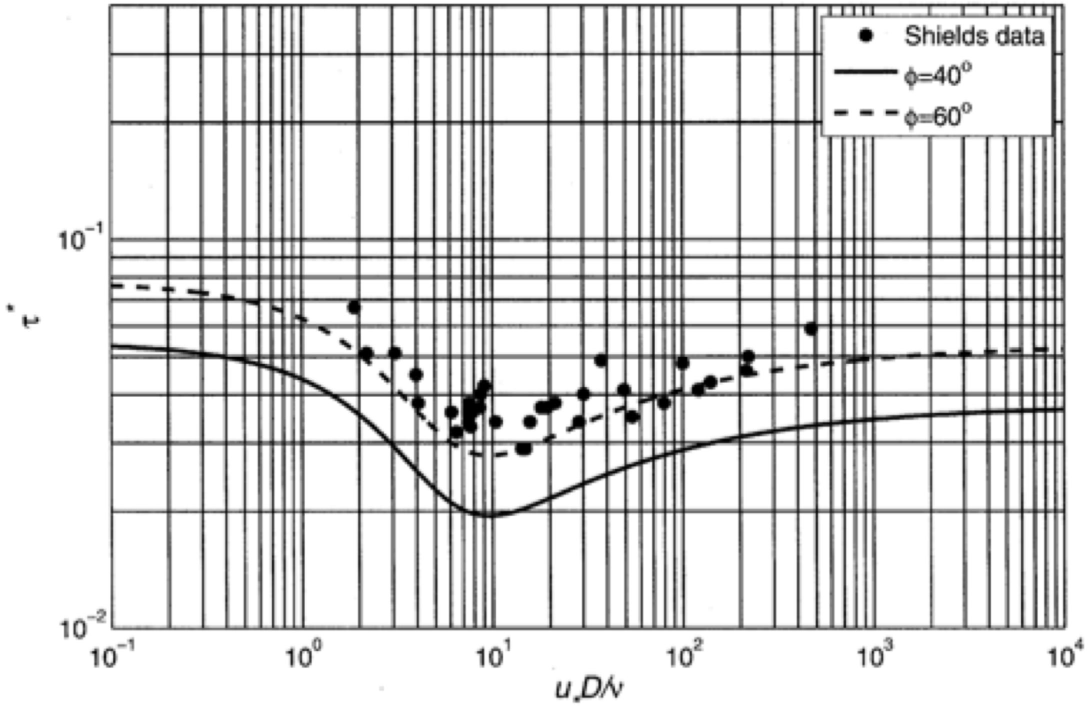
$$\frac{u_f^2}{RgD} = \frac{4}{3}\frac{\mu}{c_D + \mu c_L}$$

- Since $\frac{u_f}{u_*} = F\left(\frac{u_* D}{\nu}\right)$ and $\tau_b = \rho u_*^2$

- Ikeda-Coleman-Iwagaki

$$\tau_c^* = \frac{4}{3}\frac{\mu}{\left(c_D + \mu c_L\right)}\frac{1}{F^2\left(u_{*c} D / \nu\right)}$$

2. Critical Stress for Flow over a Granular Bed



$$\phi = 40° \; and \; 60°$$

$$k_s = 2D \quad C_L = 0.85 C_D$$

- Comparison of Ikeda-Coleman-Iwagaki model for initiatio n of motion with Shields data

3. Shields Diagram

- Shields elucidate the condition for which sediment grins would be at the verge of moving.
- He used the fundamental concepts of similarity and dime nsional analysis.
- He said shield number should be function of shear Reyn olds number

$$\tau_c^* \sim function\left(\frac{u_{*c}D}{\nu}\right)$$

- Key parameters for the system are

$$\tau_{bc},\ \ \gamma\ \ and\ \ \gamma_s, D, u_{*c} = \sqrt{\tau_{bc}/\rho}, \nu$$

- So simply

$$\tau_c^* = \frac{\tau_{bc}}{\rho g R D} = F_*\left(\frac{u_{*c}D}{\nu}\right)$$

3. Shields Diagram

$$\tau_c^* = \frac{\tau_{bc}}{\rho g R D} = F_*\left(\frac{u_{*c} D}{\nu}\right)$$

