Chapter 7

Performance Analysis

Objectives and Contents

- Predict performance of parallel programs
  Understand barriers to higher performance
  - General speedup formula
  - Amdahl’s Law
  - Gustafson-Barsis’ Law
  - Karp-Flatt metric
  - Isoefficiency metric

Speedup Formula

$$\text{Speedup} = \frac{\text{Sequential execution time}}{\text{Parallel execution time}}$$

$$\psi(n, p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p + \kappa(n, p)}$$

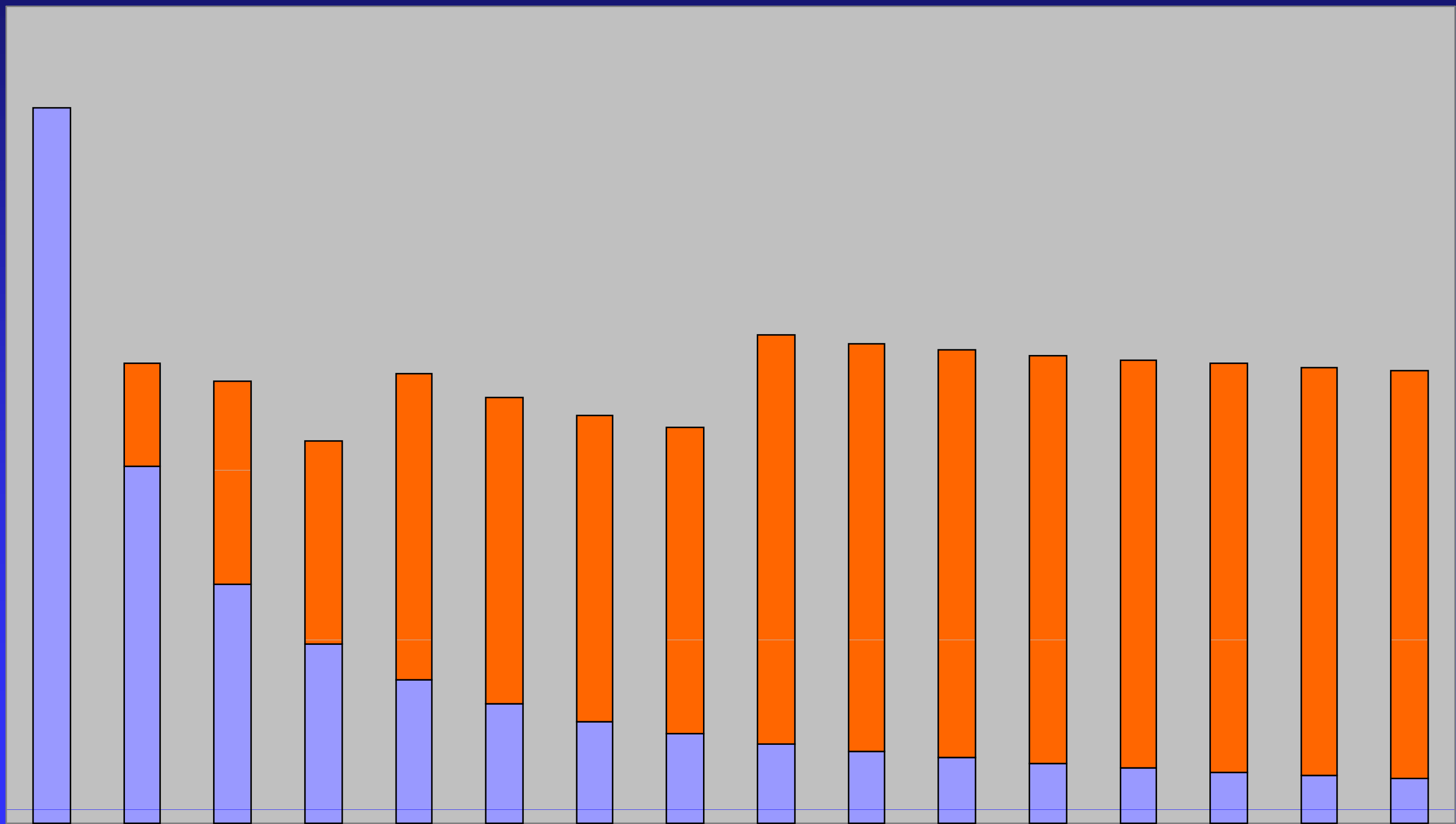
Execution Time Components

Inherently sequential computations: $\sigma(n)$

Potentially parallel computations: $\varphi(n)$

Communication operations: $\kappa(n,p)$

$\varphi(n)/p + \kappa(n,p)$



Efficiency = Speedup/ Number of processors

$0 \leq \varepsilon(n,p) \leq 1$

$$\varepsilon(n, p) \leq \frac{\sigma(n) + \varphi(n)}{p\sigma(n) + \varphi(n) + p\kappa(n, p)}$$

All terms > 0 $\Rightarrow \varepsilon(n,p) > 0$

Denominator > numerator $\Rightarrow \varepsilon(n,p) < 1$

Amdahl's Law

Amdahl's law states that the performance improvement to be gained from using some faster mode of execution is limited by the fraction of the time

the faster mode can be used.

$$\psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p+\kappa(n,p)}$$

$$\le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p}$$

Let $f = \sigma(n)/(\sigma(n) + \varphi(n))$

$$\psi \le \frac{1}{f+(1-f)/p}$$

Example 1

- 95% of a program's execution time occurs inside a loop that can be executed in parallel. What is the maximum speedup we should expect from a parallel version of the program executing on 8 CPUs?

$$\psi \leq \frac{1}{0.05 + (1 - 0.05)/8} \cong 5.9$$

Example 2

- 20% of a program's execution time is spent within inherently sequential code. What is the limit to the speedup achievable by a parallel version of the program?

$$\lim_{p\to\infty} \frac{1}{0.2 + (1-0.2)/p} = \frac{1}{0.2} = 5$$

Pop Quiz

- An oceanographer gives you a serial program and asks you how much faster it might run on 8 processors. You can only find one function amenable to a parallel solution. Benchmarking on a single processor reveals 80% of the execution time is spent inside this function. What is the best speedup a parallel version is likely to achieve on 8 processors?

Pop Quiz

- A computer animation program generates a feature movie frame-by-frame. Each frame can be generated independently and is output to its own file. If it takes 99 seconds to render a frame and 1 second to output it, how much speedup can be achieved by rendering the movie on 100 processors?

