Fusion Reactor Technology I

(459.760, 3 Credits)

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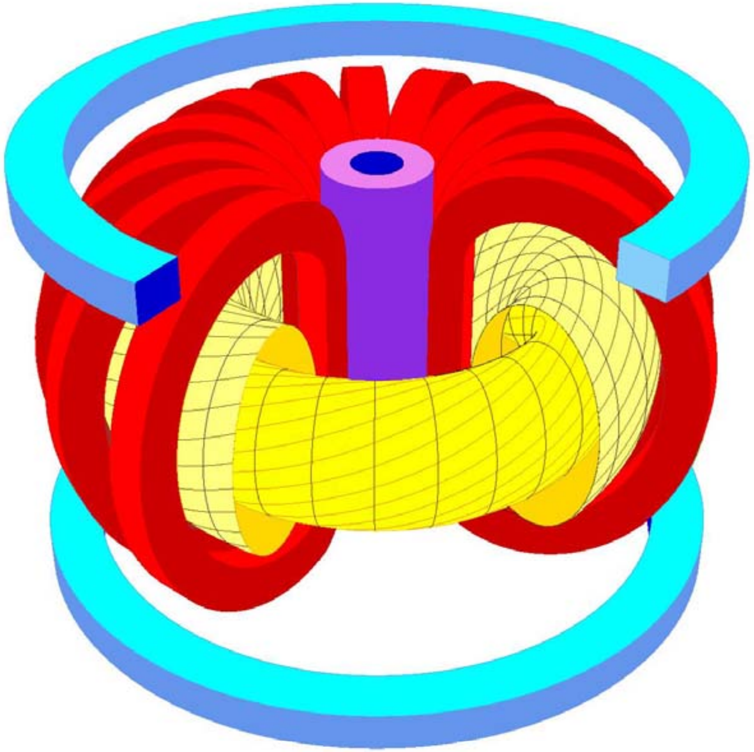
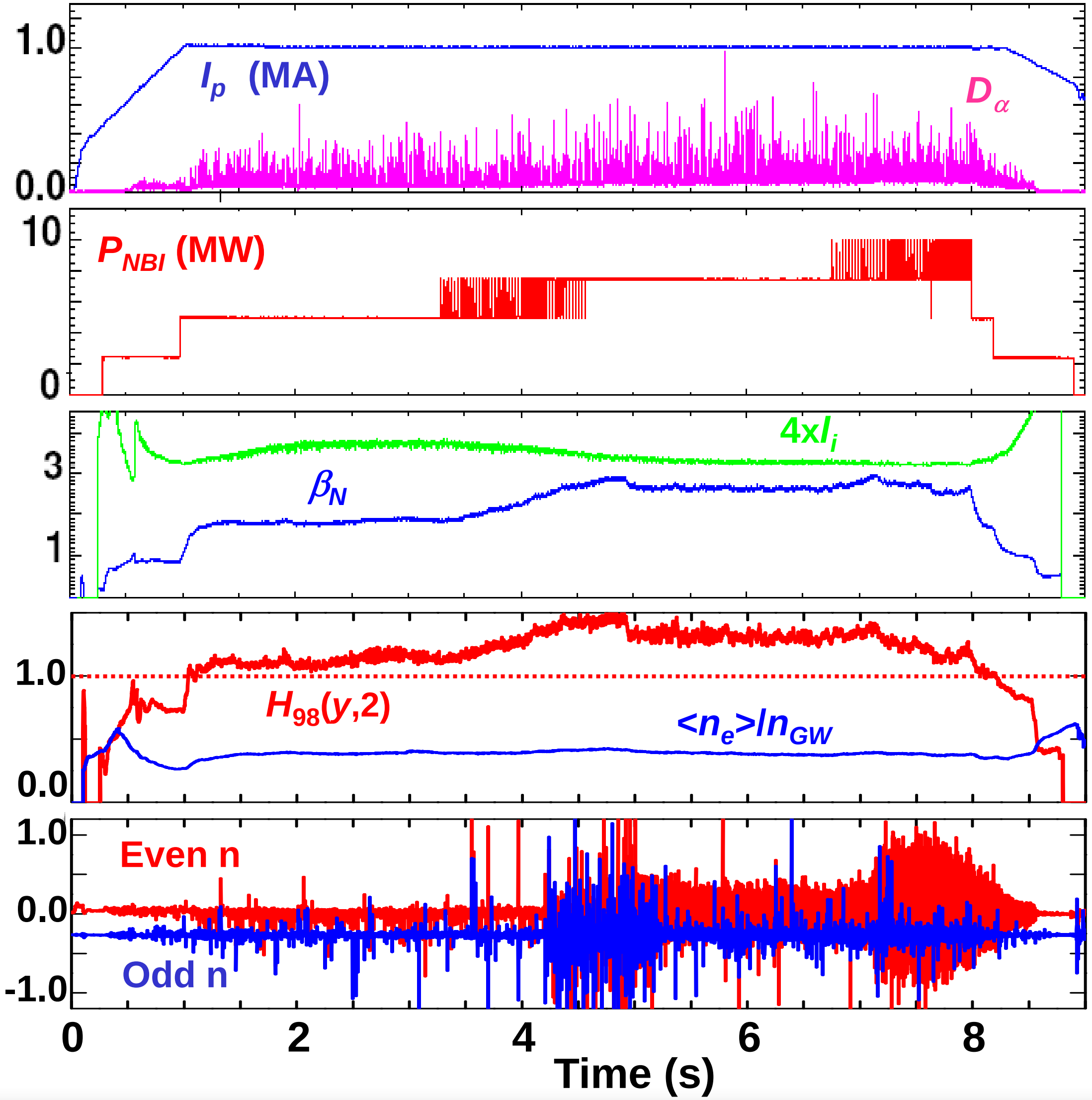


- High $\langle n_e \rangle / n_{GW}$
- High β_N
- High $H_{98}(y,2)$
- Pulse length

