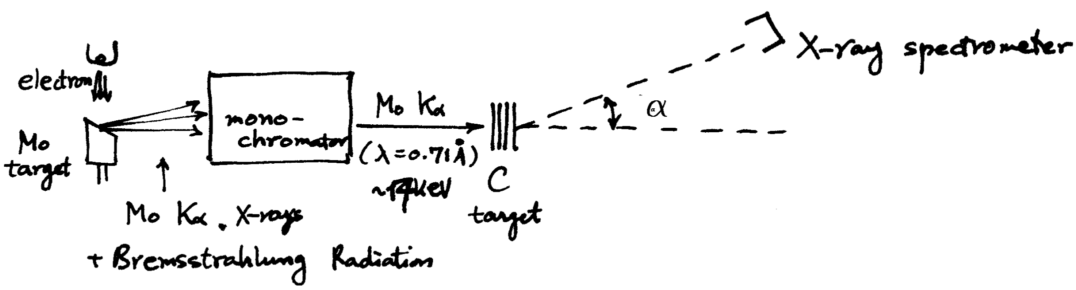
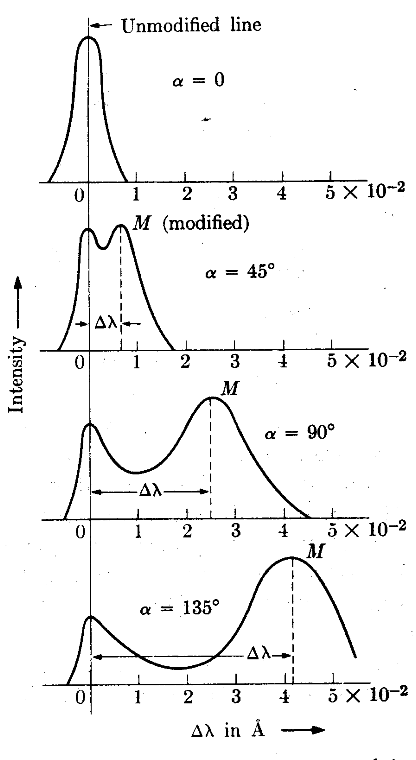
(Lecture 6)   Waves and Particles

1.   Wave -Particle Duality of Light

- EM radiation : visible light, IR, UV, x-rays, γ-rays
  wave property in interference phenomenon
  ← vivid example in two slit experiment
- photon : corpuscle property in photoelectric effect
  ← direct collision(absorption) with metallic electron

- Wave-particle duality in Compton scattering   (A.H. Compton, 1923)





Detected x-rays at any scattering angle : two components

1) A component of no change in wavelength (unmodified peak) :

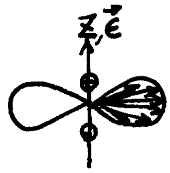
Classical electrodynamics —

Forced vibration of electron by the EM radiation(=x-ray)

→ reradiation by the same frequency (the same wavelength)

→ intensity distribution of radiation from a dipole (antenna)

→ isotropic distribution due to the random orientation of the vibrating direction(=polarization direction of the incoming EM wave)



"Thomson scattering" (=coherent scattering of x-rays by a free electron)

$$\frac{d\sigma}{d\Omega} = \tfrac{1}{2} r_0^2 (1 + \cos^2 \theta), \quad r_0 \equiv \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mc^2}$$ =2.818 fm (classical electron radius)

$$\sigma_T = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{8\pi}{3} r_0^2 = \frac{e^4}{6\pi\varepsilon_0^2 m^2 c^4}$$ =0.67 barn per electron

2) A component with a longer wavelength (modified peak) :

"Compton scattering" — no classical explanation

Photon collision with a free electron elastically

Shift of wavelength is explained by energy-momentum conservation (in relativistic mechanics)

Photon energy $E = h\nu = \dfrac{hc}{\lambda}$

Photon momentum $p = \dfrac{E}{c} = \dfrac{h\nu}{c} = \dfrac{h}{\lambda}$

Configuration before and after the collision :

Conservation of momentum : $\vec{p}_0 = \vec{p}_e + \vec{p}_1$ (1)

