4. 암석의 파괴기준

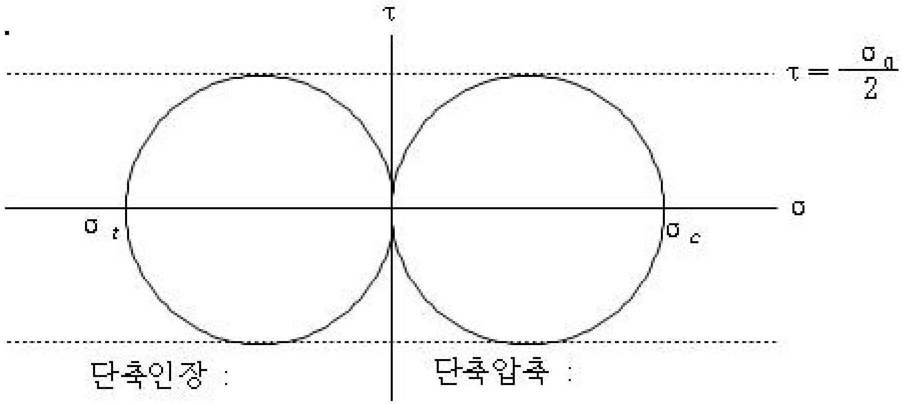
4.1 Tresca 이론

파괴기준: 더 이상의 하중증가 없이 소성변형이 일어나거나 파괴가 발생하는 조건

- 최대전단응력설: Tresca에 의해 1968년 제안
- 임의점에서 최대전단응력(τ_{max})이 일정치 (S_0)에 도달하면 (수직응력에 상관없이) 파괴가 발생한다는 이론: 중간주응력은 파괴에 영향을 미치지 않음.

$$\frac{\sigma_{\max} - \sigma_{\min}}{2} = S_0$$

- 단점: 암석의 인장파괴를 제대로 설명하지 못함.



$\sigma_1 = 0, \quad \sigma_3 = -\sigma_t$

$\tau = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t}{2}$

$\sigma_1 = \sigma_c, \quad \sigma_3 = 0$

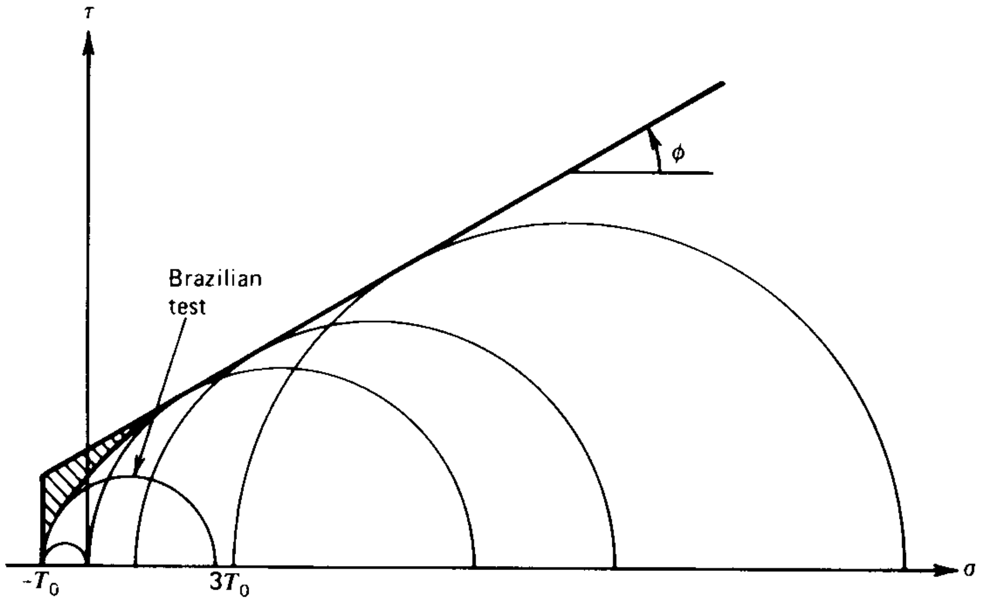
$\tau = \frac{\sigma_1 - \sigma_3}{2} = S_0 = \frac{\sigma_c}{2}$