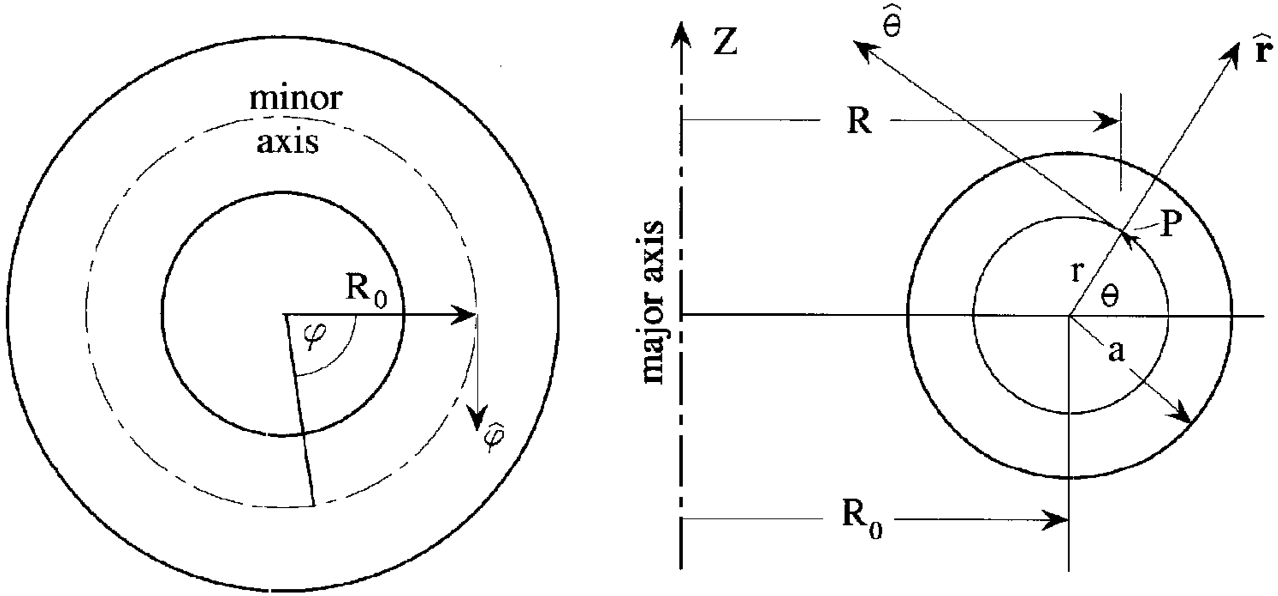
Objectives of the Tokamak Operation



- High $<n_e>/n_{GW}$
- High β_N
- High $H_{98}(y,2)$
- Pulse length

Basic Tokamak Variables

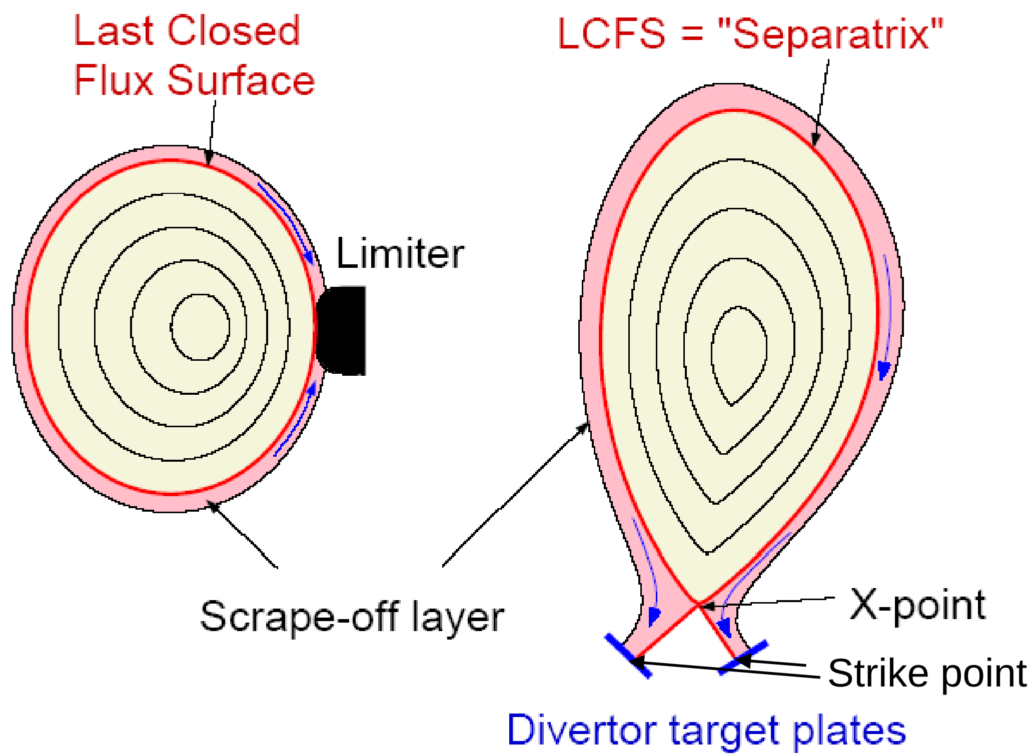
- **Cylindrical and local coordinates for a tokamak**



- Aspect ratio: R_0/a ~ 3-5
  ex) KSTAR: 3.6, ITER: 3.1

Basic Tokamak Variables

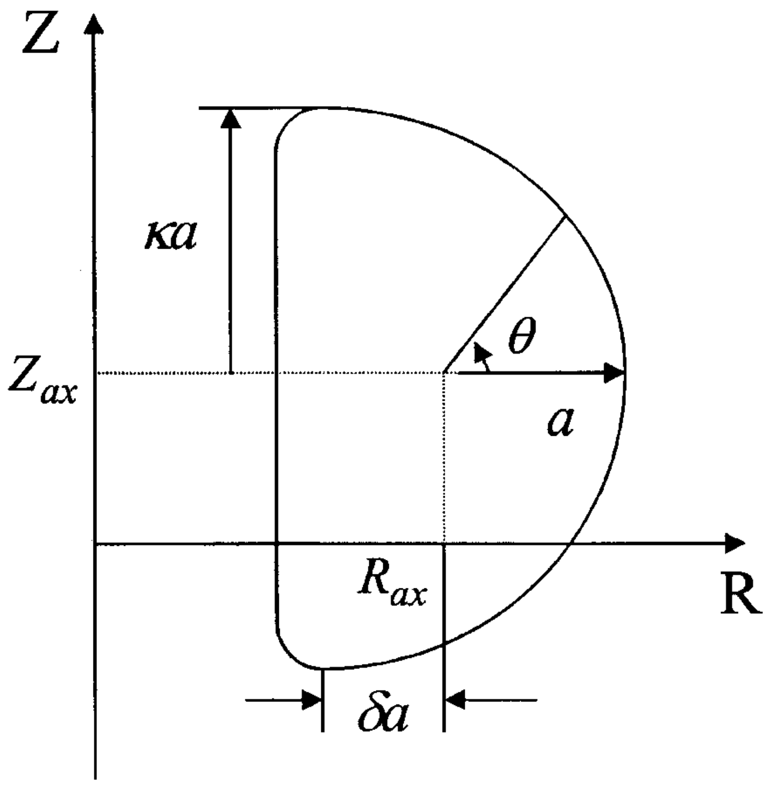
• **Separation of plasma from wall by a limiter and a divertor**



• Advantage of the divertor configuration
- First contact with material surface at a distance from plasma boundary
- Reducing the influx of ionized impurities into the interior of the plasma by diverting them into an outer „SOL“

