

Fusion Reactor Technology I

(459.760, 3 Credits)

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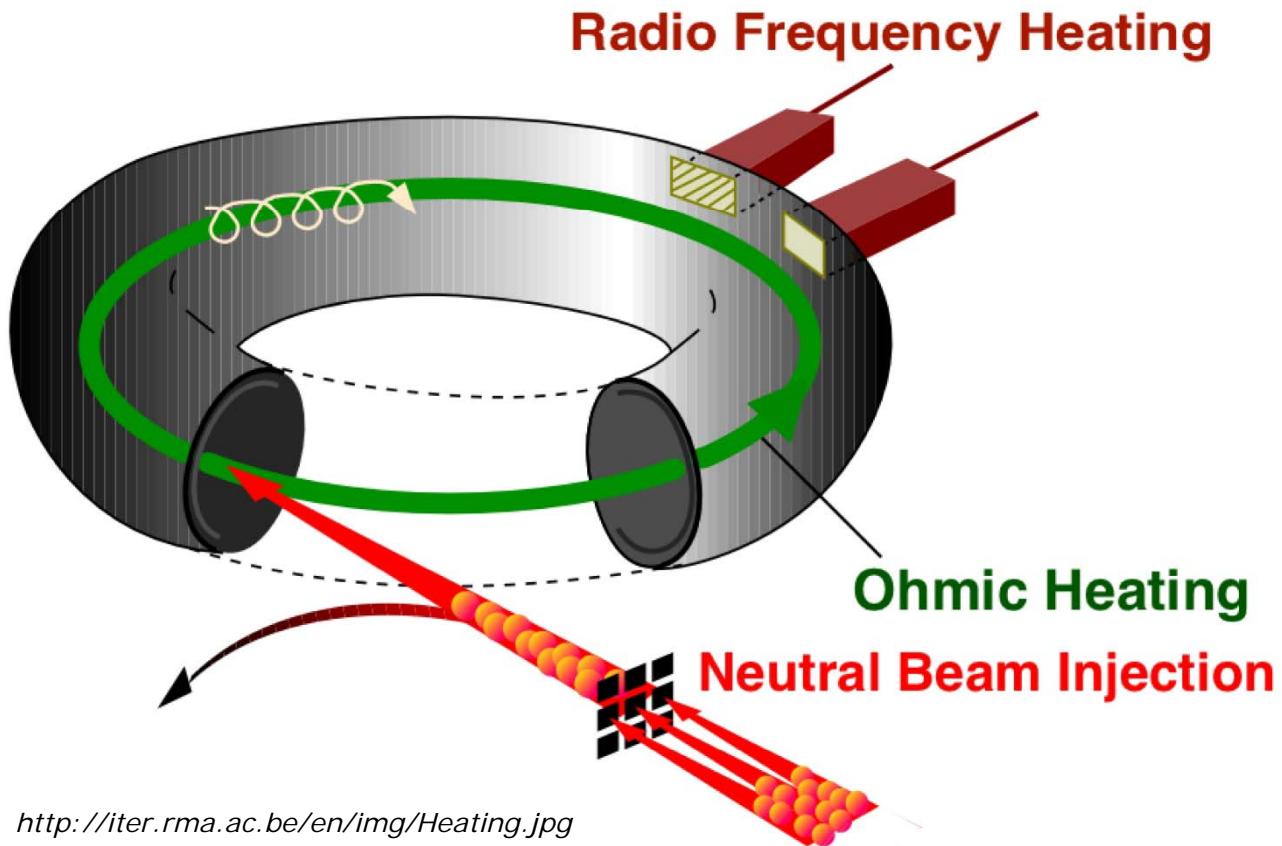
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Week 11. Energy Losses from Tokamaks

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Week 13-14. Tokamak Operation

Heating and Current Drive



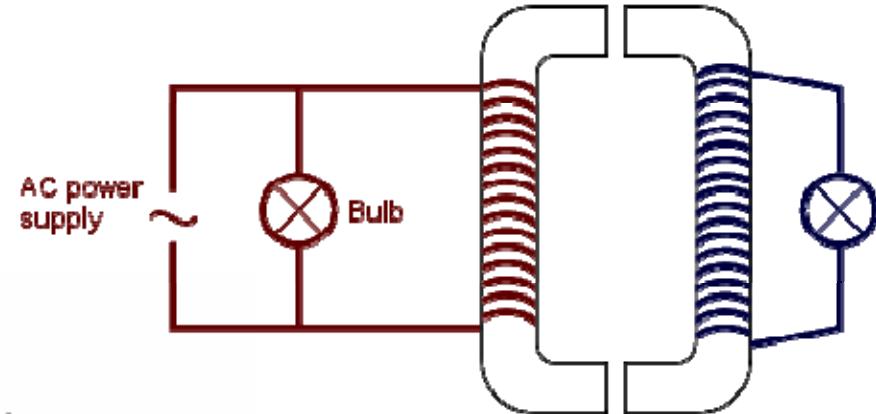
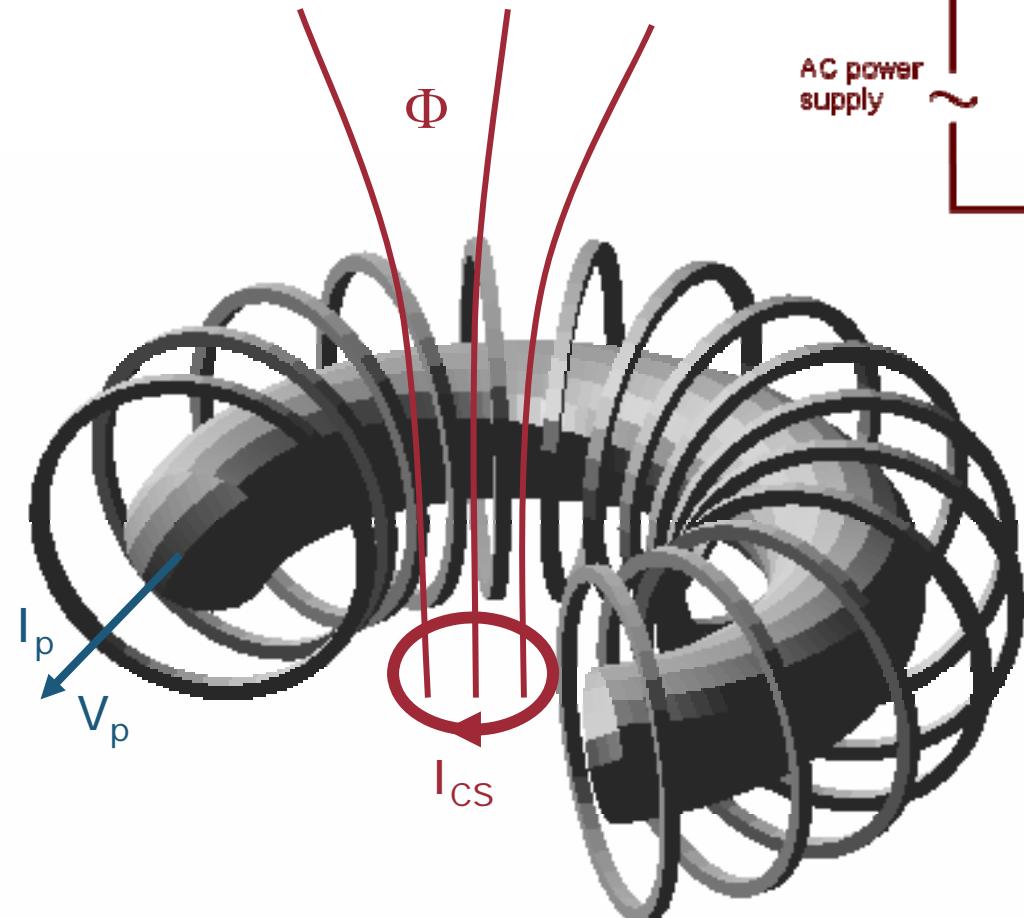
Ohmic Heating

SAMIK

Electric blanket



Ohmic Heating



I_{CS} I_p

$$L_p \dot{I}_p + I_p R_p = V_p = -\dot{\phi}$$

Ohmic Heating

$$L_p \dot{I}_p + I_p R_p = V_p = -\dot{\phi}$$

Total change in magnetic flux needed to induce a final current

$$\Delta\phi_{ind} = \int_0^{t_f} \dot{\phi} dt = L_p I_p^f \approx \mu_0 R_0 \left[\ln\left(\frac{8R_0}{a\sqrt{k}}\right) + \frac{l_i}{2} - 2 \right] I_p^f$$

$$l_i \approx \ln[1.65 + 0.89(q_{95} - 1)] \quad \text{internal inductance}$$

Additional magnetic flux needed to overcome resistive losses during start up

$$\Delta\phi_{res} = C_E \mu_0 R_0 I_p^f, \quad C_E \approx 0.4 \quad \text{Ejima coefficient}$$

Further change in magnetic flux needed to maintain I_p after start up

$$\Delta\phi_{burn} = \int_0^t I_p^f R_p dt'$$

Technological limit to the maximum value of B_{OH}

$$\Delta\phi \approx \pi r_v^2 \Delta B_{OH} \quad \text{Tokamak is inherently a pulsed device.}$$

Ohmic Heating

$$P_\Omega = \eta j^2 \quad \text{Ohmic heating density}$$

$$\eta_n = \frac{\eta_s}{\left(1 - \left(\frac{r}{R}\right)^{\frac{1}{2}}\right)^2}$$

Neoclassical resistivity

η_s : Spitzer resistivity

$$\eta \approx 8 \times 10^{-8} Z_{eff} / T_e^{\frac{3}{2}} \quad (r = a/2, R/a = 3)$$

$$j(r) = j_0 (1 - (r/a)^2)^\nu \quad B_\theta(r) = \frac{\mu_0 a^2 j_0}{2(\nu+1)r} \left[1 - \left(1 - \frac{r^2}{a^2}\right)^{\nu+1} \right]$$

Ampère's law

$$q_a = B_\phi a / B_\theta R, \quad q_a / q_0 = \nu + 1, \quad j_0 = 2B_\phi / Rq_0\mu_0$$

$$\langle j^2 \rangle = 2 \left(\frac{B_\phi}{\mu_0 R} \right)^2 \frac{1}{q_a \left(q_a - \frac{1}{2} q_0 \right)}$$

Ohmic Heating

$$P_\Omega = \eta \langle j^2 \rangle = 3nT / \tau_E = P_L$$

$$\eta \langle j^2 \rangle = 1.0 \times 10^5 Z_{eff} T^{-3/2} (B_\phi / R)^2 / q_o (q_a - q_o / 2)$$

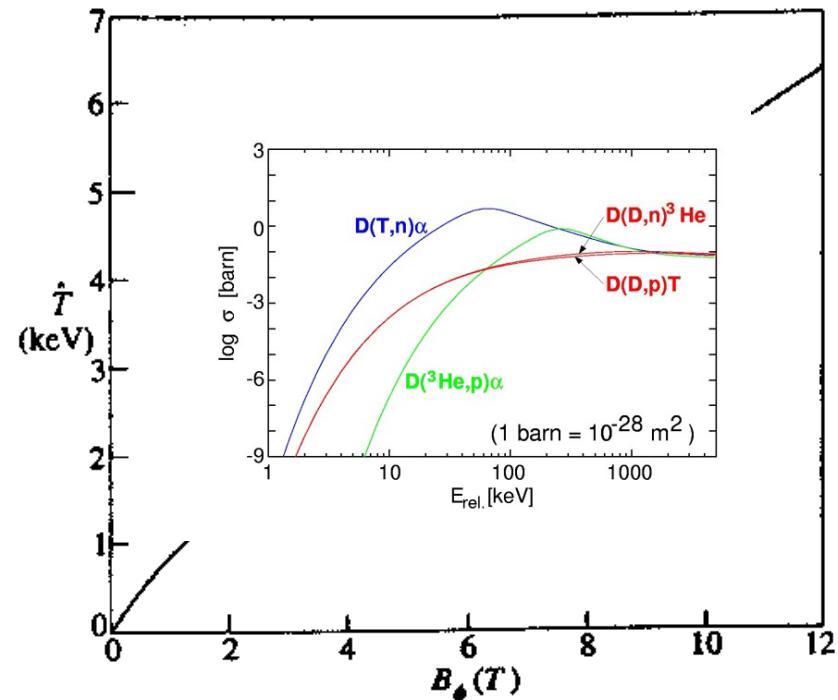
$$T = 2.7 \times 10^8 \left(\frac{Z_{eff} \tau_E}{n q_a q_0} \right)^{\frac{2}{5}} \left(\frac{B_\phi}{R} \right)^{\frac{4}{5}}$$

$$Z_{eff} = 1.5 \quad q_a q_o = 1.5$$

$$\tau_E = (n / 10^{20}) a^2 / 2$$

Alcator scaling

$$T = 0.87 B_\phi^{\frac{4}{5}}$$



Average temperatures above $\sim 7\text{keV}$ are necessary before alpha heating is large enough to achieve a significant fusion rate

Ohmic Heating

Plasma stability requires:

$$q_a = \frac{B_\phi a}{B_\theta R} = \frac{B_\phi a}{\frac{\mu_0 I_p}{2\pi a} R} = \frac{B_\phi a}{\frac{\mu_0 \langle j \rangle \pi a^2}{2\pi a} R} = \frac{2B_\phi}{\mu_0 \langle j \rangle R} > 2$$

$$\langle j \rangle < \frac{B_\phi}{\mu_0 R}$$

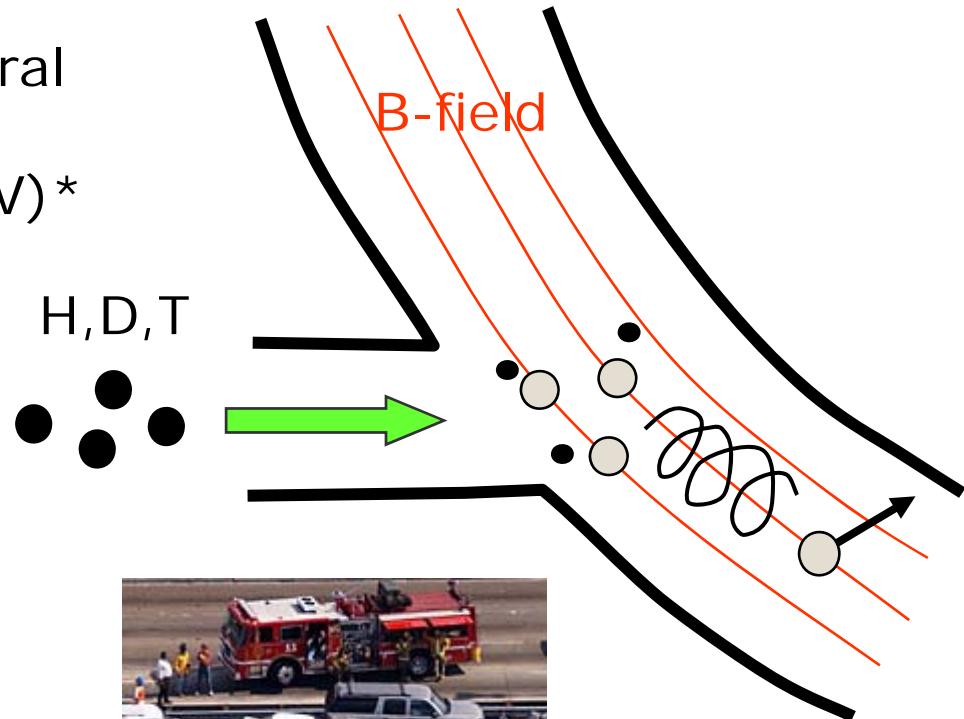
Neutral Beam Injection

Injection of a beam of neutral fuel atoms (H, D, T) at high energies ($E_b > 50$ keV)*

