CHAPTER 5. Static Magnetic Fields

Reading assignments: Cheng Ch.5, Ulaby Ch.4, Hayt Chs. 8, 9, Halliday Chs.28-30

1. Magnetostatics in Free Space (or in nonmagnetic media excluding ferromagnetic materials)

Steady-state (time-independent) magnetic phenomena caused by moving electric charges or steady currents.

Deductive Approach :
Define $\boldsymbol{B}$ and $\boldsymbol{H}$ $\Rightarrow$ Fundamental postulates
$\Rightarrow$ Derive other laws, theorems, and relations (Gauss's and Ampere's laws, Biot-Savart law, Vector magnetic potential, ...), which are verified by experiments

A. Fundamental Postulates

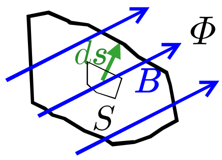
to represent the physical laws of magnetostatics in free space

1) Magnetic flux density (or Magnetic induction) $\boldsymbol{B}$

= magnetic flux per unit area (or = magnetic force per current moment)

$$\boldsymbol{B} \equiv \lim_{\Delta s \to 0} \frac{\Delta \Phi}{\Delta s} \quad (\text{or} \quad F_m / IL) \qquad (\text{T or Wb/m}^2 \text{ or N/A·m}) \qquad (1)$$

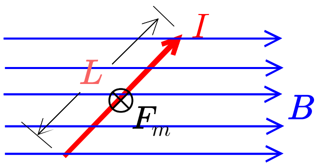
where $\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{s}$ (Wb or N·m/A) : magnetic flux (5-23)



$\boldsymbol{F}_m$ is the magnetic force

$$\boldsymbol{F}_m = q\boldsymbol{u} \times \boldsymbol{B} \quad \text{(N) on a moving charge } q \text{ in the field} \qquad (5\text{-}4)$$

or $\boldsymbol{F}_m = I\boldsymbol{L} \times \boldsymbol{B}$ (N) on a current-carrying element in the field (5-116)*



(cf) $\boldsymbol{F}_e = q\boldsymbol{E}$ $(I\boldsymbol{L} \to q,\ \boldsymbol{B} \to \boldsymbol{E})$

Notes)

i) Total electromagnetic force on a charge q

$$\boldsymbol{F} = \boldsymbol{F}_e + \boldsymbol{F}_m = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \quad \text{(N) : Lorentz's force} \qquad (5\text{-}5)$$

ii) Electromagnetic body force (= e.m. force per unit volume) in plasmas

$$\boldsymbol{f} = \boldsymbol{f}_e + \boldsymbol{f}_m = nq(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) = \rho_v \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B} \qquad (5\text{-}5)^*$$

2) Differential form of Gauss's and Ampere's laws

$\nabla \cdot \ B = 0$: Gauss's law for magnetism (5-6)

solenoidal (continuous closed loops, no magnetic flow sources)

$\nabla \times B = \mu_o J$: Ampere's law (in nonmagnetic media) (5-7)

Note) $\nabla \cdot (5-7) \ \Rightarrow \ \nabla \cdot J = 0$: Current continuity equation (5-8)

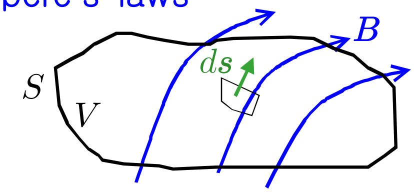
(Conservation of charge)

3) Integral form of Gauss's and Ampere's laws

$$\int_V (5\text{-}6)\ dv \ \Rightarrow \ \int_V \nabla \cdot B\, dv = 0$$

divergence theorem $(2-75)$ $\searrow$

$$\Rightarrow \ \oint_S B \cdot ds = 0 \qquad (5\text{-}9)$$



: Conservation of magnetic flux

(No total outward magnetic flux thru any closed surface,
No isolated monopoles)

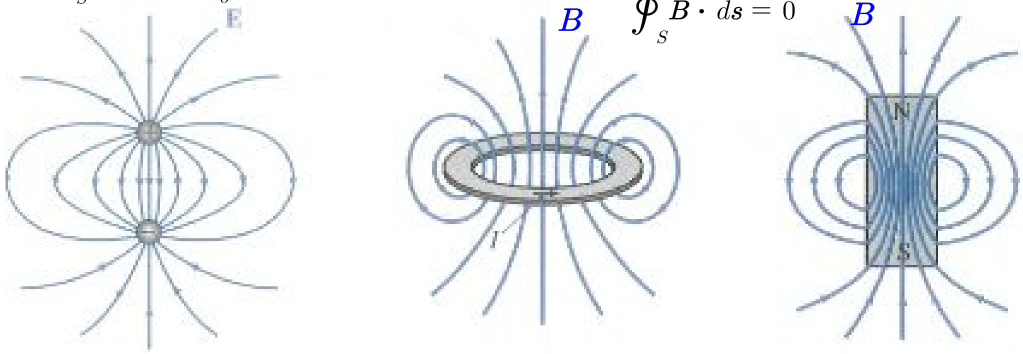
(cf)