4.2 Mohr-Coulomb 이론

- 파괴시의 Mohr 응력원을 접하는 포락선을 직선으로 표현.
- 절리면 전단강도식과 비슷한 형태이나 내부마찰각이 적용됨.

$$\tau_p = c + \sigma \tan\phi$$

- 단점: 실제 포락선은 곡선이며 특히 인장파괴부분에서 오차가 크다.
- 장점: 간단한 표현



4.2 Mohr-Coulomb 이론

- 주응력 domain으로의 변경: $(\sigma, \tau) \rightarrow (\sigma_3, \sigma_1)$

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\beta \quad (\because \tau_{13} = 0)$$

$\tau = -\dfrac{\sigma_1 - \sigma_3}{2}\sin 2\beta$ 와 $2\beta = 90^\circ + \phi$ 를 $\tau = c + \sigma \tan\phi$ 에 대입하면

$$\sigma_1 = 2c\frac{\cos\phi}{1-\sin\phi} + \sigma_3\frac{1+\sin\phi}{1-\sin\phi}$$

$$\sigma_c = 2c\frac{\cos\phi}{1-\sin\phi} = 2c\tan\left(45^\circ + \frac{\phi}{2}\right) \quad \left(\because \tan\frac{\phi}{2} = \sqrt{\frac{1-\cos\phi}{1+\cos\phi}}\right)$$

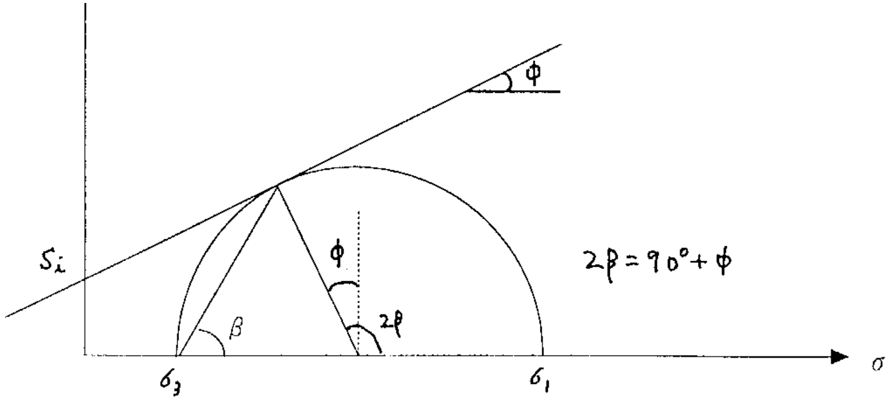


표4.1 암석의 점착강도와 내부마찰각

암 종	공극률 (%)	점착강도 (MPa)	내부마찰각 (도)	구속압의 범위 (MPa)
Berea sandstone	18.2	27.2	27.8	0 ~ 200
Bartlesville sandstone		8.0	37.2	0 ~ 203
Pottsville sandstone	14.0	14.9	45.2	0 ~ 68.9
Repetto siltstone	5.6	34.7	32.1	0 ~ 200
Muddy shale	4.7	38.4	14.4	0 ~ 200
Stockton shale		0.34	22.0	0.8 ~ 4.1
Edmonton bentonitic shale (water content 30%)	44.0	0.3	7.5	0.1 ~ 3.1
Sioux quartzite		70.6	48.0	0 ~ 203
Texas slate; loaded				
30°to cleavage		26.2	21.0	34.5 ~ 276
90°to cleavage		70.3	26.9	34.5 ~ 276
Georgia marble	0.3	21.2	25.3	5.6 ~ 68.9
Wolf Camp limestone		23.6	34.8	0 ~ 203
Indiana limestone	19.4	6.72	42.0	0 ~ 9.6
Hansmark dolomite	3.5	22.8	35.5	0.8 ~ 5.9
Chalk	40.0	0	31.5	10 ~ 90
Blaine anhydrite		43.4	29.4	0 ~ 203
Inada biotite granite	0.4	55.2	47.7	0.1 ~ 98
Stone Mountain granite	0.2	55.1	51.0	0 ~ 68.9
Nevada Test Site basalt	4.6	66.2	31.0	3.4 ~ 34.5
Schistose gneiss				
90°to schistocity	0.5	46.9	28.0	0 ~ 69
30°to schistocity	1.9	14.8	27.6	0 ~ 69