Conservation of energy : $\frac{hc}{\lambda_0} + m_e c^2 = \frac{hc}{\lambda_1} + \sqrt{c^2 p_e^2 + (m_e c^2)^2}$ (2)

(1)→ $\vec{p}_e = \vec{p}_0 - \vec{p}_1$, $p_e^2 = \vec{p}_e \cdot \vec{p}_e = (\vec{p}_0 - \vec{p}_1)^2 = p_0^2 + p_1^2 - 2 p_0 p_1 \cos\theta$ (3)

(2)식에서 근호를 없애고, (3)을 대입하여 정리하면,

$$2\frac{h}{\lambda_0}\frac{h}{\lambda_1}(1-\cos\theta) = 2m_e c\left(\frac{h}{\lambda_0} - \frac{h}{\lambda_1}\right) \quad \text{or}$$

$$\lambda_1 = \lambda_0 + \frac{h}{m_e c}(1-\cos\theta) = \lambda_0 + \lambda_C(1-\cos\theta)$$

where $\lambda_C \equiv \frac{h}{m_e c}$ =0.024 Å (Compton wavelength)

- 논의 :
  1) Thomson 산란 : energy 전달 $\to 0$, momentum 전달 $\neq 0$ (방향만 변화)
     Compton 산란의 저에너지 극한의 경우
     Compton 산란 : energy 전달 $\neq 0$, momentum 전달 $\neq 0$
     두 경우 모두 자유 전자를 대상으로 하고 있으며, 실제 실험자료들은 거의 다 원자에 구속된 전자를 대상으로 하므로 구속효과를 포함함.
  2) Compton 산란의 미분 단면적은 상대론적 양자론을 이용하여 유도되는 것으로 Klein-Nishina 공식으로 알려져 있음.
  3) Wave and particle nature of light :
     Wave nature — frequency ν, wavelength λ, phase velocity u,
     Amplitude A, intensity I
     이들의 몇몇은 상호 관련, $\lambda\nu = u$
     spreadout in space for a single frequency(or wavelength)
     Particle nature — mass m, velocity v, momentum p, energy E
     상호 관련 p=mvγ, $E = \sqrt{c^2 p^2 + m^2 c^4}$
     restricted position in space
  4) Connection between the duality of photon-EM wave
     $E = h\nu = \frac{hc}{\lambda}$, $p = \frac{h}{\lambda} = \frac{E}{c}$ ← momentum-energy for rest mass 0
     Photon momentum ↔ radiation pressure
     Photon spin ↔ angular momentum of circularly polarized light
     Both the radiation pressure and angular momentum of light were described in classical EM theory and consistent with quantum theory.

2. The de Broglie Wave

- The dual nature of light : $p = \frac{h}{\lambda} = \frac{E}{c}$

  The dual nature of matter particle ? : electrons, protons, neutrons etc.

  de Broglie's postulate of matter wave (1924) $\lambda = \frac{h}{p}$ *or* $p = \frac{h}{\lambda}$

  Experimental verification of de Broglie's matter wave :

  Wave nature of electron in the Davisson and Germer's experiment

- Matter wave (or Pilot wave)

  이 wave 가 입자(particle)의 운동을 guide(=pilot) 한다는 개념.

  Question) What is oscillating in the matter wave?

  What is the velocity of the wave?

  Simple consideration) matter wave = wave of single frequency ν

  → phase velocity u = λν

  energy analogy $E = mc^2 = h\nu$, $p = mv$, $m = m_0\gamma$

  where v is the particle velocity

  → Then, $u = \nu\lambda = \frac{E}{h}\frac{h}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v} > c \quad (v < c)$

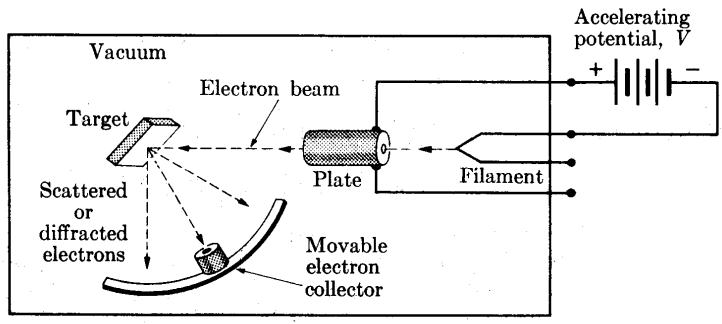
  Wavelength λ measurable

  Phase velocity u, frequency ν unmeasurable for the de Broglie wave

  Derivation of de Broglie wave postulate (de Broglie) and quantum condition of bound electron's orbital angular momentum

