Fusion Reactor Technology I
(459.760, 3 Credits)

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Fusion Reactor Energetics

- **Fundamental requirement of a fusion reactor system**

$$E^*_{net} = E^*_{out} - E^*_{in} > 0$$

$$\int_o^\tau \left(\frac{dE^*}{dt}\right)_{net} dt = \int_o^\tau \left(\frac{dE^*}{dt}\right)_{out} dt - \int_o^\tau \left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

- **Fusion Plasma Energy Balance**

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = E^*_{aux} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt$$

Time variations of power considered

$$E^*_{aux} = \eta_{in} E^*_{in}$$

$$\frac{E^*_n}{E^*_{fu}} = 1 - f_c \qquad E^*_{fu} - E^*_n = f_c E^*_{fu}$$

$$Q_p = \frac{E^*_{fu}}{E^*_{aux}} = \frac{E^*_{fu}}{\eta_{in} E^*_{in}}$$

Plasma Q-value

Fusion Reactor Energetics

$$\int_{o}^{\tau_b} \frac{dE_{th}^{*}}{dt} dt = \left(f_c + \frac{1}{Q_p} \right) E_{fu}^{*} - E_{rad}^{*} - \int_{o}^{\tau_b} \frac{E_{th}^{*}}{\tau_E^{*}} dt$$

$$\int_{o}^{\tau_b} dE_{th}^{*} dt = E_{th}^{*}(\tau_b) - E_{th}^{*}(0)$$

If, steady state

$$E_{fu}^{*} = \frac{E_{rad}^{*} + \int_{o}^{\tau_b} \frac{E_{th}^{*}}{\tau_E^{*}} dt}{f_c + \frac{1}{Q_p}}$$

- if, $Q_p \rightarrow \infty$, the fusion energy delivered to the plasma via the charged reaction products is seen to balance the total energy loss from the plasma

Fusion Reactor Energetics

Considering a D-T plasma with $Q_p \to \infty$,

$$f_{c,dt} \int_V d^3r \int_o^{\tau_b} R_{dt}(\vec{r},t) Q_{dt} dt = \int_V d^3r \left[\int_o^{\tau_b} (P_{br} + P_{cyc}^{net}) dt + \int_o^{\tau_b} \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} dt \right]$$

$$P_{br} = A_{br} N_i N_e Z^2 \sqrt{kT_e} \qquad A_{br} \approx 1.6\times 10^{-38} \left(\frac{m^3 J}{\sqrt{eV} s} \right)$$

$$P_{cyc}^{net} = A_{cyc} N_e B^2 kT_e \psi \qquad A_{cyc} \approx 6.3\times 10^{-20} (JeV^{-1} T^{-2} s^{-1})$$

Why effect of ions not included in P_{br} and P_{cyc}?

Homework

$$\int_V d^3r \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} = \frac{E_{th}^*(t)}{\tau_{E*}(t)}$$

Volume integrated

In a steady state, homogeneous plasma

$$f_{c,dt} P_{dt}(N_i, T_i) = P_{br}(N_i, N_e, T_e) + P_{cyc}^{net}(N_e, T_e) + \frac{3}{2} \frac{(N_i kT_i + N_e kT_e)}{\tau_E}$$

$$E_{th.j} = \frac{3}{2} N_j kT_j , \quad j = i, e$$

- complex interrelation between the plasma density and its temperature as required for ignition

Fusion Reactor Energetics

- **Lawson criterion**

recoverable energy from a fusion reactor must exceed the energy supplied during a representative time interval

