# Chapter 4 Two-Phase Flow

## I. Basic Definitions

(1) Void Fractions

: the time averaged volumetric fraction of vapor in a two phase mixture.

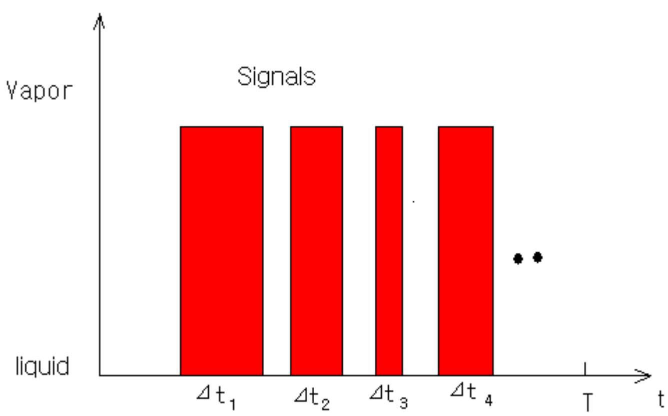
$$\alpha = \iiint_{V_g} dV / \iiint_{V} dV = V_g/(V_l+V_g) \qquad (4.1.1)$$

o Measurement

1) Needle probe method

- electrical probe
- optical probe

2) X ray absorption method (Jones and Zuber)




$$\alpha = \frac{\sum_{i}^{N} \Delta t_i}{T}$$

For differential length element

$$\alpha = \Delta z \iint_{A_g} dA / (\Delta z \iint_{A} dA)$$

$$= A_g/(A_l + A_g) \qquad (4.1.2)$$

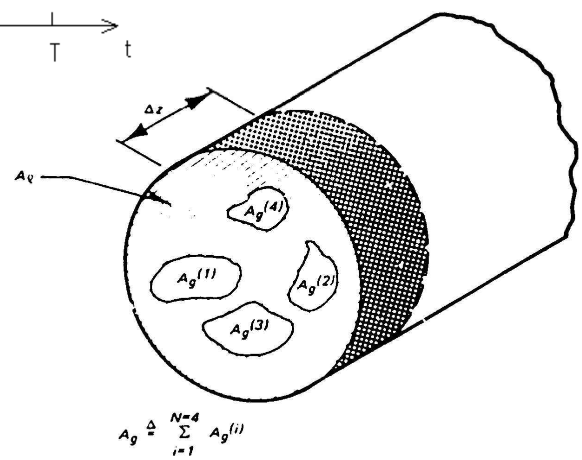



그림 1.1 Representation of void fraction.

(2) Phase Velocity

: the volumetric flow rate of the phase through its cross sectional flow area

$$u_l = Q_l / A_l$$

$$u_g = Q_g / A_g \qquad (4.1.3)$$

where $Q_i$ is the volumetric flow rate ($ft^3$/sec) of phase i

(3) Volumetric flux or superficial velocity

: the volumetric flow rate of the phase divided by the total cross sectional flow area.

$$j_l = Q_l / (A_l + A_g)$$

$$j_g = Q_g / (A_l + A_g) \qquad (4.2.4)$$

Now, using the relations given in Eqs.(4.1.2) and (4.1.3), we can write

$$j_l = u_l (1 - \alpha)$$

$$j_g = u_g \alpha \qquad (4.1.5)$$

And the total volumetric flux of the mixture, j, is

$$j = j_l + j_g \qquad (4.1.6)$$

(4) Relative velocity, $u_r$

: velocity difference between the vapor and the liquid phase

$$u_r = u_g - u_l \qquad (4.1.7)$$

For homogeneous flow

$$u_r = 0$$

Thus, from Eq. (4.1.5)

$$j_g / \alpha = j_l / (1 - \alpha)$$

By solving for the void fraction

$$\alpha = j_g / (j_l + j_g) \qquad (4.1.8)$$

(5) Slip ratio

: the ratio of the phase velocity

$$S = u_g / u_l \qquad (4.1.9)$$

$$= \frac{x}{(1-x)} \frac{(1-\alpha)}{\alpha} \frac{\rho_l}{\rho_g}$$

For homogeneous flow, $S = 1$, by definition.