3. The Davisson and Germer Experiment

1927, Davisson and Germer : scattering experiment of electrons by nickel metal



An accidental breakout of vacuum system

→ oxidation of the nickel target surface

→ reduction of the target at high temperature

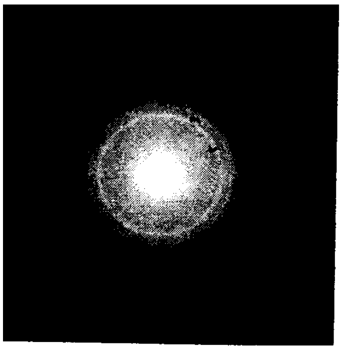
→ single crystalization happened at regions near surface

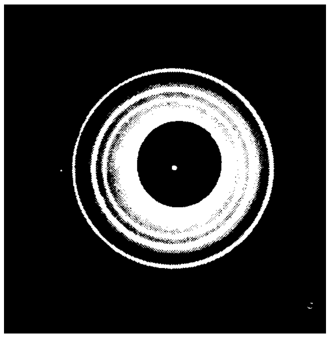
→ repeated experiment gave a totally different result

(maxima and minima shown in the scattered intensity)

→ Diffraction pattern

Calculated wavelength from the diffraction pattern by using the lattice spacing of Ni single crystal = de Broglie wavelength





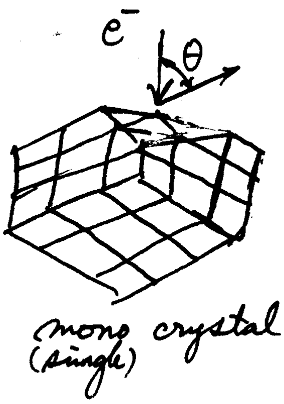
Electron energy $E_k = 75(eV) = eV = \frac{1}{2} m_e v^2$

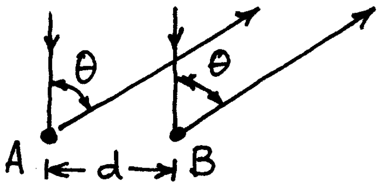
$$\rightarrow \quad \lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}} = 1.42 \text{ Å}$$

Experimental arrangement = Laue's method of x-ray diffraction

Diffraction maxima at scattering angles satisfying the condition

$$n\lambda = d \sin\theta$$





결론) 전자는 de Broglie 파장을 갖는 물질파로서도 작용함.

또한 de Broglie 물질파를 실험적으로 확인.

이후 전자 외의 다른 입자의 물질파 특성도 실험으로 확인됨

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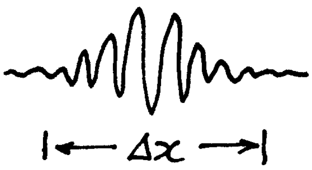
4. Wave Groups

- Dual nature of wave and particle :

  conceptual difficulty for spatial extension

  $\rightarrow$ Wave can be confined in space by means of wave group or wave packet.



Superposition of many waves of different frequency (or wavelength)

e.g. 음향학에서 진동수가 약간 다른 파들의 중첩이 일어날 경우, beat(맥놀이, 脈動) 현상 = 합성파의 진폭 변동

- 파동의 중첩 합성 :

  진동수 (또는 파장)가 다른 파동들이 여럿 중첩될수록, 합성파의 진폭이 큰 시간 영역 (또는 공간 영역)의 범위는 줄어듬.

$$y = A\sin 2\pi(\frac{x}{\lambda} - \nu t) = A\sin(kx - \omega t) \quad or \quad A\cos(kx - \omega t)$$

Choice of sin or cos function $\rightarrow$ rotating vector representation = phasor

각진동수 ω_1 - ω_2의 범위에서 $\delta\omega$ 씩 서로 다른 n개의 진동수, 동일 진폭(A)의 조화파(harmonic wave) 합성 :