– reactor criterion: energy viability of the entire plant

$$E_{out}^{*} = \eta_{fu} E_{fu}^{*} + \eta_{rad} E_{rad}^{*} + \eta_{th} E_{th}^{*}$$

$$\eta_{in} E_{in}^{*} = E_{rad}^{*} + E_{th}^{*}$$

$$\eta_{fu} E_{fu}^{*} + \eta_{rad} E_{rad}^{*} + \eta_{th} E_{th}^{*} > \frac{E_{rad}^{*} + E_{th}^{*}}{\eta_{in}}$$

$$\eta_{in}\eta_{out}(E_{fu}^{*} + E_{rad}^{*} + E_{th}^{*}) > E_{rad}^{*} + E_{th}^{*}$$

$$\eta_{out} = \frac{\sum_{l} \eta_l E_l}{\sum_{l} E_l}, \quad l = fu, rad, th$$

Average conversion efficiency

Fusion Reactor Energetics

$$E_l^* = \tau_{E*} \int_V P_l(\vec{r}) d^3r$$ Global energy terms

Assuming, Bremsstrahlung only

$$\eta_{in}\eta_{out} \int_V d^3r(\tau_{E*}P_{fu} + \tau_{E*}P_{br} + 3NT) > \int_V d^3r(\tau_{E*}P_{br} + 3NT)$$

$$E_{th}(\vec{r}) = \frac{3}{2}(N_iT_i + N_eT_e) = 3NT$$

Assuming, homogeneity throughout the plasma volume V

$$\eta_{in}\eta_{out}\left(\frac{N_aN_b}{1+\delta_{ab}} <\sigma v>_{ab} Q_{ab}\tau_{E*} + A_{br}N^2\sqrt{T}\tau_{E*} + 3NT\right) > A_{br}N^2\sqrt{T}\tau_{E*} + 3NT$$

Kronecker-d introduced to account for the case of indistinguishable reactants

$$N\tau_{E*} > \frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\frac{<\sigma v>_{ab}(T)Q_{ab}}{4(1+\delta_{ab})} - (1-\eta_{in}\eta_{out})A_{br}\sqrt{T}}$$ $\eta_{in}\eta_{out} \approx 1/3$

Fusion Reactor Energetics

- **Ignition**

Energy viability of the fusion plasma

$$f_{c,dt} P_{dt} \tau_{E*} \geq (P_{br} + P_{cyc}^{net}) \tau_{E*} + 3NT$$

$$(N\tau_{E*})_{dt} \geq \frac{3T}{f_{c,dt} Q_{dt} \frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc} B^2 \psi T}{N}}$$

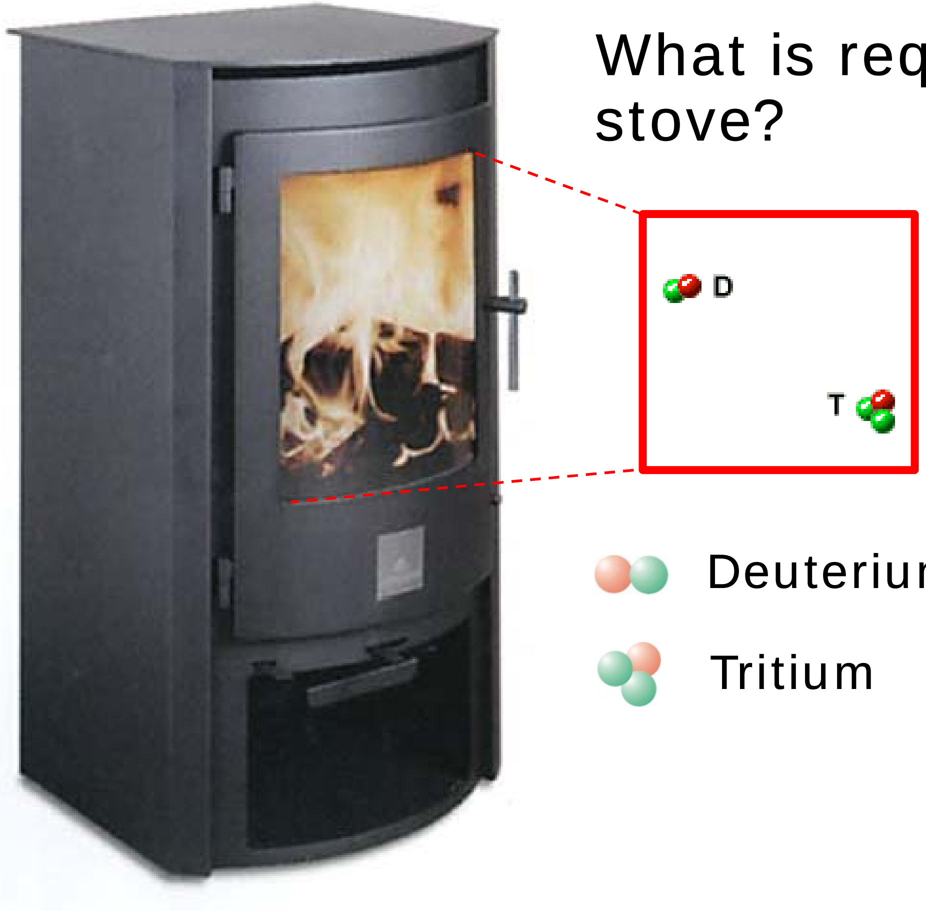
No energy conversion efficiency contained