$$\nabla \cdot \ E = \frac{\rho_v}{\epsilon_o}$$

$$\oint_S E \cdot ds = \frac{Q}{\epsilon_o}$$

$$\nabla \cdot \ B = 0$$

$$\oint_S B \cdot ds = 0$$



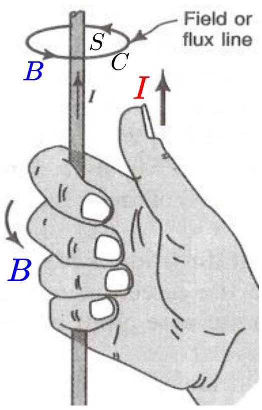
(a) Electric dipole (b) Magnetic dipole (c) Bar magnet

$$\int_S (5\text{-}7) \cdot ds \ \Rightarrow \ \int_S (\nabla \times B) \cdot ds = \mu_o \int_S J \cdot ds$$

Stoke's theorem $(2-103)$ $\searrow$

$$\Rightarrow \ \oint_C B \cdot dl = \mu_o I \qquad (5\text{-}10)$$

: Ampere's circuital law
(in nonmagnetic media)



B around any closed path

= current crossing the area bounded by the path

(Right-had rule for the directions)

(e.g. 5-1)

Infinitely long straight wire carrying a steady current I

For a cylindrically symmetric field $(\partial / \partial\phi = 0)$,

(5-10) $\Rightarrow \oint_C \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_o I$

a) Inside the conductor $(r \le b)$

$$\oint_{C_1} \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_o I_1$$

$$\Rightarrow \underbrace{\int_0^{2\pi} B(r\,d\phi)}_{2\pi r B} = \mu_o \frac{\pi r^2}{\pi b^2} I$$

$$\Rightarrow \boldsymbol{B}(r) = \hat{\phi}\frac{\mu_o I}{2\pi b^2} r \quad (r \le b) \quad \text{(5-11)}$$

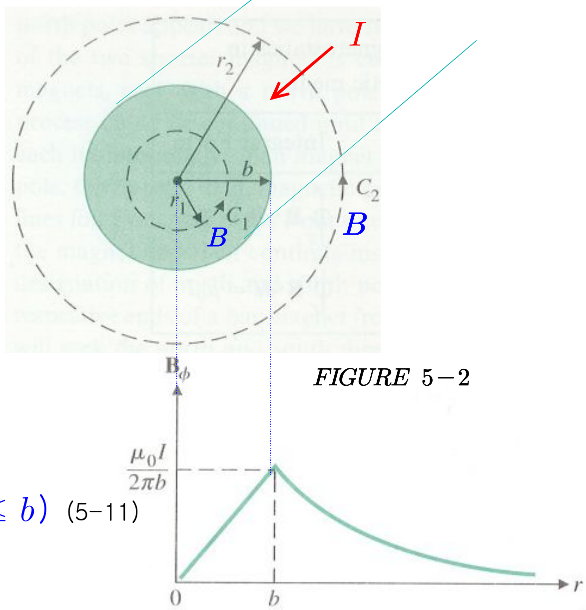


FIGURE 5−2

b) Outside the conductor $(r \ge b)$

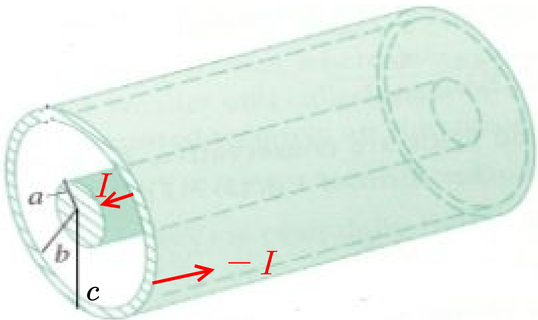
$$\oint_{C_2} \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_o I$$

$$\Rightarrow \int_0^{2\pi} B(r\,d\phi) = \mu_o I \quad \Rightarrow \boldsymbol{B}(r) = \hat{\phi}\frac{\mu_o I}{2\pi}\frac{1}{r} \quad (r \ge b) \quad \text{(5-12)}$$

(e.g.) Infinitely long coaxial line carrying a steady current I

Axially symmetric $(\partial/\partial\phi = 0)$.

(5-10) $\Rightarrow \oint_C \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_o I$



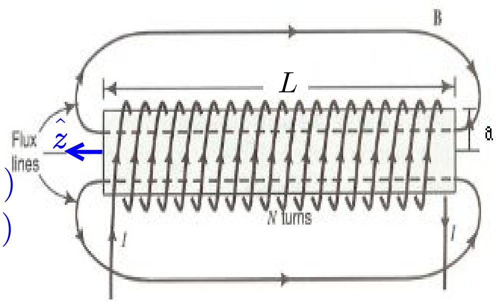
$$\boldsymbol{B}(r) = \begin{cases} \hat{\phi}\frac{\mu_o I}{2\pi a^2} r & (r \le a) \\ \hat{\phi}\frac{\mu_o I}{2\pi r} & (a \le r \le b) \\ \hat{\phi}\frac{\mu_o I}{2\pi r}\frac{c^2 - r^2}{c^2 - b^2} & (b \le r \le c) \\ 0 & (c \le r) \end{cases} \quad \text{(5-12)*}$$

