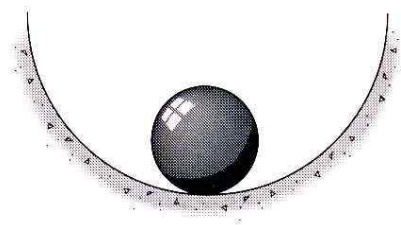


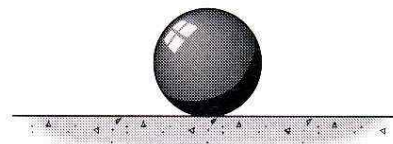
9. Analyzing Columns

9.1 Introduction

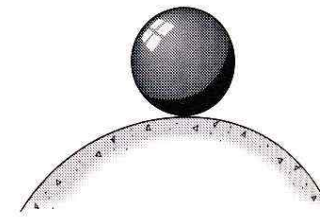
- Buckling: the sudden large deformation of a structure due to a slight increase of an existing compressive load
- Buckling of a column is caused not by failure of the column material but by deterioration of what was a stable state of equilibrium to an unstable one.
- Three equilibrium states of beam deflection:



(a)



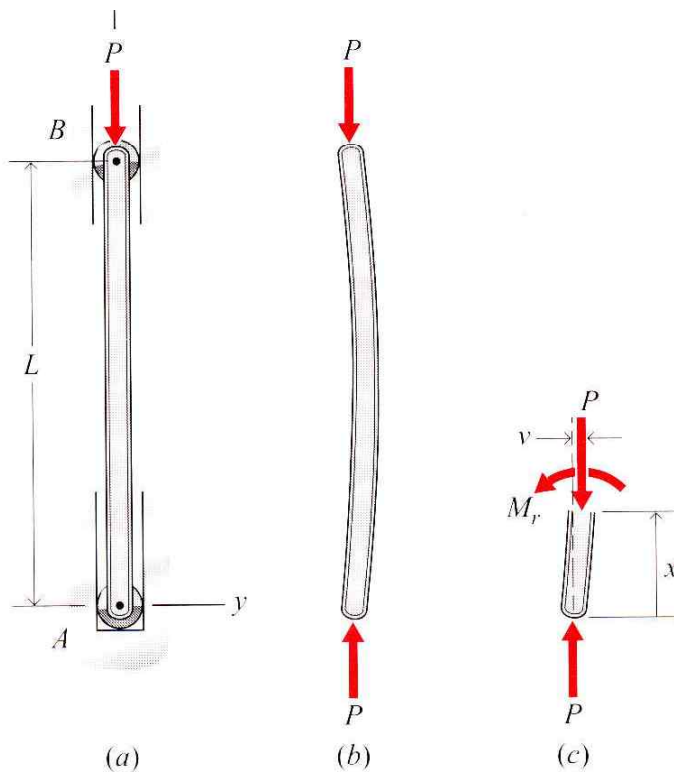
(b)



(c)

9.2 Buckling of long, straight columns

- The minimum axial compressive load for which a pin-ended column will experience lateral deflections



$$EI \frac{d^2v}{dx^2} = M_r = -Pv \rightarrow \frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

(homogeneous 2nd order linear differential equation)

$$v = A \sin px + B \cos px$$

$$\left(-p^2 + \frac{P}{EI}\right)(A \sin px + B \cos px) = 0$$

$$\therefore p^2 = \frac{P}{EI}$$

B.C.: $v = 0$ at $x = 0$ or $x = L$

$$\rightarrow v = A \sin pL = 0 \rightarrow pL = n\pi \quad (n = 1, 2, 3 \dots)$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}: \text{ Euler buckling load}$$

9.2 Buckling of long, straight columns

When the radius of gyration about the axis of bedding is set r

$$I = Ar^2, \quad \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} = \sigma_{cr}$$

$L/r = \text{slenderness ratio}$

Euler buckling condition: $L/r > 140$ (steel columns)

Short compression condition: $L/r < 40$ (steel columns)

9.2 Buckling of long, straight columns

- Example Problem 9-1

An 8-ft long pin-ended timber column [$E = 1.9(10^6)$ psi and $\sigma_e = 6,400$ psi] has a 2×4 in. rectangular cross section.