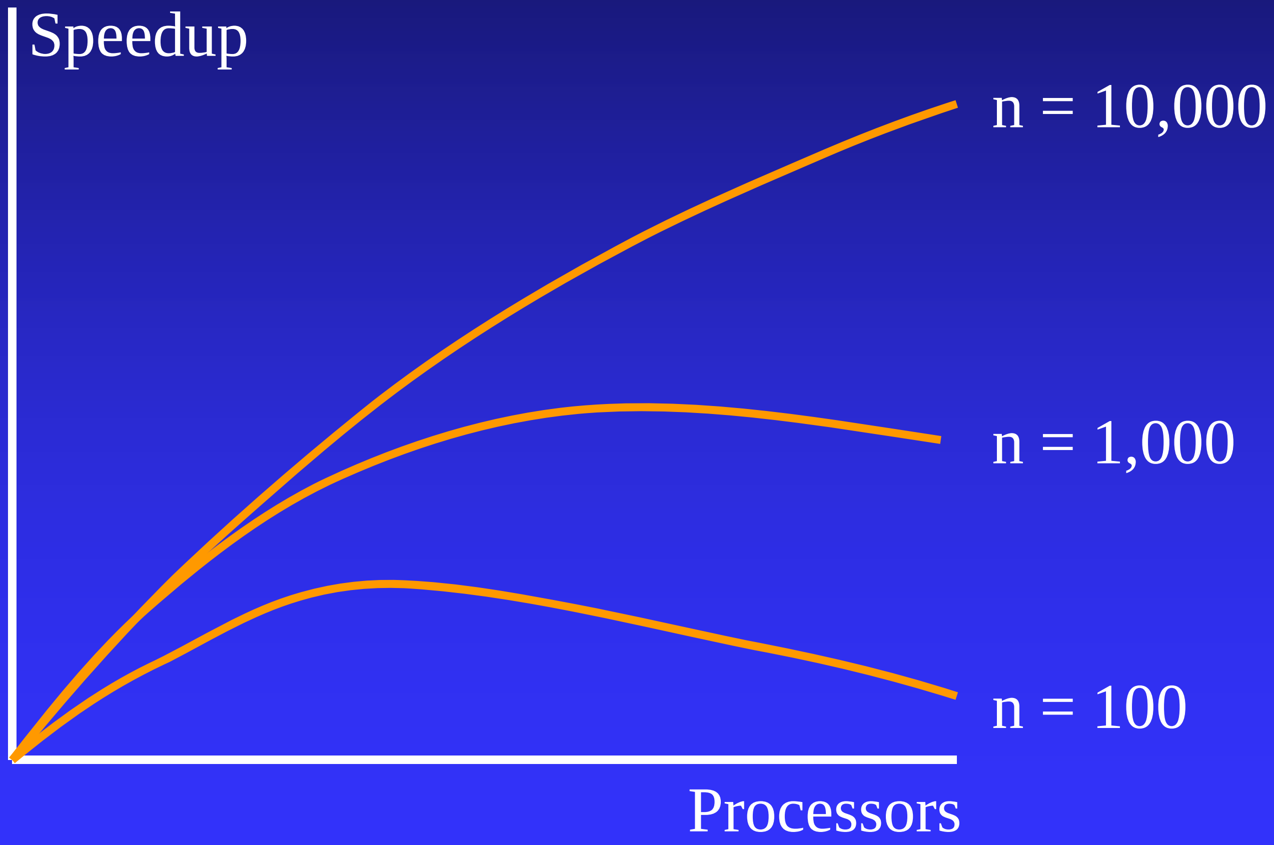
Limitations of Amdahl's Law

- Ignores $\kappa(n,p)$
- Overestimates speedup achievable

Amdahl Effect

- Typically $\kappa(n,p)$ has lower complexity than $\varphi(n)/p$
- As n increases, $\varphi(n)/p$ dominates $\kappa(n,p)$
- As n increases, speedup increases

Illustration of Amdahl Effect



Review of Amdahl’s Law

- Treats problem size as a constant
- Shows how execution time decreases as number of processors increases

Another Perspective

- We often use faster computers to solve larger problem instances
- Let's treat time as a constant and allow problem size to increase with number of processors

Gustafson-Barsis's Law

$$\psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p}$$

Let $s = \sigma(n)/(\sigma(n)+\varphi(n)/p)$

$$\psi \le p + (1-p)s$$

psi

Gustafson-Barsis's Law

- Begin with parallel execution time
- Estimate sequential execution time to solve same problem
- Problem size is an increasing function of *p*
- Predicts **scaled speedup**

Example 1

- An application running on 10 processors spends 3% of its time in serial code. What is the scaled speedup of the application?

$$\psi = 10 + (1-10)(0.03) = 10 - 0.27 = 9.73$$

…except 9 do not have to execute serial code

Execution on 1 CPU takes 10 times as long…

Example 2

- What is the maximum fraction of a program's parallel execution time that can be spent in serial code if it is to achieve a scaled speedup of 7 on 8 processors?

$$7 = 8 + (1-8)s \Rightarrow s \approx 0.14$$

Pop Quiz

- A parallel program executing on 32 processors spends 5% of its time in sequential code. What is the scaled speedup of this program?

The Karp-Flatt Metric

- Amdahl's Law and Gustafson-Barsis' Law ignore $\kappa(n,p)$
- They can overestimate speedup or scaled speedup
- Karp and Flatt proposed another metric

Experimentally Determined Serial Fraction

$$e = \frac{\sigma(n) + \kappa(n,p)}{\sigma(n) + \varphi(n)}$$

Inherently serial component of parallel computation + processor communication and synchronization overhead

Single processor execution time

$$e = \frac{1/\psi - 1/p}{1 - 1/p}$$

Experimentally Determined Serial Fraction

- Takes into account parallel overhead
- Detects other sources of overhead or inefficiency ignored in speedup model
  - Process startup time
  - Process synchronization time
  - Imbalanced workload
  - Architectural overhead

Example 1

p	2	3	4	5	6	7	8
ψ	1.8	2.5	3.1	3.6	4.0	4.4	4.7

What is the primary reason for speedup of only 4.7 on 8 CPUs?

e	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Since *e* is constant, large serial fraction is the primary reason.