↓
Ionisation in the plasma

↓
Beam particles confined

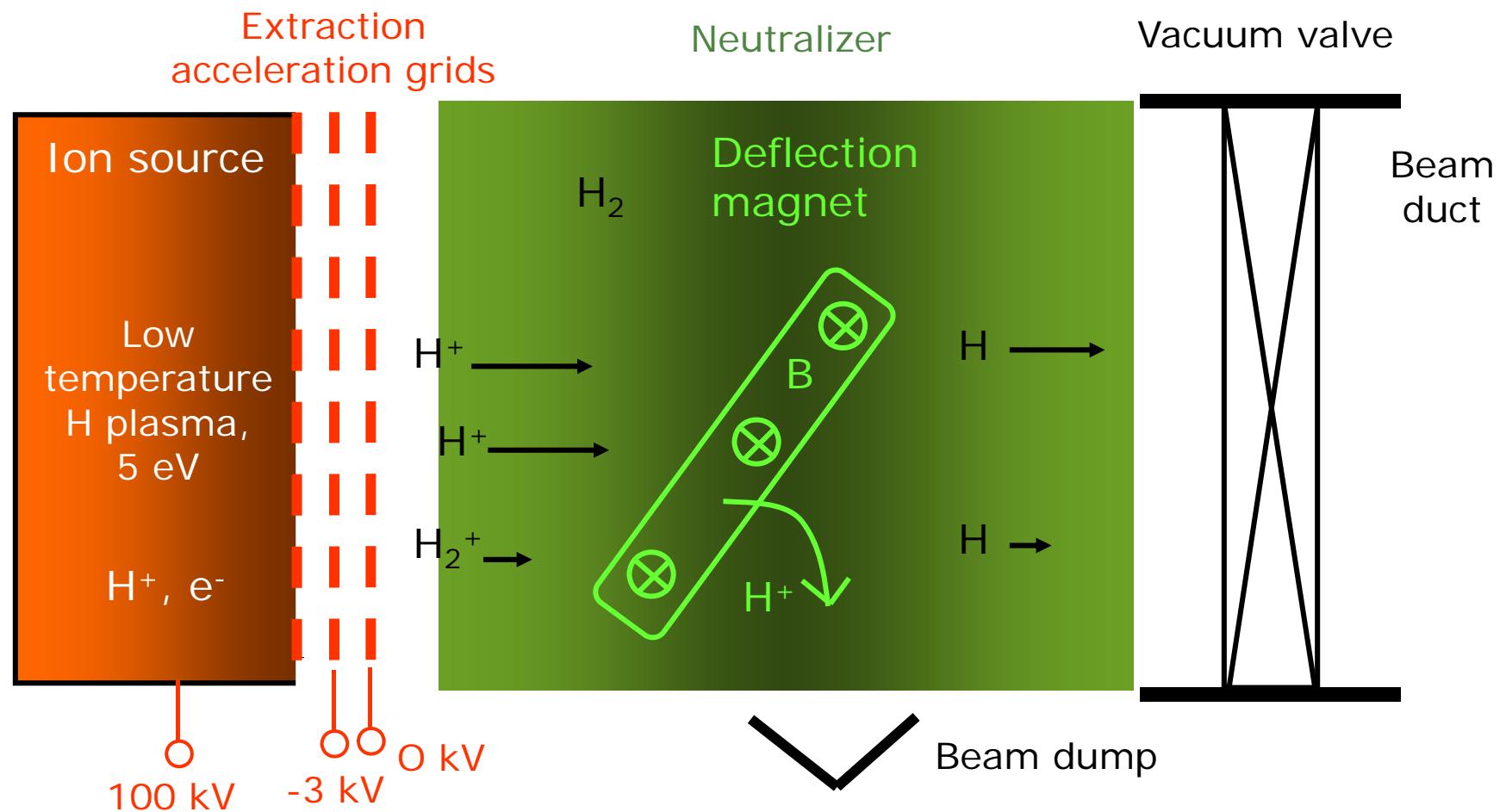
↓
Collisional slowing down



* $E_b = 120$ keV and 1 MeV for KSTAR and ITER, respectively

Neutral Beam Injection

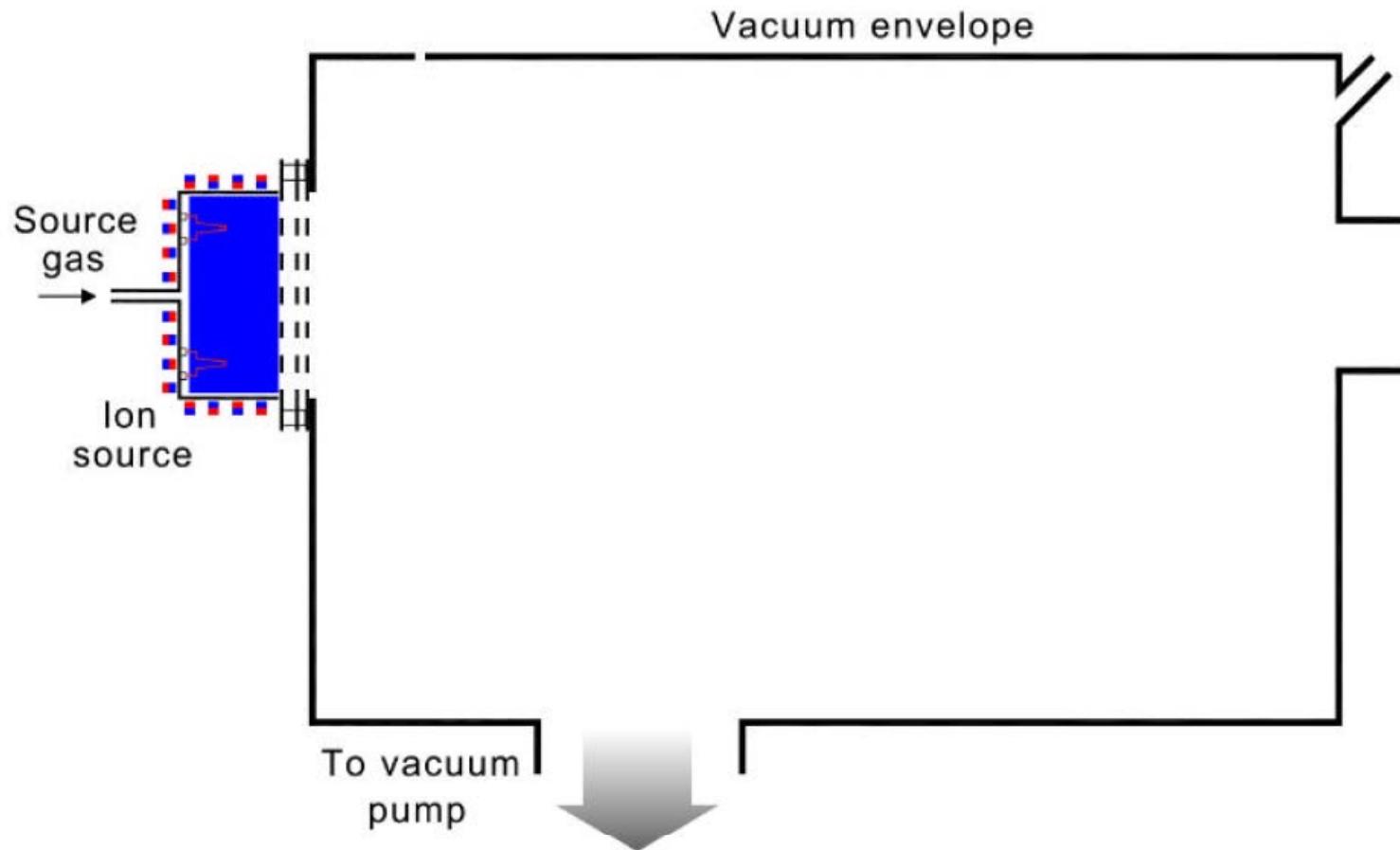
- Generation of a neutral fuel beam



Ex) W7-AS: $V=50$ kV, $I=25$ A, power deposited in plasma: 0.4 MW

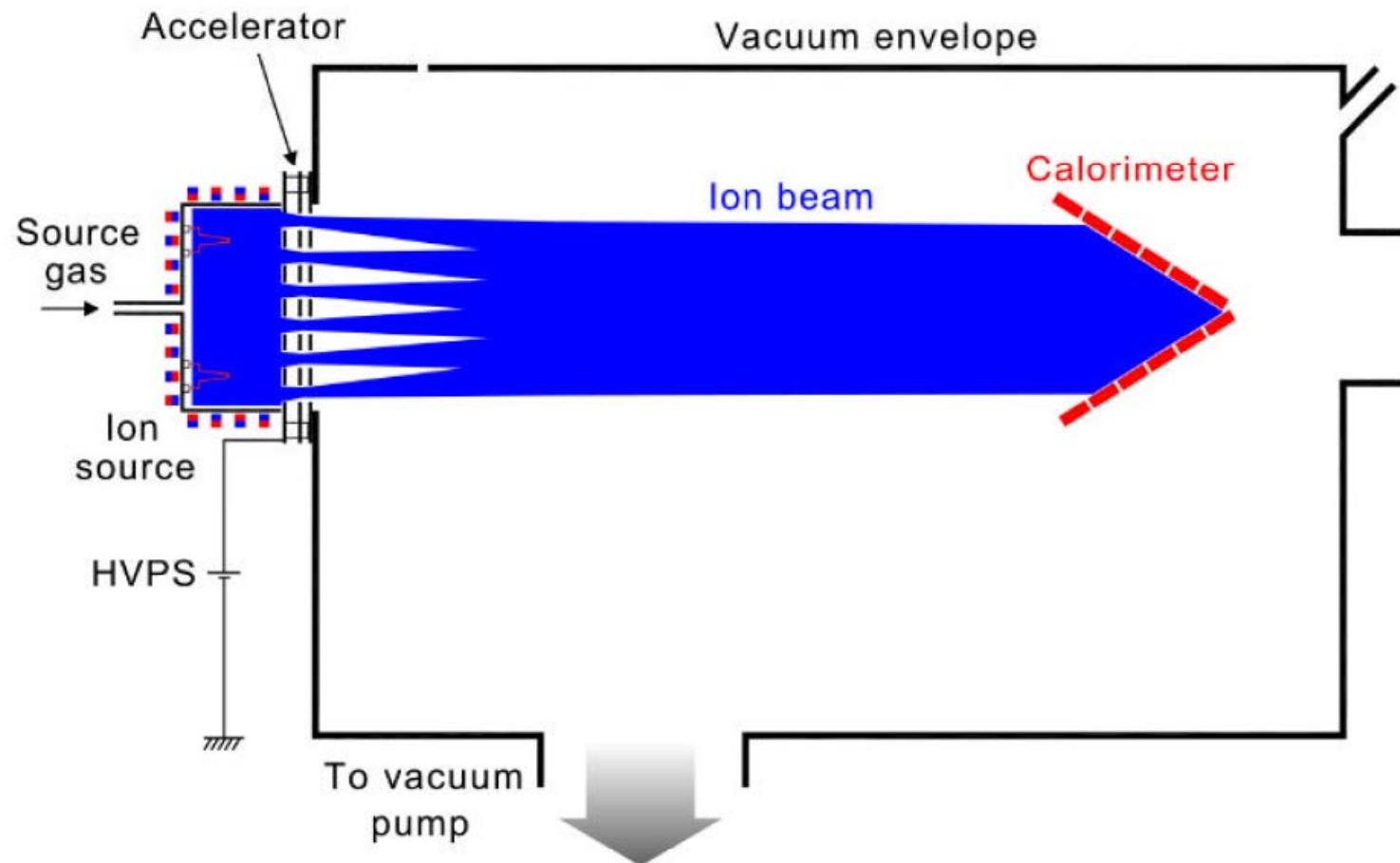
Neutral Beam Injection

- Ion source

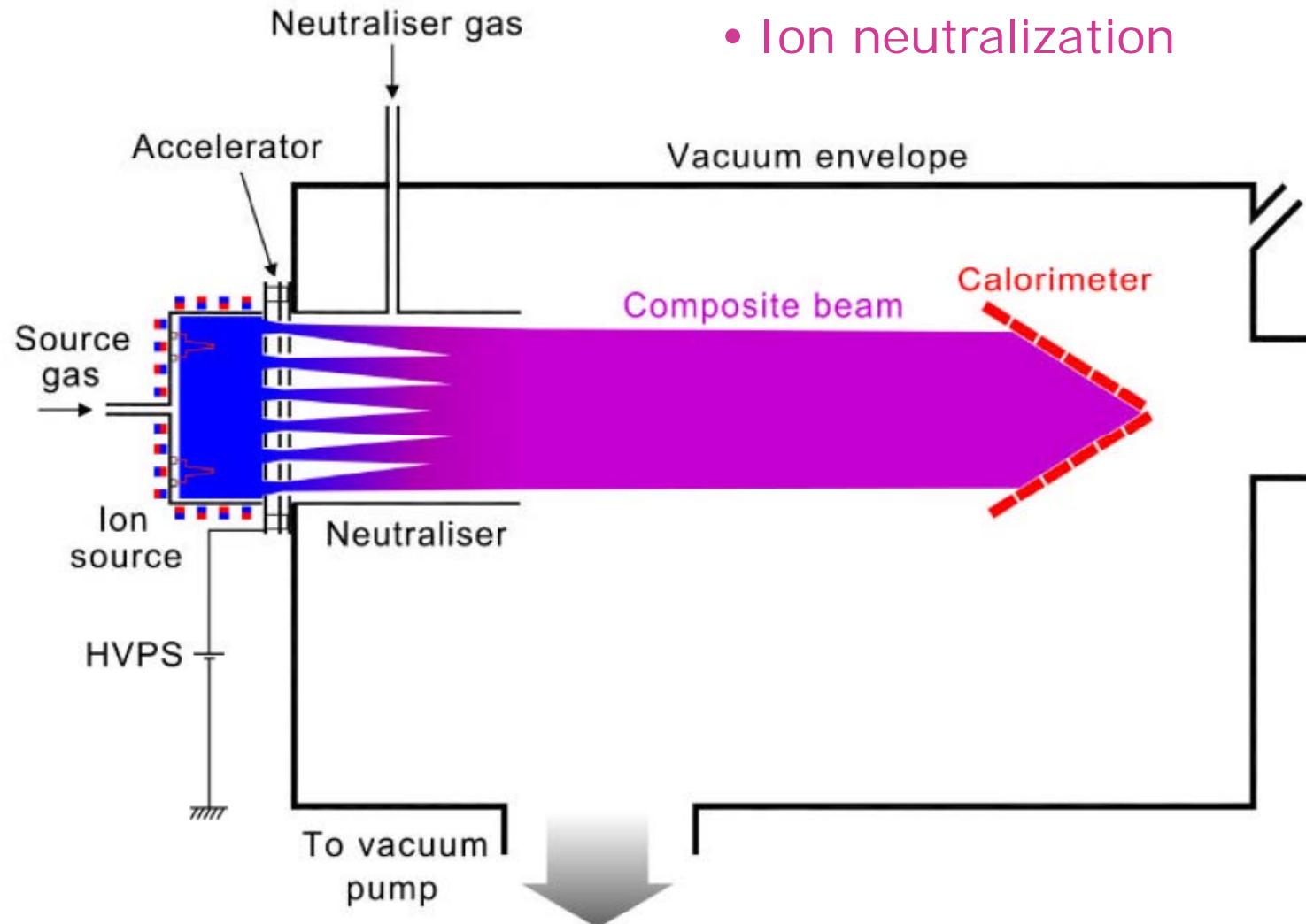


Neutral Beam Injection

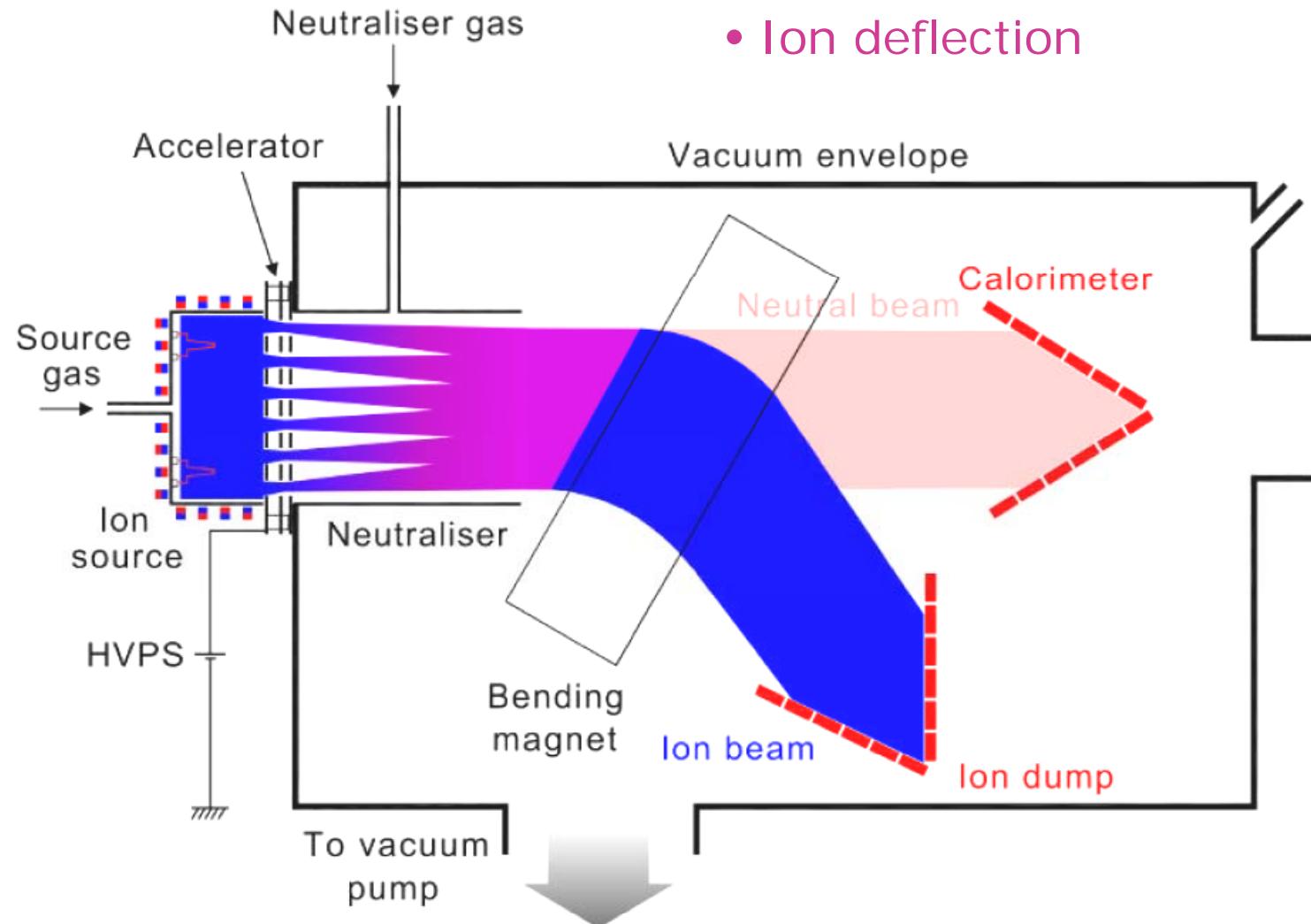
- Ion acceleration



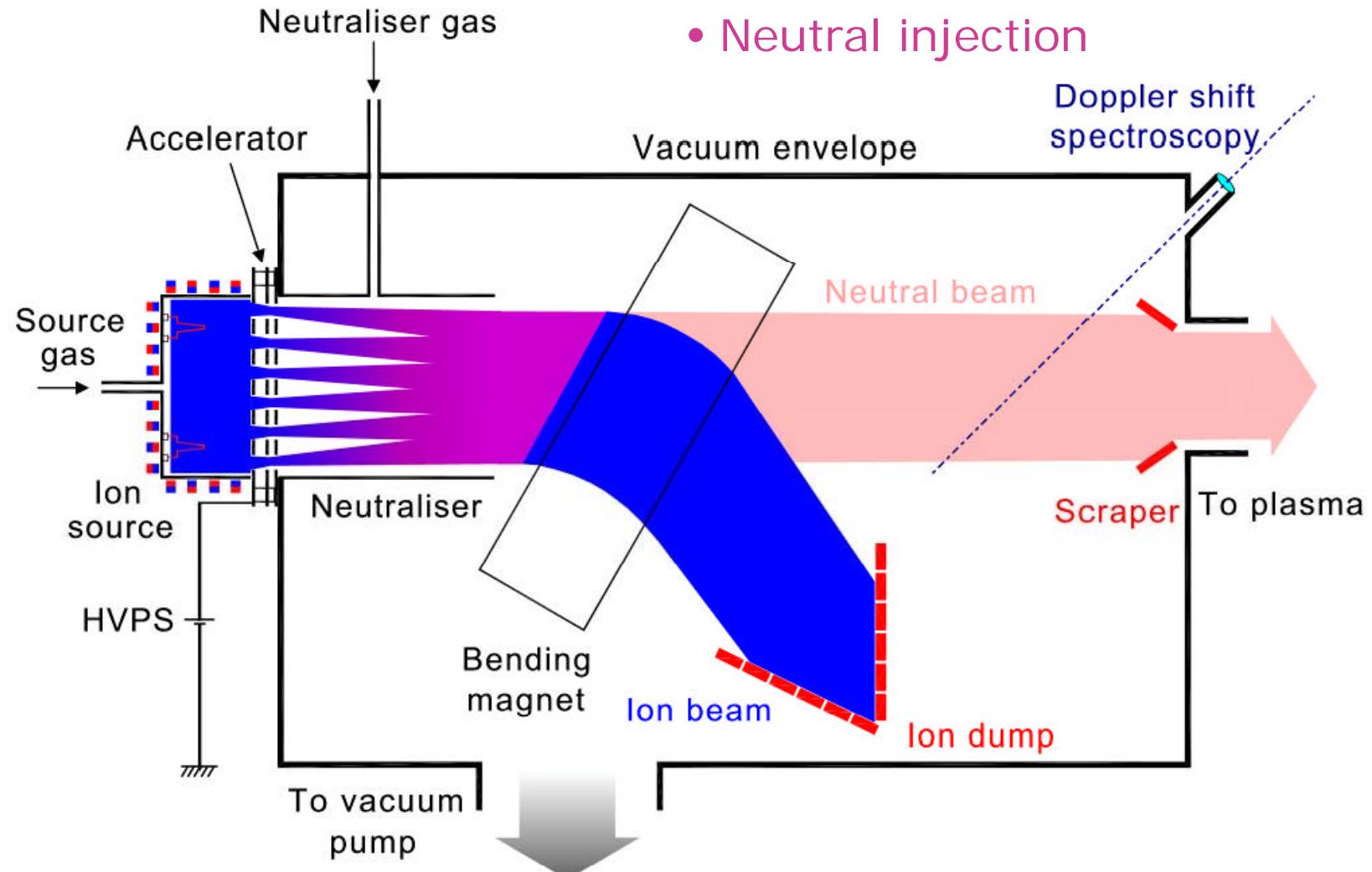
Neutral Beam Injection



Neutral Beam Injection

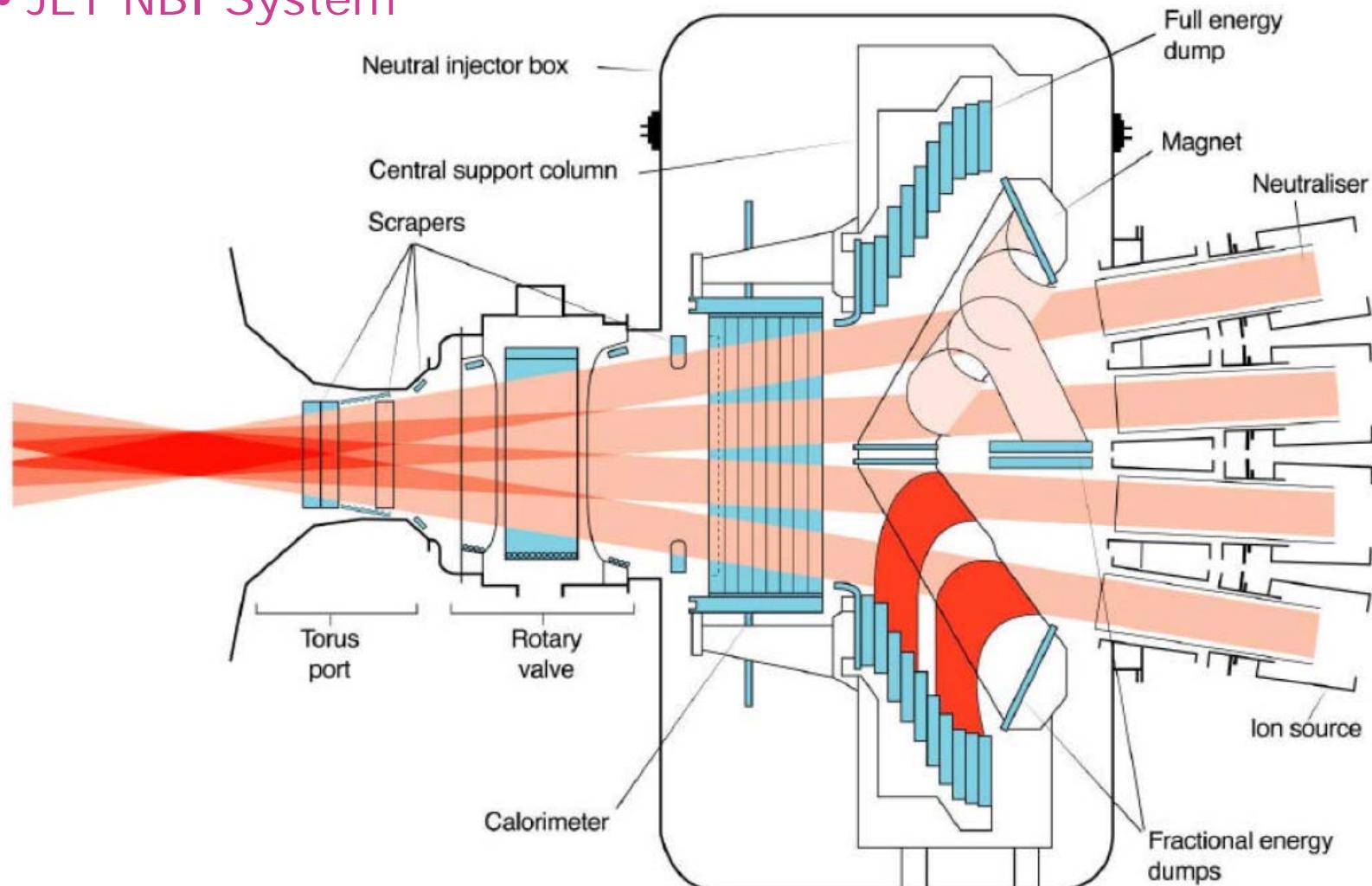


Neutral Beam Injection



