CONVECTIVE HEAT-TRANSFER COEFFICIENT
**Introduction**

In most transport processes, heat transfer in fluids is accompanied by some form of fluid motion so that the heat transfer does not occur by conduction alone.

**Forced convection:** fluid motion arises principally from a pressure gradient caused by a pump or blower.

**Natural convection:** fluid motion arises only from density differences associated with the temperature field.
Heat transfer in fluids

- Conduction
- Convection
  - Forced convection
  - Natural convection
  - Mixed convection
Introduction

Whether the heat-transfer mechanism is natural or forced convection, the fluid motion can be described by the equations of fluid mechanics.

@ low velocities → the flow is laminar throughout the system
@ high velocities → laminar near the heating surface
   + turbulent some distance away

Although fluid velocities in natural convection are usually lower than in forced convection, it is incorrect to think of natural convection as causing only laminar flow; turbulence does occur when critical Reynolds number for the system is exceeded.
Analytical Solution

All problems of convective heat transfer can be expressed in terms of the differential mass, energy, and momentum balances. However, the mathematical difficulties connected with integration of these simultaneous nonlinear partial differential equations are such that analytical solutions exist only for simplified cases.

One of the problems simplest to deal with analytically is that of

**Heat transfer between a fluid and a flat plate when the fluid is flowing parallel to the plate.**

*(Fig. 21-1)*
Heat transfer between a fluid and a flat plate when the fluid is flowing parallel to the plate.

(Fig. 21-1)

Fig. 21-1
Development of a thermal boundary layer for flow over a flat plate
Temperature profiles in a developing thermal boundary layer on a flat plate

\[ t_0, t_s \]

\[ x_0, x_1, x_2, x_3 \]

\[ y, \text{ distance normal to flat plate} \]
Development of thermal boundary layer for flow over a flat plate

Blasius flow (laminar flow)

\[ \text{Re}_x = \frac{\rho u_0 x}{\mu} \leq 5 \times 10^5 \text{ (Laminar flow)} \]

Direction of flow

- Velocity gradient
- Temperature gradient

Hydrodynamic Boundary Layer (HBL) = 0.99U_0

Thermal Boundary Layer (TBL) = 0.99\Delta t

Flat plate

\[ \text{Pr} = \frac{\nu}{\alpha} > 1 \cdots \text{HBL} > \text{TBL} \]

\[ \text{Pr} = \frac{\nu}{\alpha} < 1 \cdots \text{HBL} < \text{TBL} \]

Pr: water = 6.5, air = 0.7, Hg = 0.025

water (350° F) = 1
Heat transfer system

Heat is transferred between a fluid and the wall of a pipe

Direction of flow

Edge of thermal boundary layer

Temperature profile become flatter.
Development of thermal boundary layer for flow in a pipe

**Entrance region**

\[ \text{Re}_D = \frac{\rho u_0 x}{\mu} \leq 2100 \text{ (Laminar flow)} \]

**Thermal Entrance Length**

- **HBLT** : \( \frac{L_e}{D} \approx 0.05 \text{Re}_D \)
- **TBLT** : \( \frac{L_e}{D} \approx 0.05 \text{Re}_D \cdot \text{Pr} \)

*In turbulent flow*: \( \frac{L_e}{D} \approx 40 \sim 50 \)
Temperature profiles near the entrance of a pipe

Figure 21-4
Temperature profiles vs Velocity profiles

The temperature profile at a point in a flow system is influenced by the velocity profile.

The velocity profile is influenced by the temperature profile. (The velocity profile of an isothermal system may differ substantially from the velocity of a system in which heat is being transferred.)
Temperature profile in a fluid is influenced by the velocity profile

Differential energy balance

\[ u_x \frac{\partial t}{\partial x} + u_y \frac{\partial t}{\partial y} + u_z \frac{\partial t}{\partial z} + \frac{\partial t}{\partial \theta} = \frac{k}{\rho C_p} \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) \]  (8-11)

velocity profile \hspace{2cm} \text{Temperature profile}

Velocity profile is influenced by the temperature profile in a fluid

Navier-Stokes equation

\[ u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_z}{\partial z} + \frac{\partial u_x}{\partial \theta} = g_c X - \frac{g_c}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) \]

Note that viscosity is temp dependent!
Simplification

The non-isothermal velocity profile can be used in solving the differential momentum and energy balances for a non-isothermal system.

This simplification may introduce a serious error when the viscosity of the fluid is strongly dependent on temperature.