단일 위치에서 고려하며 따라서 편의상 kx=0 로 둠.

$$\delta\omega = \frac{\omega_2 - \omega_1}{n-1} \equiv \frac{\Delta\omega}{n-1},$$

$$\Psi(t) = A\cos\omega_1 t + A\cos(\omega_1 + \delta\omega)t + \cdots + A\cos\omega_2 t = A(t)\cos(\omega_{av} t), \qquad (1)$$

where $\omega_{av} = \dfrac{\omega_1 + \omega_2}{2} = \omega_1 + \dfrac{\Delta\omega}{2}$ and

$$A(t) = A\frac{\sin\left(\frac{n\delta\omega}{2}t\right)}{\sin\left(\frac{\delta\omega}{2}t\right)}.$$

(1)식의 합산 유도. $\rightarrow$ complex algebra 이용.

- 합성파의 진폭 A(t)의 크기 → 시간 경과에 따라 감소.

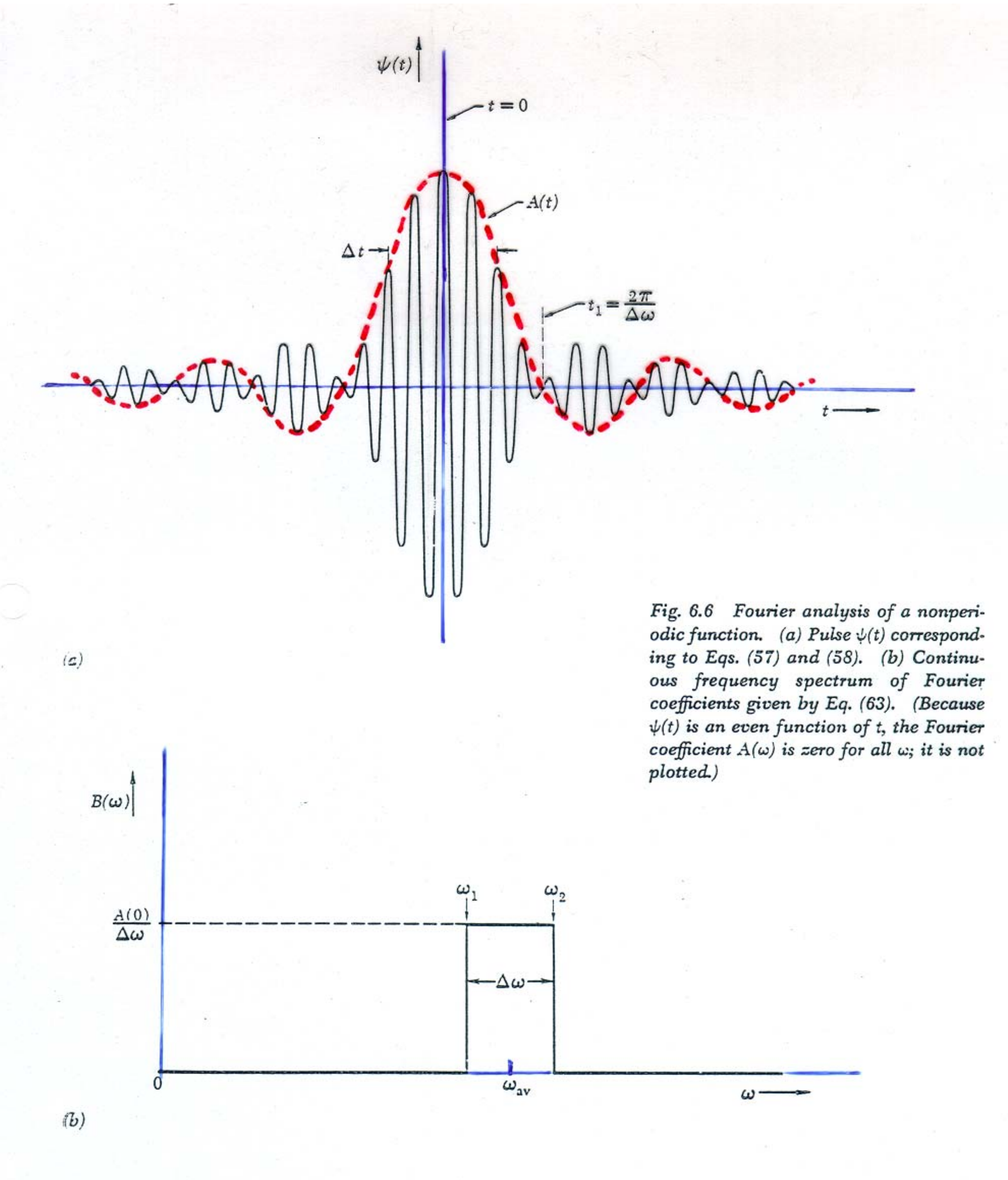


Fig. 6.6 Fourier analysis of a nonperiodic function. (a) Pulse ψ(t) corresponding to Eqs. (57) and (58). (b) Continuous frequency spectrum of Fourier coefficients given by Eq. (63). (Because ψ(t) is an even function of t, the Fourier coefficient A(ω) is zero for all ω; it is not plotted.)

→ 최초의 0 인 시각 t_1은 $t_1\delta\omega = \frac{2\pi}{n} = t_1\left(\frac{\omega_2 - \omega_1}{n-1}\right)$

or $t_1 = \left(\frac{2\pi}{\omega_2 - \omega_1}\right)\frac{n-1}{n}$

The regions of large amplitude : wave groups

The individual waves : phase waves

Wave group의 시간 폭을 $2t_1$으로 대표한다면, 연속적 중첩의 경우에

$t_1(n \to \infty) = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{\Delta\omega}$ 이며,