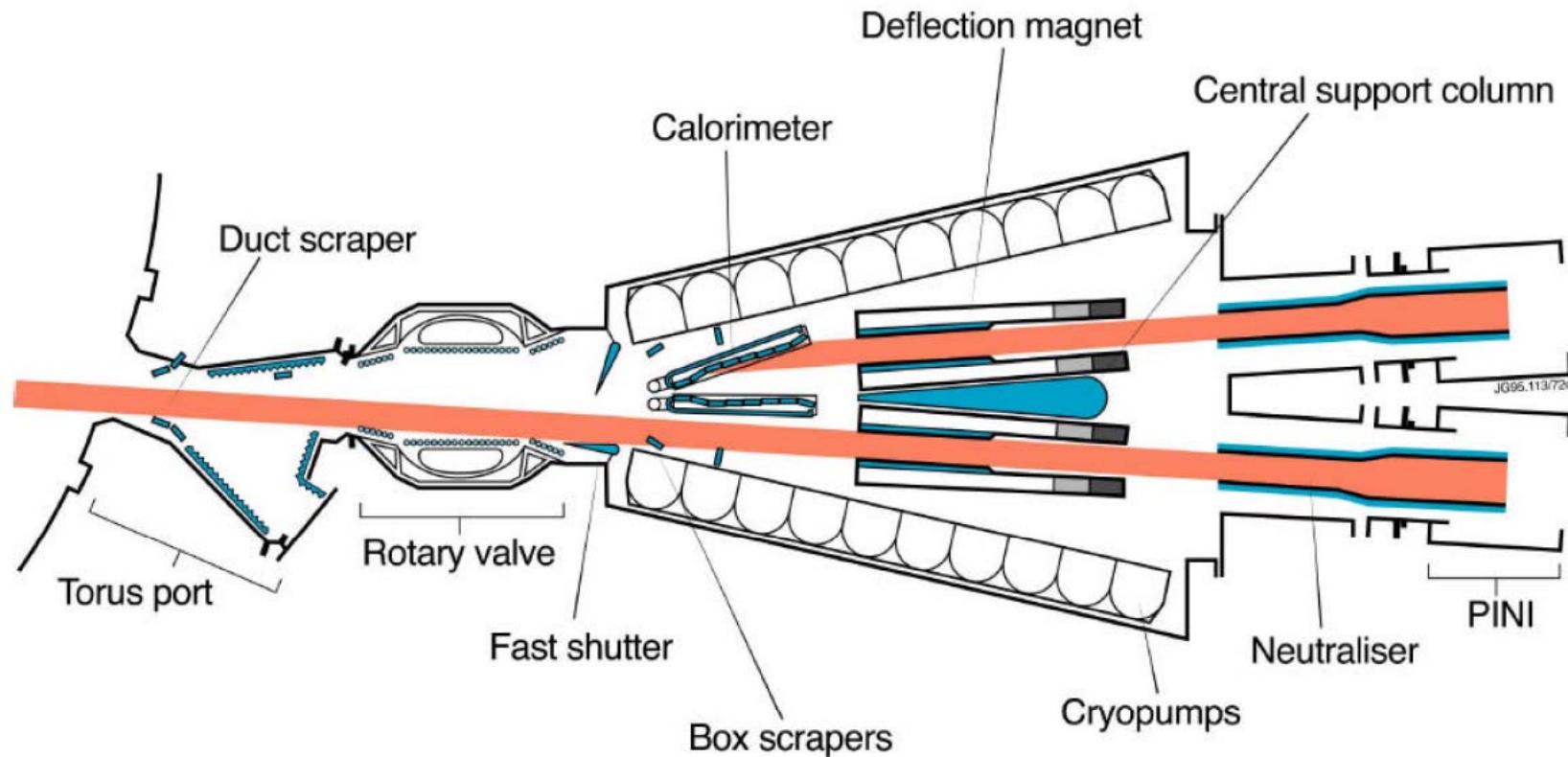
Neutral Beam Injection

- JET NBI System



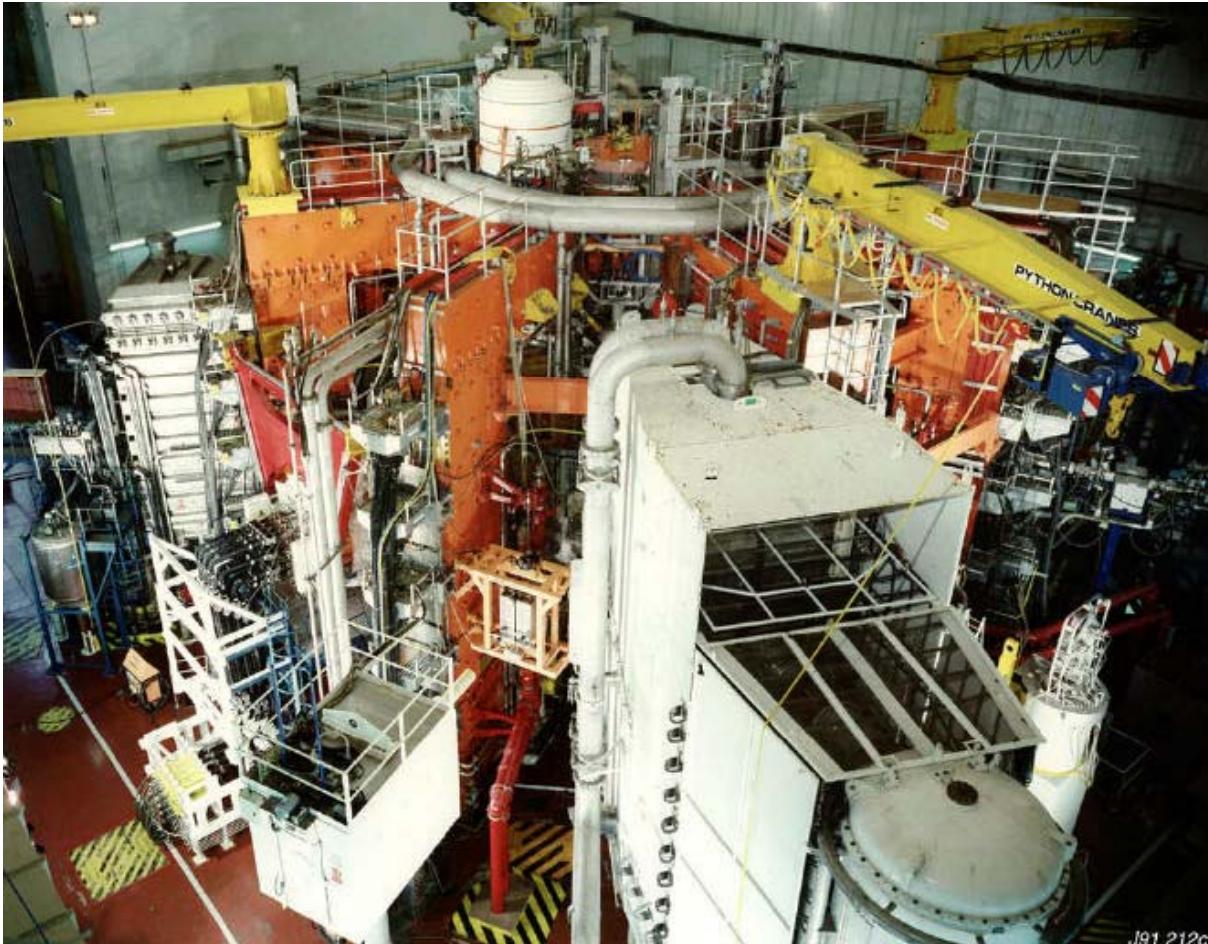
Neutral Beam Injection

- JET NBI System



Neutral Beam Injection

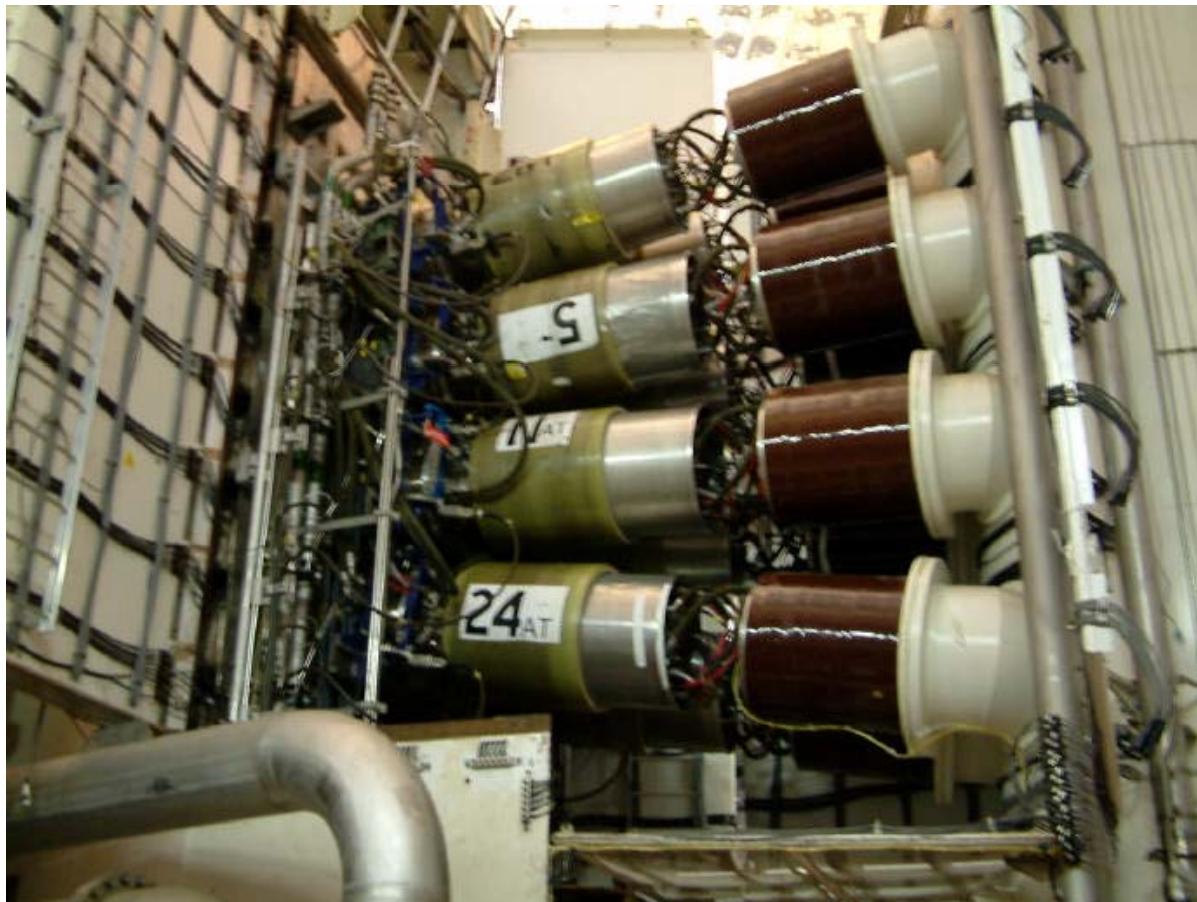
- JET NBI System



JET with machine and Octant 4 Neutral Injector Box

Neutral Beam Injection

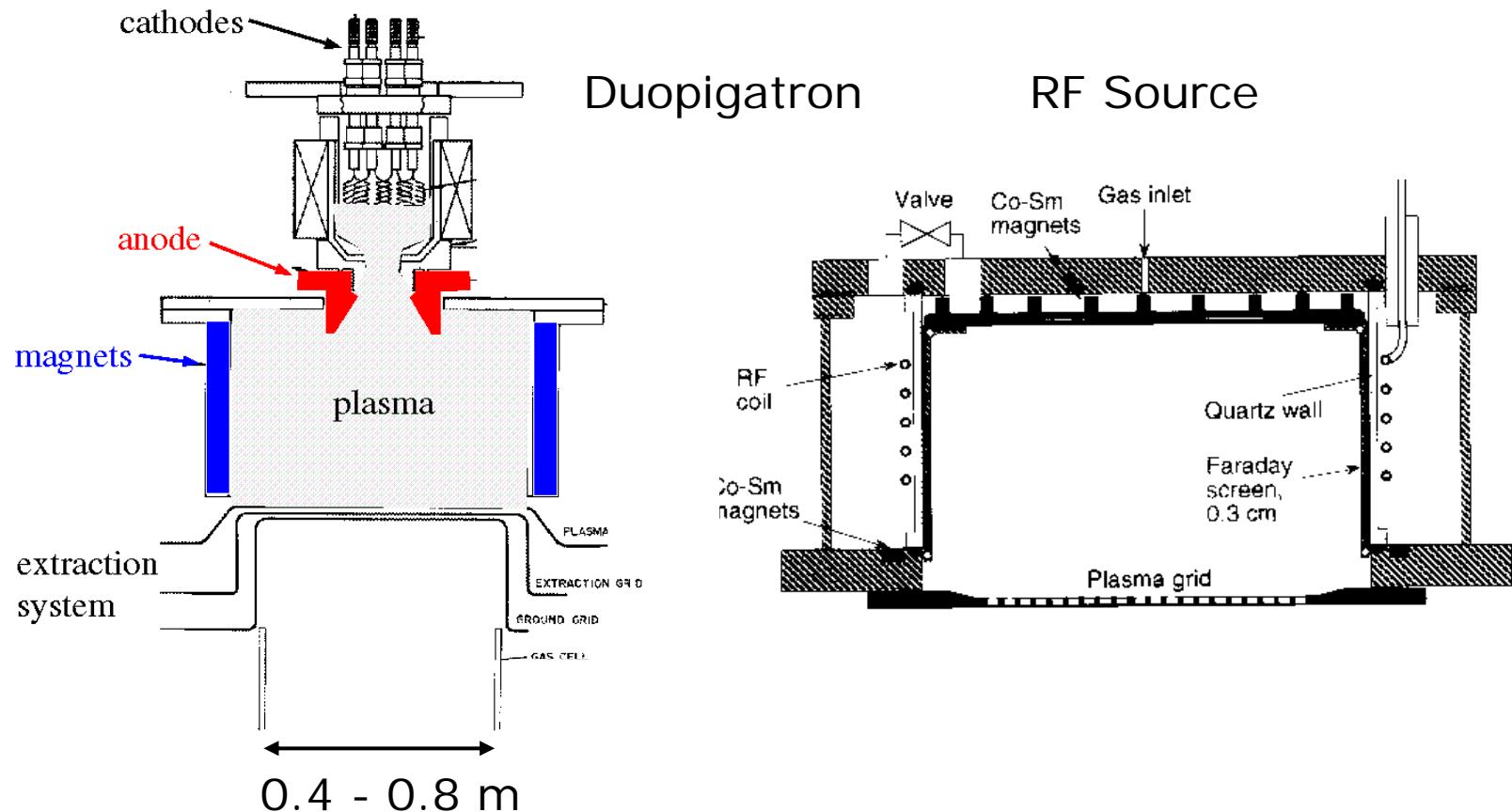
- JET NBI System



Octant 4 Neutral Injector Box

Neutral Beam Injection

- Ion sources

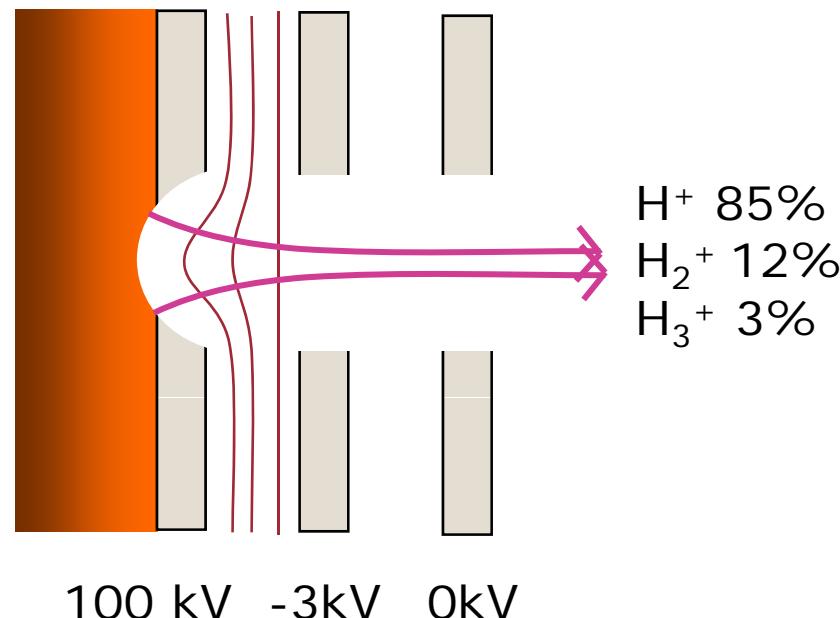


Cathodes difficult to replace, finite life time

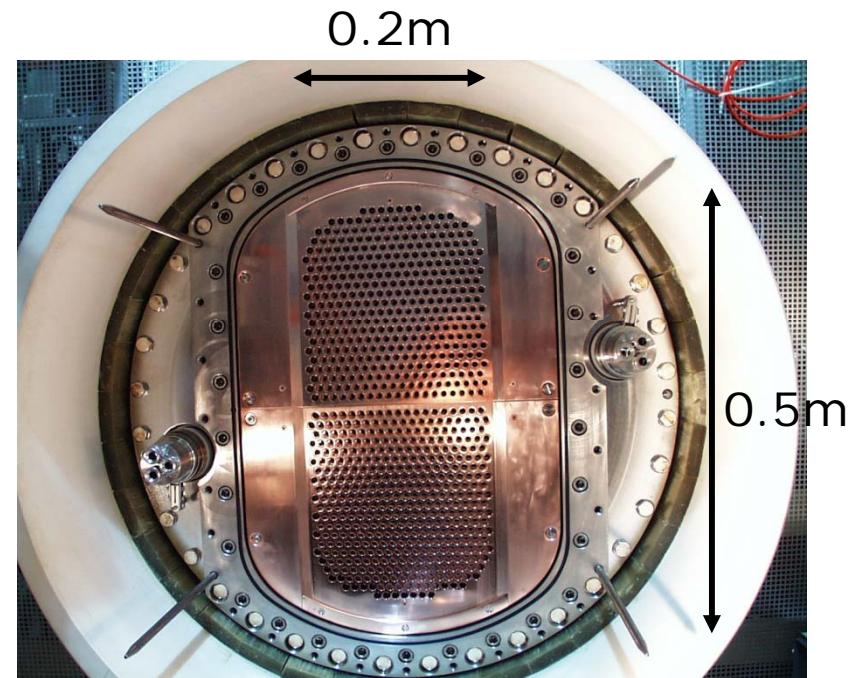
Neutral Beam Injection

- Extraction and steering

- 3-lens system
potential lines



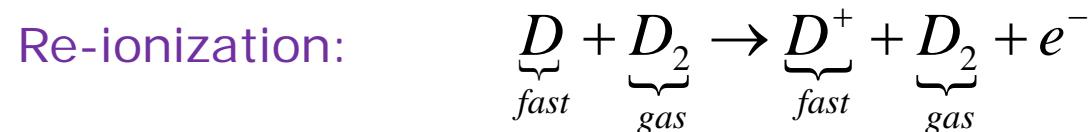
Typically 800 extraction holes per source