- Slenderness ratio
- Euler buckling load
- The ratio of the axial stress under the action of the buckling load to the elastic strength σ_e of the material

9.2 Buckling of long, straight columns

- Example Problem 9-2

A 12-ft long pin-ended column is made of 6061-T6 aluminum alloy [$E = 10(10^6)$ psi and $\sigma_y = 40,000$ psi]. The column has a hollow circular cross section with an outside/inside diameter of 5 in./4in.

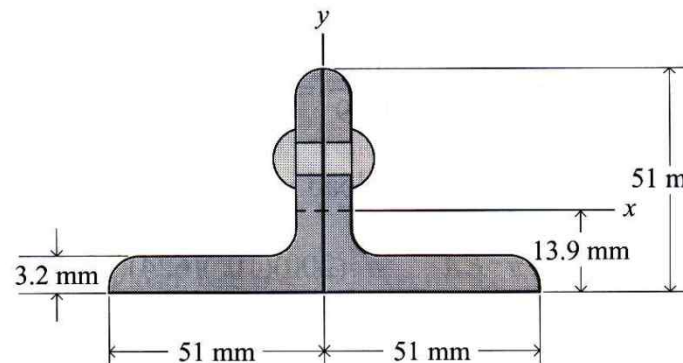
- The smallest slenderness ratio for which the Euler buckling load equation is valid
- Critical buckling load

9.2 Buckling of long, straight columns

- Example Problem 9-3

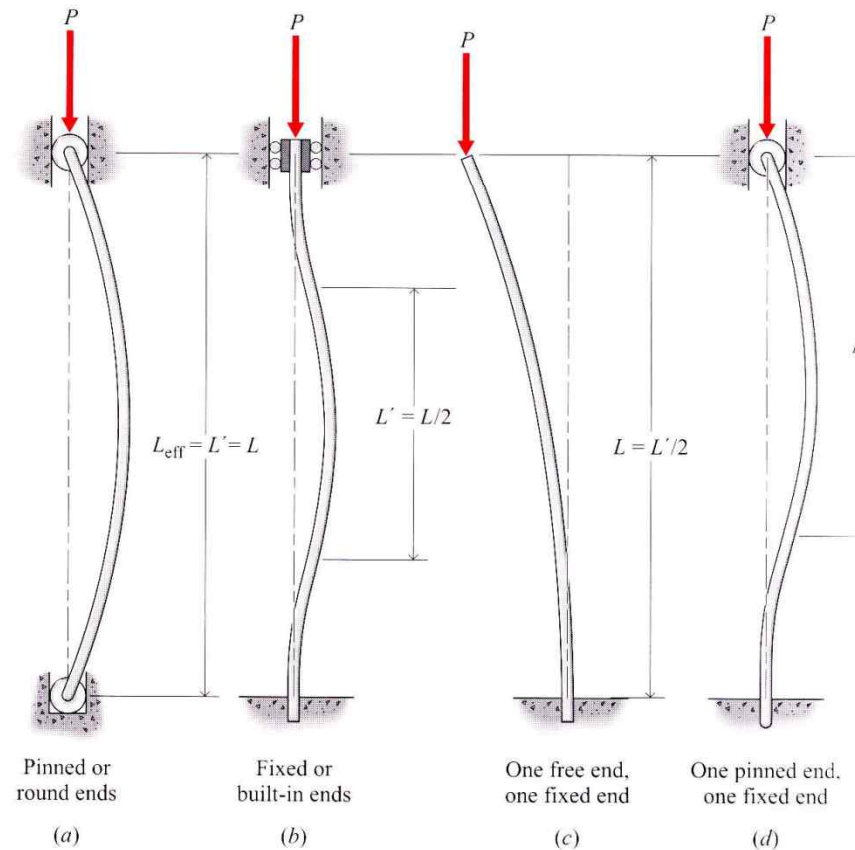
Two $51 \times 51 \times 3.2$ mm structural angles (gyration radius: $r_z = 10.1$ mm, $r_x = 15.9$ mm $A = 312$ mm², $E = 200$ GPa) 3m long will be used as a pin-ended column. Determine the slenderness ratio and the Euler buckling load if