Example 2

p	2	3	4	5	6	7	8
ψ	1.9	2.6	3.2	3.7	4.1	4.5	4.7

What is the primary reason for speedup of only 4.7 on 8 CPUs?

e	0.070	0.075	0.080	0.085	0.090	0.095	0.100

Since *e* is steadily increasing, overhead is the primary reason.

Pop Quiz

p	4	8	12
ψ	3.9	6.5	?

- Is this program likely to achieve a speedup of 10 on 12 processors?

Isoefficiency Metric

- Parallel system: parallel program executing on a parallel computer
- Scalability of a parallel system: measure of its ability to increase performance as number of processors increases
- A scalable system maintains efficiency as processors are added
- Isoefficiency: way to measure scalability
  - •Program A shows 90% efficiency with P=20
  - •Program B shows 80% efficiency with P=20
  - •Is program A better for parallel computation regardless of the problem size?

Isoefficiency Derivation Steps

- Begin with speedup formula
- Compute total amount of overhead
- Assume efficiency remains constant
- Determine relation between sequential execution time and overhead

Deriving Isoefficiency Relation

Determine overhead

$$T_o(n,p) = (p-1)\sigma(n) + p\kappa(n,p)$$

Substitute overhead into speedup equation

$$\psi(n,p) \le \frac{p(\sigma(n)+\varphi(n))}{\sigma(n)+\varphi(n)+T_0(n,p)}$$

Substitute T(n,1) = σ(n) + φ(n). Assume efficiency is constant.

