A sample paper for class review

Title: Reservoir Characterization using EnKF and nonparametric approach for highly non-Gaussian permeability fields

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Abstract

An invserse scheme is developed for reservoir characterization using ensemble Kalman filter (EnKF) and non-parametric approach. EnKF has been studied by many researchers due to its novelties on easy access to parallel processing, recursive data processing, and quantifying the amount of uncertainties on its results. Previous studies have shown poor characterization results with non-Gaussian permeability distributions.

In this study, non-parametric approach is used to characterize permeability distribution with strong non-Gaussian characteristics. Normal score transform can be used to transform a distribution, which is explained by non-parametric approach, to a Gaussian distribution. The Gaussian assumption of EnKF in the assimilation step can be satisfied by using normal score transformation.

For characterization effect of initial ensembles from different distributions, initial ensembles with higher similarity to the reference distribution would have more successful characterization results than with less similarity. Additional improvement in reservoir characterization results is obtained by using normal score transformation to normalize permeability values in the EnKF assimilation step.

Keywords: ensemble Kalman filter, reservoir characterization, non-parametric approach,

normal score transformation, highly non-Gaussian distribution

Introduction

Making a reliable production forecasting of given reservoir is essential for having an optimum production schedule, deciding further development plan and accessing reserves. For having reliable production forecasting, all available dynamic data, such as production data and pressure data of each well, are needed to be fully utilized in reservoir characterization. Conventional characterization methods, which are based on gradient of objective function, have strengths on quick convergence. However they have deficiencies on converge on local minimum and they should re-process all procedures to update and honor additional dynamic reservoir data acquired as the reservoir produce more oil, gas, and water.

Ensemble Kalman Filter (EnKF) is a characterization method based on stochastic approach which can conduct characterization by using the difference between estimations on ensemble members, equi-potential realizations of parameters including uncertainties, and actual observations with covariance and uncertainties of observation. It is characterized by its suitability on real-time characterization and to give quantitative assessment on uncertainties on produced values by EnKF (Evensen, 1994). It has been mainly studied and utilized on Earth Science, especially meteorology on weather forecasting and oceanography on oceanic current analysis. Since EnKF is introduced in reservoir engineering as a reservoir characterization method in 2002, by Naevdal and Vefring, it is actively discussed and researched for real-time reservoir characterization.

In the previous studies, there are two distinguished problems on reservoir characterization using EnKF. First problem is overshooting and undershooting problem which is having unreasonable big or small value on permeability or porosity from the result of EnKF applications. Second one is the filter divergence problem, forecasting of dynamic values, like production rate or well flowing pressure, is diverged from actual values.

In the previous studies, there were researches on the effect of number of ensemble members on characterization results (Wen and Chen, 2005; Park et al., 2005; Evensen et al., 2007). However there are little studies on the effect of generation of ensemble members and characterization results. There are also no studies for the EnKF application scheme on parameters which have distribution characteristics far from Gaussian, which is the main assumption on parameter distribution on EnKF.

In this study, EnKF application for reservoir characterization on a permeability field which has highly non-Gaussian distribution on its parameter values is assessed. It mainly consists of generation of initial ensemble members which are generated by considering distribution characteristic of parameters to characterization and using proper transformation of parameters to meet Gaussian assumption on EnKF.

Ensemble Kalman Filter

Kalman Filter (KF) algorithm is a recursive process which minimizes estimation error in linear models and suitable for real time noise filtering (Kalman, 1960). Ensemble Kalman Filter was developed to adapt KF algorithm for nonlinear filtering problems (Evensen, 1994). EnKF uses ensemble members, multiple equi-potential realizations generated by using available data, such as observations, for independent forecasting. It assumes the mean of ensemble members as a true value of parameter and considers their covariance as a measure of the estimation error. EnKF is consisted in two steps, forecasting step which gives estimation and measure of estimation error and assimilation step which gives corrected value by considering observations and correction error estimation. Forecasting step can be stated by following equations.

$$\hat{\mathbf{x}}^{-}_{k} = f(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_{k-1}) \tag{1}$$

$$\mathbf{P}_k^- = \mathrm{E}[\mathbf{e}_k^- \mathbf{e}_k^{-\mathrm{T}}] = \frac{1}{N_e - 1}\sum_{j=1}^{N_e} \mathbf{e}_{k,j}^- \mathbf{e}_{k,j}^{-\ \mathrm{T}} \tag{2}$$

$$\mathbf{e}_{k,j}^- = \hat{\mathbf{x}}_{k,j} - \overline{\mathbf{x}}_k \tag{3}$$