(6) Quality

1) Thermal equilibrium quality : the flow fraction of vapor in the thermal equilibrium condition

$$x_e = (h - h_f)/h_{fg} \qquad (4.1.10)$$

2) Flow quality : the true flow fraction of vapor regardless of whether thermal equilibrium exists or not

$$x = \rho_g u_g A_g/(\rho_f u_f A_f + \rho_g u_g A_g)$$
$$= w_g/w \qquad (4.1.11)$$

3) Static quality : the mass fraction of vapor

$$x_s = \rho_g A_g/(\rho_f A_f + \rho_g A_g) \qquad (4.1.12)$$

From the definitions given in Eqs.(4.1.11) and (4.1.12), $x$ and $x_s$ can be related through the following identity

$$x/(1-x) = (u_g/u_f)\, x_s/(1-x_s) = S x_s/(1-x_s) \qquad (4.1.13)$$

(7) Cross-sectional Average Notation, ⟨ ⟩

$$\langle \zeta \rangle = \frac{1}{A_{x-s}} \iint_{A} \zeta \, dA$$

$$\langle \zeta_f \rangle_f = \frac{1}{A_{x-s}(1-\langle \alpha \rangle)} \iint_{A_f} \zeta_f (1-\alpha)\, dA = \frac{\langle (1-\alpha)\zeta_f \rangle}{\langle 1-\alpha \rangle}$$

$$\langle \zeta_g \rangle_g = \frac{1}{A_{x-s}\langle \alpha \rangle} \iint_{A_g} \zeta_g \alpha \, dA = \frac{\langle \alpha \zeta_g \rangle}{\langle \alpha \rangle} \qquad (4.1.14)$$

Note : $\langle f^2 \rangle \neq \langle f \rangle^2$

(8) Mass flux, G

$$G = w/A_{x-s} = \rho_f \langle u_f \rangle_f (1-\langle \alpha \rangle)/(1-\langle x \rangle) = \rho_g \langle u_g \rangle_g \langle \alpha \rangle/\langle x \rangle \qquad (4.1.15)$$

where the last two equalities come from the identities

$$G(1-\langle x \rangle)A_{x-s} = \rho_f \langle u_f \rangle_f (1-\langle \alpha \rangle)A_{x-s}$$

and

$$G\langle x \rangle A_{x-s} = \rho_g \langle u_g \rangle_g \langle \alpha \rangle A_{x-s}$$

which also imply

$$\langle u_f \rangle_f = G(1-\langle x \rangle)/[\rho_f(1-\langle \alpha \rangle)] \qquad (4.1.16)$$

$$\langle u_g \rangle_g = G\langle x \rangle/(\rho_g \langle \alpha \rangle) \qquad (4.1.17)$$

(9) The fundamental Void Quality Relation

The fundamental void quality relation is obtained as,

$$\langle \alpha \rangle = \langle x \rangle / [\langle x \rangle - S(\rho_g/\rho_f)(1-\langle x \rangle)] \qquad (4.1.18)$$

With respect to the static quality, Eq.(4.1.18) will be

$$\langle \alpha \rangle = \langle x_e \rangle \rho_f / [\langle x_e \rangle (\rho_f - \rho_g) - \rho_g] \qquad (4.1.19)$$

and, from $\langle \bar{\rho} \rangle = (1-\alpha)\rho_f + \alpha\rho_g$

$$\langle \bar{\rho} \rangle = \rho_f \rho_g / [\langle x_e \rangle \rho_f - (1-\langle x_e \rangle)\rho_g] = 1/(v_f - \langle x_e \rangle v_{fg}) \qquad (4.1.20)$$

For homogeneous flow condition, $S=1$

$$\langle \rho_H \rangle = \frac{G}{\langle j \rangle} = \frac{w}{Q} = 1/(v_f + \langle x \rangle v_{fg}) \qquad (4.1.21)$$

but for slip flow condition

$$\langle \bar{\rho} \rangle \neq 1/(v_f - \langle x \rangle v_{gf})$$

(10) Drift velocity, $V_{gj}$

: the void weighted average velocity of vapor phase with respect to the velocity of the center of volume of the mixture.

$$V_{gj} = \langle \alpha (u_g - j) \rangle / \langle \alpha \rangle \qquad (4.1.22)$$