Basic Tokamak Variables

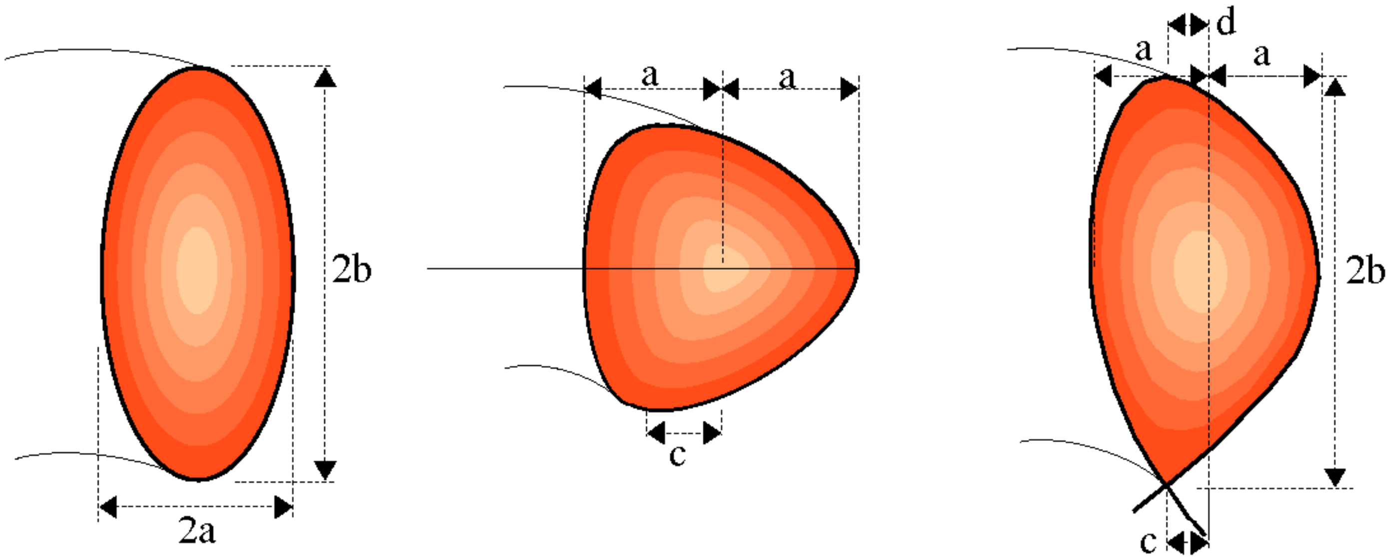
- **Plasma equilibrium parameters**



- Elongation: k
- Triangularity: d
- Squareness: ζ

Basic Tokamak Variables

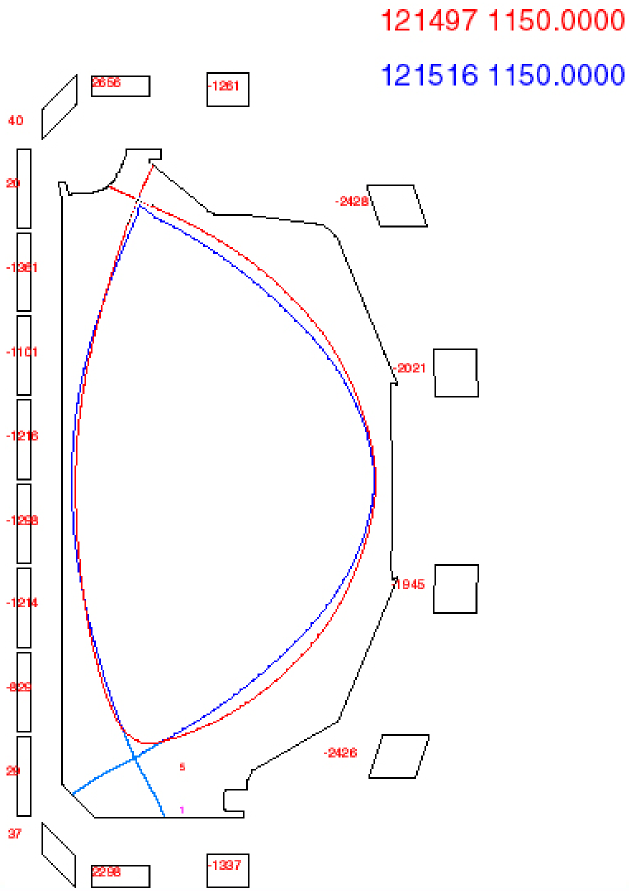
- **Plasma equilibrium parameters**



$$\kappa = \frac{b}{a} \qquad \delta = \frac{c+d}{2a}$$

Basic Tokamak Variables

- **Plasma equilibrium parameters**



- Outer and inner squareness: $\zeta_{o,i}$

What is the squreness?

$$R = R_0 + a\cos(\theta + \sin^{-1}\delta \sin\theta)$$

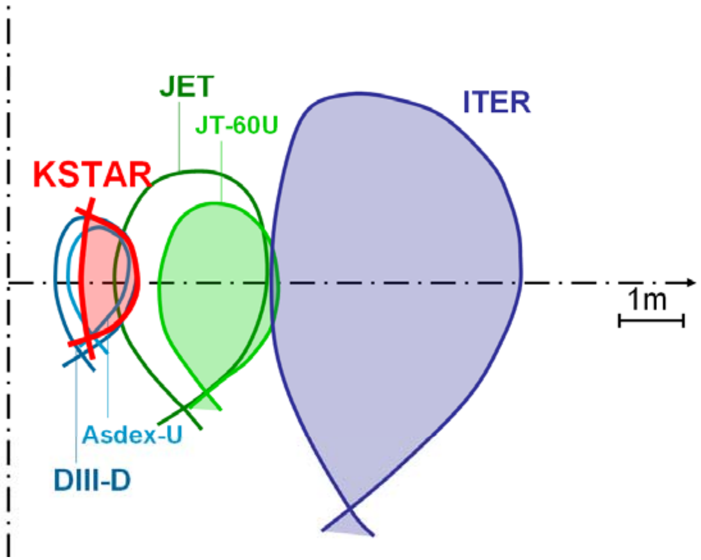
$$Z = \kappa a \sin(\theta + \zeta_{o,i} \sin 2\theta)$$

Basic Tokamak Variables

- **Plasma equilibrium parameters**

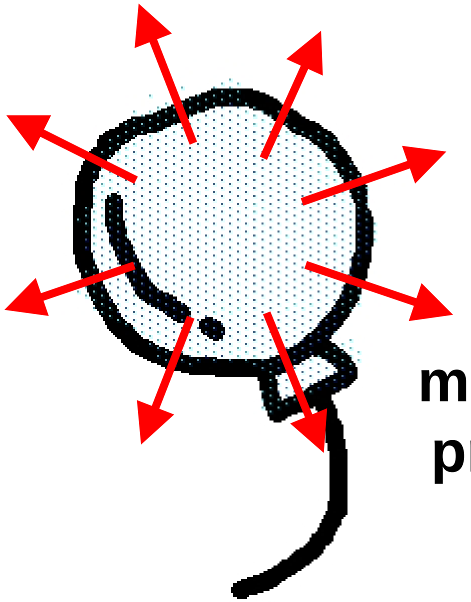
Parameters	KSTAR	ITER
Major Radius, R_0	1.8 m	6.2 m
Minor Radius, a	0.5 m	2.0 m
Plasma Current, I_P	2.0 MA	15 MA
Elongation, κ_x	2.0	1.85
Triangularity, δ_x	0.8	0.5
Toroidal Field, B_0	3.5 T	5.3 T
Pulse Length	300 s	500 s
Fuel	H, D	D, T