Equation (1) means forecasting by forward model. In this study, forward model is 2 Phase Black Oil IMPES simulator that considers oil and water for two phases. Subscript k is the time step, x is the state variable, u is the control variables such as boundary conditions. Postscript "^" means the variables which are input in forward model and comes out from forward models are not true values, but estimations. Priori covariance matrix of estimation is expressed by equation (2). N_e means the numbers of ensemble members. Postscript (-) means it is priori state vector. Equation (3) is estimate error of ensemble member. Subscript j means its identification of j-th ensemble and $\overline{x}_k$ means average of all ensemble members. In the circumstance that it is impossible to know the true value of concerning object, considering the average of all ensemble as true is one of the most important assumptions on EnKF.

As observations are available in time step k, EnKF generate updated parameter by combining estimated value and observation values and gives quantitative assessment of its uncertainties. Equation (4) is Kalman Gain which is calculated by correlation of priori estimation error and observation error covariance. Matrix H is consisted by 0 and 1 and it is used to correspond priori estimation error matrix and observation error matrix in the situation which sizes are not the same.

$$\mathbf{K} = \mathbf{P}_k^- \mathbf{H}^{\mathrm{T}} \left(\mathbf{H}\mathbf{P}_k^- \mathbf{H}^{\mathrm{T}} + \mathbf{R}\right)^{-1} \tag{4}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_k^-) \tag{5}$$

$$\mathbf{P}_k = \mathrm{E}[\mathbf{e}_k \mathbf{e}_k^{\ \mathrm{T}}] = \frac{1}{N_e - 1}\sum_{j=1}^{N_e} \mathbf{e}_{k,j} \mathbf{e}_{k,j}^{\ \mathrm{T}} \tag{6}$$

By combining Kalman Gain and equation (5), assimilation that reflects observation is achieved. As can be seen is equation (5), the assimilation will be small if the difference between estimation and observation is small, and assimilation will be achieved in the way to reflect observation if estimation error is comparatively bigger than observation error. Left had side of equation (5) has no superscript (-) means it is posteriori state vector that is generated as assimilation. Equation (6) is posteriori covariance matrix which assesses the amount of uncertainties after assimilation. This whole procedure of EnKF can be summarized in Figure 1.

Reservoir Characterization using EnKF

Many of previous studies generate synthetic permeability fields which has lognormal distributions under the assumption that the distribution of permeability values would follow log-normal distribution (Naevdal, 2003; Wen and Chen, 2005; Park, 2006; Evensen, 2007). They generate initial ensemble members by using sample values of permeability from injection and production wells by assuming they are acquired by coring from each well. The generation of initial ensemble members can be described as Figure 2. To make permeability of the generated initial ensemble members following the lognormal distribution, they took the logarithm value of sampled permeability to generate the permeability field by geostatistics such as Sequential Gaussian Simulation (SGS) and then convert those logarithm values to real permeability by using exponential.

In the assimilation step, to meet the Gaussian assumption on distribution of priori state of state vector in EnKF, logarithm permeability values of all ensemble members are used in equations (2)~(6) to update parameters to honor the observations on the given time step. After the assimilation step, all assimilated parameter should be converted to actual value by exponential to be used in forecasting step. The EnKF update procedure is explained by Figure 3.

If the distribution of permeability value is highly non-Gaussian in a given reservoir, every initial ensemble has less similarity with actual field in terms of distribution characteristics. It also uses logarithm value of permeability on the assimilation step which possibly misses the distribution characteristics of given permeability field. Those problems could be possible reasons of overshooting problems and filter divergence on previous studies.

Non-parametric Approach and lognormal distribution in EnKF for reservoir with highly non-Gaussian distribution of permeability value

Parametric approaches explain the distribution by mathematical assumptions and some parameters, such as mean and variance. In case the distribution could not be simplified by parameter approaches, we can explain the distribution by couple of discrete values of it and their cumulative possibilities for the given values, nonparametric approach. If we have empirical understanding of shape of parameter distribution, we can explain the distribution by using non-parametric approach. To apply those parameters explained by non-parametrically to methods based on Gaussian assumption, it is possible to use normal score transformation to meet the assumption. The procedure for transformation a value explained non-parametrically to Gaussian distribution can be explained by Figure 4. Left graph is the empirical CDF of the distribution which parameters are explained non-parametrically. Right graph is CDF of standard normal distribution. The actual transformation is linked by using cumulative proportion as seen in Figure 5.