## II. Two phase Model

o Number of unknowns for three dimensional flow

| Single Phase | | Two Phase | |
|---|---|---|---|
| Parameter | No | Parameter | No |
| Velocity Vector | 3 | Void Fraction | 1 |
| Pressure | 1 | Velocity Vector of Liquid | 3 |
| Temperature | 1 | Velocity Vector of Vapor | 3 |
| Density | 1 | Density of Liquid | 1 |
| | | Density of Vapor | 1 |
| | | Pressure | 1 |
| | | Temperature of each Phase | 2 |
| Total number of unknowns | 6 | Total number of unknowns | 12 |

o Number of governing equations

| Single Phase | | Two Phase | |
|---|---|---|---|
| Parameter | No | Parameter | No |
| Conservation of Mass | 1 | Mass Eq. for Liquid | 1 |
| Conservation of Momentum | 3 | Mass Eq. for Vapor | 1 |
| Conservation of Energy | 1 | Momentum Eq. for Liquid | 3 |
| Equation of State | 1 | Momentum Eq. for vapor | 3 |
| | | Energy Eq. for each Phase | 2 |
| | | Eq. of State for each Phase | 2 |
| Total number of equations | 6 | Total number of equations | 12 |

o Constitutive Equations

- interfacial transfer (jump condition)
- various correlations dependent on the flow regime

o Various forms for two phase flow modeling

- Local Instantaneous Formula
- Instantaneous Space - or Local Time Averaged Equations
- Homogeneous Model
- Two Fluid Model (Drift Flux Model)

# 1. Local Instantaneous Formula

- balance equations for each phase
- interfacial condition(i.e. jump condition) as well as BC & IC

  F = F($\underline{r}$, t)

■ Fundamental Importances in the Local Instantaneous Formula

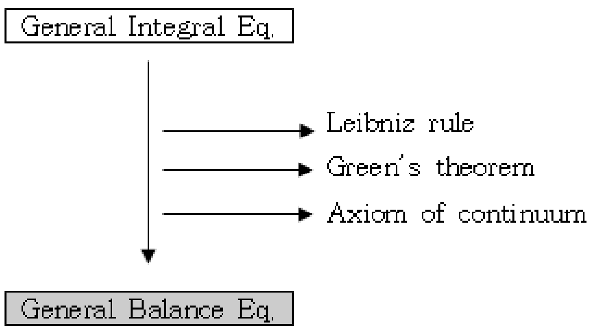
a) direct application to study separated flow

  : Δp calculation, instability analysis, bubble dynamics, etc

b) fundamental base of all two phase models

  perform overall calculations using averaging techniques

  void fraction calculation, development of drift flux model, etc




The general integral balance can be written as,

$$\frac{d}{dt}\int_{V}\rho_k\phi_k dV = -\int_{A}\overline{n_k}\cdot\overline{J_k}dA + \int_{V}\rho_k\Phi_k dV \qquad (4.2.1)$$

Using the Renold transport theorem to obtain the differential form of the balance equation. i.e

$$\frac{d}{dt}\int_{V}F_k dV - \int_{V}\frac{\partial F_k}{\partial t}dV - \int_{A}F_k\overline{v_k}\cdot\overline{n}dA \qquad (4.2.2)$$

the differential equation is,

$$\frac{\partial\rho_k\phi_k}{\partial t} + \nabla\cdot(\overline{v_k}\rho_k\phi_k) = -\nabla\cdot\overline{J_k} + \rho_k\Phi_k \qquad (4.2.3)$$

| Balance | $\psi_k$ | $\bar{J}_k$ | $\phi_k$ |
|---|---|---|---|
| Mass | 1 | 0 | 0 |
| Momentum | $v_k$ | $p_k \bar{I} - \bar{\tau}_k$ | $\bar{g}_k$ |
| Energy | $u_k + \frac{v_k^2}{2}$ | $q_k - \bar{T}_k \cdot v_k$ | $\bar{g}_k \cdot \bar{v}_k + \frac{Q_k}{\rho_k}$ |

(1) Interfacial balance (Jump condition)

Consider the interface as a singular surface across which the fluid density, energy and velocity suffer from jump discontinuity, although mass, momentum and energy must be conserved.