(e.g.) Solenoid consisting of N turns of fine wire carrying a steady current I

$$\underbrace{\oint_C \boldsymbol{B} \cdot d\boldsymbol{l}}_{BL} = \begin{cases} \mu_o NI & (r < a) \\ 0 & (r > a) \end{cases}$$

$\Rightarrow$

$$\boldsymbol{B}(r) = \hat{z}\begin{cases} \underbrace{\mu_o (N/L) I}_{constant} & (r < a: \ inside) \\ 0 & (r > a: \ outside) \end{cases} \quad \text{(5-12)**}$$



(e.g. 5-2)

Current-carrying toroidal coil $\Rightarrow$ Toroidal-field coil in fusion devices

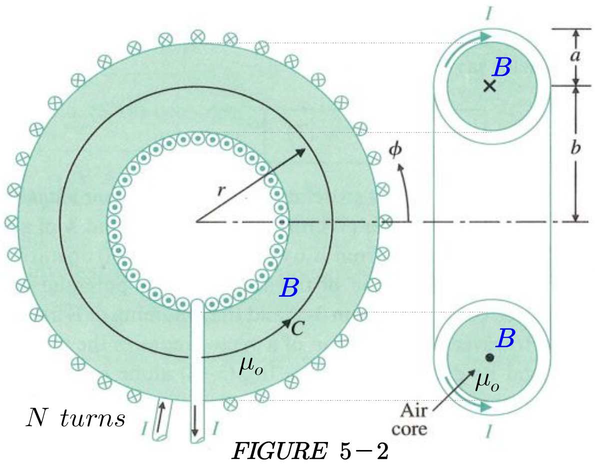
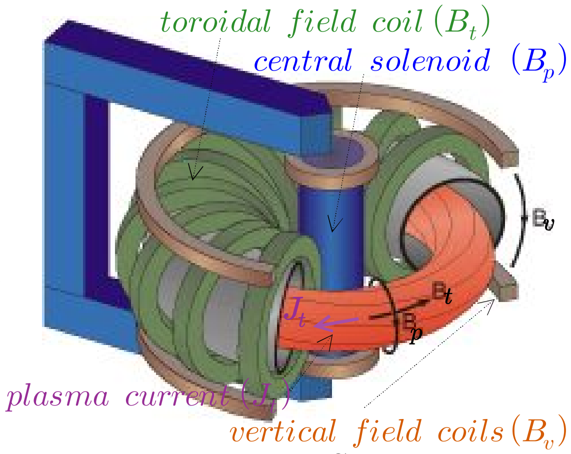


FIGURE 5−2

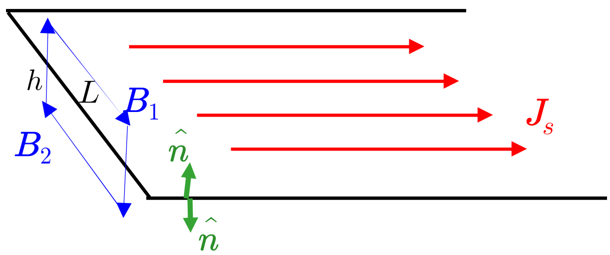


Tokamak Structure

$$\oint_C \overbrace{B\cdot dl}^{2\pi r B} = \begin{cases} \mu_o NI, & (b-a)<r<(b+a) \\ 0, & r<(b-a),\ r>(b+a) \end{cases}$$

$$\Rightarrow \quad B(r) = \hat{\phi}\begin{cases} \dfrac{\mu_o NI}{2\pi}\dfrac{1}{r}, & (b-a)<r<(b+a) \\ 0, & r<(b-a),\ r>(b+a) \end{cases} \qquad (5\text{-}13)$$

(e.g.) Infinite current sheet with a uniform surface current density J_s

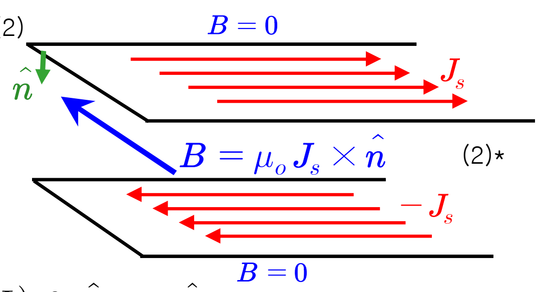


$$(5-10) \Rightarrow \oint_C B\cdot dl = \mu_o I \quad \Rightarrow \int_{\Box} B\cdot dl = \mu_o \int_L J_s \cdot dl$$

$$\xRightarrow{h\to 0} B_1 L + B_2 L = \mu_o J_s L \quad \Rightarrow \quad 2B = \mu_o J_s$$