- **Break-even**

The total fusion energy production amounts to a magnitude equal to the effective plasma energy input.

$$\frac{E_{fu}^*}{\eta_{in} E_{in}^*} = Q_p = 1$$

Fusion Reactor Energetics

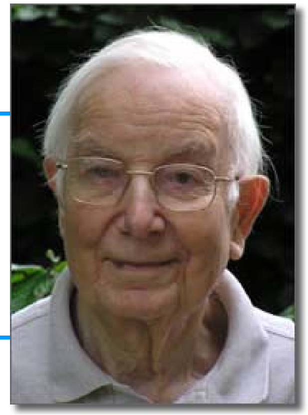


What is required to light a fire in a stove?

- Fuel: D, T
- Amount/density: n
- Heat insulation: τ
- Ignition temperature: T

Deuterium

Tritium



$\geq ?$

quired

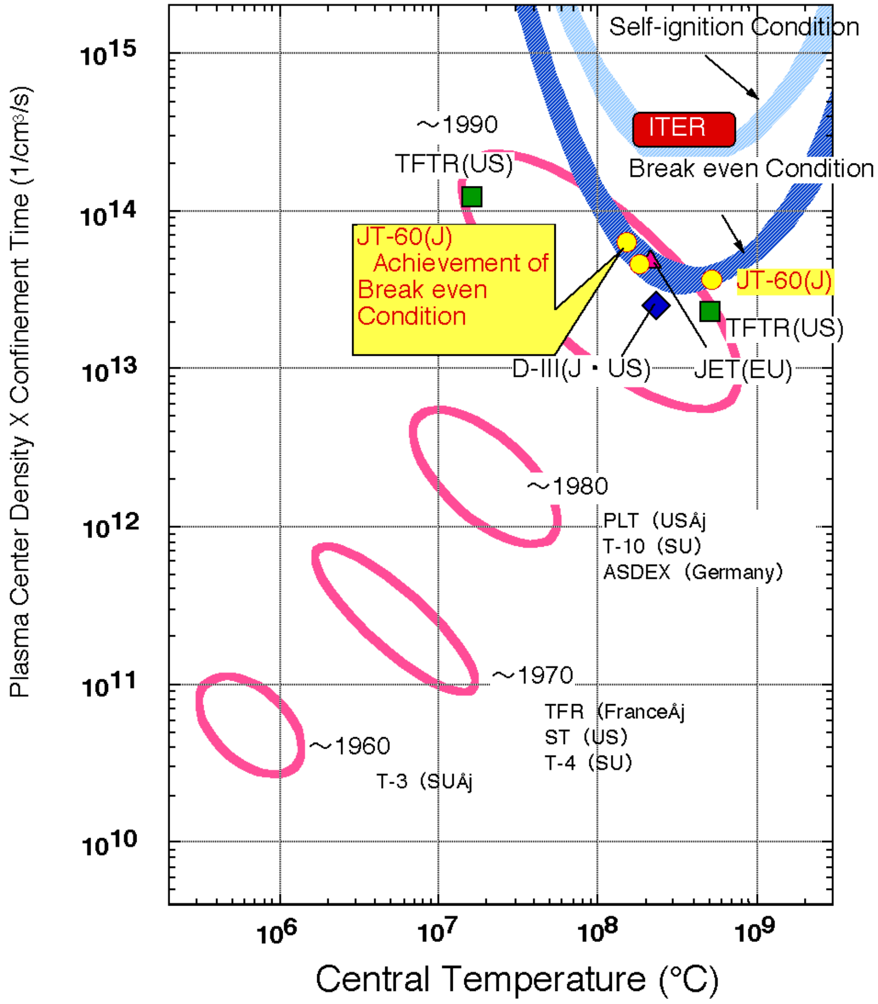
Lawson Criterion

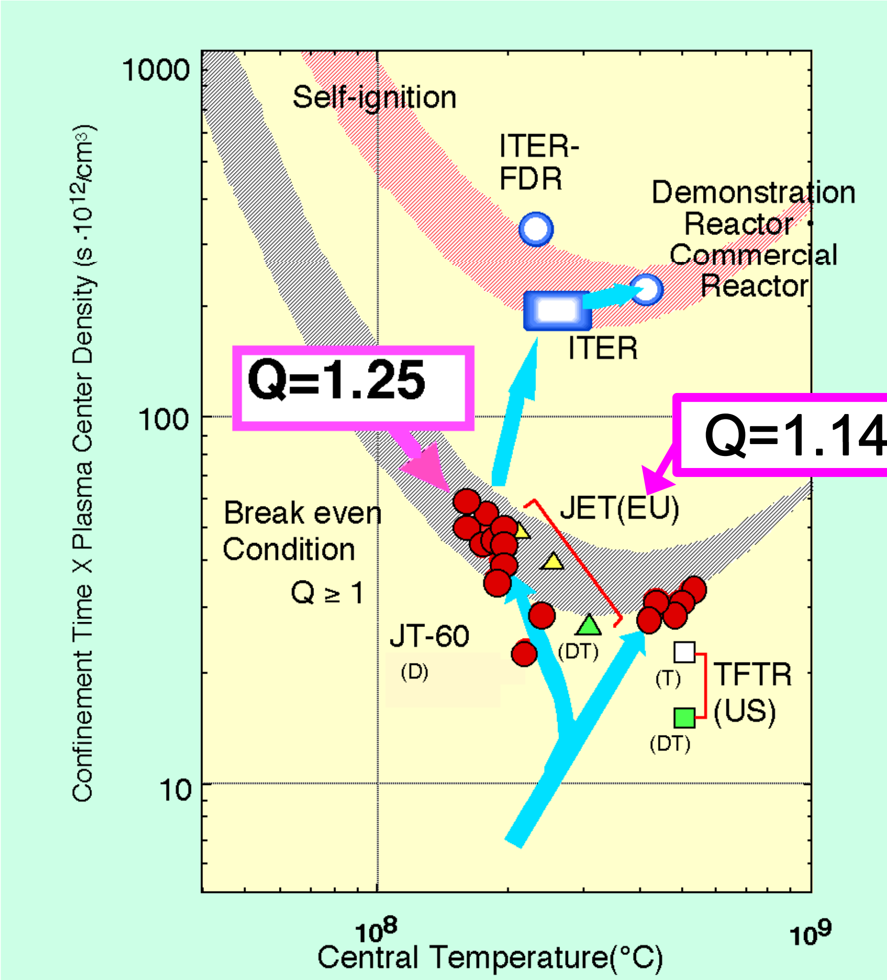
Fusion Reactor Energetics

- A.A. Harms, K.F. Schoepf, G.H. Miley, D.R. Kingdon, „Principles of Fusion Energy“, Ch. 8