$$T(n,1) \ge CT_0(n,p)$$

Isoefficiency Relation

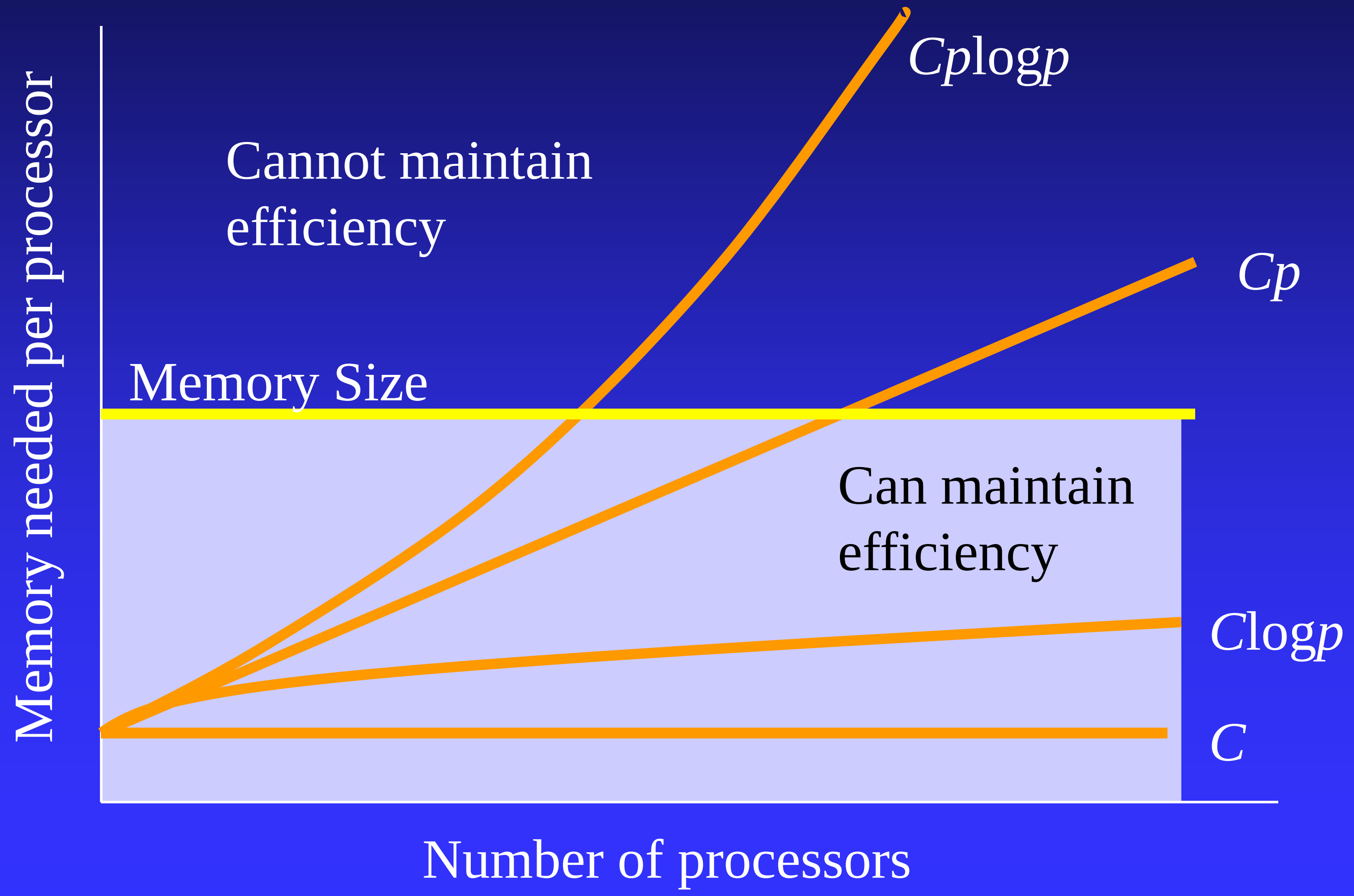
Scalability Function

- Suppose isoefficiency relation is $n \geq f(p)$
- Let $M(n)$ denote memory required for problem of size n
- $M(f(p))/p$ shows how memory usage **per processor** must increase to maintain same efficiency
- We call $M(f(p))/p$ the scalability function

Meaning of Scalability Function

- To maintain efficiency when increasing *p*, we must increase *n*
- Maximum problem size limited by available memory, which is linear in *p*
- Scalability function shows how memory usage per processor must grow to maintain efficiency
- Scalability function a constant means parallel system is perfectly scalable

Interpreting Scalability Function



Example 1: Reduction

- Sequential algorithm complexity
  $T(n,1) = \Theta(n)$
- Parallel algorithm
  - Computational complexity = $\Theta(n/p)$
  - Communication complexity = $\Theta(\log p)$
- Parallel overhead
  $T_0(n,p) = \Theta(p \log p)$

Reduction (continued)

- Isoefficiency relation: $n \geq C\, p \log p$
- We ask: To maintain same level of efficiency, how must n increase when p increases?
- $M(n) = n$

$$M(Cp\log p)/p = Cp\log p/p = C\log p$$

- The system has good scalability

Example 2: Floyd's Algorithm