- Plasma shape



Basic Tokamak Variables

- **Beta**

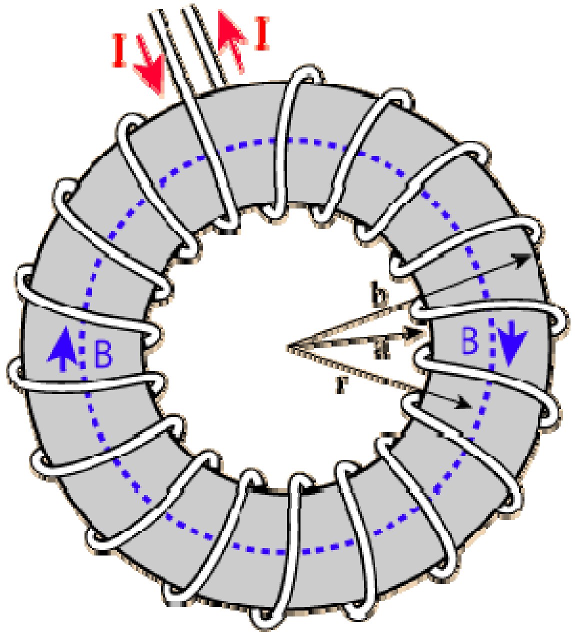


$$\beta = 2\mu_0 p / B^2$$

- The ratio of the plasma pressure to the magnetic field pressure
- A measure of the degree to which the magnetic field is holding a non-uniform plasma in equilibrium.

Basic Tokamak Variables

- **Beta**



$$B = \frac{\mu N I}{2\pi r}$$

$$\beta = 2\mu_0 p / B^2$$

- The power output for a given magnetic field and plasma assembly is proportional to the square of beta.
- In a reactor it should exceed 0.1 – economic constraint

Basic Tokamak Variables

- **Beta**

Assuming that the magnetic surfaces have concentric, circular CXs and that conditions are independent of φ.

$$\langle p \rangle = \int p dS / \int dS = \frac{2\pi}{\pi a^2} \int_0^a p(r) r dr$$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$ Ampère's law

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_0 j_\varphi \,, \quad B_\theta = \frac{\mu_0}{r}\int_0^r j_\varphi(r') r' dr'$$

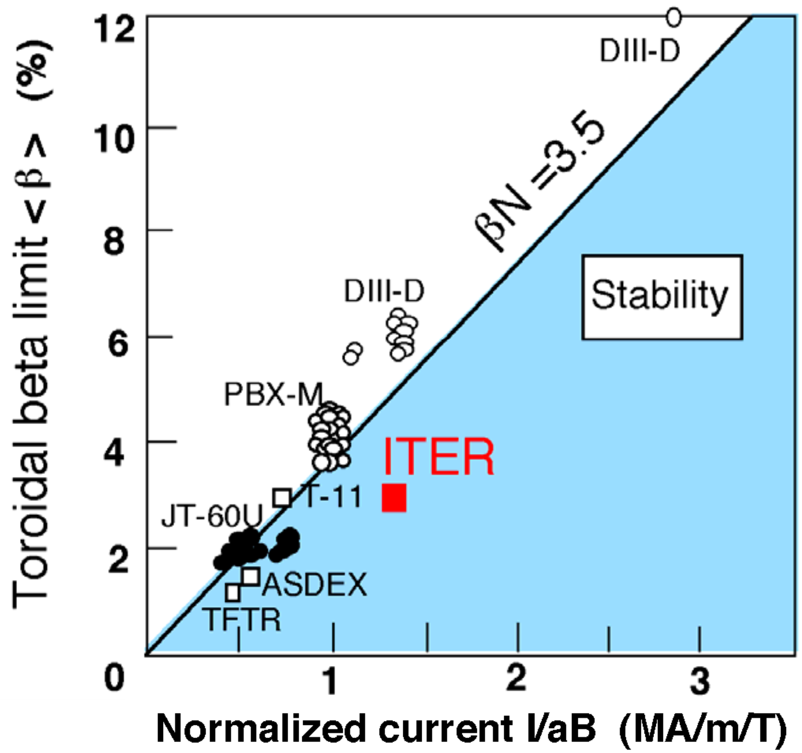
$$I_p = 2\pi \int_0^a j_\varphi r dr = 2\pi a B_{\theta a} / \mu_0$$

$$\beta_t = \frac{2\mu_0 \langle p \rangle}{B_\varphi^2}, \quad \beta_p = \frac{2\mu_0 \langle p \rangle}{B_{\theta a}^2} = \frac{8\pi^2 a^2 \langle p \rangle}{\mu_0 I_p^2}$$