Homework: Problems 8.6

Status of the Tokamak Research





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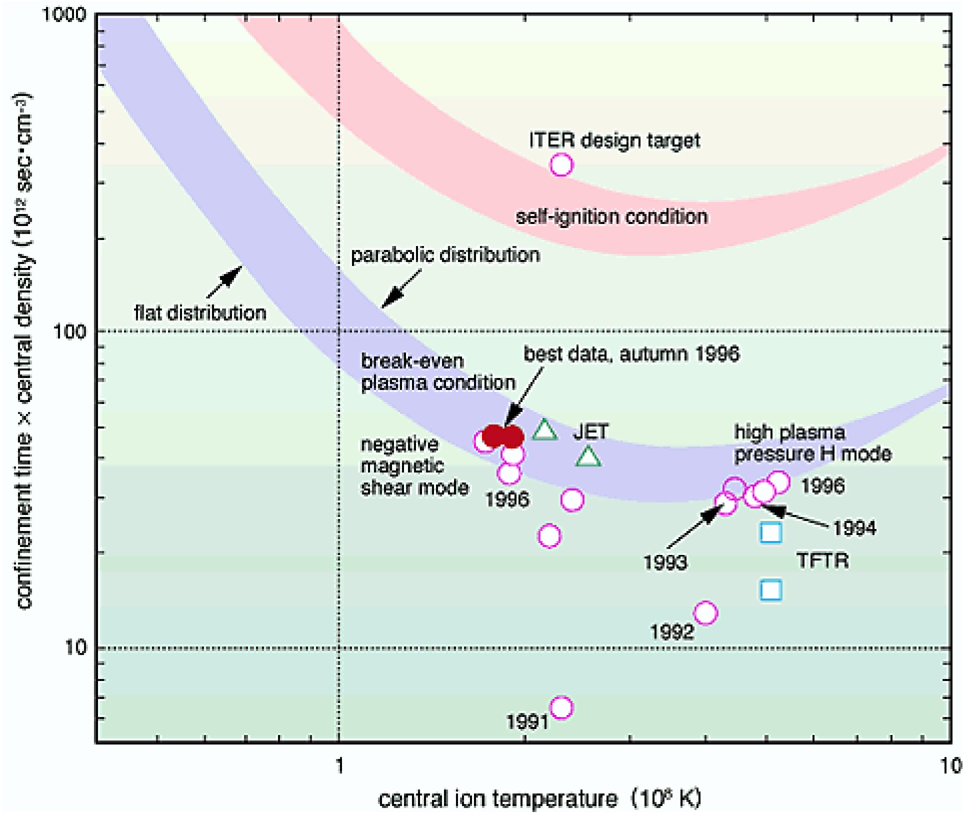