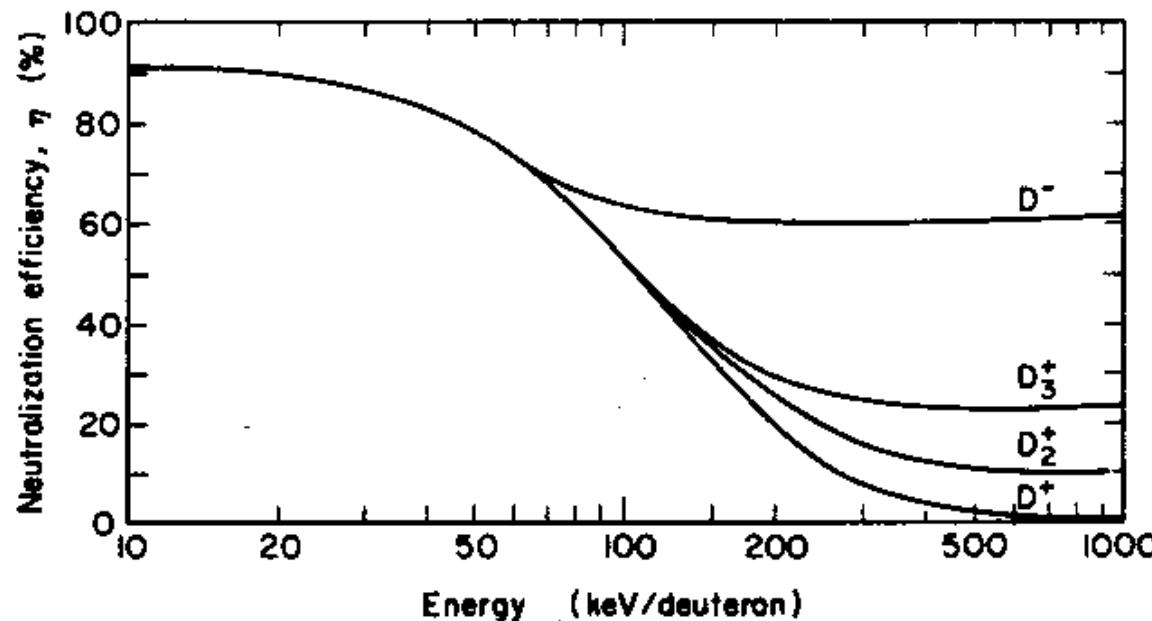
Grid system at ASDEX Upgrade

Neutral Beam Injection

- Neutralizer

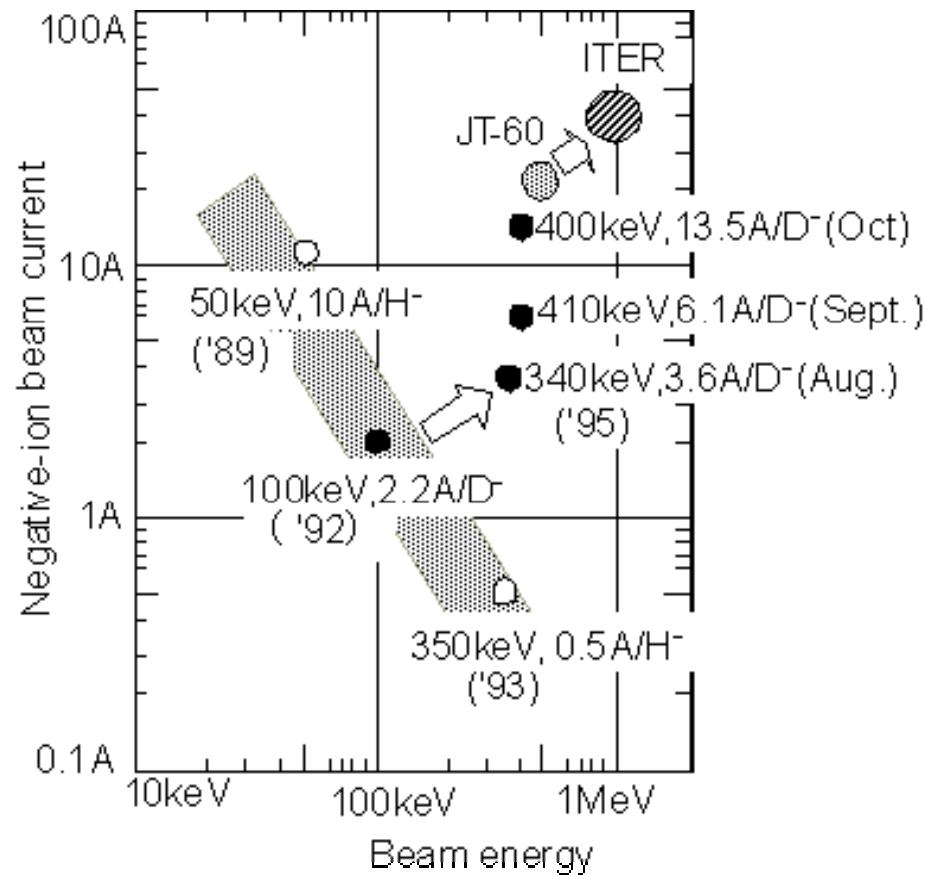


Efficiency:



Neutral Beam Injection

- Negative ion beam development in JT-60U



Neutral Beam Injection

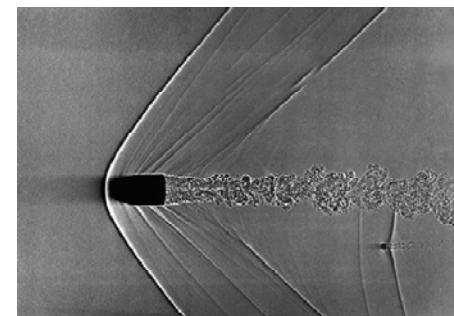
- Ionisation in the plasma



Attenuation of a beam of neutral particles in a plasma



n : density
 σ : cross section



beam energy



NBI

Andy Warhol

Neutral Beam Injection

- Ionisation in the plasma



Attenuation of a beam of neutral particles in a plasma

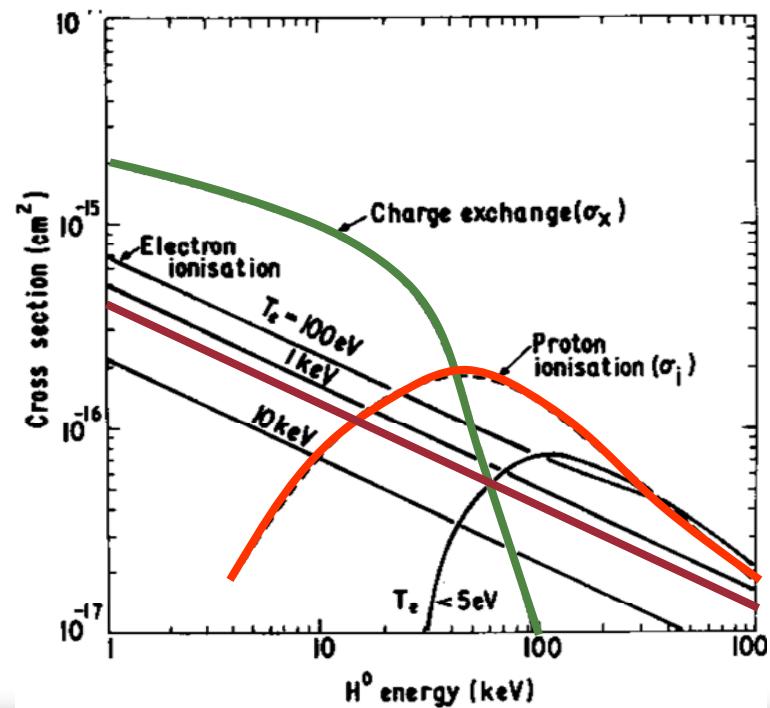
$$\frac{dN_b(x)}{dx} = -N_b(x)n(x)\sigma_{tot}$$

Ex) beam intensity: $I(x) = I_0 \cdot \exp(-x/\lambda)$

$$E_{b0} = 70 \text{ keV} \quad \sigma_{tot} = 5 \cdot 10^{-20} \text{ m}^2$$

$$n = 5 \cdot 10^{20} \text{ m}^{-3} \quad \lambda = \frac{1}{n\sigma_{tot}} \approx 0.4 \text{ m}$$

In large reactor plasmas,
beam cannot reach core!



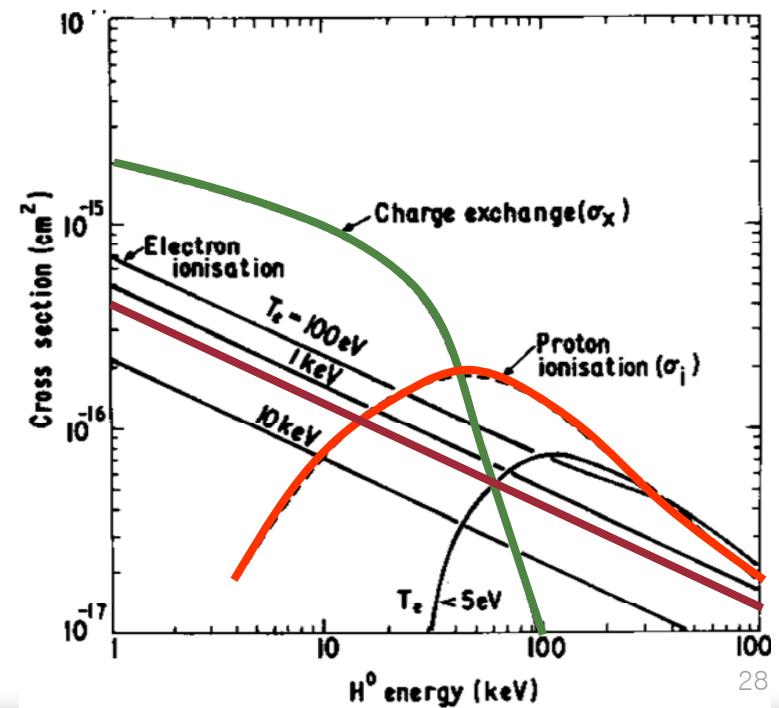
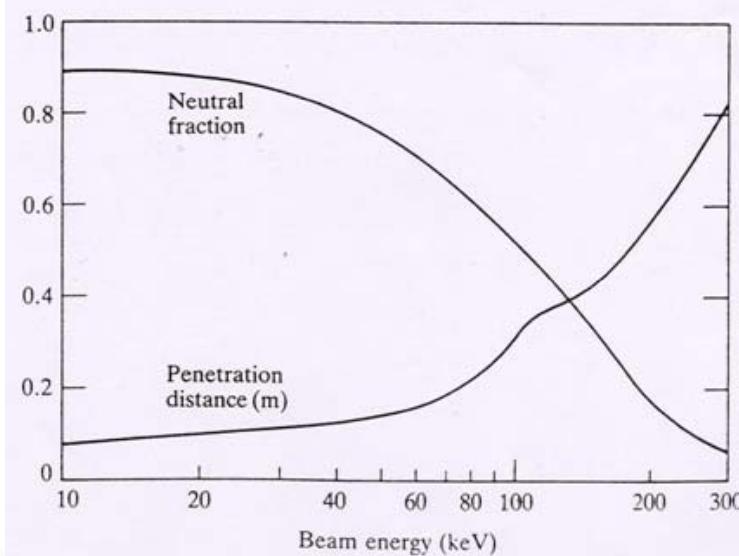
Neutral Beam Injection

- Ionisation in the plasma



Attenuation of a beam of neutral particles in a plasma

$$\frac{dN_b(x)}{dx} = -N_b(x)n(x)\sigma_{tot}$$



Neutral Beam Injection

- Slowing down

$$\frac{dW_b}{dt} = -\frac{2^{\frac{1}{2}} n_e Z_b^2 e^4 m_e^{\frac{1}{2}} \ln \Lambda}{6\pi^{\frac{3}{2}} \epsilon_0^2 M_b} \left(\frac{W_b^{\frac{3}{2}}}{T_e^2} + \frac{C}{W_b^{\frac{1}{2}}} \right), \quad C = 3\pi^{\frac{1}{2}} Z^2 M_b^{\frac{3}{2}} / 4m_e^{\frac{1}{2}} m_i \approx 81$$

$$\frac{W_{b,crit}}{T_e} = C^{\frac{2}{3}} \approx 19$$

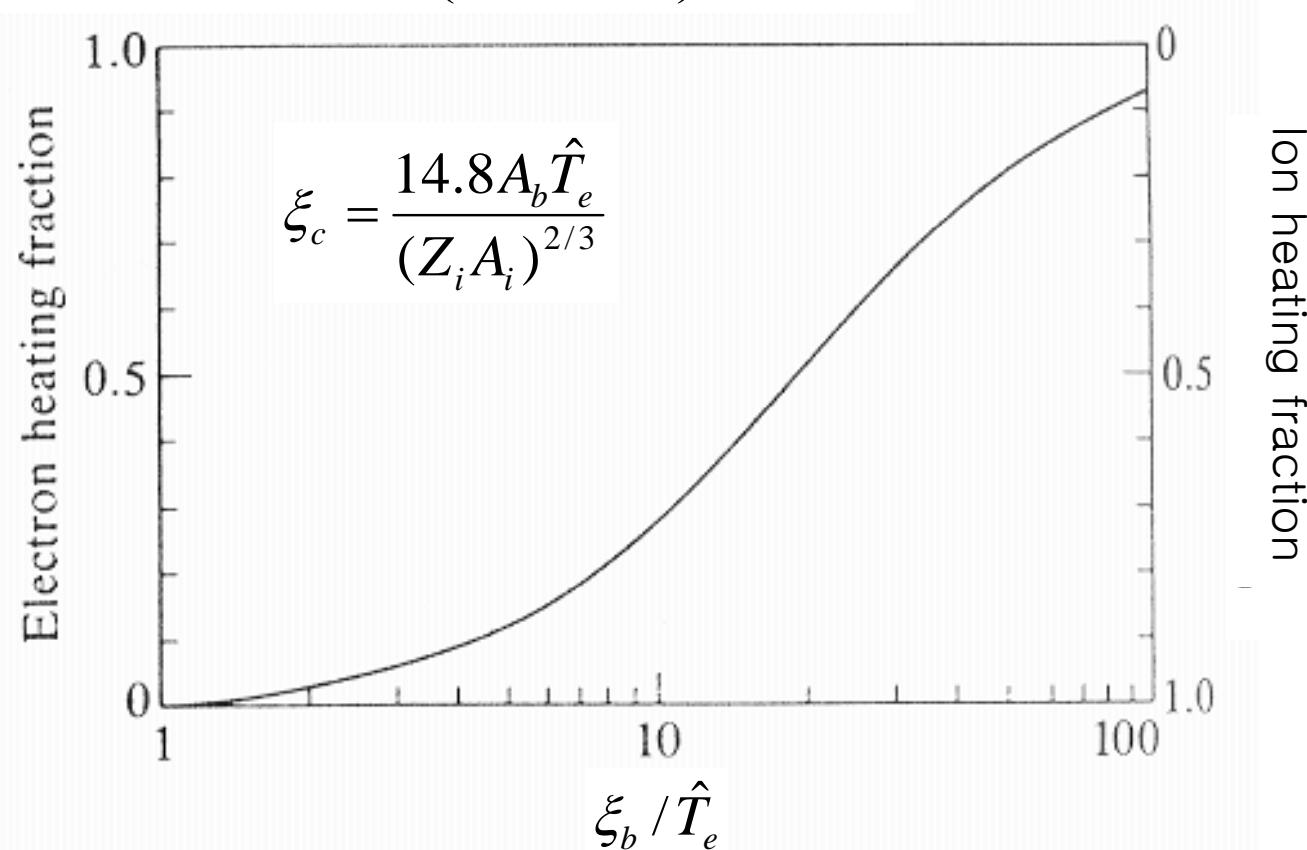
$$\text{Critical energy } W_{b,crit}: \quad W_{b,crit} = 14.8 \cdot \frac{A_b}{A_i^{2/3}} \cdot T_e [\text{keV}]$$

The electron and ion heating rates are equal.