Basic Tokamak Variables

- **Normalized beta – stability limit**

$$\beta_N = \beta_t \frac{aB_t}{I_p}$$

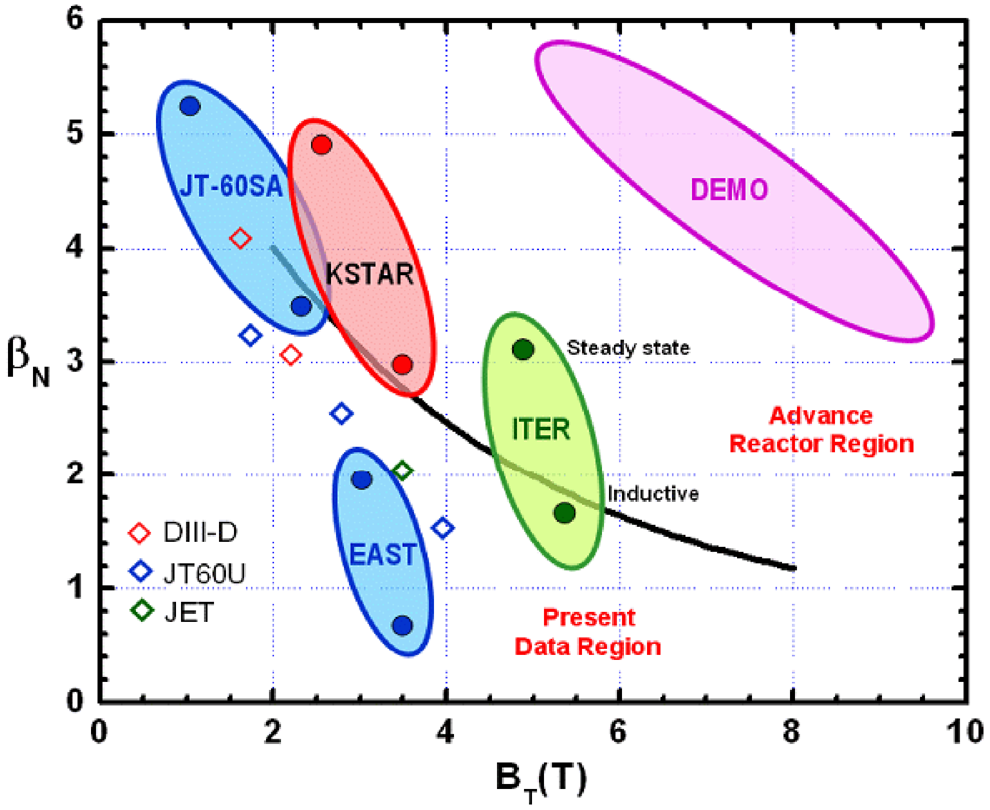


- Fundamental elements for the β_N-limit

1. Current profile
2. Pressure profile
3. Plasma shape
4. Stabilising wall
5. Resistive instability

Basic Tokamak Variables

- **Normalized beta – stability limit**

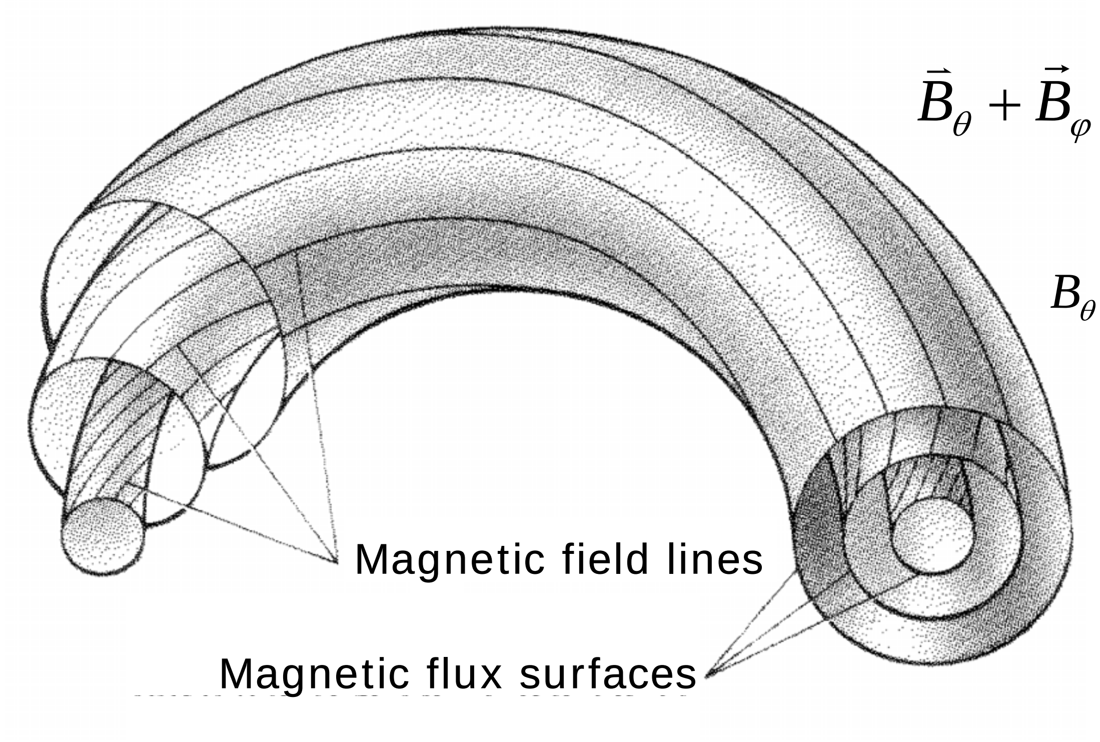


- High β_N in KSTAR

1. Strong plasma shaping
2. Passive stabilizers
3. PF/CS system capability
4. High heating power
5. RWM control coils

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**



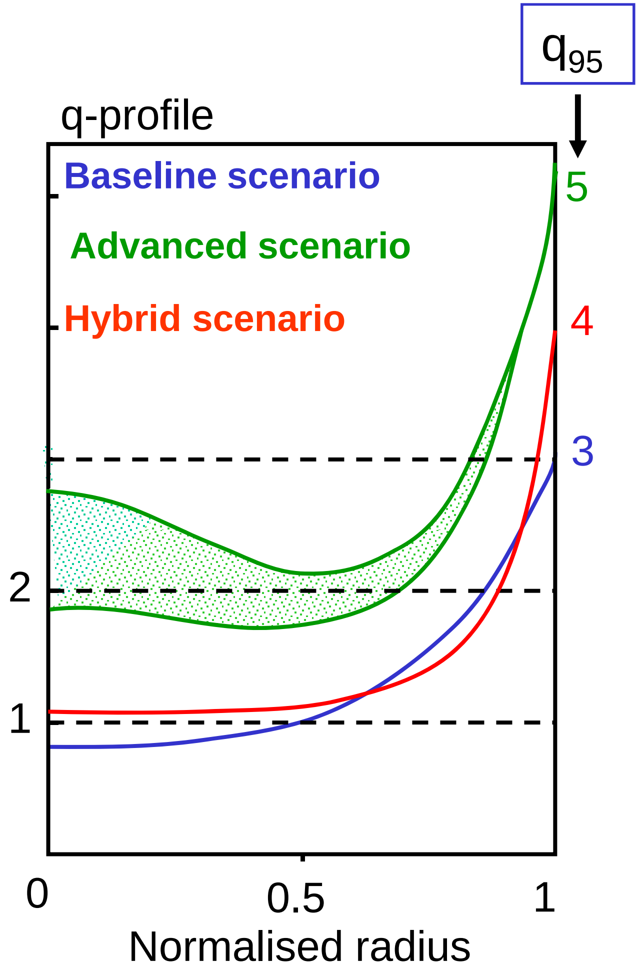
$$\vec{B}_\theta + \vec{B}_\varphi$$

$$B_\theta = \frac{\mu_0}{r} \int_0^r j_\varphi(r')r'dr'$$

$$q = \frac{\text{number of toroidal windings}}{\text{number of poloidal windings}}$$

Basic Tokamak Variables

• **Safety factor *q* = number of toroidal orbits per poloidal orbit**



Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**

Large aspect ratio tokamak with a circular CX

$$q(r) = \frac{rB_\varphi}{R_0 B_\theta} = \frac{\varepsilon}{s}$$

$$\varepsilon \equiv \frac{r}{R_0}, \quad s \equiv \frac{B_\theta}{B_\varphi} = \frac{\mu_0}{rB_\varphi}\int_0^r j_\varphi(r')r'dr'$$

$$q_a = \frac{aB_\varphi}{R_0 B_{\theta a}} = \frac{2\pi a^2 B_\varphi}{\mu_0 I_p R_0}, \quad \langle j_\varphi \rangle = \frac{I_p}{\pi a^2}$$

$$\mu_0 \langle j_\varphi \rangle = \frac{2B_\varphi}{R_0 q_a}, \quad q_0 = \frac{2B_\varphi}{\mu_0 j_{\varphi 0} R_0}$$

Derive this!

Homework

$$\frac{q_a}{q_0} = \frac{j_{\varphi 0}}{\langle j_\varphi \rangle}$$

Current profile peakedness

- General definition

$$q = \oint \frac{B_\varphi}{R_0 B_\theta} ds$$

Integral is along a closed path enclosing the minor axis and lying on a specific magnetic surface; thus *q* is a surface quantity.