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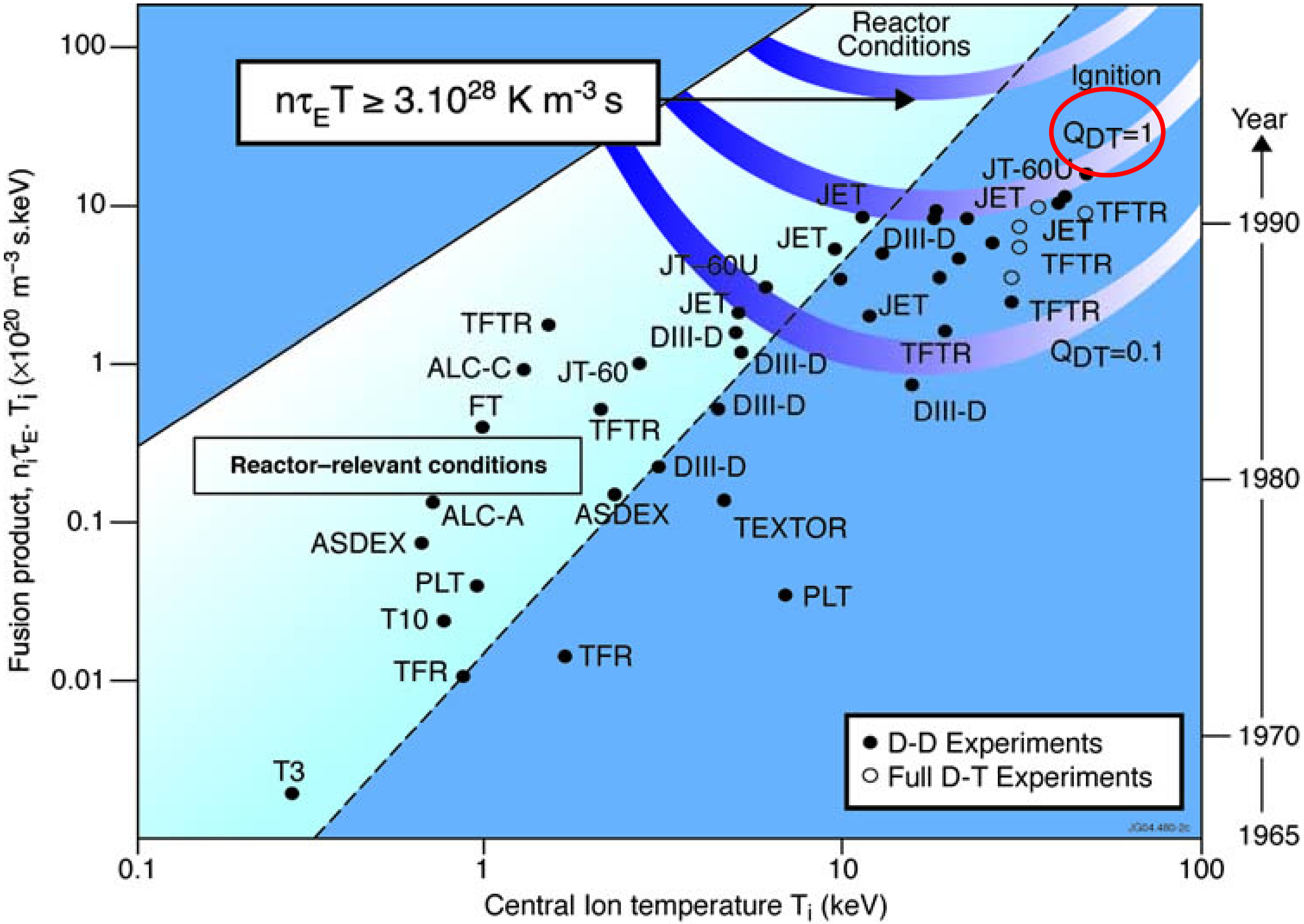