- The two angles are not connected, and each acts as an independent member.
- The two angles are fastened together to act as a unit.



9.3 Effects of different idealized end conditions

- The effective column length can be used for different end conditions: the distance between successive inflection points or points of zero moments.

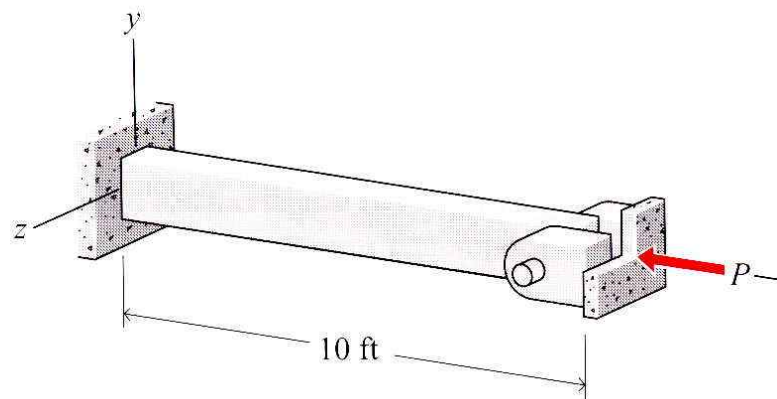


9.3 Effects of different idealized end conditions

- Example Problem 9-4

A 10-ft long structural steel column [$E = 29,000$ ksi] must support an axial compressive load P . The column has a 1×2 -in. rectangular cross section.

- Max. safe load for the column if a factor of safety of 2 with respect to failure by buckling is specified.



9.4 Empirical column formulas-Centric loading

- Euler's formula is valid when the axial compressive stress for a column is less than the yield strength.

Euler buckling condition: $L/r > 140$ (steel columns)

Short compression condition: $L/r < 40$ (steel columns)

- Intermediate range: the range between the compression block and the slender ranges