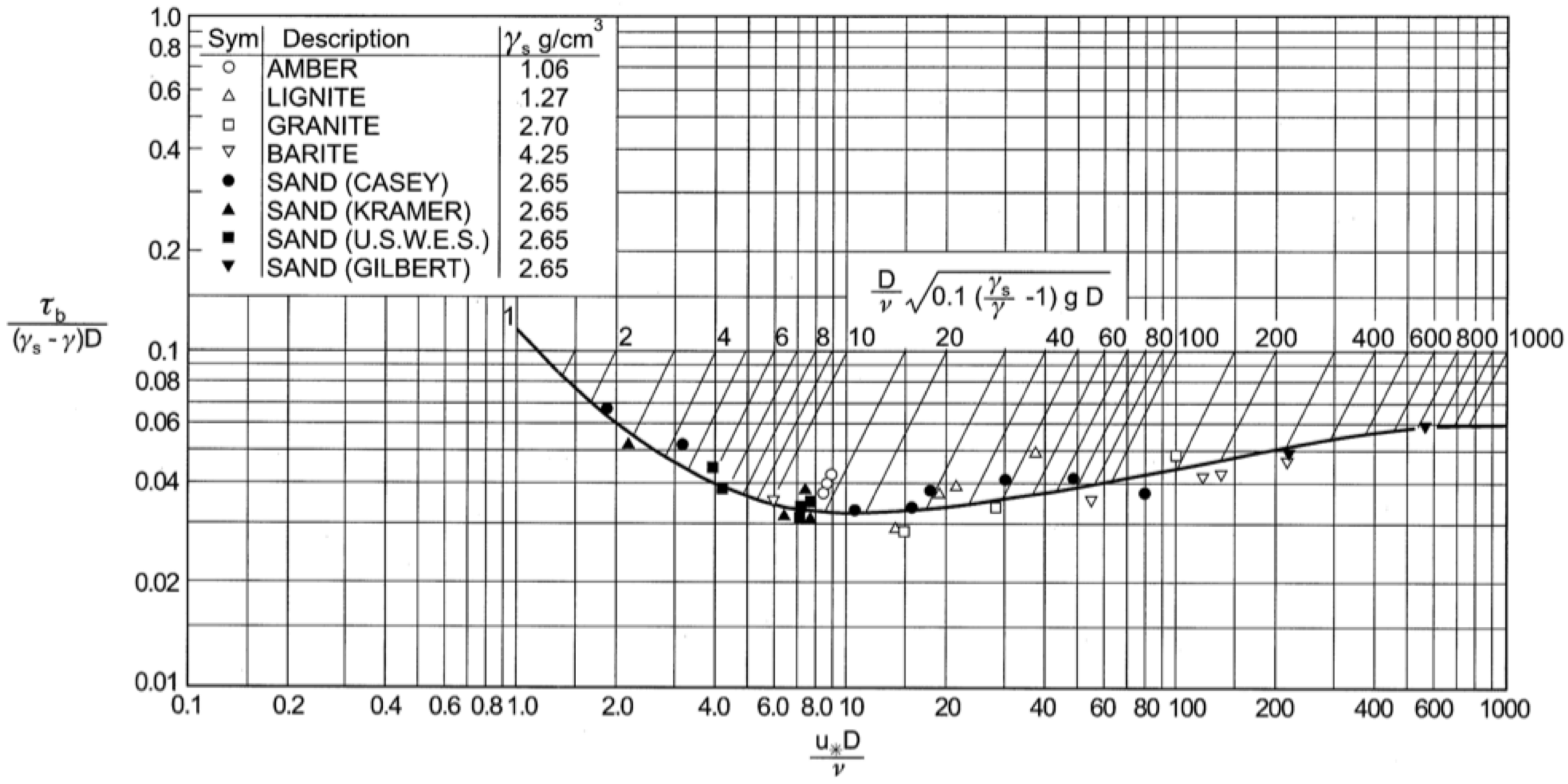
- But this form is not practical,
- Therefore, Vanoni (1964) proposed

$$\frac{u_* D}{\nu} = \frac{u_*}{\sqrt{gRD}} \frac{\sqrt{gRD}D}{\nu} = \left(\tau^*\right)^{1/2} \mathrm{R}_{ep}$$

- Which was used in falling velocity relations

3. Shields Diagram

- .



4. Granular Sediment on a Sloping Bed

- The work of Shields on initiation of motion applied only to the case of nearly horizontal slopes.
- Most streams, particularly in mountain areas, have steep gradients, creasing a need to account for the effect of th e downslope component of gravity on the initiation motio n.
- Wiberg and Smith (1987)

$$\tau_c^* = \frac{4}{3}\frac{\left(\tan\phi_0\cos\alpha - \sin\alpha\right)}{\left(C_D + \tan\phi_0 C_L\right)}\frac{1}{F^2(z/z_0)}$$

$\alpha =$ Bed slope angle

$\phi_0 =$ Angle of repose of the grains

4. Granular Sediment on a Sloping Bed

- The effect of the streamwise bed slope on incipient sediment motion can be illustrated by considering the forces active on a particle lying in a bed consisting of similar particles over which water flows.
- Chiew and Parker (1994)

$$\frac{\tau^*_{c\alpha}}{\tau^*_{co}} = \cos\alpha\left(1 - \frac{\tan\alpha}{\tan\phi}\right)$$

ϕ : Repose angle

$\tau^*_{c\alpha}$: critical shear stress for sediment on a bed with a longitudinal slope angle α