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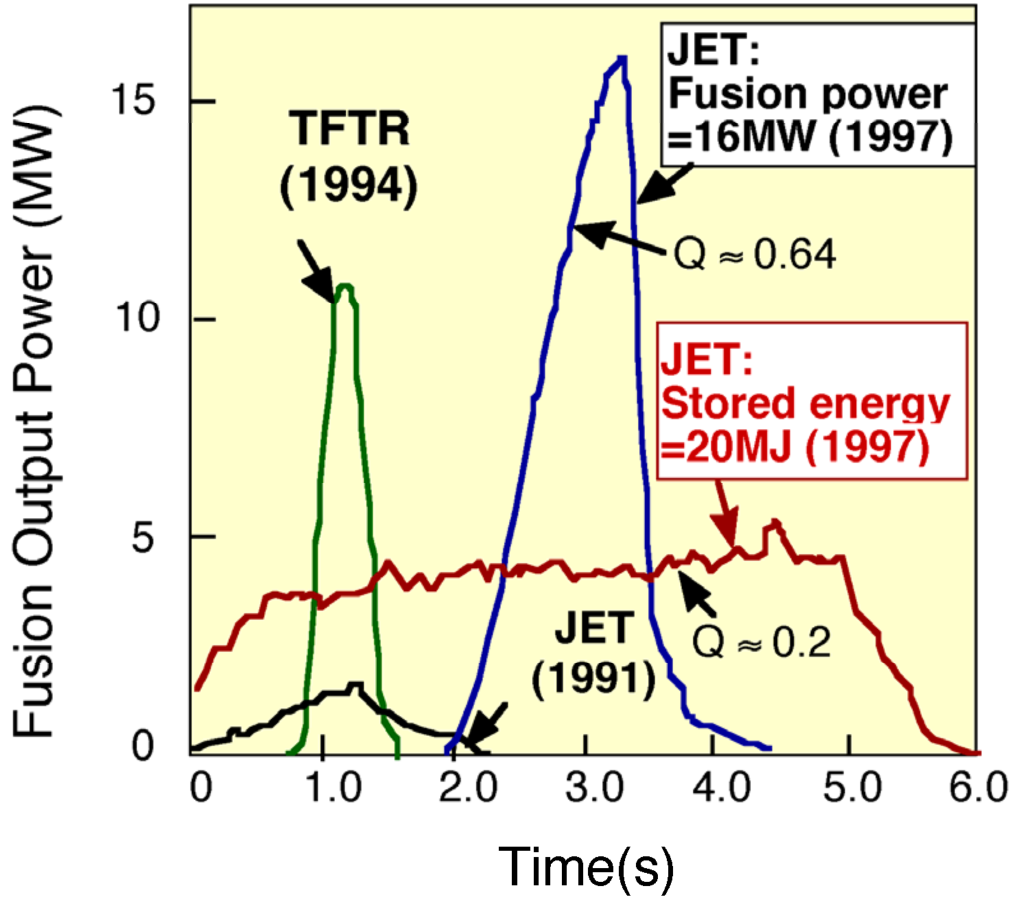
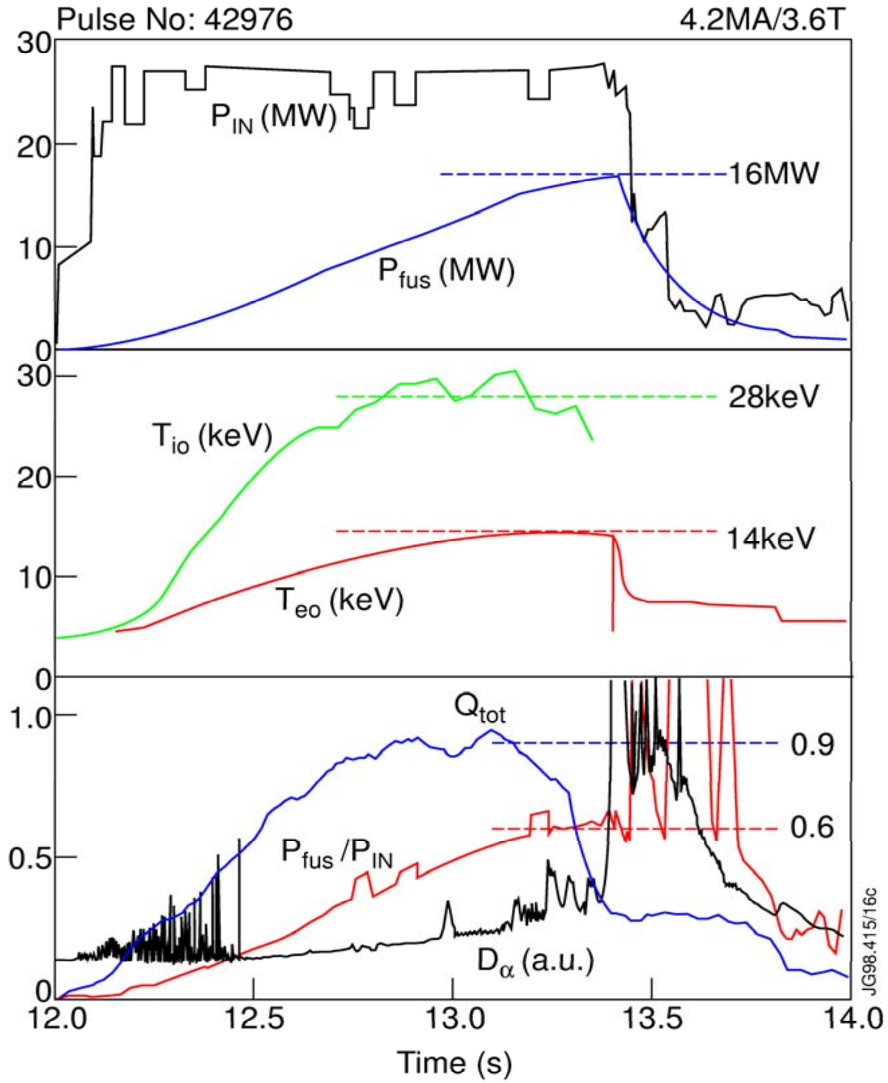