If we know the rough characteristics of permeability distribution for the reference field, empirical CDF which are more representative than normal distribution, it is possible to generate initial ensemble members which contain the distribution characteristics of reference field. The process to generation of initial ensembles by normal score transformation is as same as conventional generation of initial ensembles as explain in Figure 2, except it uses normal score transformation, rather than log and exponential transformation.

If the ensemble members have non-Gaussian permeability distribution, those parameters can be transformed to meet Gaussian assumption in the assimilation step by normal score transformation, replacing log and exponential transformation to normal score transformation in Figure 3.

Results

A synthetic reservoir is generated as a reference field with highly non-Gaussian permeability distribution. It is 2D synthetic permeability field with size of 1,000 ft by 1,000 ft with 25 ft height. It consists by 20 by 20, total 400 grid blocks. Average permeability is 49.5 md, with standard deviation of 23.9md, 5md as minimum and 100md as its maximum. Porosity for all blocks is 0.2. Figure 6(a) is the generated reference field and Figure 7(a) is its probability distribution. It can be seen that the reference field is far from normal distribution. Table 1 contains default field data of the reference field. The reference field is produced by inverted five-spot pattern with waterflooding which has one water injection well inter center and four producing wells in the corners. Triangle in the center of Figure 6(a) is injector and 4 circles in the corners of Figure 6(a) are producers. Observations are made for 7 times, 50, 100, 200, 300, 400, 500, 600 days from 1st production.

For the generation of initial ensembles, 9 sampled permeability values from 9 points in Figure 6(a) are used in SGS. For the assimilation step, oil and water production rates from 4 producers and well flowing pressure (Pwf) from injector, total 9 dynamic data, are used. The reference production data and Pwf are on Figure 8(a) and (b).

4 different sets of initial ensembles are generated, Base case which use logarithm values (conventional approaches from previous studies), Case 1 with triangle pdf, Case 2 with CDF from core samplings, Case 3 with CDF of the reference field. Base case is naturally follows lognormal distribution.

To see the effect of initial ensembles on EnKF characterization results, 4 different initial ensembles are used separately in conventional EnKF reservoir characterizations explained in

Figures 2 and 3. Characterized fields are shown in Figures 6(b), (c), (e), and (g). Result of Base case shows that EnKF figure out the main trend of high-low permeability, but it shows typical problems like overshooting on parameter values as seen too many red grids which means grids they have bigger permeability than 100md. Case 1 shows worse result than Base case. Case 2 and Case 3 show improved results than Base case. It shows less number of grids with overshooting. Table 2 compares the sum of square errors (SSE) of characterized permeability of each case and the reference values. It can be seen that Case 1 has the worst result, Case 2 has 22.4% and Case 3 has 17 % SSE from Base case, which means Case 3 as the best result.

To see the result of meeting the Gaussian assumption on EnKF in the assimilation steps, comparisons were made Case A, Case B and Case C which used normal score transformation in the assimilation steps. Assimilation steps are conducted as seen in Figure 7 with normal score transformation which is explained in Figure 4 and 5. Figures 6(d), (f) and (h) are the results of Case A, B, and C, respectively. Table 2 has the SSEs from each case. There are additional reductions on SSE which directly mean additional improvements on characterization results. Distinguishable reduction on red grids which means the results has much less number of grids with overshooting in as a characterization results can be noticed. Figure 7 shows the pdf of characterization results. It can be seen that with initial ensembles which show higher resemblance with the reference field and proper transformation which leads to meet the Gaussian assumption of EnKF could lead improved characterization results.

To compare the characterization results in terms of dynamic data, such as production rates, reservoir simulation by using updated permeability fields of each case is conducted. Table 3 is SSE of OPR and WPR between those of the reference field and updated fields of each case. As same as SSEs of permeability in Table 2, updated field with initial ensembles which has more resemblance with probability distribution of the reference field permeability shows less

SSE. It also shows additional improved results when it use normal score transformation in assimilation steps to meet Gaussian assumption.

Conclusions

This study presents the EnKF applications using non-parametric approach and normal score transformation for fields which have parameters with highly non-Gaussian distribution. From the study, it is possible to get improved characterization results.