Neutral Beam Injection

- Slowing down

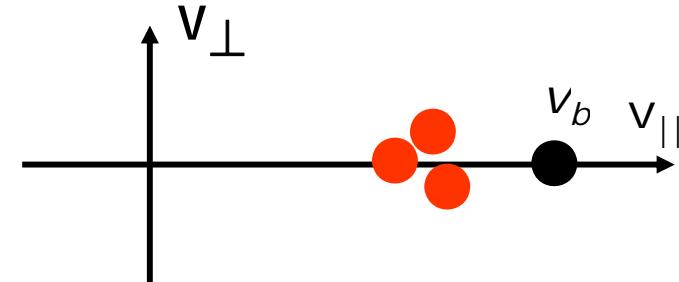
$$P = 1.71 \times 10^{-18} \frac{n_e \xi_b}{A_b \hat{T}_e^{3/2}} \left(1 + \left(\frac{\xi_c}{\xi_b} \right)^{3/2} \right) [\text{keV s}^{-1}] \quad \xi_b = \frac{1}{2} m_b v_b^2$$



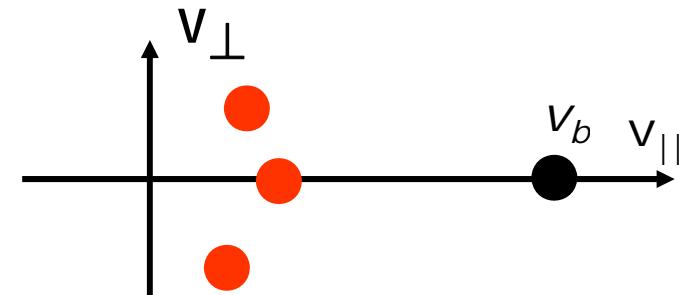
Neutral Beam Injection

- Slowing down

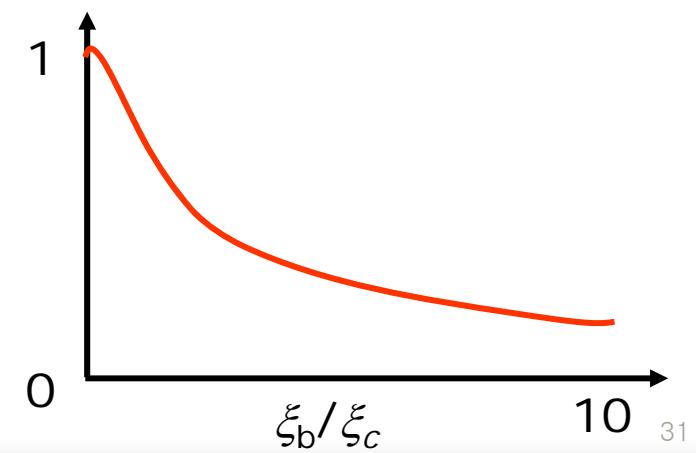
1. $\xi_b > \xi_c$: Slowing down on electrons
no scatter



2. $\xi_b < \xi_c$: Slowing down on ions
scattering of beams



Fraction of initial beam energy
going to ions

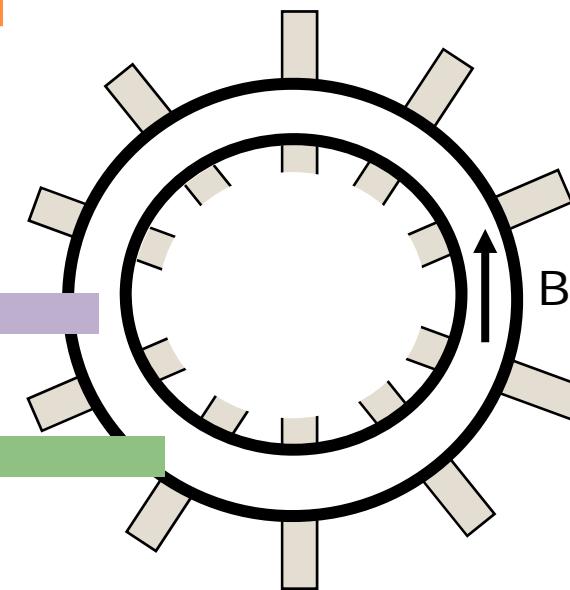


Neutral Beam Injection

- Injection angle

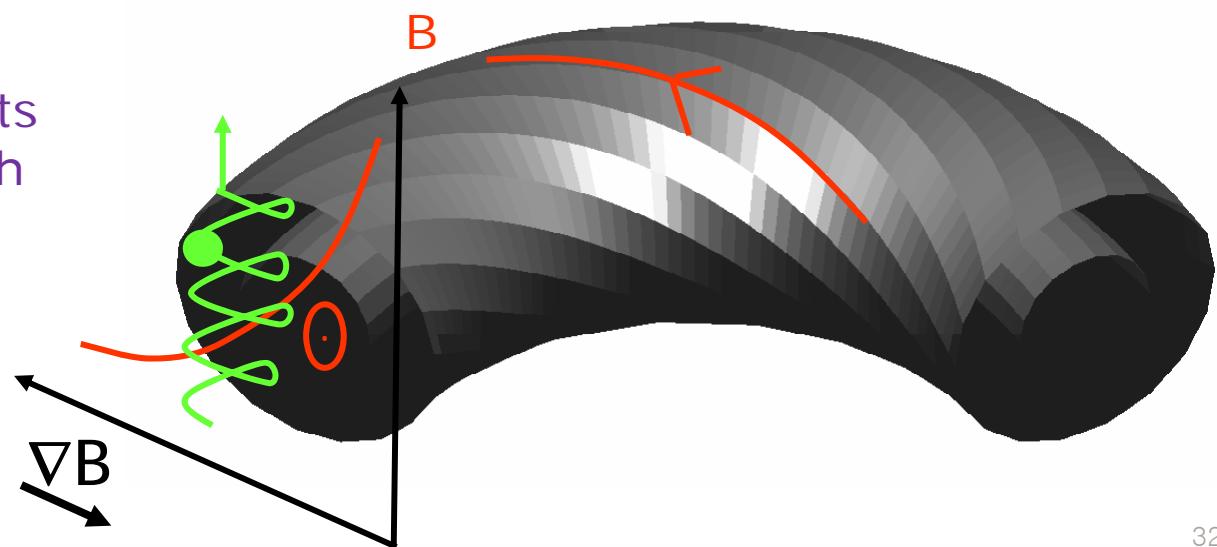
Radial (perpendicular, normal) injection

Tangential injection



Radial injection:

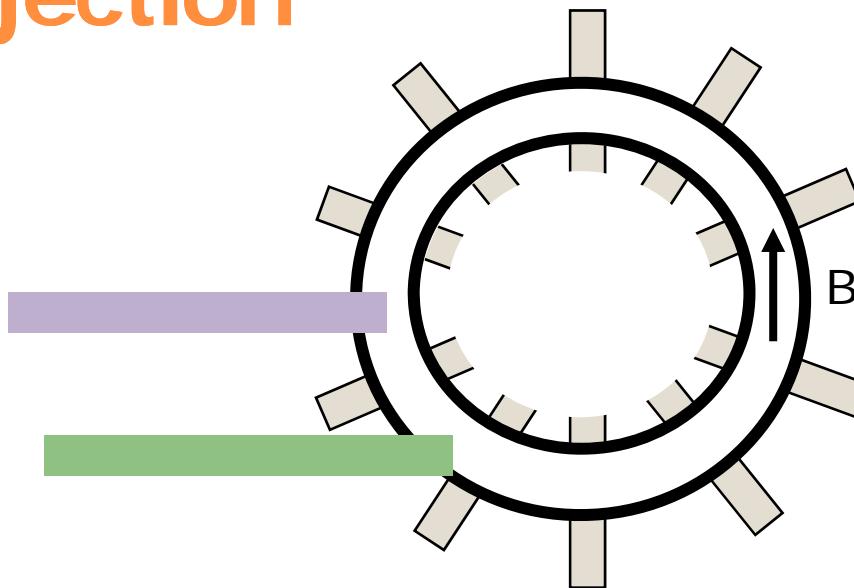
- standard ports
- shine-through
- particle loss



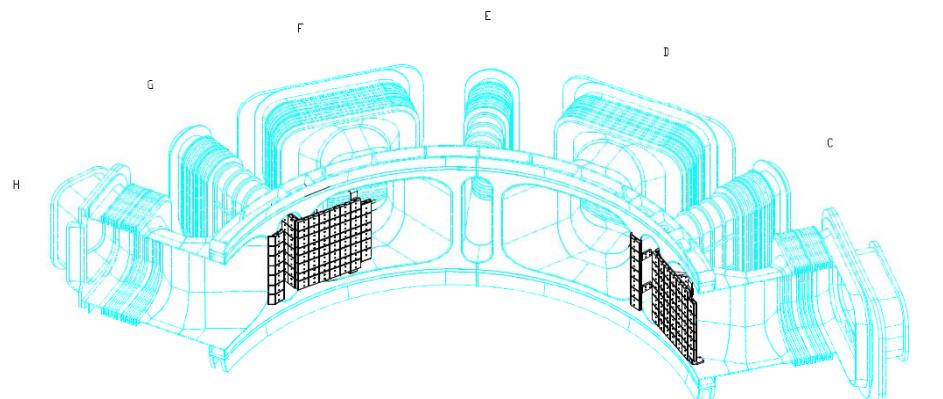
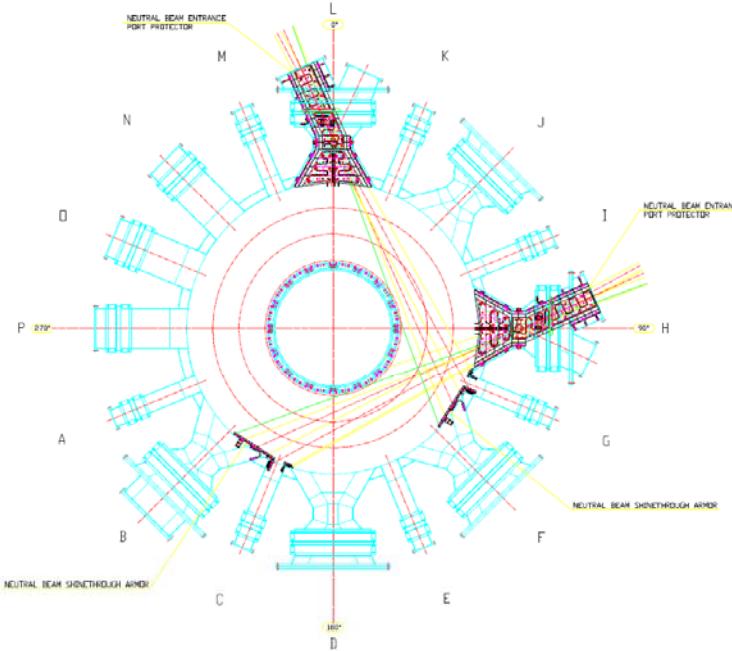
Neutral Beam Injection

- Injection angle

Radial (perpendicular, normal) injection



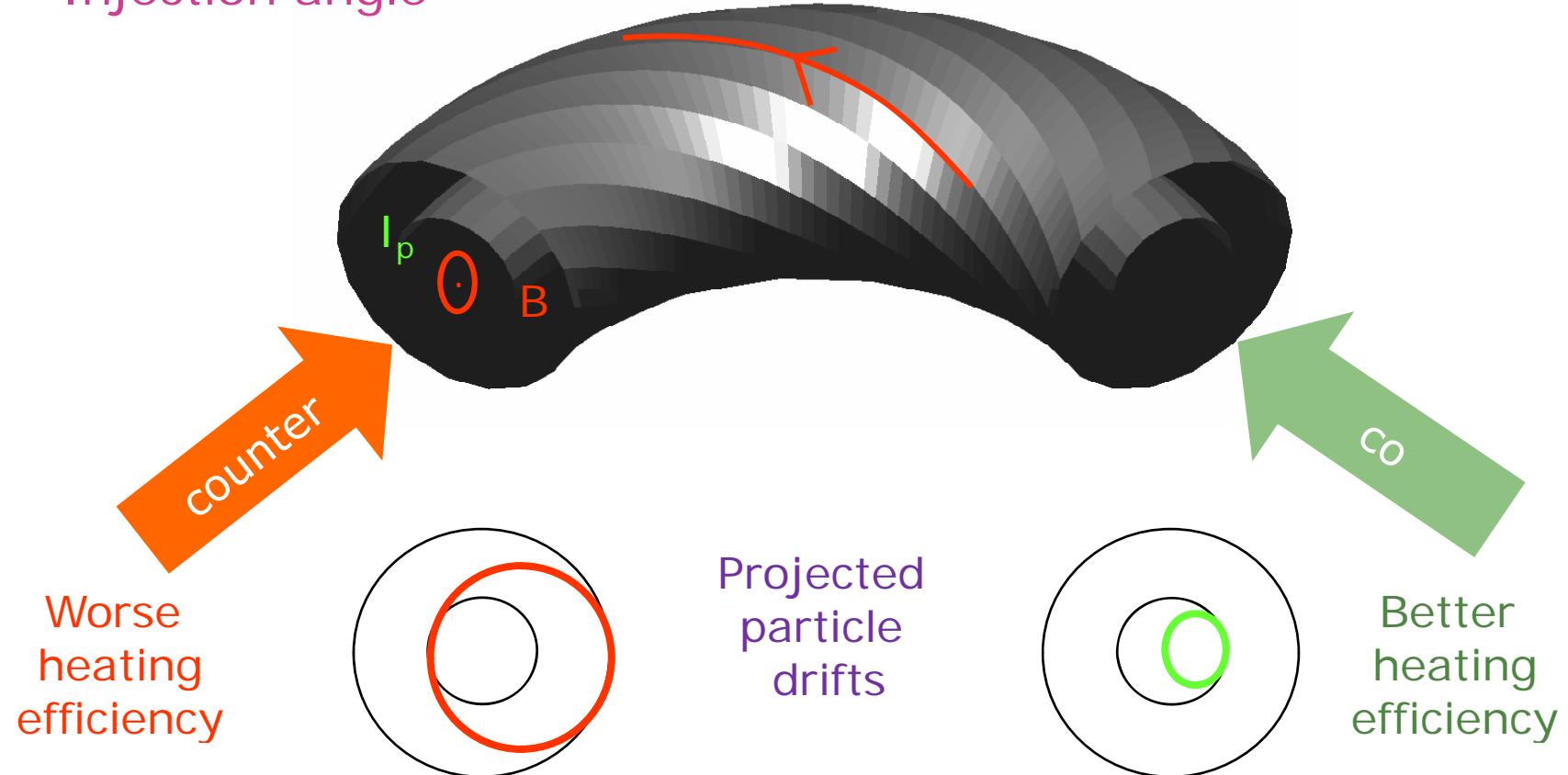
Tangential injection



KSTAR NB shine-through armor

Neutral Beam Injection

- Injection angle

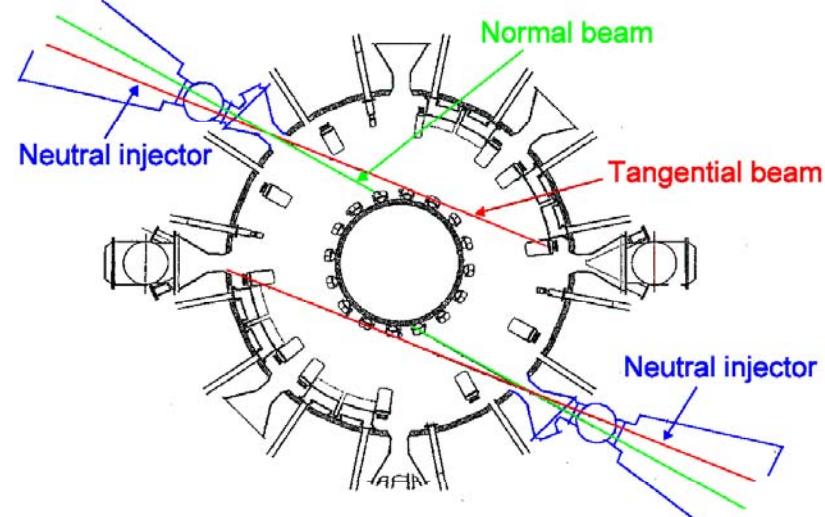
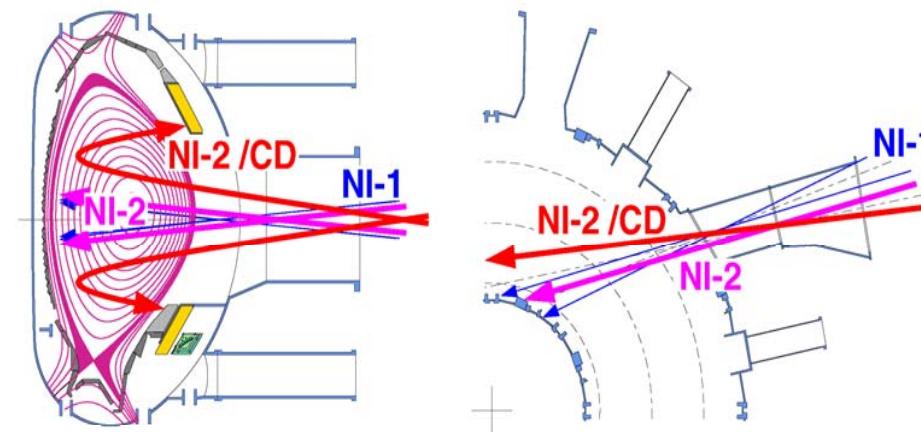
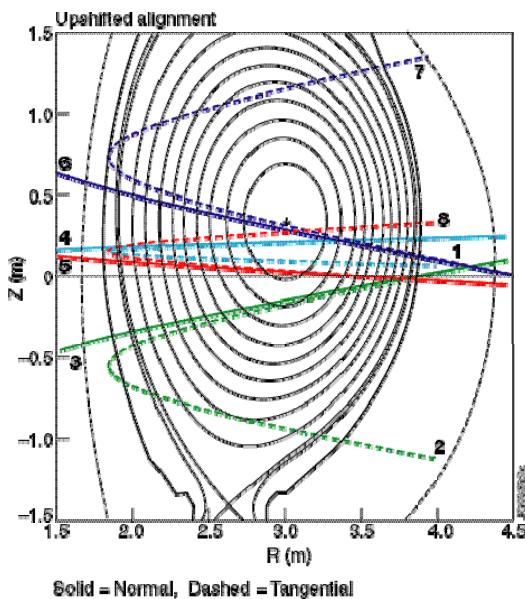


At low magnetic fields heating efficiency depends on NBI direction.