$$\Rightarrow \quad B = \frac{\mu_o}{2} J_s \times \hat{n} \quad (2)$$

(cf) Two parallel current sheets



Note) Solenoid (5-12)** with $J_s = \hat{\phi}(NI/L)$ & $\hat{n} = -\hat{r}$

$$B(r) \overset{(2)^*}{=} \mu_o J_s\, \hat{\phi}\times(-\hat{r}) = \hat{z}\,\mu_o (N/L) I \quad (r<a:\ inside\ the\ solenoid)$$

$$B(r) = 0 \quad (r>a:\ outside\ the\ solenoid)$$

B. Vector Magnetic Potential $\boldsymbol{A}$

1) Definition of $\boldsymbol{A}$

The fundamental postulate (5-6) in magnetostatics: $\nabla \cdot \boldsymbol{B} = 0$

Null vector identity (2-109): $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$

$\boldsymbol{B}$ can be found by defining an vector magnetic potential $\boldsymbol{A}$ such that

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \quad \text{(T)} \tag{5-14}$$

Note) $\boldsymbol{A}$ has no physical meaning, only intermediate mathematical step for $\boldsymbol{B}$.

2) Vector Poisson's equation

(5-14) in $\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{J}$ (5-7) using $\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$:

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_o \boldsymbol{J}$$

$$\Rightarrow \quad \nabla^2 \boldsymbol{A} = -\mu_o \boldsymbol{J} \quad \text{: vector Poisson's equation} \tag{5-20}$$

where the Lorentz condition (or Coulomb condition) is imposed as

$$\nabla \cdot \boldsymbol{A} = 0 \tag{5-19}$$

Note) Lorentz gauge transformation:

$$\boldsymbol{A} \;\rightarrow\; \boldsymbol{A}' = \boldsymbol{A} + \nabla R \quad (R = \textit{gauge function}) \tag{5-19)*}$$

does not affect B in (5-14): Gauge invariance

(Proof) $\boldsymbol{B} = \nabla \times \boldsymbol{A}' = \nabla \times \boldsymbol{A} + \cancel{\nabla \times \nabla R} = \nabla \times \boldsymbol{A}$

Choose R in (5-19)* so that $\nabla \cdot \boldsymbol{A} = 0$ (Lorentz condition)

$$\Rightarrow \quad \nabla^2 R = 0 \tag{5-19)**}$$

3) Solution of the vector Poisson's equation

In view of the solution, $V(\boldsymbol{R}) = \dfrac{1}{4\pi\epsilon_o} \displaystyle\int_{V'} \dfrac{\rho_v(\boldsymbol{R}')}{|\boldsymbol{R} - \boldsymbol{R}'|} dv'$, of the scalar

Poisson's equation $\nabla^2 V = -\rho_v / \epsilon_o$ in electrostatics,

the vector Poisson's equation (5-20) has the solution

$$\boldsymbol{A}(\boldsymbol{R}) = \frac{\mu_o}{4\pi} \int_{V'} \frac{\boldsymbol{J}(\boldsymbol{R}')}{|\boldsymbol{R} - \boldsymbol{R}'|} dv' \quad \text{(Wb/m)} \tag{5-22}$$

Notes) $\boldsymbol{A}(\boldsymbol{R}) = \dfrac{\mu_o}{4\pi} \displaystyle\int_{S'} \dfrac{\boldsymbol{J}_s(\boldsymbol{R}')}{|\boldsymbol{R} - \boldsymbol{R}'|} ds'$ for surface current density (5-22)*

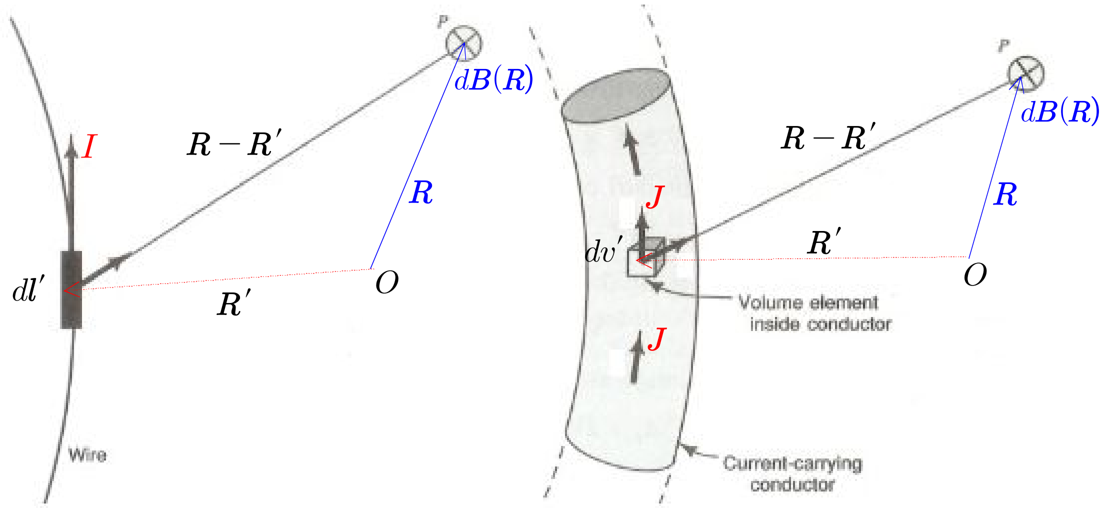
$\boldsymbol{A}(\boldsymbol{R}) = \dfrac{\mu_o}{4\pi} \displaystyle\int_{C'} \dfrac{I}{|\boldsymbol{R} - \boldsymbol{R}'|} d\boldsymbol{l}'$ for line current (5-22)**