1. Using initial ensembles which reflect the distribution characterization of the reference field would improve the characterization results. The higher resemblance would lead the better results on characterization.
2. By using non-parametric approach and normal score transformation in the assimilation steps, additional improvement on characterization can be achieved. It could prevent the tendency of becoming lognormal distribution after adapting conventional EnKF to permeability field characterization.
3. Presented approach shows improved results even in the case when only a part of reference distribution is used. Higher similarities on assumed distributions lead better characterization results in terms of both characterized permeability values and simulated dynamic performances of reservoir, OPR and WPR. It seems possible to assess properness of assumed distribution features by comparing the resemblance of simulated dynamic data and those of the reference field.

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Table 1
The Default data of the reference field

Initial water saturation	0.25
Initial pressure, psia	2,000
Productivity index, bbl/day-psi	0.75
Oil viscosity, cp	3
Water viscosity, cp	1
Injection well condition, stb/day	500
Production well condition, psia	1,000

Table 2
Sum of square errors(SSE) of permeability values between the reference and updated permeability by each cases

	SSE, md^2	Percentage, %
Base case	15.8E+5	100
Case 1	132.0E+5	835.3
Case A	8.8E+5	55.8
Case 2	3.5E+5	22.4
Case B	2.8E+5	17.7
Case 3	2.7E+5	16.0
Case C	2.1E+5	13.1

Table 3
Sum of square errors(SSE) of production data between the reference and updated permeability by each cases

	SSE of OPR, $[{}^{bbl}/_{day}]^2$	SSE of WPR, $[{}^{bbl}/_{day}]^2$
Base case	7.5E+5 (100%)	1.3E+5 (100%)
Case 1	19.5E+5 (258.4%)	6.0E+5 (471.16%)
Case A	9.5E+5 (126.5%)	2.2E+5 (172.09%)
Case 2	1.9E+5 (25.8%)	0.1E+5 (10.76%)
Case B	0.7E+5 (9.8%)	0.05E+5 (4.66%)
Case 3	1.5E+5 (19.3%)	0.07E+5 (5.45%)
Case C	0.2E+4 (3.3%)	0.03E+5 (2.14%)