Neutral Beam Injection

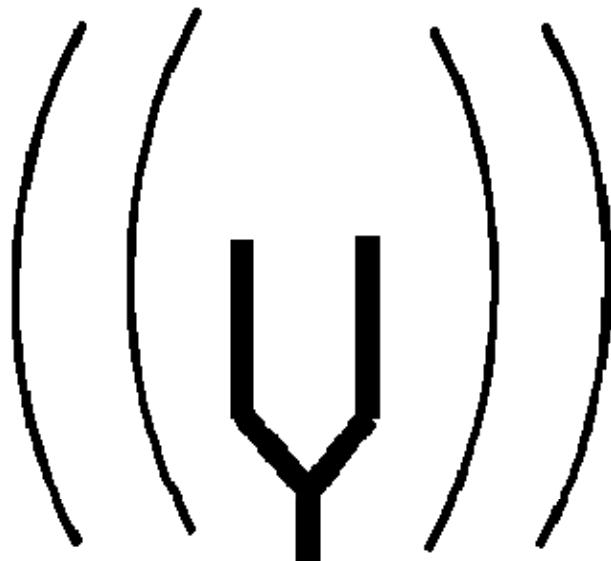
ASDEX Upgrade

JET

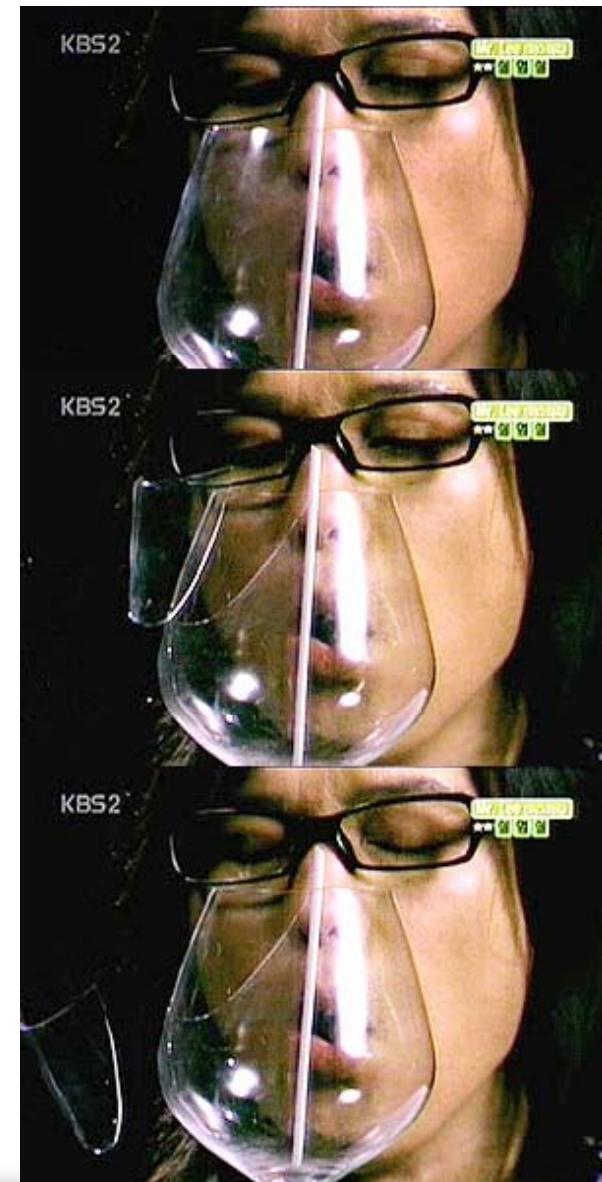


Electromagnetic Waves

Tuning fork



Resonance

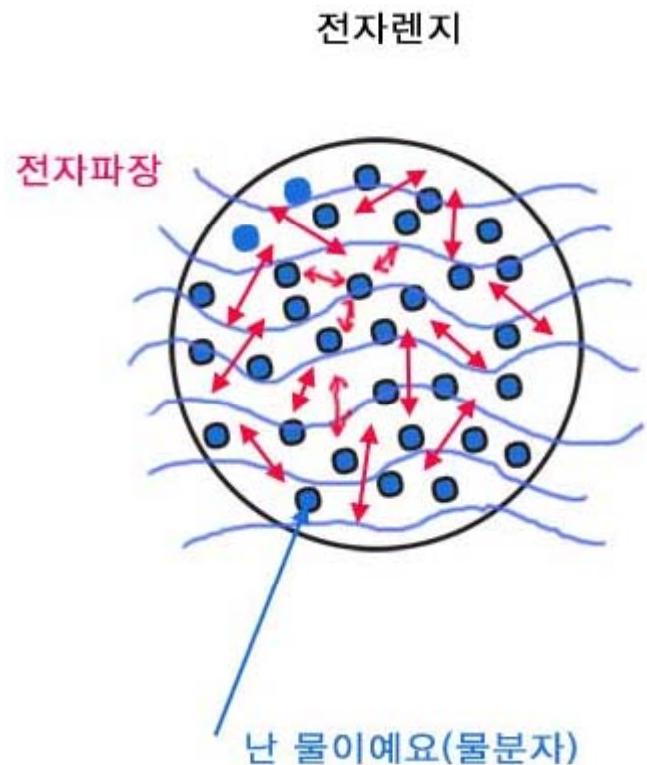


Electromagnetic Waves



Tacoma Narrows Bridge
(1940. 11. 4)

Electromagnetic Waves



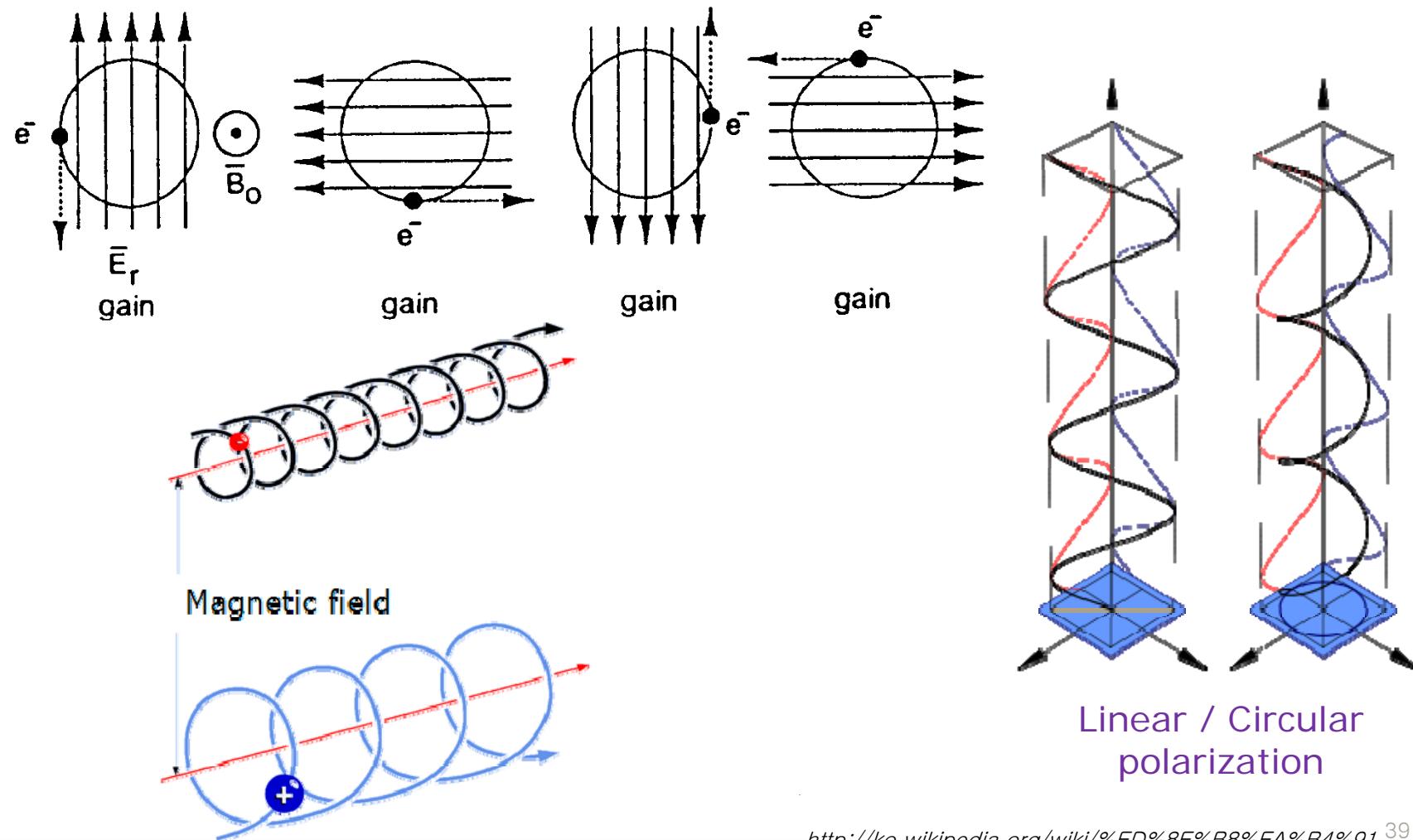
Microwave oven

http://cafe.naver.com/nadobaker.cafe?iframe_url=/ArticleRead.nhn%3FarticleId=82

<http://blog.naver.com/r1hyuny27?Redirect=Log&logNo=30029307561>

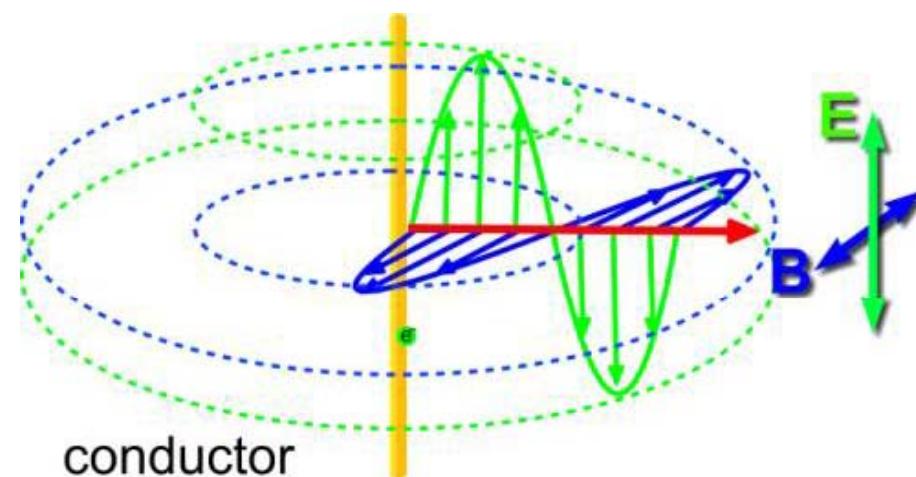
Electromagnetic Waves

- Electron Cyclotron Resonance Heating (ECRH)

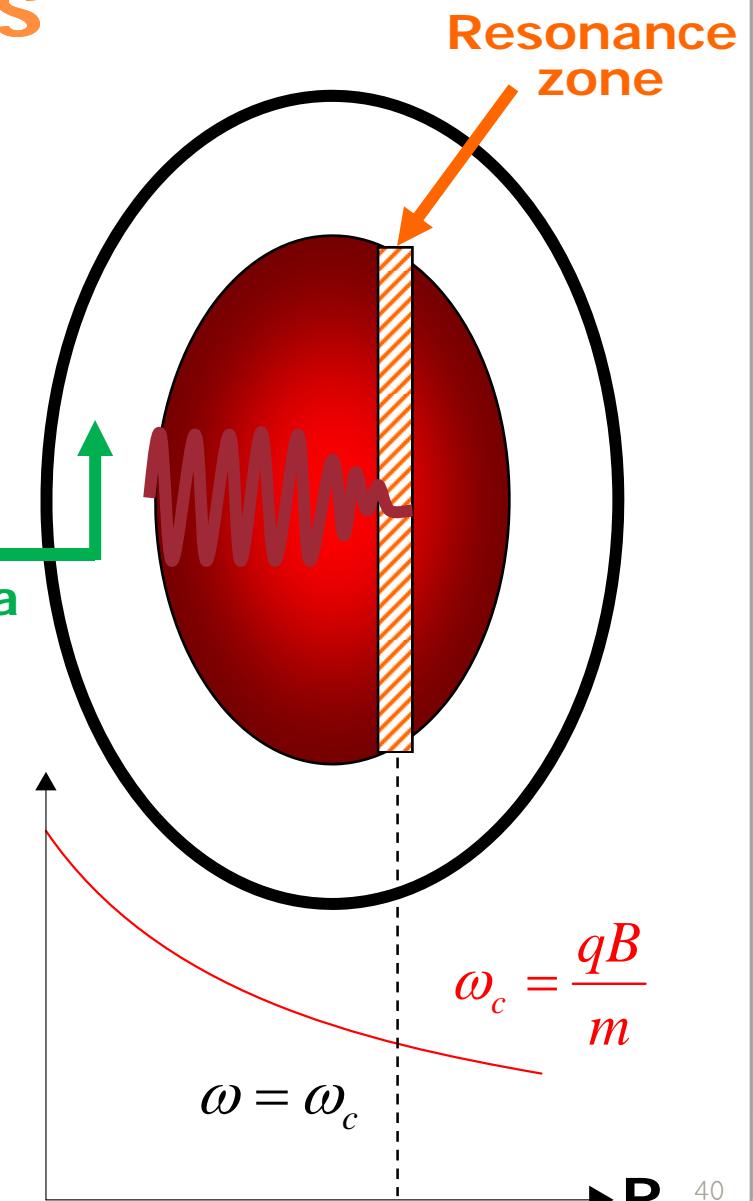


Electromagnetic Waves

Excitation of plasma wave
(frequency ω) near plasma edge



Antenna



Electromagnetic Waves

Excitation of plasma wave
(frequency ω) near plasma edge



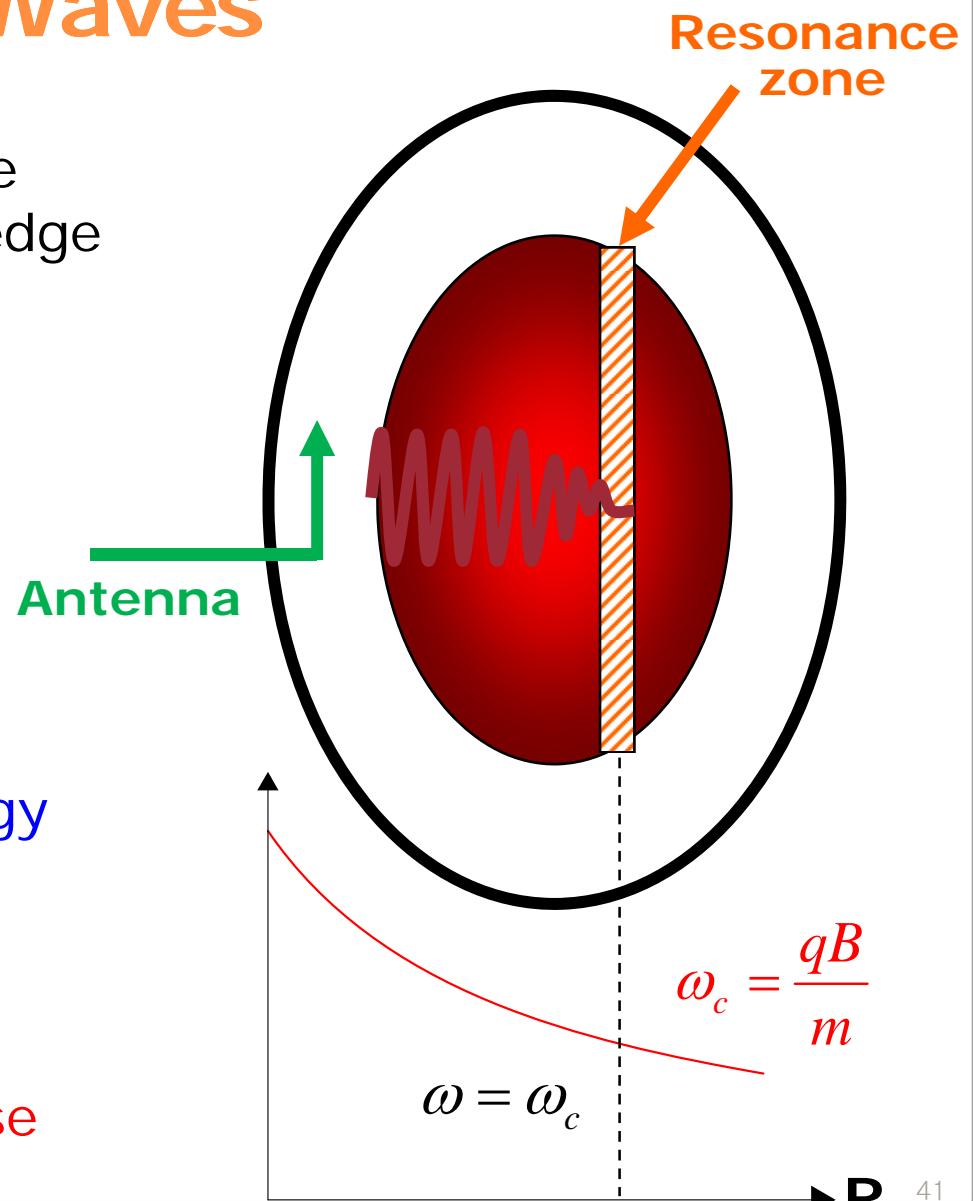
wave transports power
into the plasma center



absorption near resonance,
e.g. $\omega \approx \omega_c$,
i.e. conversion of wave energy
into kinetic energy of
resonant particles



Resonant particles thermalise



Plasma Waves

- Considering externally driven perturbations in the magnetic and electric fields and in the current, relative to an equilibrium condition for a cold plasma w/o external magnetic fields

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Maxwell equation

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial \vec{j}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$j \equiv \sum_i n_i e u_i$$

$$m_i n_i \left(\frac{\partial \vec{u}_i}{\partial t} + (\vec{u}_i \cdot \nabla) \vec{u}_i \right) = n_i e (\vec{E} + \vec{u}_i \times \vec{B}) - \nabla p_i$$

Equations of motion:
isotropic pressure assumed

$$\Rightarrow \frac{\partial \vec{u}_i}{\partial t} = \frac{e}{m_i} \vec{E}$$

Plasma Waves

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \left(\sum_i \frac{n_i e^2}{m_i} \right) \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

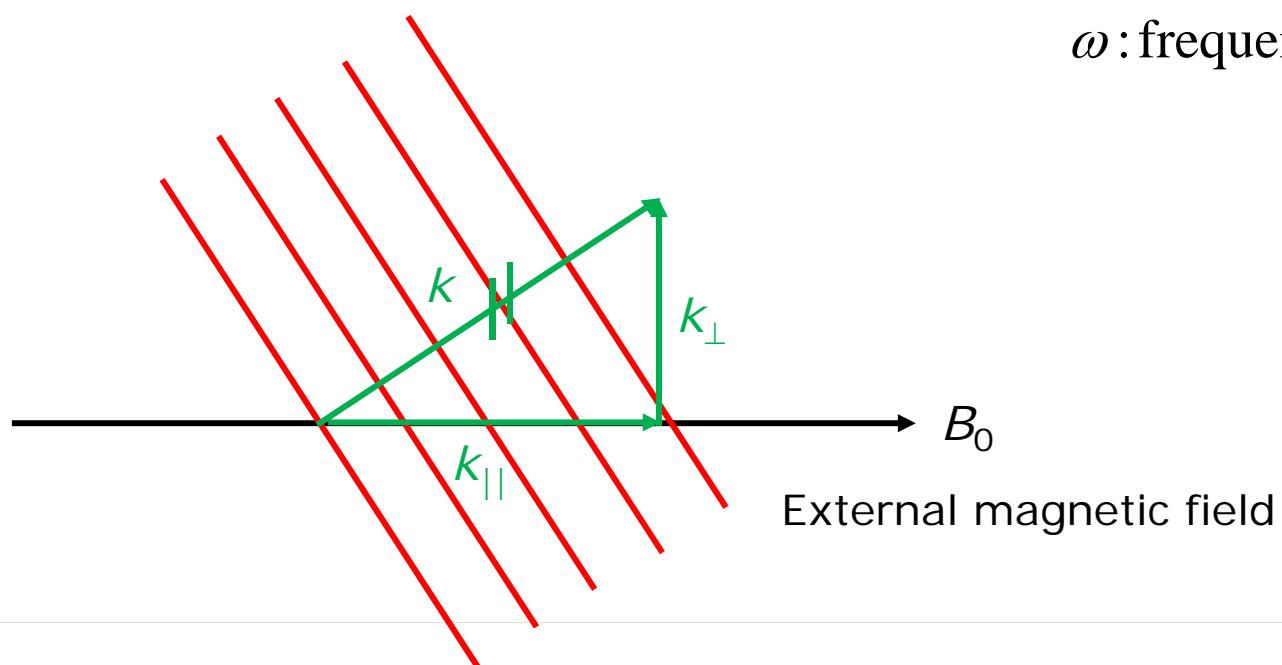
$$-\vec{k}(\vec{k} \cdot \vec{E}) + k^2 \vec{E} = \frac{\omega^2}{c^2} \vec{E} - \mu_0 \left(\sum_i \frac{n_i e^2}{m_i} \right) \vec{E}$$