Basic Tokamak Variables

- **Z-effective: a convenient measure of the extent to which the plasma contaminated**

$$n_e Z_{eff} = \sum_s n_s Z_s^2\ ,\quad n_e = \sum_s n_s Z_s$$

Z_s : charge number for the s-type ion

- Z_{eff} = 1 in a pure hydrogen plasma
- Method to determine Z_{eff}

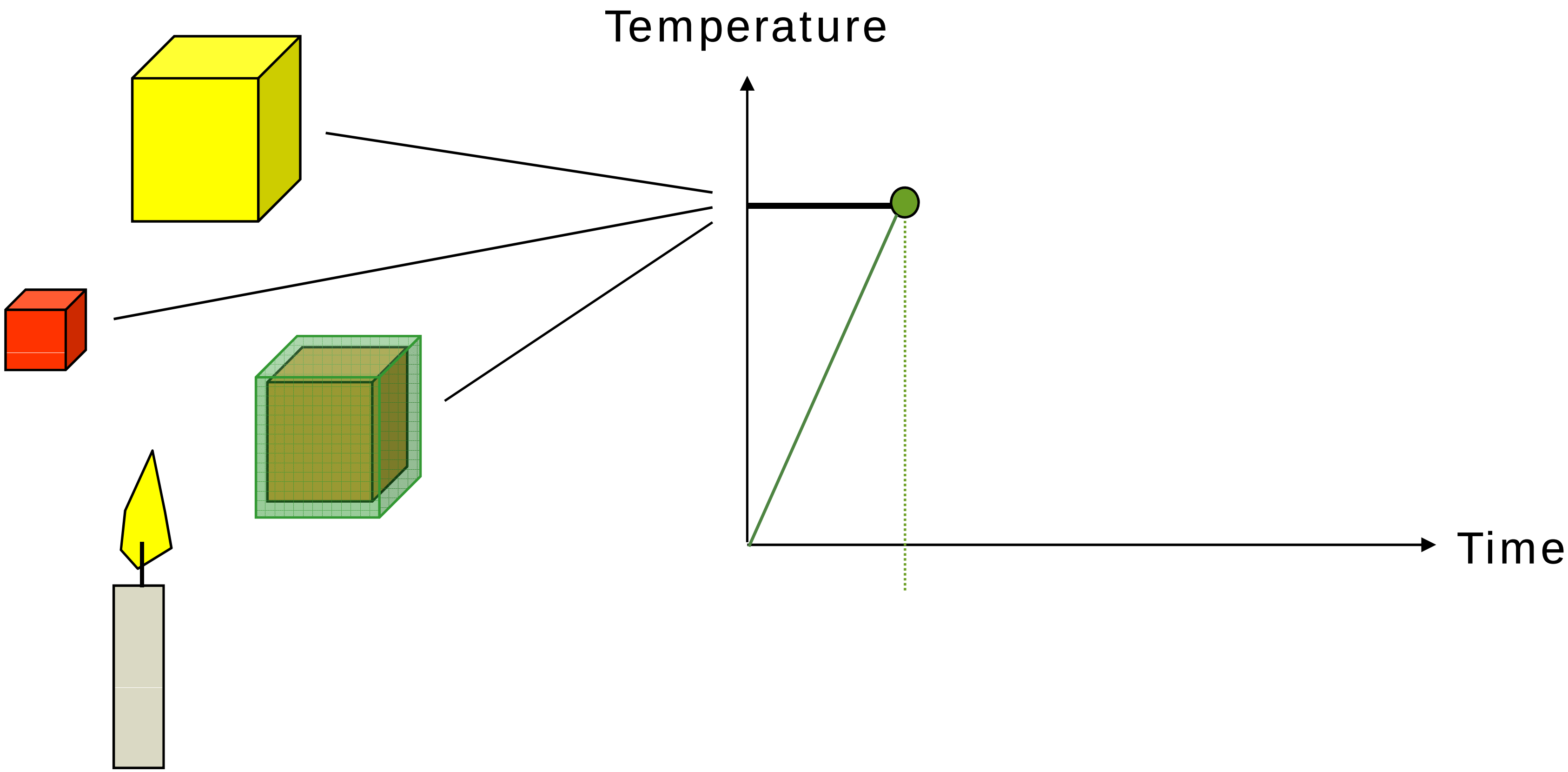
-Impurity concentration determined by analyzing resonance line intensities in the vacuum UV, supplemented by measurements of soft X-ray spectra; this data, coupled with a theory for ionization rates

- visible Bremsstrahlung radiation

- Spitzer's formula for the parallel resistivity

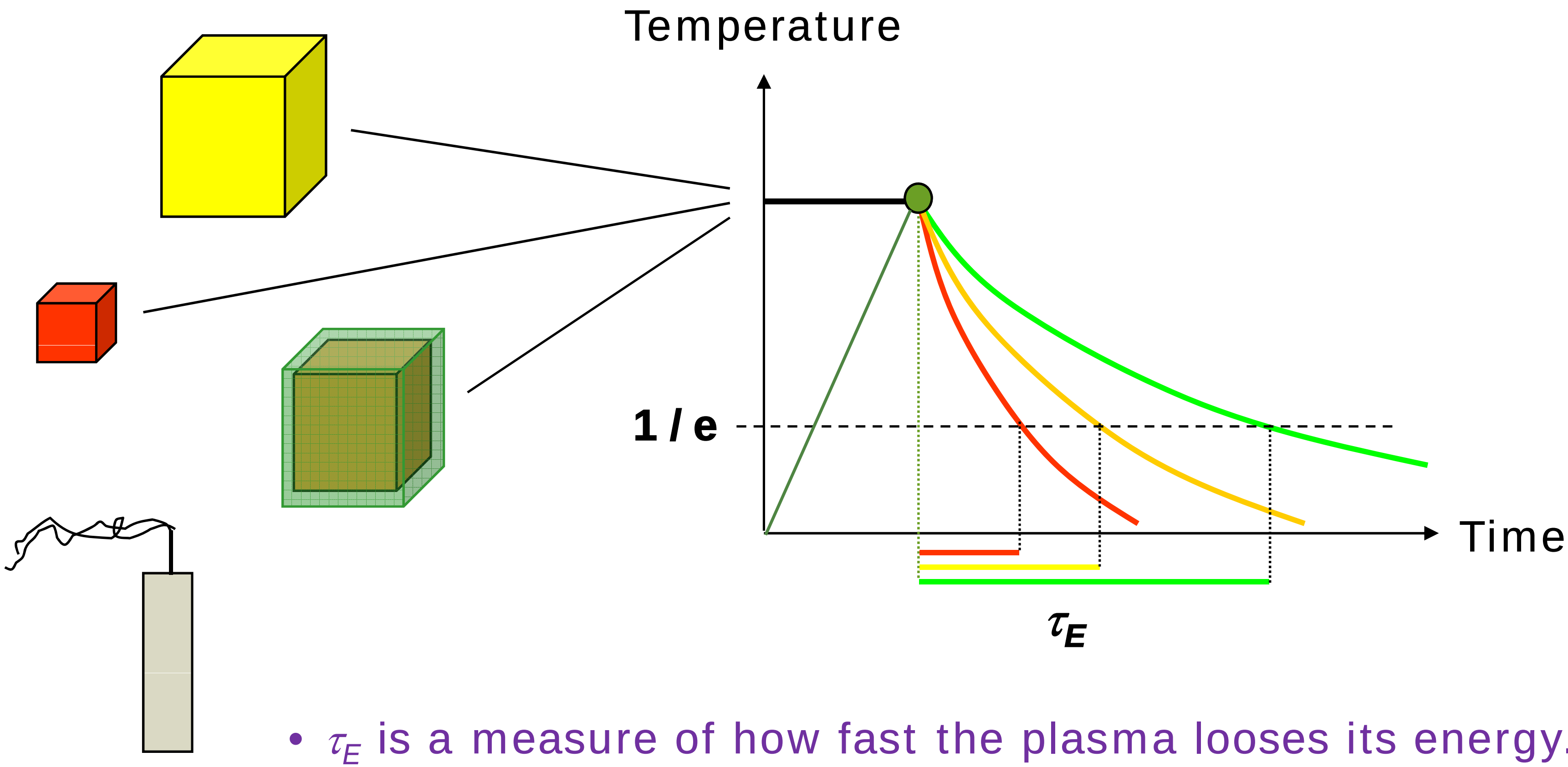
Basic Tokamak Variables

- **Energy confinement time**



Basic Tokamak Variables

- **Energy confinement time**



- τ_E is a measure of how fast the plasma looses its energy.
- The loss rate is smallest, τ_E largest if the fusion plasma is big and well insulated.