4.3 Griffith 이론

- 균열의 존재로 인해 감소한 변형률에너지의 일부가 균열의 성장에 사용됨.
- 균열주변 에너지(변형률에너지감소분−균열표면에너지:$W_e - W_s$)가 최대가 될 때 균열이 성장함.

$$W = W_e - W_s = \frac{\pi c^2 \sigma_0^2}{E} - 4c\alpha \quad (\alpha : \text{단위균열표면에너지})$$

$$\frac{\partial W}{\partial c} = 2\frac{\pi c \sigma_0^2}{E} - 4\alpha = 0$$

$$\sigma_t = \sqrt{\frac{2\alpha E}{\pi c}} \quad (plane \;\; stress)$$

$$\sigma_t = \sqrt{\frac{2\left(1-\nu^2\right)\alpha E}{\pi c}} \quad (plane \;\; strain)$$

- 파괴조건식: $(\sigma_1 - \sigma_3)^2 = 8\sigma_t(\sigma_1 + \sigma_3)$ $(\sigma_1 + 3\sigma_3 > 0$ 일 때)

  $\sigma_3 = \sigma_t$ $(\sigma_1 + 3\sigma_3 < 0$ 일 때)
- 단점: 압축강도가 인장강도의 8배로 나타남

4.4 전단변형에너지설

- 단축압축상태하 단위부피 미소직육면체의 변형률에너지:

$$W = \frac{1}{2}\sigma\,\varepsilon\,dx\,dy\,dz \rightarrow W = \frac{1}{2}\sigma\,\varepsilon$$

- 삼축압축상태하 단위부피 미소직육면체의 변형률에너지:

$$W = \frac{1}{2}\left(\sigma_1\,\varepsilon_1 + \sigma_2\,\varepsilon_2 + \sigma_2\,\varepsilon_2 - \left(\sigma_2\nu\varepsilon_1 + \sigma_3\nu\varepsilon_1 + \sigma_1\nu\varepsilon_2 + \sigma_3\nu\varepsilon_2 + \sigma_1\nu\varepsilon_3 + \sigma_2\nu\varepsilon_3\right)\right)$$

$$= \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right)$$

- W의 분석: 체적변형에너지 + 전단변형에너지

$$W = W_v + W_s = \frac{1-2\nu}{6E}\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2 + \frac{1+\nu}{6E}\left(\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right)$$

$$= \frac{1}{2}pe_v + \frac{1}{12G}\left(\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right)$$

$$\text{where } p\left(= \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right) = ke_v = \frac{E}{3(1-2\nu)}e_v\,,\ \tau = G\gamma = \frac{E}{2(1+\nu)}\gamma$$

4.4 전단변형에너지설

– 전단변형에너지를 이용한 파괴기준

- Hencky–Huber: $\left((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right)=12GW_s$
- Von Mises: $\left((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right)=2\tau_0^2$
- Nadai: $\left((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right)=9\tau_{oct}^2$