Plane waves with space
and time dependences

$$\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$$

\vec{k} : wave vector

ω : frequency



Plasma Waves

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \left(\sum_i \frac{n_i e^2}{m_i} \right) \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$-\vec{k}(\vec{k} \cdot \vec{E}) + k^2 \vec{E} = \frac{\omega^2}{c^2} \vec{E} - \mu_0 \left(\sum_i \frac{n_i e^2}{m_i} \right) \vec{E}$$

- $\vec{k} \parallel \vec{E}$

$$\omega^2 = \frac{1}{\epsilon_0} \sum_i \frac{n_i e^2}{m_i} \equiv \omega_p^2 \quad \text{Plasma frequency}$$

- $\vec{k} \perp \vec{E}$

$$\omega^2 = c^2 k^2 + \omega_p^2$$

Plane waves with space
and time dependences

$$\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$$

\vec{k} : wave vector

ω : frequency

Plasma Waves

- Waves applied to the edge of a plasma tend to be “shielded” out of the highly conducting plasma.

$$\partial \vec{B} / \partial t = -\nabla \times \vec{E} = (\eta / \mu_o) \nabla^2 \vec{B}$$
$$\vec{B} \approx \vec{B}_o e^{-x/\delta}, \delta \approx \sqrt{\eta / \mu_o \omega}$$

- Externally applied RF waves can penetrate the plasma only by coupling to natural waves in the plasma determined by the plasma dielectric tensor.

$$\nabla \times (\nabla \times \vec{E}) = (\omega / c)^2 \vec{K} \cdot \vec{E}$$

Dielectric tensor

Determinant = 0  Disperison relation $D(\omega, k) = 0$

Dispersion Relation

Plasma waves are solutions of dispersion relation $D(\omega, k) = 0$.

generally:

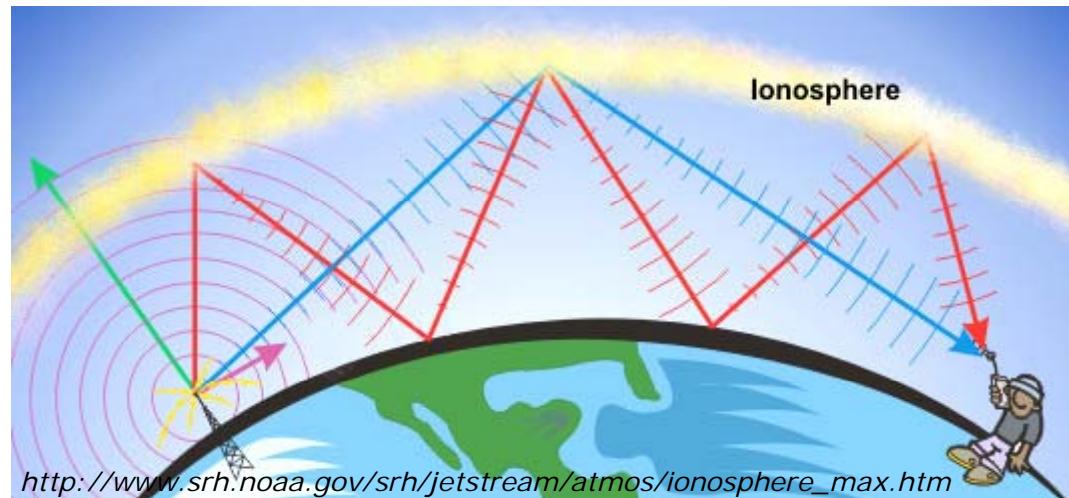
ω given by generator
 $k_{||}$ given by antenna
where $k_{||} > k_{||, \text{vacuum}}$



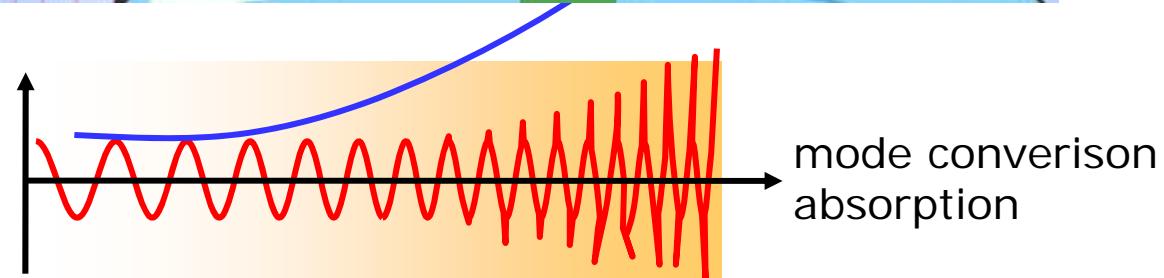
solution: $k_{\perp} = k_{\perp}(\omega, k_{||})$

Special cases:

1. $k_{\perp} \rightarrow 0$ „cutoff“



2. $k_{\perp} \rightarrow \infty$ „resonance“



Electromagnetic Waves

- Ion Cyclotron Resonance Heating (ICRH):

$$\omega \sim \omega_{ci}, \text{ 30 MHz} - \text{120 MHz} (\sim 10 \text{ m})$$

Ion-ion resonance frequency

$$\omega_{ii}^2 = \frac{\omega_{c1}\omega_{c2}(1 + n_2m_2/n_1m_1)}{(m_2Z_1/m_1Z_2 + n_2Z_2/n_1Z_1)}, \quad \omega_{ci} = \frac{z_i e B}{m_i}$$

- Lower Hybrid (LH) Resonance Heating:

$$\omega_{ci} < \omega < \omega_{ce}, \text{ 1 GHz} - \text{8 GHz} (\sim 10 \text{ cm})$$

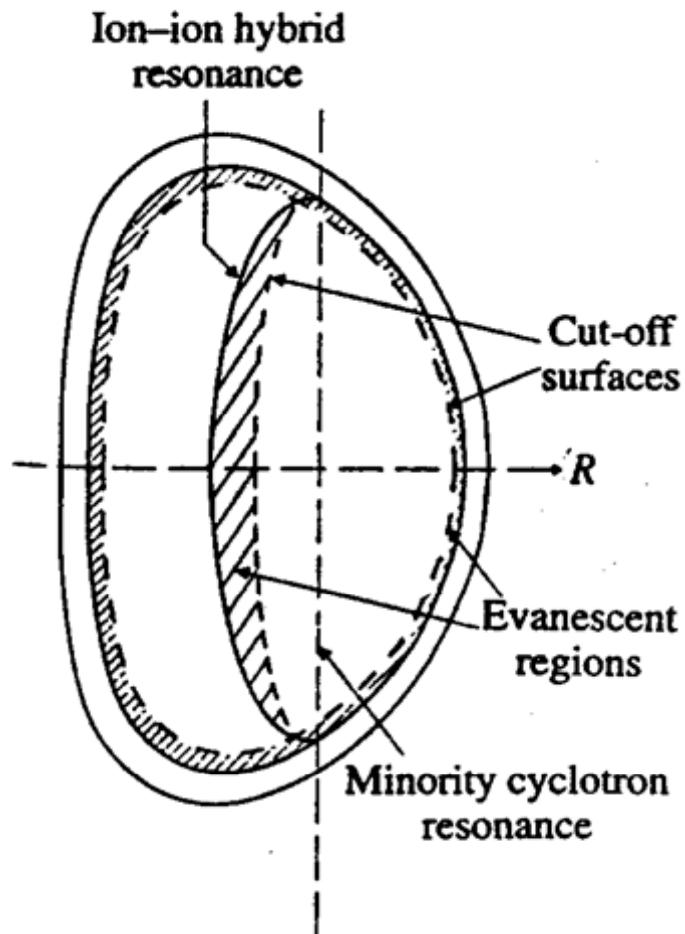
$$\omega_{LH}^2 \approx \omega_{pi}^2 / (1 + \omega_{pi}^2 / \omega_{ce}^2), \quad \omega_{pi}^2 \gg \omega_{ci}^2$$

- Electron Cyclotron Resonance Heating (ECRH):

$$\omega \sim \omega_{ce}, \text{ 100 GHz} - \text{200 GHz} (\sim \text{mm})$$

$$\omega_{UH}^2 \approx \omega_{pe}^2 + \omega_{ce}^2$$

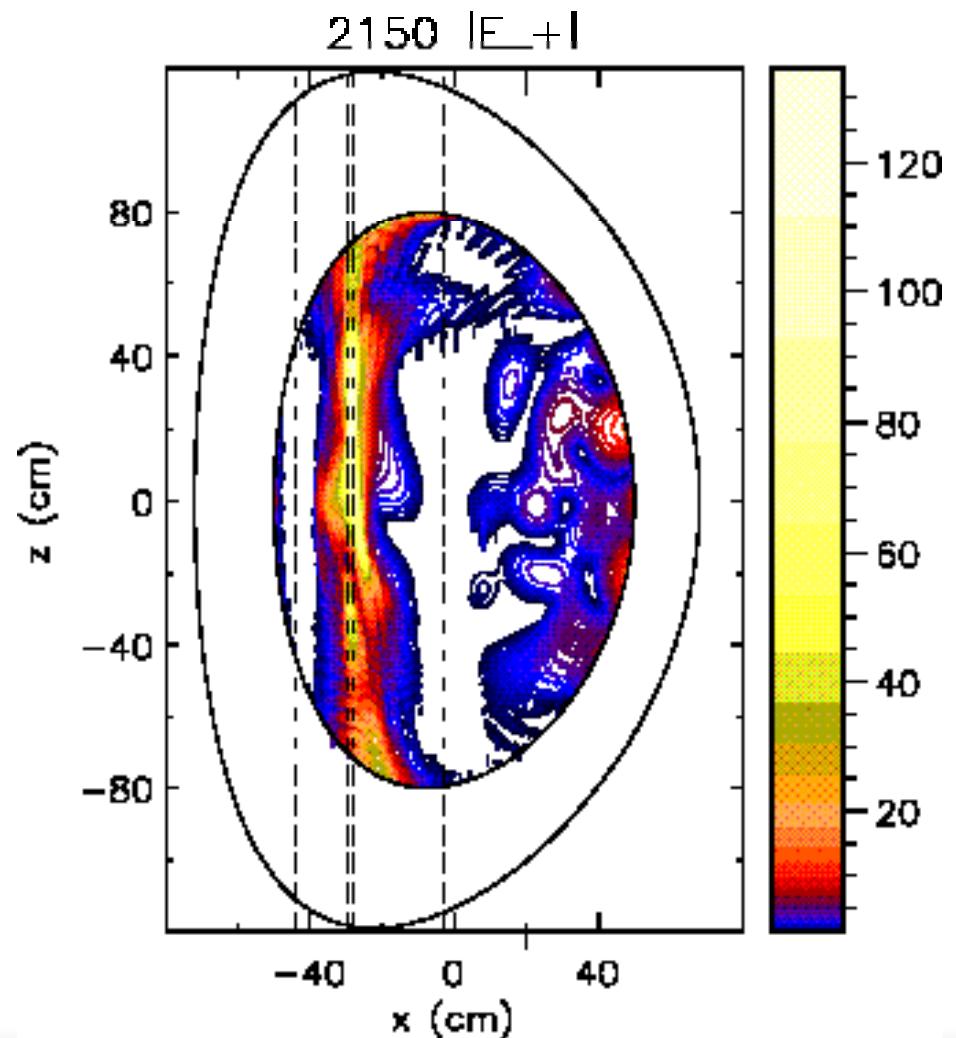
ICRH – Wave Propagation



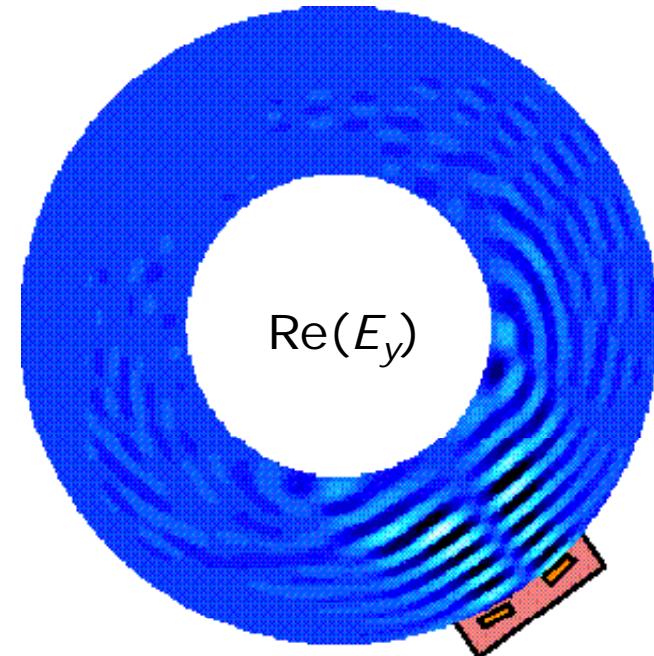
Loci of cut off and resonances in the poloidal cross section of a tokamak. The fast wave is evanescent in the hatched region.