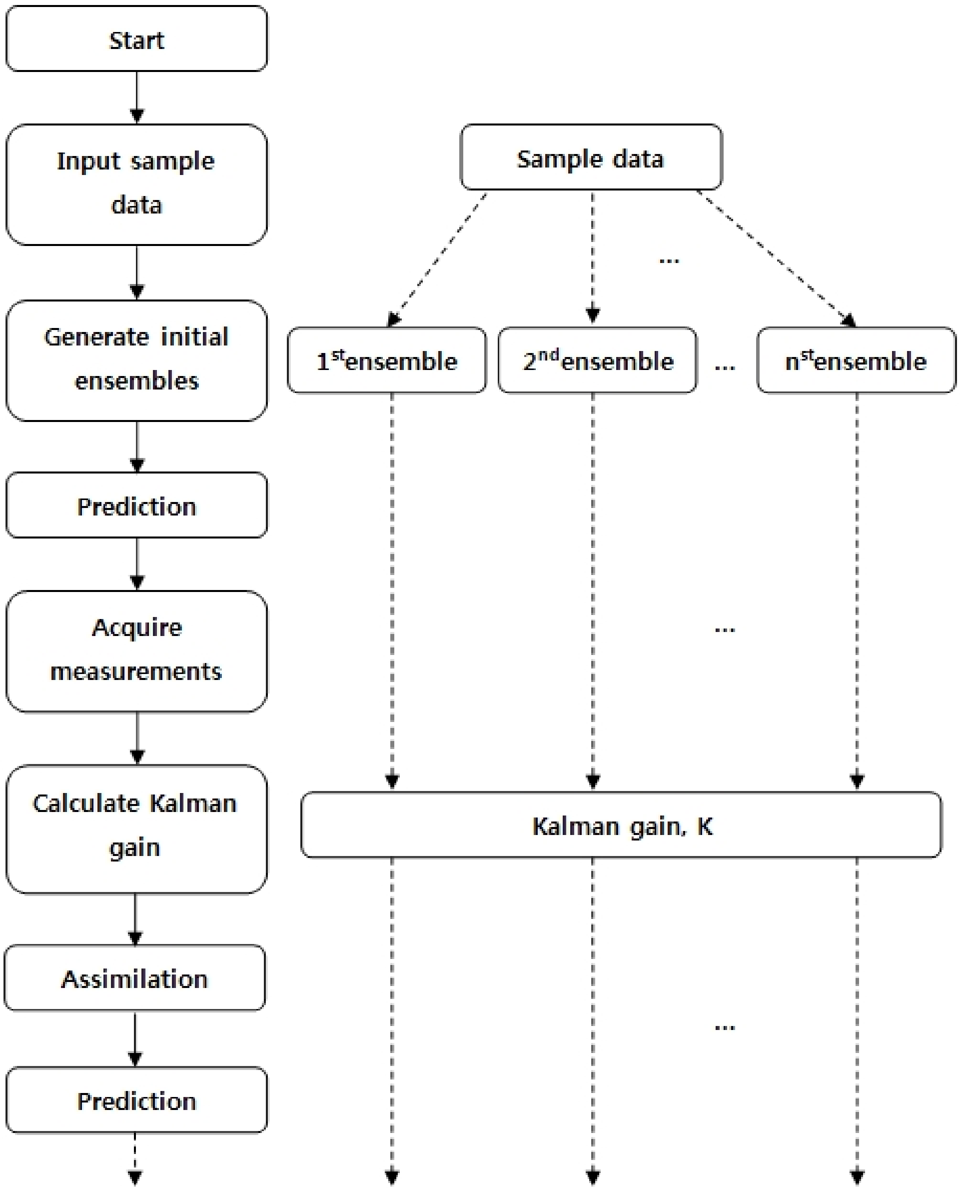

Figure 1. The flow diagram of Ensemble Kalman filter

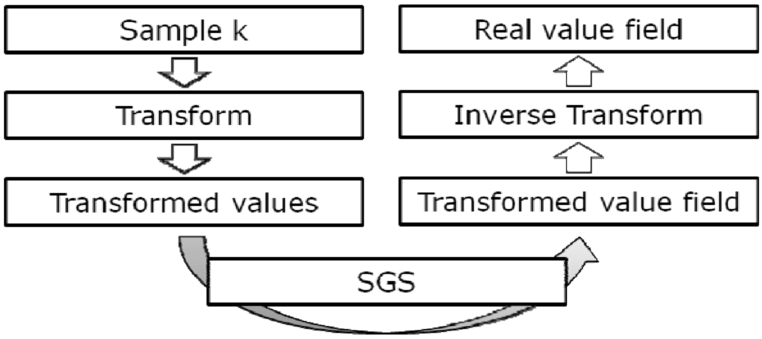

Figure 2. Flow diagram of Ensemble generation.

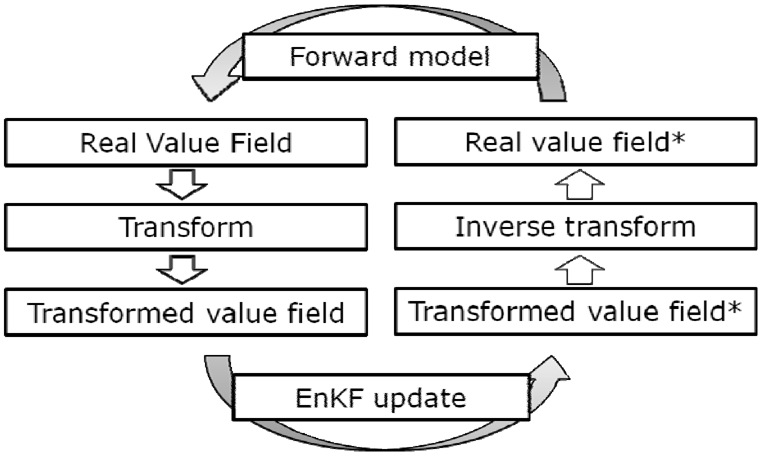

Figure 3. Flow diagram of EnKF update.