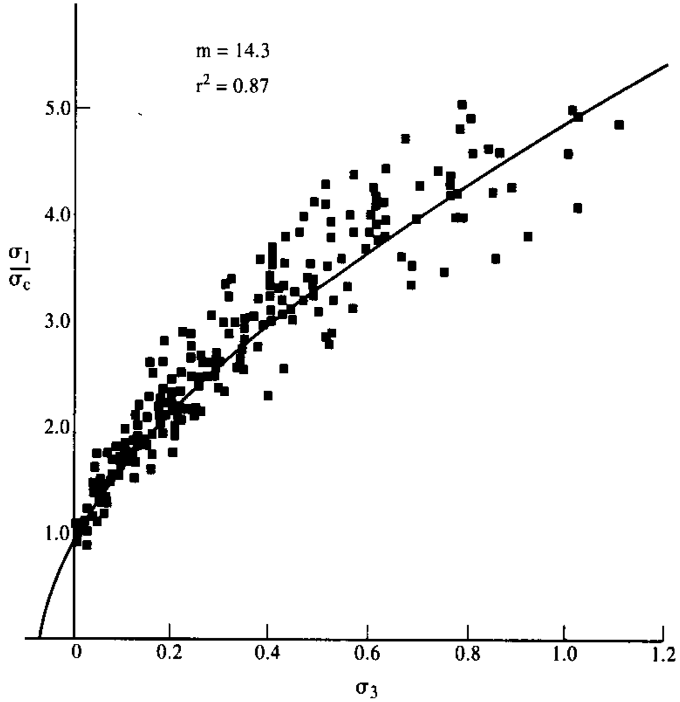
– 특징: 중간주응력 σ_2가 파괴거동에 영향을 미치는 것으로 나타남.
– 단점: 압축강도와 인장강도가 같은 값을 갖는 것으로 나타남.

4.5 Hoek-Brown의 경험식

- 암석의 파괴시 σ_1과 σ_3의 경험적 관계식 제안 (1990)

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$$

여기서, s는 암석의 파쇄도 (무결암에서 1), m은 암석 입자간 결합도

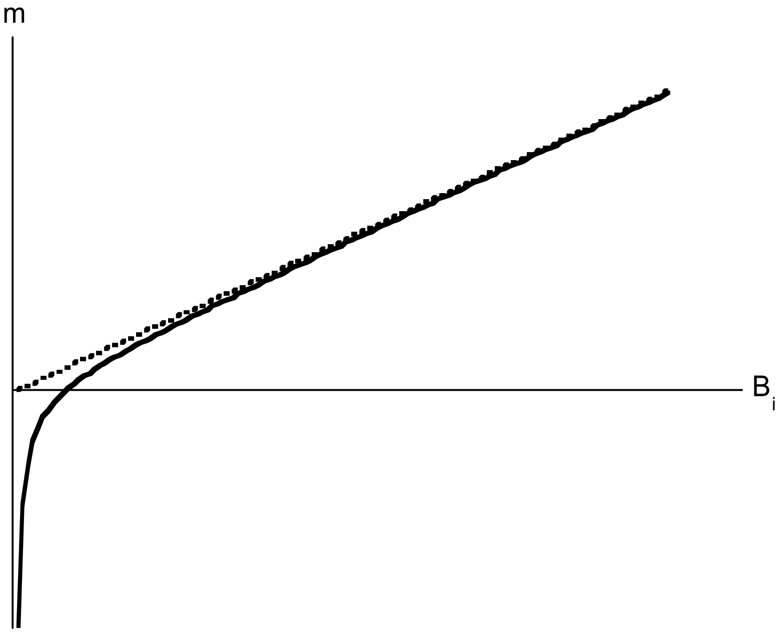


4.5 Hoek-Brown의 경험식

- 인장강도, 취성도(B_i)

$$\sigma_t = -\frac{\sigma_c\left(\sqrt{m^2+4s}-m\right)}{2}$$

$$\rightarrow m - \frac{2\sigma_t}{\sigma_c} = \sqrt{m^2+4s}\ ,\ m = \frac{\sigma_t}{\sigma_c} - \frac{s\sigma_c}{\sigma_t} = sB_i - \frac{1}{B_i}$$



Approximate relationship between rock mass quality and material constants

Disturbed rock mass m and s values *undisturbed rock mass m and s values*

EMPIRICAL FAILURE CRITERION

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$$

σ_1 = major principal stress
σ_3 = minor principal stress
σ_c = uniaxial compressive strength of intact rock, and
m and s are empirical constants.