The magnetic flux linking a surface S bounded by a contour C is

$$\Phi \underset{\text{(5-23)}}{=} \int_S \boldsymbol{B} \cdot d\boldsymbol{s} \underset{\text{(5-14)}}{=} \int_S (\nabla \times \boldsymbol{A}) \cdot d\boldsymbol{s} \underset{\text{Stokes's theorem}}{=} \oint_C \boldsymbol{A} \cdot d\boldsymbol{l} \quad \text{(Wb)} \tag{5-24}$$

C. Biot-Savart Law

Determination of magnetic field due to a current-carrying wire:



For a thin wire,

$$J dv' = JS\,dl' = I\,dl' \tag{5-25}$$

Then, (5-22) becomes (5-22)**

$$A\,(R) = \frac{\mu_o}{4\pi}\int_{C'}\frac{I\,dl'}{|R-R'|} \tag{5-26}$$

$$\therefore \quad B = \nabla\times A = \nabla\times\left[\frac{\mu_o I}{4\pi}\int_{C'}\frac{dl'}{|R-R'|}\right] = \frac{\mu_o I}{4\pi}\int_{C'}\nabla\times\left(\frac{dl'}{|R-R'|}\right)$$

$$= \frac{\mu_o I}{4\pi}\int_{C'}\left[\frac{1}{|R-R'|}\underbrace{\cancel{\nabla\times dl'}}_{0} + \underbrace{\nabla\left(\frac{1}{|R-R'|}\right)}_{=-\frac{R-R'}{|R-R'|^3}}\times dl'\right]$$

$$= \frac{\mu_o I}{4\pi}\int_{C'}\left(-\frac{R-R'}{|R-R'|^3}\right)\times dl'$$

$$\Rightarrow \quad B(R) = \frac{\mu_o}{4\pi}\int_{C'}\left(\frac{I\,dl'}{|R-R'|^2}\times\underbrace{\frac{R-R'}{|R-R'|}}_{=a_R}\right) \quad (\mathrm{T}) \tag{5-31}$$

: Biot-Savart Law

$$B(R) = \int_{C'} dB \tag{5-32a}$$

where

$$dB(R) = \frac{\mu_o}{4\pi}\frac{I\,dl'\times a_R}{|R-R'|^2} = \frac{\mu_o}{4\pi}\frac{I\,dl'\times(R-R')}{|R-R'|^3} \tag{5-32b,c}$$

D. *Calculations of A and B*

1) A current-carrying straight wire of a finite length 2L *(e.g. 5-1)*

a) By applying calculation of A

(5-26):

$$A(R) = \frac{\mu_o}{4\pi}\int_{C'}\frac{I\,dl'}{|R-R'|}$$

$$= \hat{z}\,\frac{\mu_o}{4\pi}\int_{-L'}^{L}\frac{I\,dz'}{\sqrt{r^2+z'^2}}$$

$$= \hat{z}\,\frac{\mu_o I}{4\pi}ln\left(\frac{\sqrt{r^2+z'^2}+L}{\sqrt{r^2+z'^2}-L}\right)$$

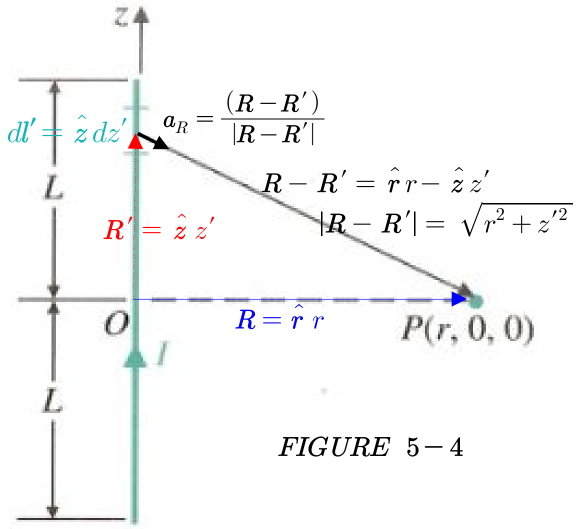


FIGURE 5-4

(5-14):

$$B = \nabla\times A$$

$$= \nabla\times(\hat{z}A_z) = \hat{r}\frac{1}{r}\cancelto{0}{\frac{\partial A_z}{\partial\phi}} - \hat{\phi}\frac{\partial A_z}{\partial r}$$

$$= -\hat{\phi}\,\frac{\partial}{\partial r}\left[\frac{\mu_o I}{4\pi}ln\left(\frac{\sqrt{r^2+z'^2}+L}{\sqrt{r^2+z'^2}-L}\right)\right] = \hat{\phi}\left(\frac{\mu_o IL}{2\pi r\sqrt{r^2+L^2}}\right) \qquad (5\text{-}34)$$