- Sequential time complexity: $\Theta(n^3)$
- Parallel computation time: $\Theta(n^3/p)$
- Parallel communication time: $\Theta(n^2 \log p)$
- Parallel overhead: $T_0(n,p) = \Theta(pn^2 \log p)$

Floyd's Algorithm (continued)

- Isoefficiency relation
  $n^3 \geq C(p\, n^2 \log p) \Rightarrow n \geq C\, p \log p$
- $M(n) = n^2$

$$M(Cp\log p)/p = C^2 p^2 \log^2 p / p = C^2 p \log^2 p$$

- The parallel system has poor scalability

Example 3: Finite Difference

- Sequential time complexity per iteration: $\Theta(n^2)$
- Parallel communication complexity per iteration: $\Theta(n/\sqrt{p})$
- Parallel overhead: $\Theta(n\sqrt{p})$

Finite Difference (continued)

- Isoefficiency relation
  $n^2 \geq Cn\sqrt{p} \Rightarrow n \geq C\sqrt{p}$
- $M(n) = n^2$

$$M(C\sqrt{p})/p = C^2 p/p = C^2$$

- This algorithm is perfectly scalable

Summary (1/3)

- Performance terms
  - Speedup
  - Efficiency
- Model of speedup
  - Serial component
  - Parallel component
  - Communication component

Summary (2/3)

- What prevents linear speedup?
  - Serial operations
  - Communication operations
  - Process start-up
  - Imbalanced workloads
  - Architectural limitations

Summary (3/3)

- Analyzing parallel performance
  - Amdahl's Law
  - Gustafson-Barsis' Law
  - Karp-Flatt metric
  - Isoefficiency metric