		CARBONATE ROCKS WITH WELL DEVELOPED CRYSTAL CLEAVAGE *dolomite, limestone and marble*	LITHIFIED ARGILLACEOUS ROCKS *mudstone, siltstone, shale and slate (normal to cleavage)*	ARENACEOUS ROCKS WITH STRONG CRYSTALS AND POORLY DEVELOPED CRYSTAL CLEAVAGE *sandstone and quartzite*	FINE GRAINED POLYMINERALLIC IGNEOUS CRYSTALLINE ROCKS *andesite, dolerite, diabase and rhyolite*	COARSE GRAINED POLYMINERALLIC IGNEOUS & METAMORPHIC CRYSTALLINE ROCKS – *amphibolite, gabbro gneiss, granite, norite, quartz-diorite*
INTACT ROCK SAMPLES						
Laboratory size specimens free	m	7.00	10.00	15.00	17.00	25.00
from discontinuities	s	1.00	1.00	1.00	1.00	1.00
CSIR rating: RMR = 100	*m*	*7.00*	*10.00*	*15.00*	*17.00*	*25.00*
NGI rating: Q = 500	*s*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*
VERY GOOD QUALITY ROCK MASS						
Tightly interlocking undisturbed rock	m	2.40	3.43	5.14	5.82	8.56
with unweathered joints at 1 to 3m.	s	0.082	0.082	0.082	0.082	0.082
CSIR rating: RMR = 85	*m*	*4.10*	*5.85*	*8.78*	*9.95*	*14.63*
NGI rating: Q = 100	*s*	*0.189*	*0.189*	*0.189*	*0.189*	*0.189*
GOOD QUALITY ROCK MASS						
Fresh to slightly weathered rock, slightly	m	0.575	0.821	1.231	1.395	2.052
disturbed with joints at 1 to 3m.	s	0.00293	0.00293	0.00293	0.00293	0.00293
CSIR rating: RMR = 65	*m*	*2.006*	*2.865*	*4.298*	*4.871*	*7.163*
NGI rating: Q = 10	*s*	*0.0205*	*0.0205*	*0.0205*	*0.0205*	*0.0205*
FAIR QUALITY ROCK MASS						
Several sets of moderately weathered	m	0.128	0.183	0.275	0.311	0.458
joints spaced at 0.3 to 1m.	s	0.00009	0.00009	0.00009	0.00009	0.00009
CSIR rating: RMR = 44	*m*	*0.947*	*1.353*	*2.030*	*2.301*	*3.383*
NGI rating: Q = 1	*s*	*0.00198*	*0.00198*	*0.00198*	*0.00198*	*0.00198*
POOR QUALITY ROCK MASS						
Numerous weathered joints at 30-500mm,	m	0.029	0.041	0.061	0.069	0.102
some gouge. Clean compacted waste rock	s	0.000003	0.000003	0.000003	0.000003	0.000003
CSIR rating: RMR = 23	*m*	*0.447*	*0.639*	*0.959*	*1.087*	*1.598*
NGI rating: Q = 0.1	*s*	*0.00019*	*0.00019*	*0.00019*	*0.00019*	*0.00019*
VERY POOR QUALITY ROCK MASS						
Numerous heavily weathered joints spaced	m	0.007	0.010	0.015	0.017	0.025
<50mm with gouge. Waste rock with fines.	s	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
CSIR rating: RMR = 3	*m*	*0.219*	*0.313*	*0.469*	*0.532*	*0.782*
NGI rating: Q = 0.01	*s*	*0.00002*	*0.00002*	*0.00002*	*0.00002*	*0.00002*