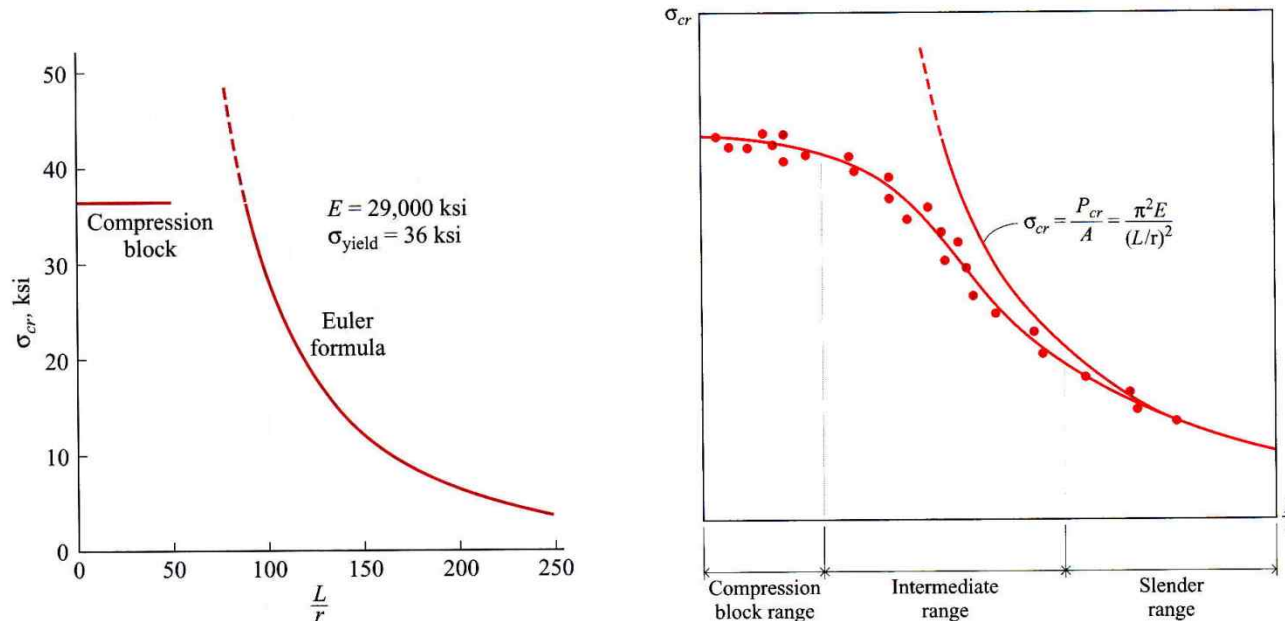


Table 9-1 Some Representative Column Codes for Centric Loading

Code No.	Source	Material	Compression-Block and/or Intermediate-Range Formulas and Limitations (L/r is the effective ratio L/r)	Slender Range
1	a	Structural steel with a yield point σ_y	$0 \leq \frac{L}{r} \leq C_c$ $\sigma_{\text{all}} = \frac{\sigma_y}{\text{FS}} \left[1 - \frac{1}{2} \left(\frac{L/r}{C_c} \right)^2 \right]$ $C_c^2 = \frac{2\pi^2 E}{\sigma_y}$ $\text{FS} = \frac{5}{3} + \frac{3}{8} \left(\frac{L/r}{C_c} \right) - \frac{1}{8} \left(\frac{L/r}{C_c} \right)^3$	$\frac{L}{r} \geq C_c$ $\sigma_{\text{all}} = \frac{\pi^2 E}{1.92(L/r)^2}$
2	b	2014-T6 (Alclad) Aluminum alloy	$\frac{L}{r} \leq 12$ $\sigma_{\text{all}} = 28 \text{ ksi}$ $= 193 \text{ MPa}$ $12 \leq \frac{L}{r} \leq 55$ $\sigma_{\text{all}} = \left[30.7 - 0.23 \left(\frac{L}{r} \right) \right] \text{ ksi}$ $= \left[212 - 1.585 \left(\frac{L}{r} \right) \right] \text{ MPa}$	$\frac{L}{r} \geq 55$ $\sigma_{\text{all}} = \frac{54,000}{(L/r)^2} \text{ ksi}$ $= \frac{372(10^3)}{(L/r)^2} \text{ MPa}$
3	b	6061-T6 Aluminum alloy	$\frac{L}{r} \leq 9.5$ $\sigma_{\text{all}} = 19 \text{ ksi}$ $= 131 \text{ MPa}$ $9.5 \leq \frac{L}{r} \leq 66$ $\sigma_{\text{all}} = \left[20.2 - 0.126 \left(\frac{L}{r} \right) \right] \text{ ksi}$ $= \left[139 - 0.868 \left(\frac{L}{r} \right) \right] \text{ MPa}$	$\frac{L}{r} \geq 66$ $\sigma_{\text{all}} = \frac{51,000}{(L/r)^2} \text{ ksi}$ $= \frac{351(10^3)}{(L/r)^2} \text{ MPa}$
4	c	Timber with a rectangular cross section $b \times d$ where $d < b$	$\frac{L}{d} \leq 11$ $\sigma_{\text{all}} = F_c^*$ $11 \leq \frac{L}{d} \leq k$ $\sigma_{\text{all}} = F_c \left[1 - \frac{1}{3} \left(\frac{L/d}{k} \right)^4 \right]$ $k = 0.671 \sqrt{E/F_c}$	$k \leq \frac{L}{d} \leq 50$ $\sigma_{\text{all}} = \frac{0.30E}{(L/d)^2}$