For $L \gg r$ (infinitely long wire),

$$(5\text{-}34) \quad\Rightarrow\quad B(r) = \hat{\phi}\,\frac{\mu_o I}{2\pi}\frac{1}{r} \qquad (5\text{-}35)\equiv(5\text{-}12)$$

For $L \ll r$ (short wire),

$$(5\text{-}34) \quad\Rightarrow\quad B(r) = \hat{\phi}\,\frac{\mu_o IL}{2\pi}\frac{1}{r^2} \qquad (5\text{-}35)*$$

(cf) (3-36) $V(R) = \dfrac{p\cdot\hat{R}}{4\pi\epsilon_o R^2}$ for electric dipole : $p = 2Lq \rightarrow 2LI$ in (5-35)*

b) By applying Biot-Savart law

$dl' = \hat{z}\,dz'$ & $R - R' = \hat{r}\,r - \hat{z}\,z'$, $|R-R'| = \sqrt{r^2+z'^2}$

$$\Rightarrow \quad dl'\times(R-R') = \hat{\phi}\,r\,dz' \text{ in (5-32c) } dB(R) = \frac{\mu_o}{4\pi}\frac{I\,dl'\times(R-R')}{|R-R'|^3}:$$

$$\Rightarrow \quad dB(r) = \hat{\phi}\,\frac{\mu_o I}{4\pi}\frac{r\,dz'}{(r^2+z'^2)^{3/2}} \text{ in (5-32a)}$$

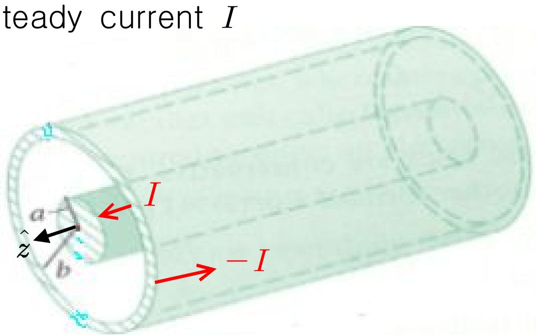
$$\Rightarrow \quad B(r) = \int_{-L}^{L}dB = \hat{\phi}\,\frac{\mu_o I}{4\pi}\int_{-L}^{L}\frac{r\,dz'}{(r^2+z'^2)^{3/2}} = \hat{\phi}\left(\frac{\mu_o IL}{2\pi r\sqrt{r^2+L^2}}\right)$$

$\equiv$ (5-34)

2) Infinitely long coaxial line carrying a steady current I

Axially symmetric $(\partial/\partial\phi = 0)$

No edge effect $(\partial/\partial z = 0)$



$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}} + \cancel{\frac{\partial^2}{\partial z^2}}$$

$$= \frac{1}{r}\frac{d}{dr}\left(r\frac{\partial}{dr}\right)$$

$\nabla^2 \boldsymbol{A} = -\mu_o \boldsymbol{J}$: vector Poisson's equation (5-20)

BVP in a current-free $(\boldsymbol{J}=0)$ region $(a<r<b)$ $\Rightarrow$ $\nabla^2 \boldsymbol{A} = 0$

$$\frac{d}{dr}\left(r\frac{dA_z}{dr}\right) = 0 \qquad ①$$

BCs:

$$A_z(r)|_{r=b} = 0 \qquad ②$$

$$\oint_C \boldsymbol{B}\cdot dl = \mu_o I \;\Rightarrow\; \oint_C (\nabla\times \boldsymbol{A})\cdot dl = \mu_o I$$

$$\Rightarrow\; \oint\left(-\frac{\partial A_z}{\partial r}\hat{\phi}\right)\cdot(\hat{\phi}\, r\, d\phi) = \mu_o I$$

$$\Rightarrow\; -\int_0^{2\pi}\left(\frac{\partial A_z}{\partial r}\right)(r\, d\phi) = \mu_o I \qquad ③$$

Integrating ① twice,

$$r\frac{dA_z}{dr} = C_1 \;\Rightarrow\; dA_z = C_1\frac{dr}{r} \;\Rightarrow\; A_z(r) = C_1 \ln r + C_2 \qquad ④$$