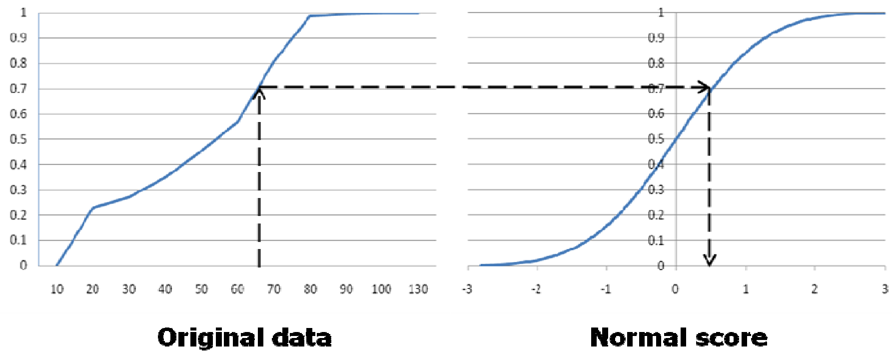

Figure 4. Graphical procedure for transformation values of one distribution into those of another.

Figure 5. Two step procedures for transformation values from one distribution to another.

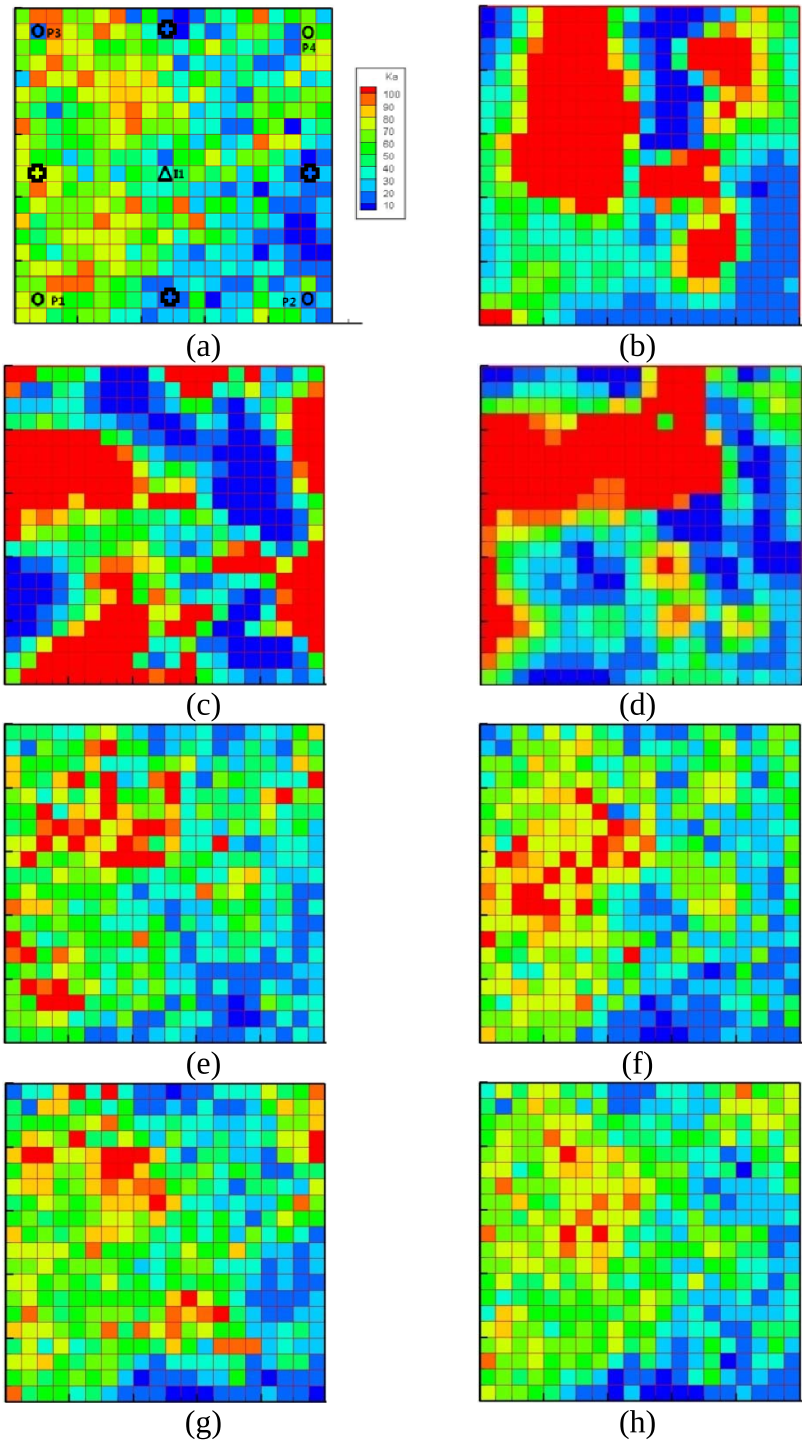

Figure 6. Permeability distribution of the reference field and the results of each case: (a) The reference field, (b) Base case, (c) Case 1, (d) Case A, (e) Case 2, (f) Case B, (g) Case 3, (h) Case C.