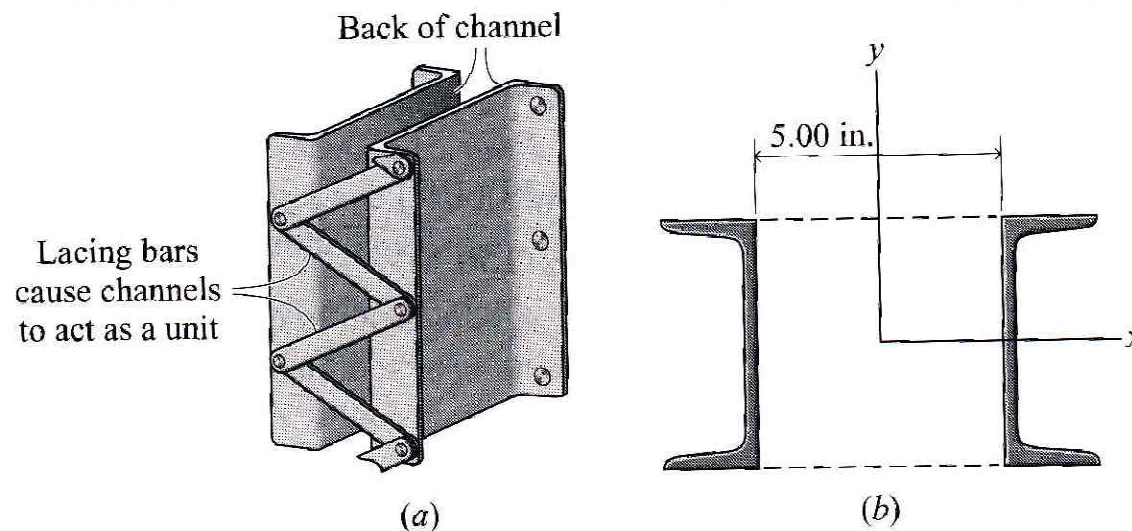
- The strength of materials has little effect on the allowable stress in long columns.

9.4 Empirical column formulas-Centric loading

- Example Problem 9-5

Two structural steel C10 × 25 [$r_x = 3.52$ in., $r_y = 0.676$ in., $x_c = 0.617$ in., $A = 7.35$ in.², $\sigma_y = 36$ ksi, $E = 29,000$ ksi] channels are latticed 5 in. back to back to form a column.

- Max. allowable load for effective lengths of 25 ft and 40 ft. (Use Code 1 for structural steel)



9.4 Empirical column formulas-Centric loading

- Example Problem 9-6

A Douglas fir [$E = 11$ GPa, and $F_c = 7.6$ MPa] timber column with an effective length of 3.5 m has a 150×200 -mm rectangular cross section.

- Max. compressive load permitted by Code 4

4	c	Timber with a rectangular cross section $b \times d$ where $d < b$	$\frac{L}{d} \leq 11 \quad \sigma_{\text{all}} = F_c^*$ $11 \leq \frac{L}{d} \leq k \quad \sigma_{\text{all}} = F_c \left[1 - \frac{1}{3} \left(\frac{L/d}{k} \right)^4 \right]$ $k = 0.671 \sqrt{E/F_c}$	$k \leq \frac{L}{d} \leq 50$ $\sigma_{\text{all}} = \frac{0.30E}{(L/d)^2}$
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9.4 Empirical column formulas-Centric loading

- **Home work problems: 9-4, 15, 25, 40, 53**