Basic Tokamak Variables

- **Energy confinement time**

$$W = \frac{1}{2\pi}\int \frac{3}{2} p dS = \int_0^a \frac{3}{2} k_B (n_e T_e + n_i T_i) r dr$$ ~ total thermal energy in the torus

total heat flux radiation energy loss rate

$$\frac{\partial(\rho u)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\{r(\rho h v_D + Q_r)\} = j_\varphi E_\varphi - L \quad (\rho u = \frac{3}{2} p, \quad \rho h = \frac{5}{2} p)$$

internal energy enthalpy density

$$\frac{\partial}{\partial t}(\ln W) + \frac{1}{\tau_E} = \frac{1}{\tau_E^*} - \frac{1}{\tau_E^R}$$

$$\tau_E \equiv \frac{W}{\left[r\left(\frac{5}{2} p v_D + Q_r\right)\right]_{r=a}}, \quad \tau_E^* \equiv \frac{W}{\int_0^a j_\varphi E_\varphi r dr}, \quad \tau_E^R \equiv \frac{W}{\int_0^a L r dr}$$

Energy confinement time Energy replacement time Radiation loss time

Basic Tokamak Variables

- **Energy confinement time**

$$\tau_E = \frac{W}{\dfrac{W}{\tau_E^*} - \dfrac{\partial W}{\partial t}} = \frac{W}{P_{in} - \dfrac{\partial W}{\partial t}} \approx \frac{W}{P_{in}} = \frac{\text{stored energy}}{\text{applied heating power}}$$

In steady conditions, neglecting radiation loss, Ohmic heating replaced by total input power

- To predict the performance of future devices, the energy confinement time is one of the most important parameter.
- Since tokamak transport is anomalous, empirical scaling laws for energy confinement are necessary.
- ***Empirical scaling laws***: regression analysis from available experimental database.

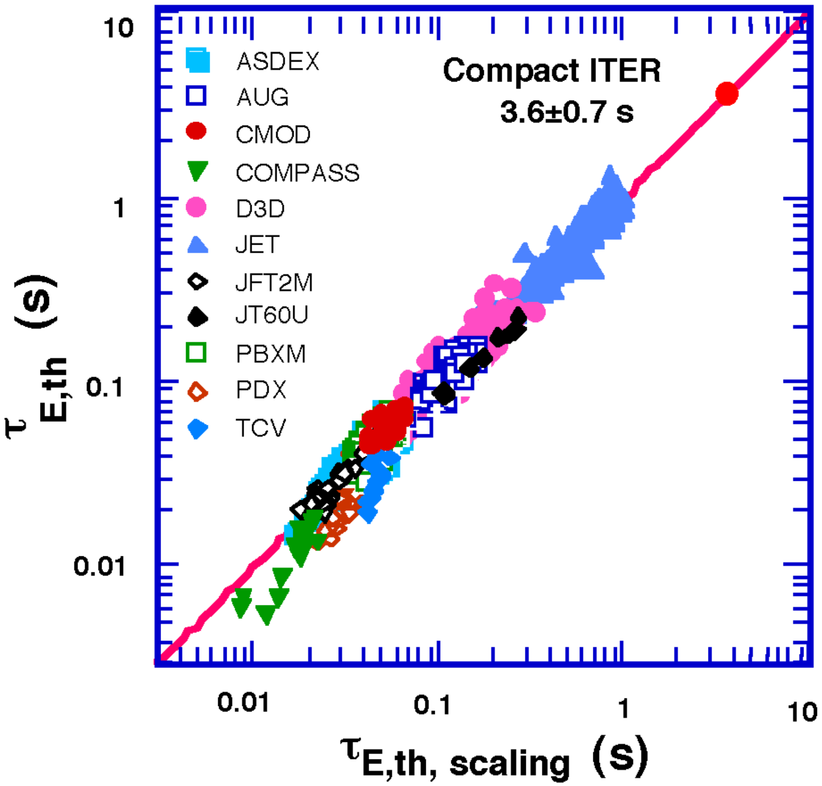
$$\tau_{th,E}^{fit} = C I^{\alpha I} B^{\alpha B} P^{\alpha P} n^{\alpha n} M^{\alpha M} R^{\alpha R} \varepsilon^{\alpha \varepsilon} \kappa^{\alpha \kappa}$$

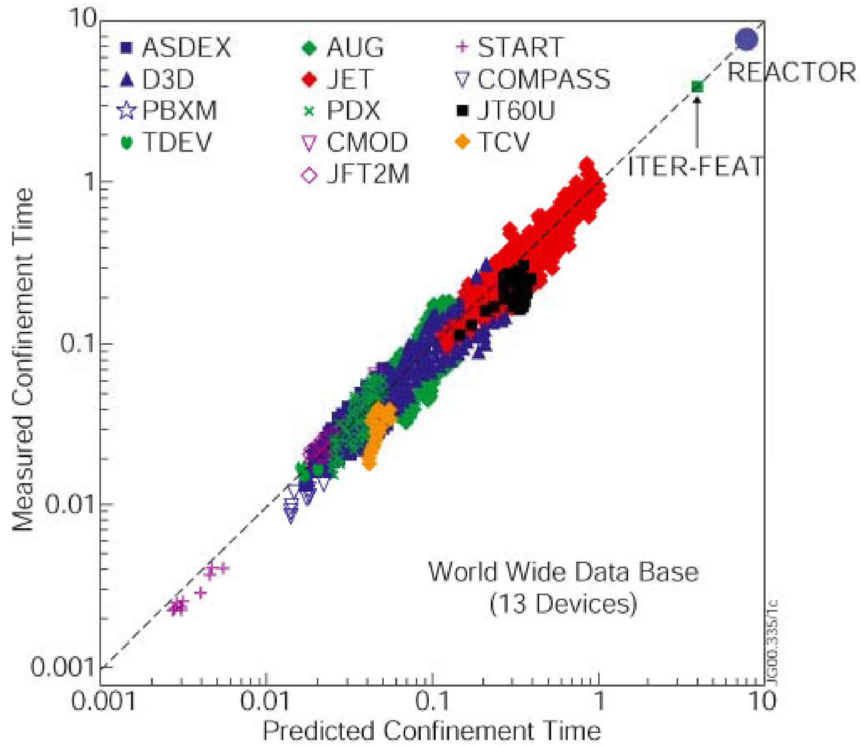
in engineering variables

Basic Tokamak Variables

- **Energy confinement time**

$$\tau_{th,E}^{IPB98(y,2)} = 0.0562\, I^{0.93} B^{0.15} P^{-0.69} n^{0.41} M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}$$





τ_E in KSTAR and ITER?