② in ④: $\qquad C_2 = C_1 \ln b \qquad ⑤$

⑤ in ④: $\qquad A_z(r) = C_1 \ln\frac{r}{b}, \qquad ⑥$

⑥ in ③: $\qquad -\int_0^{2\pi}\left(\frac{C_1}{r}\right)(r\, d\phi) = \mu_o I \;\Rightarrow\; -2\pi C_1 = \mu_o I$

$$\Rightarrow\; C_1 = -\mu_o I/2\pi \qquad ⑦$$

⑦ in ⑥: $\qquad A_z(r) = \hat{z}\frac{\mu_o I}{2\pi}\ln\frac{b}{r}$

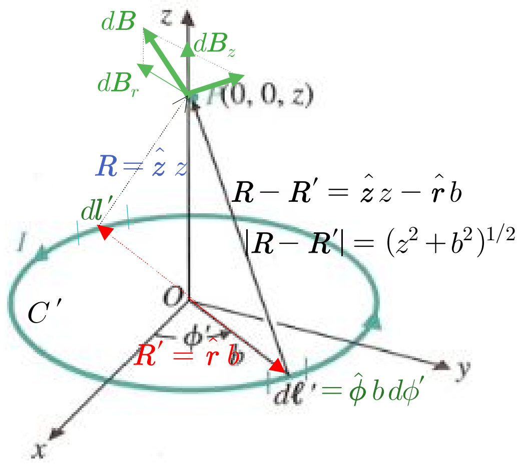
Consequently, $\boldsymbol{B}(r) = \nabla\times\boldsymbol{A} = -\hat{\phi}\frac{dA_z}{dr}$

$$= \hat{\phi}\frac{\mu_o I}{2\pi r}\left(\frac{1}{r}\right), \quad (a<r<b)$$

: same result as (5-12)*

3) Circular loop carrying a steady current I

Biot-Savart Law (5-31):



$$B(R) = \frac{\mu_o}{4\pi}\int_{C'}\left(\frac{I\,dl'}{|R-R'|^2}\times\frac{R-R'}{|R-R'|}\right)$$

$$= \frac{\mu_o I}{4\pi}\int_{C'}\left(\frac{b\,d\phi'\,\hat{\phi}}{z^2+b^2}\times\frac{\hat{z}\,z-\hat{r}\,b}{(z^2+b^2)^{1/2}}\right)$$

$$= \frac{\mu_o I}{4\pi}\left[\hat{r}\,b\int_0^{2\pi}\frac{z\,d\phi'}{(z^2+b^2)^{3/2}} + \hat{z}\int_0^{2\pi}\frac{b^2\,d\phi'}{(z^2+b^2)^{3/2}}\right)$$

$$= \frac{\mu_o I}{4\pi(z^2+b^2)^{3/2}}\left(\hat{x}\,bz\underbrace{\int_0^{2\pi}\cos\phi'\,d\phi'}_{0} + \hat{y}\,bz\underbrace{\int_0^{2\pi}\sin\phi'\,d\phi'}_{0} + \hat{z}\,b^2\underbrace{\int_0^{2\pi}d\phi'}_{=2\pi}\right)$$

$\hat{r}\,b = \hat{x}\,b\cos\phi' + \hat{y}\,b\sin\phi'$

$$\therefore \quad B(0, 0, z) = \hat{z}\,\frac{\mu_o I b^2}{2\,(z^2+b^2)^{3/2}} \qquad \text{(5-37)}$$

For $z = 0$ (at the center of the loop),

$$B(0,0,0) = \hat{z}\,\frac{\mu_o I}{2\,b} \qquad \text{(5-37)*}$$

For $z \to \infty$, $i.e.$, $R \to \infty$ (at a distant point),

$$B(R) = \hat{z}\,\frac{\mu_o I}{2}\,\frac{b^2}{R^3} = \hat{z}\,\frac{\mu_o}{2\pi}\,\frac{m}{R^3}$$: magnetic dipole field (5-37)**

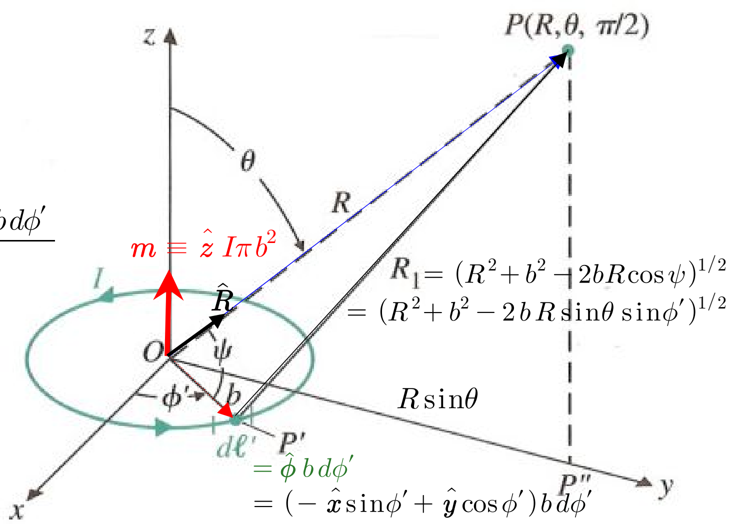
magnetic moment $m \equiv IS = I\pi b^2$