- Present machines produce significant fusion power:
  - TFTR (USA) ~10 MW in 1994
  - JET (EU) 16 MW (Q=0.64) in 1997

Status of the Tokamak Research

- DT-Experiments only in
- JET
- TFTR

- with world records in JET:
- P_{fusion} = 16 MW
- Q = 0.65



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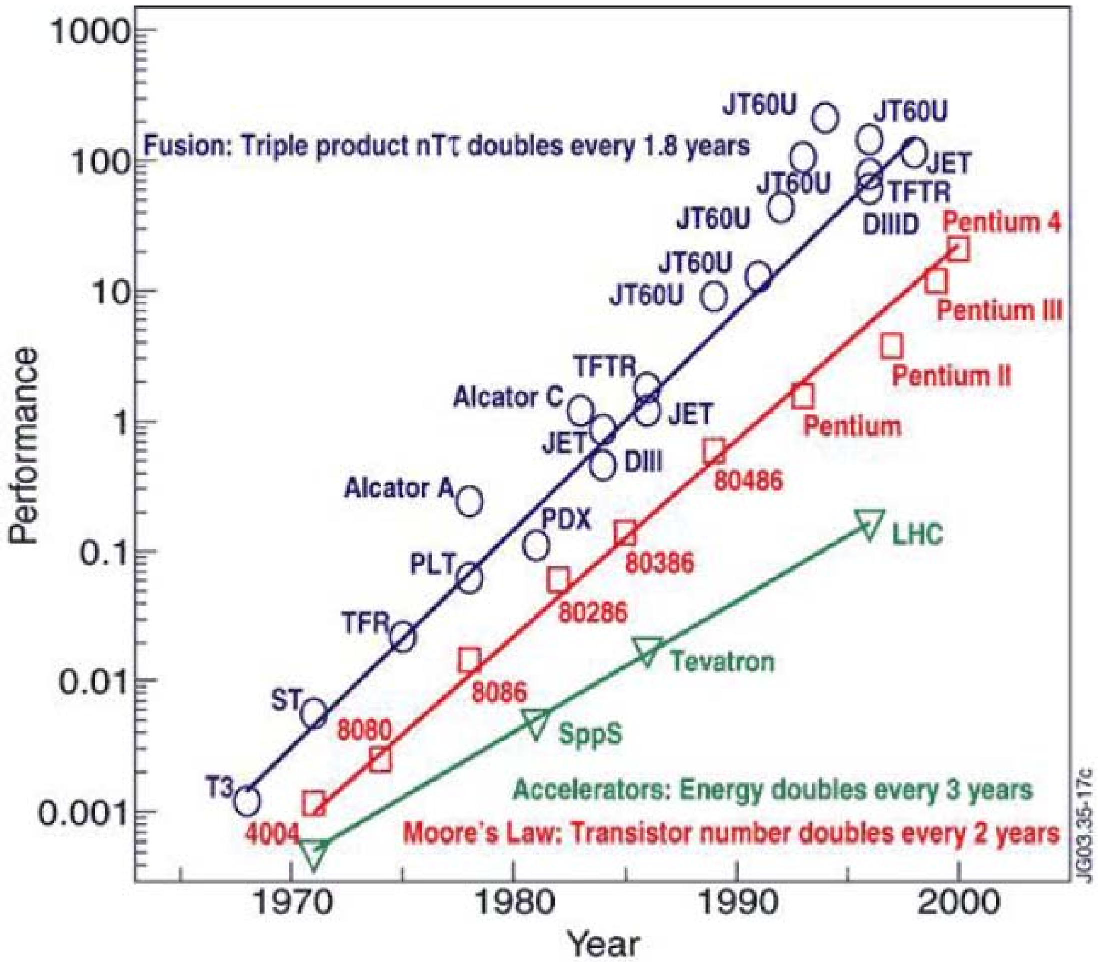

• Progress in fusion can be compared with the computing power and particle physics accelerator energy.