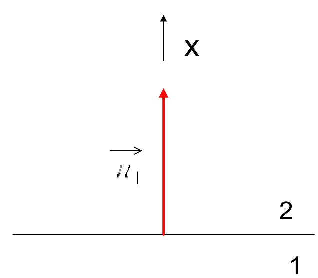
Mass Jump

$$\rho_1(\vec{v}_1 - \vec{v}_i) \cdot \vec{n}_1 + \rho_2(\vec{v}_2 - \vec{v}_i) \cdot \vec{n}_2 = 0$$

or $\dot{m}_1 = \dot{m}_2 = 0$ (4.2.4)

If no mass transfer, $\dot{m}_1 = \dot{m}_2 = 0$

$\therefore \; (\vec{v}_1 - \vec{v}_2) \cdot \vec{n} = 0 \quad or \quad \vec{v}_1 = \vec{v}_2$




Linear Momentum Jump

$$\dot{m}_1 \vec{v}_1 + \dot{m}_2 \vec{v}_2 - \vec{n}_1 \cdot \vec{T}_1 - \vec{n}_2 \cdot \vec{T}_2 = 0 \quad (4.2.5)$$

From Eq. (4.2.5), $\dot{m}_1(\vec{v}_1 - \vec{v}_2) - \vec{n}_1 \cdot \vec{T}_1 - \vec{n}_2 \cdot \vec{T}_2 = 0$

a) for inviscid phase ( $\vec{T} = -p$ )

$$\dot{m}_1(\vec{v}_1 - \vec{v}_2) - \vec{n}_1(p_1 - p_2) = 0$$

b) for viscous flow (1D flow which interface perpendicular to flow direction

$$\dot{m}_1(\vec{v}_1 - \vec{v}_2) + \vec{n}_1 \cdot (p_1 - p_2) + \vec{n}_1 \cdot (\tau_2 - \tau_1) = 0$$

For 2 dimensional case with surface tension

$$\dot{m}_1 \vec{v}_1 - \dot{m}_2 \vec{v}_2 - \vec{n}_1 \cdot \vec{T}_1 - \vec{n}_2 \cdot \vec{T}_2 + \frac{d\sigma}{dl} \vec{\tau} - \frac{\sigma}{R} \vec{n}_1 = 0 \quad (4.2.6)$$

If no mass transfer and the phase are inviscid,

$$\vec{n}_1 \cdot (p_1 - p_2) + \frac{d\sigma}{dl} \vec{\tau} - \frac{\sigma}{R} \vec{n}_1 = 0 \quad (4.2.7)$$

If the surface tension is constant,

$$p_1 - p_2 = \frac{\sigma}{R} \quad (4.2.8)$$

Total Energy Jump

$$\dot{m}_1\left(u_1+\frac{1}{2}v_1^2\right)-\dot{m}_2\left(u_2-\frac{1}{2}v_2^2\right)+\vec{q_1}\cdot\vec{n_1}+\vec{q_2}\cdot\vec{n_2}-(\vec{n_1}\cdot\vec{T_1})\cdot\vec{v_1}-(\vec{n_2}\cdot\vec{T_2})\cdot\vec{v_2}=0 \tag{4.2.9}$$

(2) Interfacial boundary conditions

: 각 phase 간 kinematical, dynamical 혹은 thermal relation을 계산할 수 있는 경계조건

a) thermal boundary condition

$$T_1 = T_2 = T_I \tag{4.2.10}$$

b) mechanical boundary condition (No slip condition)

$$\vec{v}_{t1} = \vec{v}_{t2} = \vec{v}_{tI} \tag{4.2.11}$$

c) phase change boundary condition

$$g_1 - g_2 = \left\{\frac{|\vec{v_2}-\vec{v_I}|^2}{2} - \frac{|\vec{v_1}-\vec{v_I}|^2}{2}\right\}-\left\{\frac{1}{\rho_2}(\vec{\tau_2}\cdot\vec{n_2})\cdot\vec{n_2}-\frac{1}{\rho_1}(\vec{\tau_1}\cdot\vec{n_1})\cdot\vec{n_1}\right\} \tag{4.2.12}$$

Note that in thermodynamic equilibrium,

$$T_1 - T_2 = 0$$

$$p_1 - p_2 = 0$$

$$g_1 - g_2 = 0$$