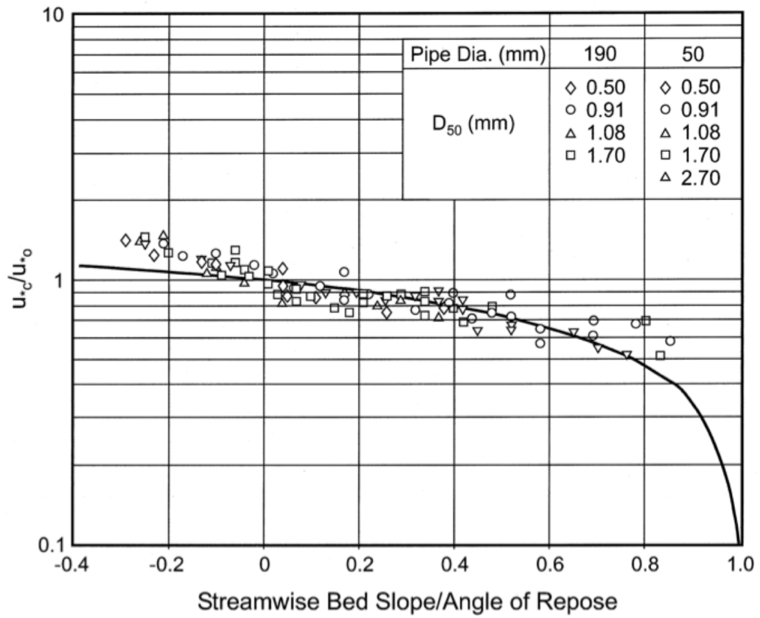
τ^*_{co} : critical shear stress for a bed with very small slope

4. Granular Sediment on a Sloping Bed

- In terms of shear velocities,

$$\frac{u_{*c}}{u_{*o}} = \sqrt{\cos\alpha\left(1-\frac{\tan\alpha}{\tan\phi}\right)}$$

- Right figure was done by Chiew and Parker in duct laboratory experiment



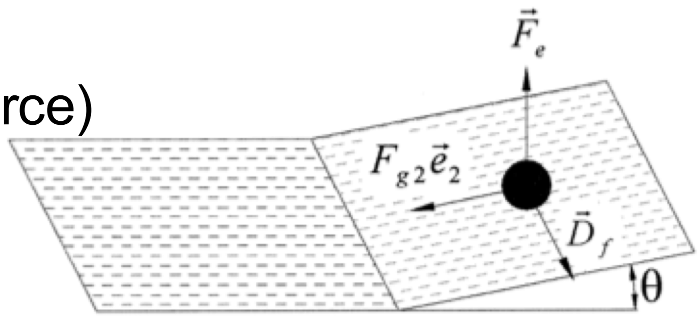
5. Threshold Condition on Side Slopes

- Until now, we just talk the horizontal in the transverse dir ection.
- In engineering application, side wall slope must be impor tant.
- Simplification of the analysis:
  - Logarithmic upward normal from the bed.

$$D_f = \rho \frac{1}{2}\pi\left(\frac{D}{2}\right)^2 c_D u_f^2 \vec{e}_1 \quad \text{(Drag force)}$$

$$F_g = F_{g2}\vec{e}_2 + F_{g3}\vec{e}_3 \quad \text{(Gravitational force)}$$

$$\left(F_{g2}, F_{g3}\right) = -\rho R g \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \left(\sin\theta, \cos\theta\right)$$



5. Threshold Condition on Side Slopes

- The lift force is given as

$$L_f = \rho \frac{1}{2} \pi \left(\frac{D}{2} \right)^2 c_L u_f^2 \vec{e}_3$$

- Coulomb resistive force must balance the sum of the imp elling forces du to flow and transverse downslope pull of gravity at the critical conditions

$$\mu^2 \left| F_{g3} \vec{e}_3 + L_f \right|^2 = \left| D_f \right|^2 + \left| F_{g2} \vec{e}_3 \right|^2$$

- With the previous terms

$$\left[\left(\frac{u_f^2}{RgD} \right)^2 + \left(\frac{4}{3c_D} \sin\theta \right)^2 \right]^{1/2} = \mu \left(\frac{4}{3c_D} \cos\theta - \frac{c_L}{c_D} \frac{u_f^2}{RgD} \right)$$

5. Threshold Condition on Side Slopes

- With some manipulations

$$\left[\left(\tau_c^*\right)^2+\left(\frac{4}{3c_D F^2}\sin\theta\right)^2\right]^{1/2}=\frac{4\mu}{3c_D F^2}\cos\theta-\mu\frac{c_L}{c_D}\tau_c^*$$

- The case of a transversely horizontal bed is recovering b y setting θ=0. The critical Shields stress if found as in th e previous

$$\tau_c^*=\frac{4}{3}\frac{\left(\mu cos\alpha-\sin\alpha\right)}{\left(c_D+\mu c_L\right)}\frac{1}{F^2\left(u_{*c}D/\nu\right)}$$

- And denotes with the subscript of o.

5. Threshold Condition on Side Slopes

- Then

$$\left[\left(\frac{\tau_c^*}{\tau_{co}^*}\right)^2 + \left(\frac{(1+\mu c_L/c_D)}{\mu}\sin\theta\right)^2\right]^{1/2} = \left(1+\mu\frac{c_L}{c_D}\right)\cos\theta - \mu\frac{c_L}{c_D}\left(\frac{\tau_c^*}{\tau_{co}^*}\right)$$

When $\mu = 0.84\ (\phi = 40°), c_L = 0.85 c_D$.

(c_D can be found in diagram)