$2t_1 = \Delta t = \frac{4\pi}{\Delta\omega}$, 즉 frequency spread($\Delta\omega$)에 의해 time duration(시간폭, Δt)이 결정됨. ♣

유사한 방식으로 고정된 시각에 대해 공간적으로 밀집된 형태의 파동 중첩을 만들려면, 분산된 파장 λ(또는 파동수 k=2π/λ)의 조화파들의 중첩이 필요함. → group wave 의 spatial size (Δx)는 $\Delta x = \frac{4\pi}{\Delta k}$ 로서 앞서와 동일한 형태의 식으로 주어지며 즉, wave number spread (Δk)에 의해 공간적 크기(Δx)가 결정됨.

- Group velocity : wave group 의 전파 속도.

Wave of a single frequency and a single wave number —

$y_1 = A\cos(k_1 x - \omega_1 t)$

constant phase : $k_1 x - \omega_1 t$ = const

constant phase 의 진행 속도 : $k_1 dx - \omega_1 dt = 0$ 로부터

phase velocity $v_p = \frac{dx}{dt} = \frac{\omega_1}{k_1}$

Wave of composition of two different frequencies and wave numbers—

$y_1 = A\cos(k_1 x - \omega_1 t)$, $y_2 = A\cos(k_2 x - \omega_2 t)$

$v_{p1} = \frac{\omega_1}{k_1}$, $v_{p2} = \frac{\omega_2}{k_2}$

합성파

$$y = y_1 + y_2 = 2A\cos\tfrac{1}{2}[(k_2 - k_1)x - (\omega_2 - \omega_1)t]\cos\tfrac{1}{2}[(k_2 + k_1)x - (\omega_2 + \omega_1)t]$$

이때 k_1, k_2 및 ω_1, ω_2가 각각 작은 양으로만 다른 경우

$k_2 = k_1 + dk, \quad \omega_2 = \omega_1 + d\omega$ 이라면

$\frac{1}{2}(k_1 + k_2) \cong k_1$, $\frac{1}{2}(\omega_1 + \omega_2) \cong \omega_1$ 이며

$$y = 2A\cos\tfrac{1}{2}[xdk - td\omega]\cos[k_1 x - \omega_1 t] = 2\cos\tfrac{1}{2}[xdk - td\omega]\cdot(\text{original wave})$$