Why should ITER be large?

Basic Tokamak Variables

- **Particle confinement time**

$$\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r n_e v_D) = S_e(r)$$ electron number density source

$$\tau_p = \tau_p^*$$ In steady state

$$\tau_p \equiv \frac{\int_0^a n_e r dr}{\left[r n_e v_D\right]_{r=a}}, \quad \tau_p^* \equiv \frac{\int_0^a n_e r dr}{\int_0^a S_e r dr}$$

particle confinement time

particle replacement time

Basic Tokamak Variables

- **Momentum confinement time**

$$\frac{\partial}{\partial t}(\rho v_\varphi) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r v_\varphi) + \nabla\cdot\Pi\cdot\widehat{\varphi} = F_b\cdot\widehat{\varphi}$$

Momentum equation having the toroidal component

$$\tau_\varphi = \tau_\varphi^*$$

In steady state

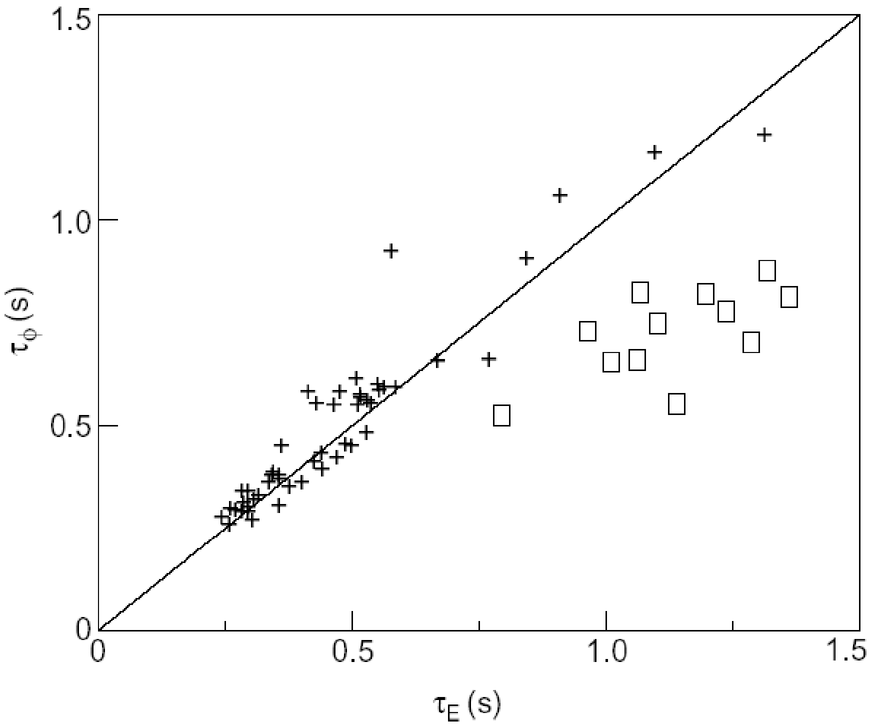
$$\tau_\varphi \equiv \frac{H_\varphi}{\int_0^a \nabla\cdot\Pi\cdot\widehat{\varphi}rdr + \left[r\rho v_r v_\varphi\right]_{r=a}}, \quad \tau_\varphi^* \equiv \frac{H_\varphi}{\int_0^a F_b\cdot\widehat{\varphi}rdr} = \frac{2\pi^2 R_0^2 a^2 H_\varphi}{\text{Beam torque}}, \quad H_\varphi \equiv \int_0^a \rho v_\varphi rdr$$

Toroidal momentum confinement time

Toroidal momentum replacement time

Basic Tokamak Variables

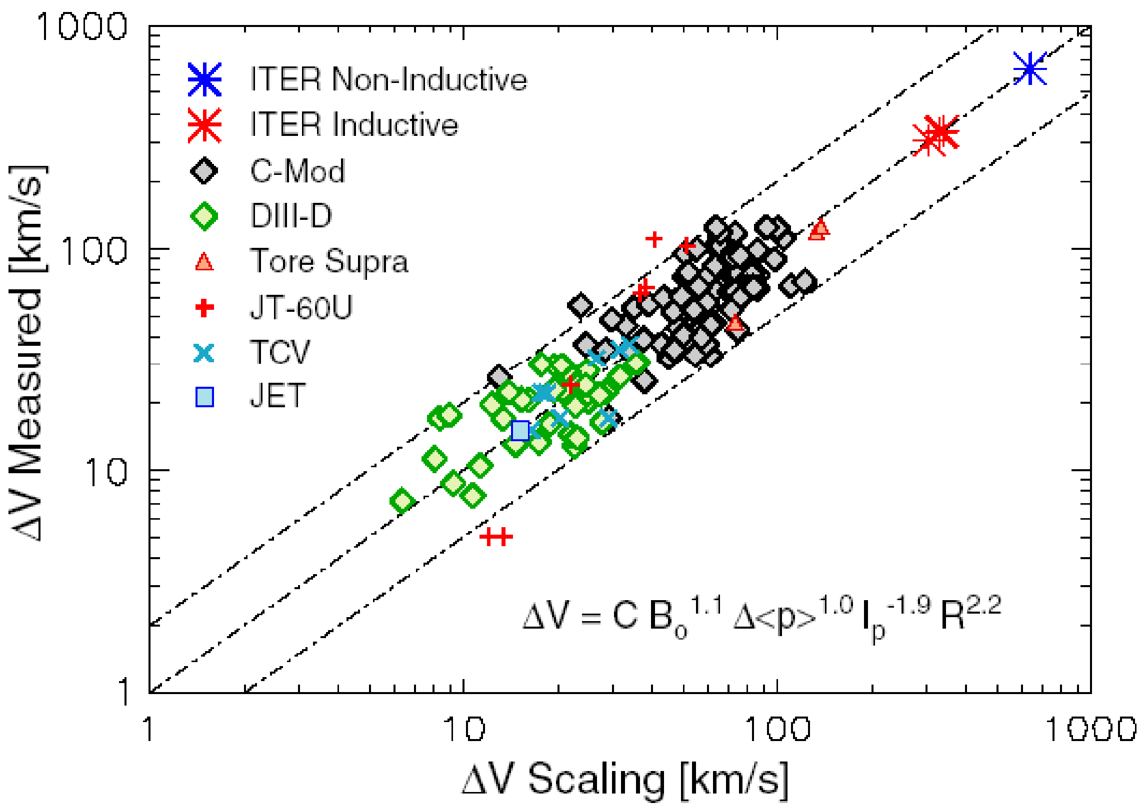
- **Momentum confinement time**



- Toroidal angular momentum confinement time of thermal particles during NBI versus simultaneously measured energy confinement time for steady state L-mode and ELMy H-mode discharges (crosses), and for transient ELM free phase of hot ion H-mode discharges (squares) in JET

Basic Tokamak Variables

- **Intrinsic rotation**



J. E. Rice *et al*, *Nucl. Fusion* **47** 1618 (2007)