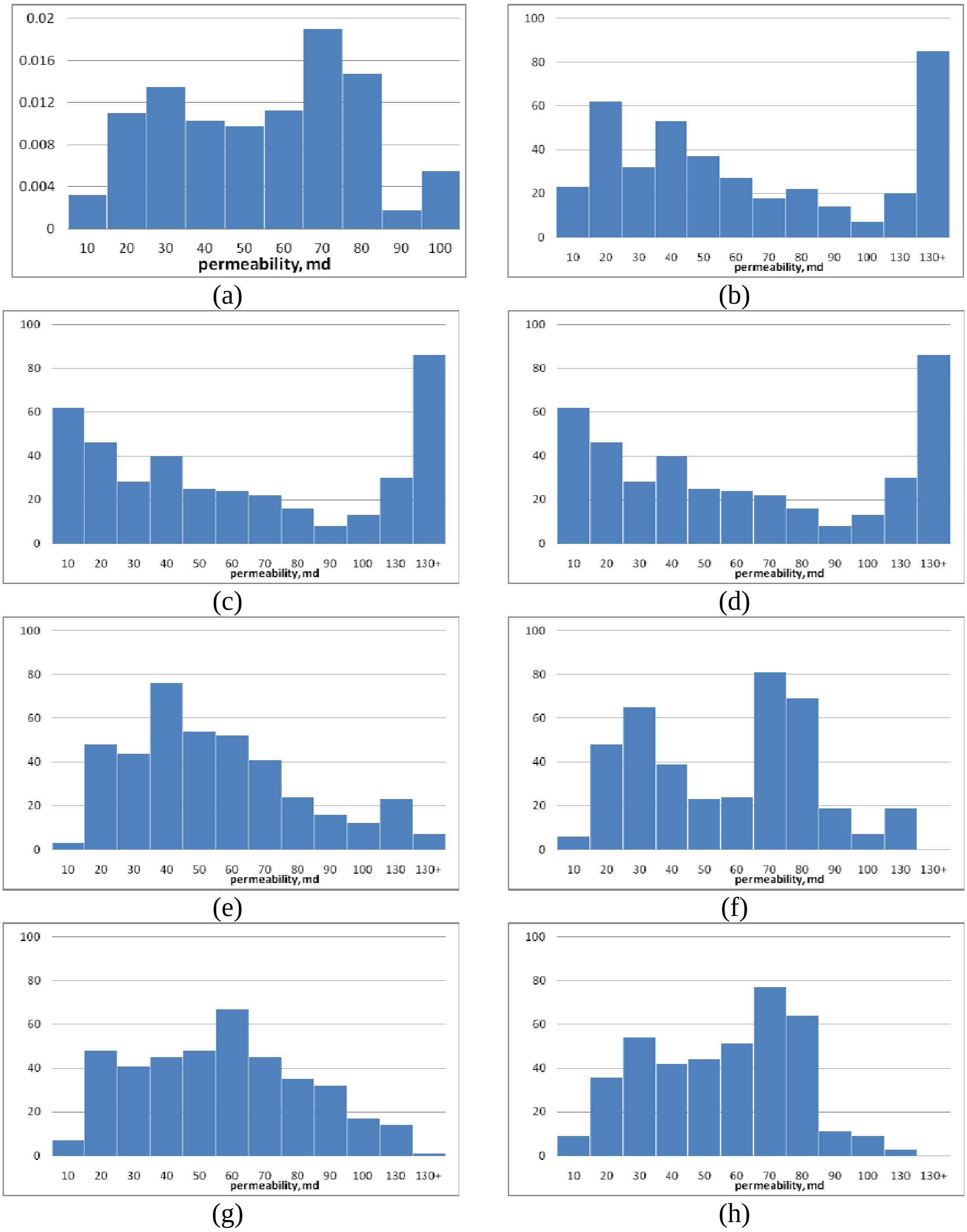

Figure 7. Histogram of permeability values of the reference field and results of each case: (a) the reference field, (b) Base case, (c) Case 1, (d) Case A, (e) Case 2, (f) Case B, (g) Case 3, (h) Case C.