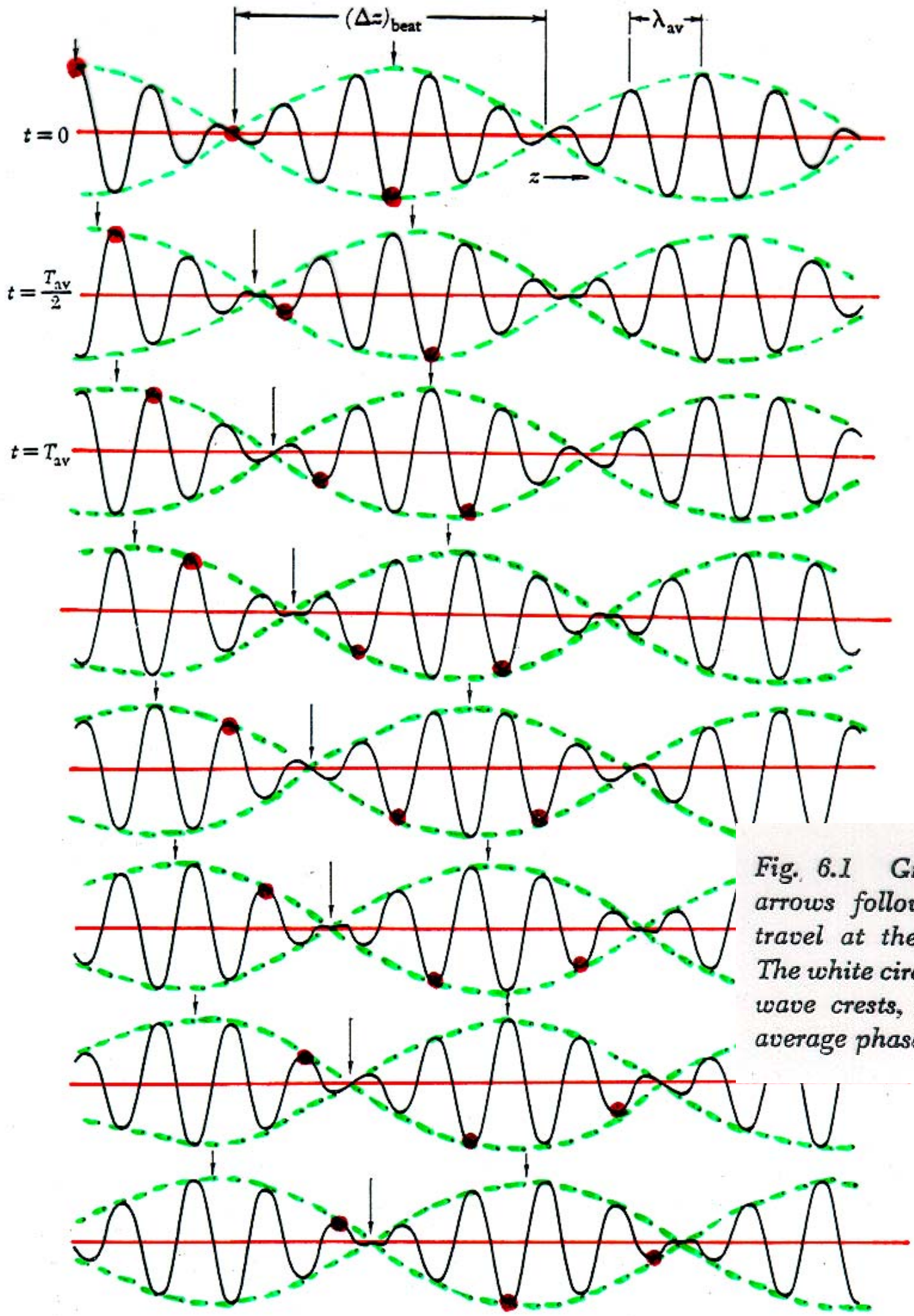


Fig. 6.1 Group velocity. The arrows follow the beats, which travel at the group velocity v_g. The white circles follow individual wave crests, which travel at the average phase velocity v_{av}.

여기서 modulation factor $2\cos\frac{1}{2}[xdk - td\omega]$ 은 original wave 의 진폭의 envelope 즉 group wave 를 형성하며, 이 envelope 의 전파속도는 group velocity 에 해당 → $[xdk - td\omega]$=constant 인 (x, t)들의 진행

$$\rightarrow \quad dxdk - dtd\omega = 0 \quad \rightarrow \quad v_g = \frac{dx}{dk} = \frac{d\omega}{dk} \quad \left(\neq \frac{\omega}{k} \quad \textit{in general}\right)$$

- Dispersion relation : $\omega = \omega(k)$.