ICRH – Wave Propagation

ASDEX Upgrade

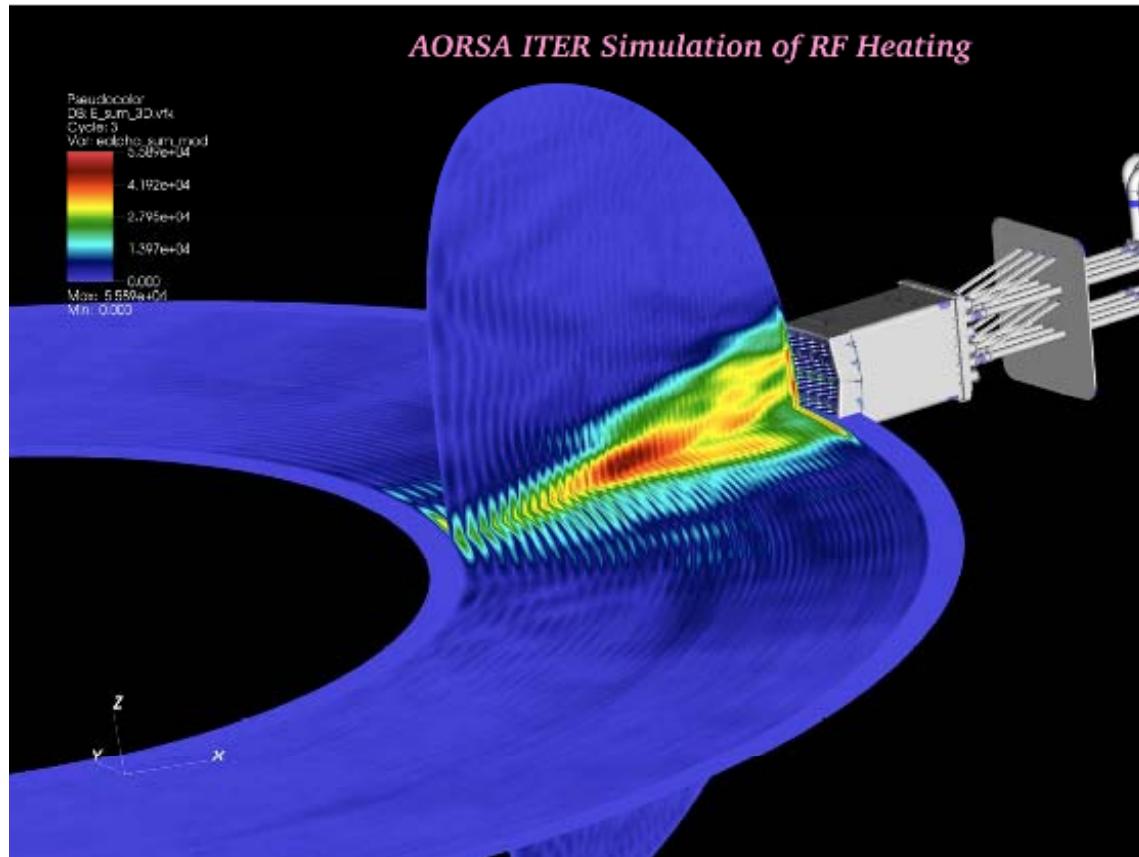


Alcator C-Mod

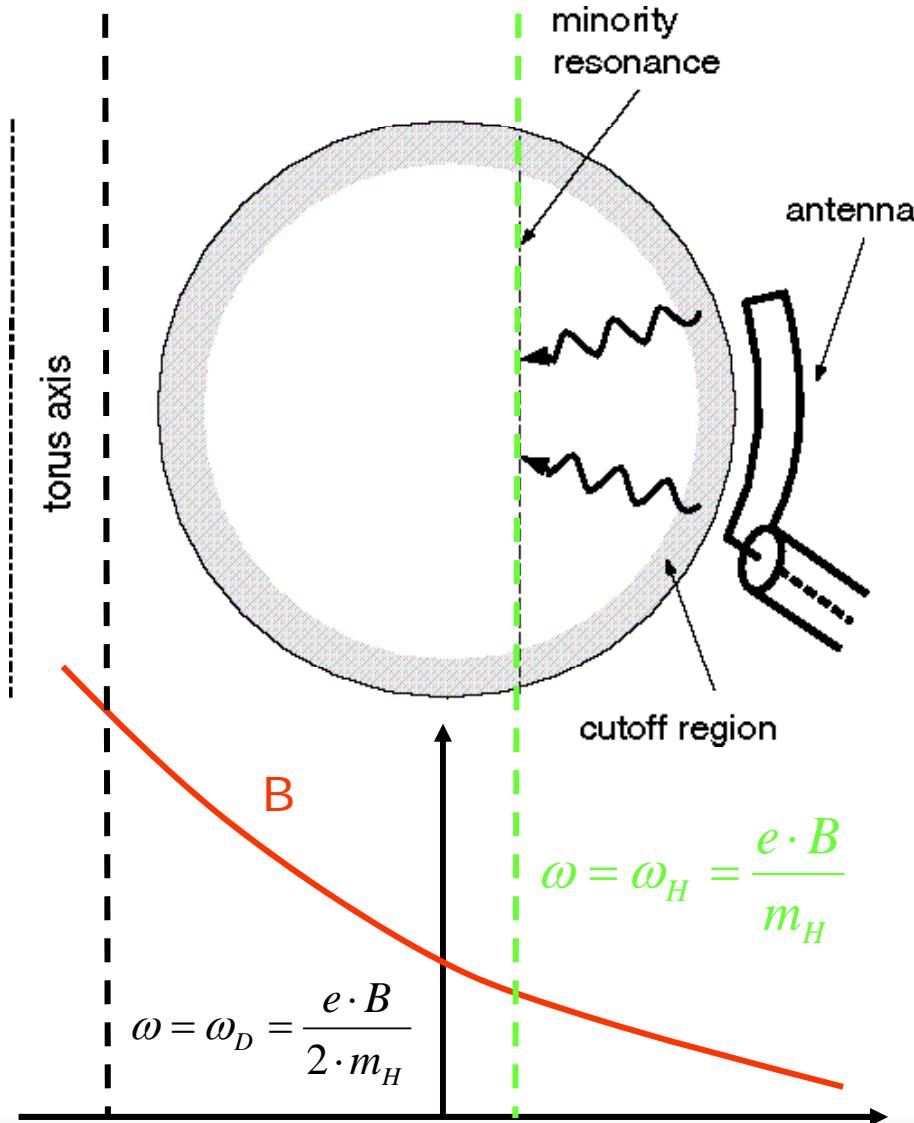


Multiple current straps

ICRH – Wave Propagation



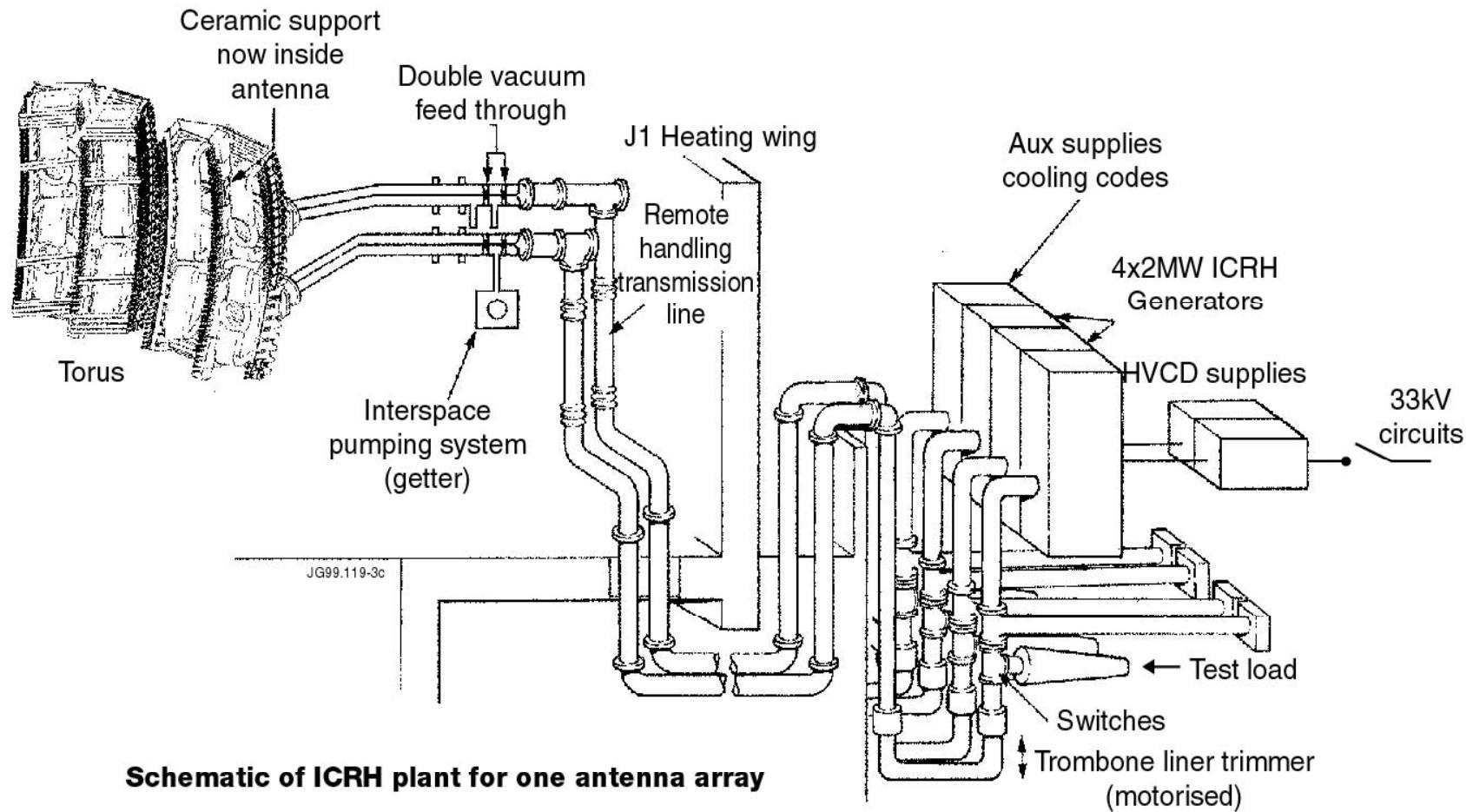
ICRH – Minority Heating



- Plasma with mixture of H and D with $n_H \ll n_D$
- $n_D \rightarrow$ polarization propagation absorption
- $n_H \rightarrow$ absorption
- Production of tail in H velocity distribution function
- Good single pass absorption

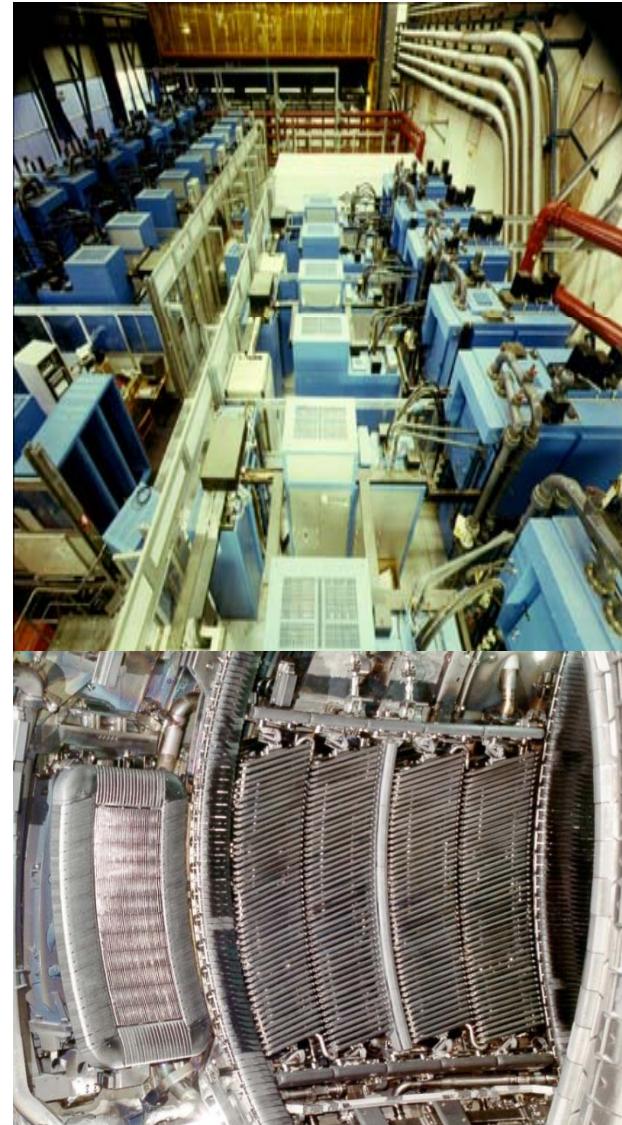
Ion Cyclotron Resonance Heating

- JET ICRH System



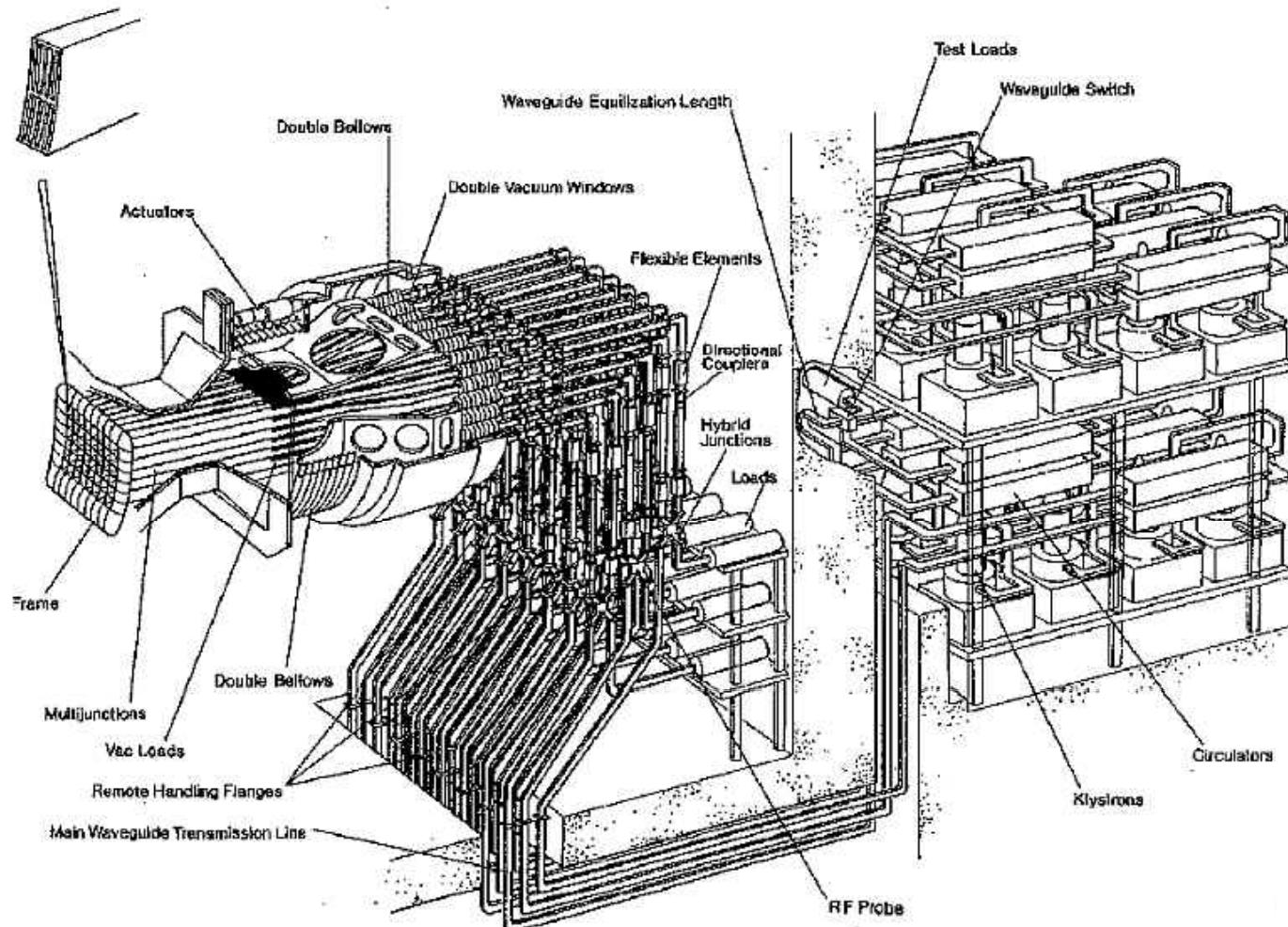
Ion Cyclotron Resonance Heating

- JET ICRH System
 - 8 x 4 MW RF generators, each one has two 2 MW outputs giving a possible 32 MW total.
 - Frequency range 23 MHz -57 MHz (excluding 39-41MHz)
 - 8 HVDC Power supplies.
 - 4 x 4 strap antenna system with vacuum interspace, Getter pumping and Penning gauge protection.



Lower Hybrid Heating

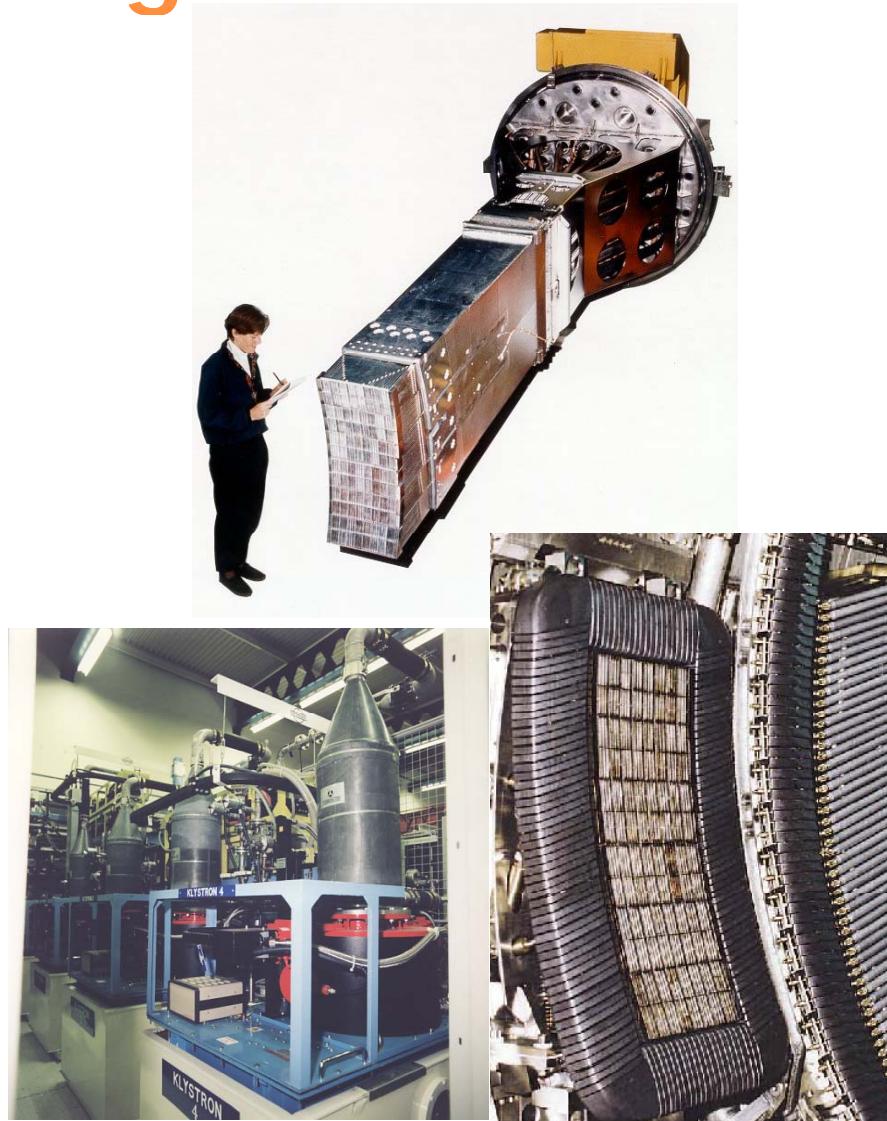
- JET LH System



Lower Hybrid Heating

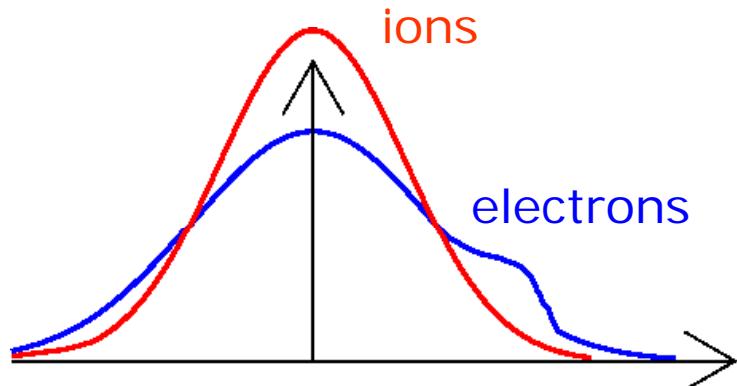
- JET LH System
 - 24 klystrons @ 3.7 GHz,
650 kW/10s or 500 kW/20s
 - For control purposes, klystrons grouped in 6 modules of 4 klystrons (modules A to F)
 - High voltage power supplies:
Two 33 kV circuit breaker, LH01 & LH02, each feeds 3 modules (12 klystrons)
 - PS limits long pulse operation
→ Total power available at generators: 12 MW for 10s
4.8 MW for 20s

Power coupled to plasma depends on launcher power handling and plasma conditions.



Non-inductive Current Drive

- Asymmetric velocity distribution can be a side effect of plasma heating.



$$j = \sum_s q_s n_s \int v_{\parallel} f(v_{\parallel}) dv$$

- Needed for:
 - Steady-state tokamak current profile control in tokamaks
 - bootstrap current compensation in stellarators

- Efficiency

Theory:

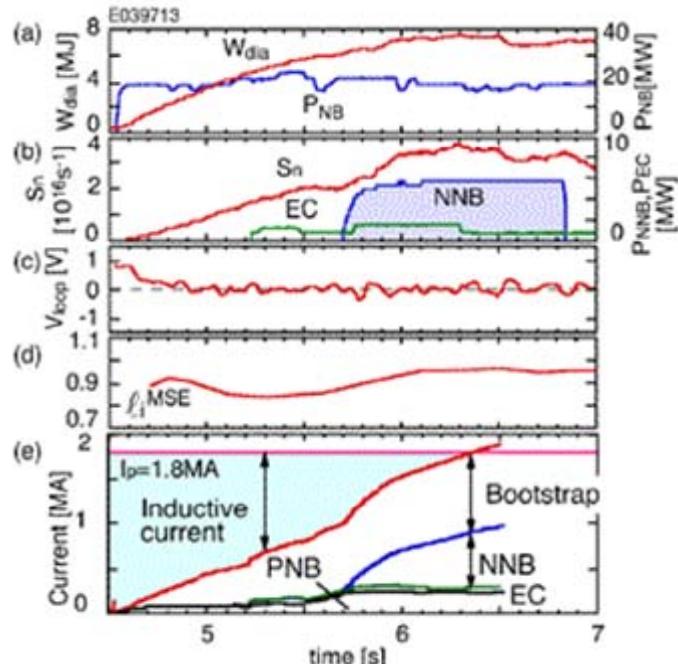
$$\eta_{th} = \frac{j}{p} = \frac{e \cdot n_{e\parallel} \cdot v_{\parallel}}{(n_{ell} \cdot m_e v_{ll}^2 / 2) \cdot v_{coll}} \propto \frac{1}{v_{\parallel} \cdot v_{coll}}$$

Experiment (Figure of merit):

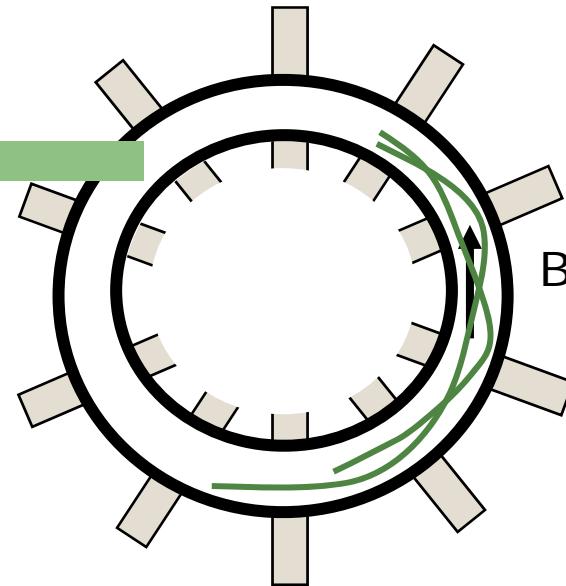
$$\gamma = \frac{n_e [10^{20} m^{-3}] \cdot R[m] \cdot I[A]}{P[W]} \propto \eta_{th}$$

Neutral Beam Current Drive

Tangential injection



JT-60U high β_p ELMy H-mode



- Circulating ions carry current partially compensated by
 - concurrent electron drift
 - trapping
 - dependence on collision frequency

Total driven current

$$\frac{I}{I_f} = 1 - \frac{Z_f}{Z_{eff}} + 1.46 \varepsilon^{\frac{1}{2}} \frac{Z_f A(Z_{eff})}{Z_{eff}}$$

Friction by
trapped
electron

Reverse electron current

Non-inductive Current Drive

	Efficiency
LHCD	0.35-0.4
ICCD	$0.1 \times T_e$ [10keV]
ECCD	$< 0.1 \times T_e$ [10keV]
NBCD	$0.2 \times T_e$ [10keV]

$$\eta_{th} = \frac{j}{p} = \frac{e \cdot n_{e\parallel} \cdot v_{\parallel}}{(n_{ell} \cdot m_e v_{ll}^2 / 2) \cdot v_{coll}} \propto \frac{1}{v_{\parallel} \cdot v_{coll}}$$

$$\gamma = \frac{n_e [10^{20} m^{-3}] \cdot R[m] \cdot I[A]}{P[W]} \propto \eta_{th}$$

Heating and Current Drive

Heating Scheme	Advantages	Disadvantages
OH	Efficient	Cannot reach ignition
NBI	Reliable	Close to torus Negative ions necessary
LH	Efficient current drive	Antenna close to plasma off-axis
ECRH	Reliable Flexible	Electron heating (density limit)
ICRH	Ion heating Central heating	Antenna close to plasma Antenna coupling