E. Magnetic Dipole

1) Far field at a distance point of a small circular loop

At $P(R, \theta, \pi/2)$,

(5-26) $\Rightarrow$



$$A = \frac{\mu_o I}{4\pi}\int_{C'}\frac{dl'}{R_1}$$

$$= \frac{\mu_o I}{4\pi}\int_0^{2\pi}\frac{(-\hat{x}\sin\phi' + \hat{y}\cos\phi')\,b\,d\phi'}{R_1}$$

$$= -\hat{x}\,\frac{\mu_o I}{4\pi}\int_0^{2\pi}\frac{b\sin\phi'}{R_1}\,d\phi'$$

$$= \hat{\phi}\,\frac{\mu_o I b}{2\pi}\int_{-\pi/2}^{\pi/2}\frac{\sin\phi'}{R_1}\,d\phi' \qquad \text{(5-40)}$$

$$\frac{1}{R_1} = \frac{1}{R}\left(1 + \frac{b^2}{R^2} - \frac{2b}{R}\sin\theta\sin\phi'\right)^{-1/2} \cong \frac{1}{R}\left(1 - \frac{2b}{R}\sin\theta\sin\phi'\right)^{-1/2}$$

$$\cong \frac{1}{R}\left(1 + \frac{b}{R}\sin\theta\sin\phi'\right) \qquad (5\text{-}41)$$

(5-41) in (5-40) :

$$\boldsymbol{A} = \hat{\phi}\frac{\mu_o I b}{2\pi R}\int_{-\pi/2}^{\pi/2}\left(\cancel{1} + \frac{b}{R}\sin\theta\sin\phi'\right)\sin\phi'\, d\phi'$$

$$\Rightarrow \quad \boldsymbol{A} = \hat{\phi}\,\frac{\mu_o I b^2}{4R^2}\sin\theta = \hat{\phi}\,\frac{\mu_o (I\pi b^2)}{4\pi R^2}\sin\theta \qquad (5\text{-}42)$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = \nabla\times(\hat{\phi}A_\phi)$$

$$= \hat{\boldsymbol{R}}\frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial R}(RA_\phi)$$

$$\Rightarrow \quad \boldsymbol{B} = \frac{\mu_o I b^2}{4R^3}(\hat{\boldsymbol{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta) \qquad (5\text{-}43)$$

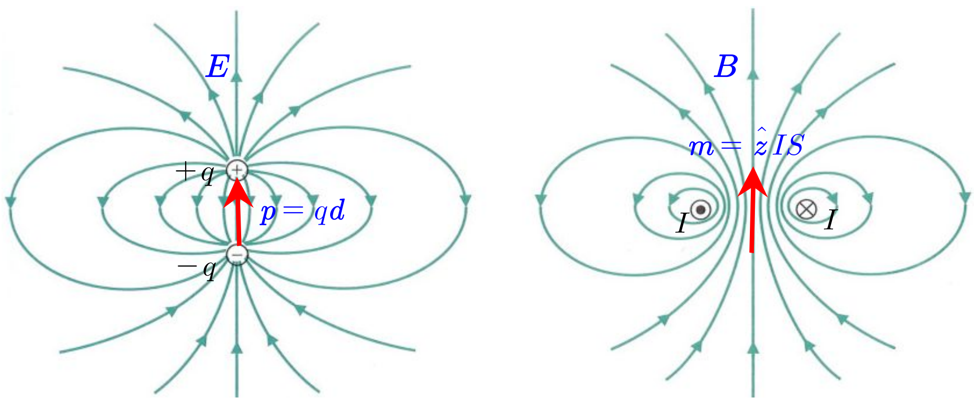
2) Magnetic dipole field

(5-42) $\Rightarrow$ $$\boldsymbol{A} = \hat{\phi}\,\frac{\mu_o(I\pi b^2)}{4\pi R^2}\sin\theta = \frac{\mu_o \boldsymbol{m}\times\hat{\boldsymbol{R}}}{4\pi R^2} \qquad (5\text{-}44)$$

(5-43) $\Rightarrow$ $$\boldsymbol{B} = \frac{\mu_o m}{4\pi R^3}(\hat{\boldsymbol{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta) \qquad (5\text{-}47)$$

where $\boldsymbol{m} \equiv \hat{z}\,IS = \hat{z}\,I\pi b^2$: magnetic dipole moment (5-45)

(cf) Electric dipole | Magnetic dipole



Electric dipole		Magnetic dipole
$V = \dfrac{\boldsymbol{p}\cdot\hat{\boldsymbol{R}}}{4\pi\epsilon_o R^2}$		$\boldsymbol{A} = \dfrac{\mu_o \boldsymbol{m}\times\hat{\boldsymbol{R}}}{4\pi R^2}$
$\boldsymbol{E} = \dfrac{p}{4\pi\epsilon_o R^3}(\hat{\boldsymbol{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta)$		$\boldsymbol{B} = \dfrac{\mu_o m}{4\pi R^3}(\hat{\boldsymbol{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta)$
$1/4\pi\epsilon_o$	$\Leftrightarrow$	$\mu_o/4\pi$
$\boldsymbol{p} = q\boldsymbol{d}$	$\Leftrightarrow$	$\boldsymbol{m} = I\boldsymbol{S}$

Homework Set 6

1) P.5-2
2) P.5-3
3) P.5-6
4) P.5-7
5) P.5-9