  주어진 wave 종류(e.g. 전자기파, 탄성파, 물질파 등)에 대해, 전파(propagation) 매질의 특성에 의해 $\omega = \omega(k)$의 관계가 성립함.

  → Dispersion relation

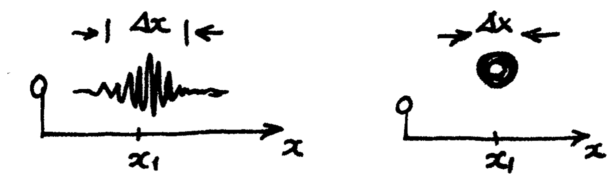
  $\omega = \mathrm{const}\times k$ 인 매질에서는 서로 다른 frequency의 wave들이 동일한 phase velocity로써 전파됨 → $\omega = v_p\times k$, $v_g = d\omega/dk = v_p$

  → wave group 은 각 phase wave 들과 동일한 속도로 진행

  → 이 매질은 non-dispersive medium

  Dispersive medium 에서는 일반적으로 $\omega = \omega(k)$의 관계가 있으므로 wave group 과 각 phase wave 들의 진행 속도가 다름.

- Analogy between a matter wave group and a particle :



velocity	group velocity v_g	particle velocity v
relation	$p = \hbar k$ (de Broglie)	$v = \frac{p}{m} = \frac{c^2 p}{mc^2} = \frac{c^2 p}{E}$ (relativity)
	$E = \hbar\omega$ (Einstein)	

$$\rightarrow \quad v_g = \frac{d\omega}{dk} = \frac{\hbar d\omega}{\hbar dk} = \frac{dE}{dp},$$

또한 $E^2 = c^2 p^2 + (mc^2)^2$ 에서 $2EdE = 2c^2 pdp$

$$\text{or} \quad v_g = \frac{dE}{dp} = \frac{c^2 p}{E} = v$$

♣ Hence group velocity of matter wave = particle velocity.

5. The Heisenberg Uncertainty Principle

- Dual nature of wave and particle :

  Wave group 의 공간적 국한성 (Δx)은 상이한 wave number 의 파동들이 무한히 중첩됨으로써 가능 →

  이때 중첩되는 파동의 wave number 분포폭 (Δk)는 (Δx)와 간에

  $$\Delta x \cdot \Delta k = 1$$

  중첩에 관련되는 각 조화파의 phase wave 는 운동량 p 와의 관계

  $$p = \frac{h}{\lambda} = \hbar k \qquad \text{i.e.} \quad \Delta p = \hbar \Delta k$$

  → $\Delta x \cdot \Delta p = \hbar$ (for gaussian wave packet, and Δ=std. dev.)

  → $\Delta x \cdot \Delta p_x \geq \hbar$ in general

  → 3 차원 경우 $\Delta y \cdot \Delta p_y \geq \hbar$, $\Delta z \cdot \Delta p_z \geq \hbar$

  These are the uncertainties originating from the dual nature of wave-particle. 이러한 wave-particle dual nature 에 근원하여 측정의 정확도에 대해서도 동일한 영향을 일으킴. 또한 시간좌표에 대해서는 각진동수가 대응되어 유사한 식이 성립.→ $\Delta\omega \cdot \Delta t = 1$

  $$E = h\nu = \hbar\omega \qquad \text{i.e.} \quad \Delta E = \hbar\Delta\omega \;\rightarrow\; \Delta E \cdot \Delta t \geq \hbar \quad \text{in general}$$

Heisenberg uncertainty principle

점입자에 대해 위치를 정확히 결정하기 위해서는 파동으로 보아서는 무한히 밀집된 위치의 wave packet 이 형성되어야 하며 이는 무한히 서로 다른 파동수 즉, 서로 다른 운동량의 파동이 중첩되어야 하므로 위치와 운동량을 동시적으로 무한히 정확하게 결정하는 일이 불가능함.

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