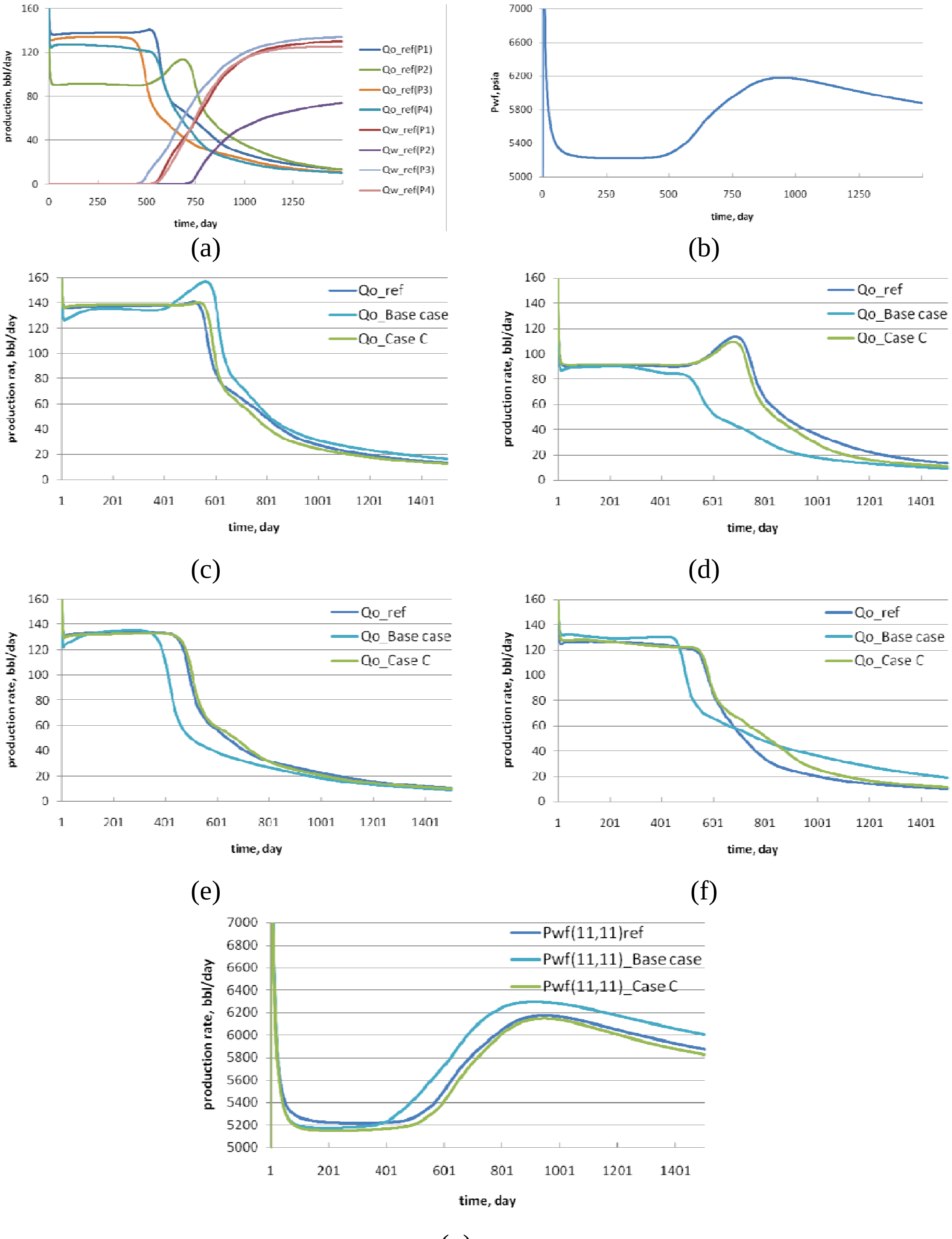

Figure 8. Production data from the reference field and comparisons of production data from the results of Base case and Case C: (a) Water and oil production from 4 producers, (b) Well flowing pressure from 1 injector, (c) ~ (f) Comparisons of production data from the results of Base case, Case C, with the reference results from producer 1 to 4 of OPR and well flowing pressure of injector in the center, respectively.