Frequently the temperature gradient is greatest near the wall, and it is in this region that the velocity gradient is also greatest. The effect of temperature on the viscosity of the fluid at the wall may therefore have a pronounced effect on both the velocity and temperature profiles of the system.
Individual Heat-transfer Coefficients

Heat transfer coefficient

Rate of Heat transfer

Surface temperature

bulk temperature

Newton’s law of cooling

Fourier’s law

At the surface, there is no fluid motion and energy transfer can only by conduction

\[ q_w = hA(t_s - t_m) = -k_f A \frac{\partial t}{\partial y} \bigg|_{y=0} \]
$h$ is a function of

the properties of the fluid,
the geometry and roughness of the surface, and
the flow pattern of the fluid.
Evaluation of individual heat transfer coefficient, $h$

Several methods are available for evaluating $h$

Laminar flow systems $\rightarrow$ Analytical methods

Turbulent systems $\rightarrow$ Integral methods
$\rightarrow$ Mixing-length theory
$\rightarrow$ Dimensional analysis
Evaluation of individual heat transfer coefficient, $h$

$$dq = kdA \left( \frac{dt}{dy} \right)_{y=0} = h(t_0 - t_s) dA$$

$$h = \frac{k}{t_0 - t_s} \left( \frac{dt}{dy} \right)_{y=0} = k \left\{ \frac{d\left[(t - t_s)/(t_0 - t_s)\right]}{dy} \right\}_{y=0}$$

Velocity of flow past the heated surface ↑
Temperature gradient at the wall ↑
Heat transfer coefficient, $h$ ↑
Individual Heat-transfer Coefficients

\[ h = \frac{-k_f \frac{\partial t}{\partial y}|_{y=0}}{t_s - t_0} \]

- The thermal boundary layer strongly influence the wall temperature gradient \( \frac{\partial t}{\partial y}|_{y=0} \)
- The wall temperature gradient will determine the rate of heat transfer across the boundary layer
- Since \((t_s-t_0)\) is a constant, independent of \(x\), while \(\delta_t\) increases with increasing \(x\), temperature gradients in the boundary layer must decrease with increasing \(x\).
- Accordingly, the magnitude of \( \frac{\partial t}{\partial y}|_{y=0} \) decreases with increasing \(x\), and if follows that \(q\) and \(h\) decreases with increasing \(x\).
If the resistance to heat flow is through of as existing only in a laminar film, $h$ is $k/\Delta x_e$ where $\Delta x_e$ is the equivalent thickness of a stationary film just thick enough to offer the resistance corresponding to observed value of $h$. Since there is often an appreciable resistance in the turbulent core and since the stationary film has not even an approximate physical counterpart in laminar flow, boiling and radiation, we shall refer $h$ as an individual heat transfer coefficient.
Individual Heat-transfer Coefficients

Definition of $h$

Flow through a heated conduit

$$q = hA(t_s - t_b)$$

$$t_b = \frac{1}{Au_{av}} \int_A u_x t dA$$

Mixing cup temp

Natural convection

$$q = hA(t_s - t_\infty)$$

$t_\infty$ = Temp far from surface

Condensation

$$q = hA(t_{sv} - t_s)$$

$t_{sv}$ = Temp of sat vapor

Boiling fluid

$$q = hA(t_s - t_{sl})$$

$t_{sl}$ = Temp of sat liquid

Radiation

$$q = h_r A(t_{s1} - t_{s2})$$
Individual Heat-transfer Coefficients

Convection in Circular pipes, Laminar : Eq. (22-40) pp352
Convection in Circular pipes, turbulent : Eq. (24-4) pp384
Convection from spheres : Eq. (24-18) pp395
Convection from a plane surface : Eq. (24-22)- Eq.(24-26)
Boiling fluid : Eq. (25-5) pp412
Condensation on vertical tubes : Eq. (25-29) pp420
Condensation on horizontal tubes : Eq. (25-32) pp421
## Individual heat transfer coefficients

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<tr>
<th>Material/Condition</th>
<th>h, Btu/(h)(ft²)(°F)</th>
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<td>Steam, dropwise condensation</td>
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<td>Air, natural convection</td>
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Multiply values of coefficients given in table by 5.678 to get W/(m²)(K)
The analysis of unsteady-state heating or cooling of solid objects can be often be simplified by assuming that the resistance to heat conduction inside the solid is negligible compared to the convective heat-transfer resistance in the surrounding fluid.

The circumstances necessary to justify this assumption are a high thermal conductivity for the solid and a low convective heat transfer coefficient in the adjacent fluid. A metallic object being heated or cooled in air often constitute such a system.
Example 21-1

To illustrate, we shall calculate the time required for a mercury thermometer initially at 70°F, placed in an oven at 400°F, to reach 279°F. The thermometer bulb has a diameter of 0.24 in and will be assumed to be adequately represented as an infinitely long cylinder with negligible resistance to heat transfer either in the glass which is very thin or in the mercury, which has an adequately high thermal conductivity \(k=5.6 \text{ Btu/(h)(ft)(°F)}\) at 140°F. The mean convective heat-transfer coefficient between the outside of the thermometer and the air in the oven will be taken as \(h=2 \text{ Btu/(h)(ft}^2\)(°F)\).
Example 21-1: time required for a mercury thermometer

thermometer bulb ➞ infinite long cylinder with negligible resistance to heat transfer either in the glass or in the mercury

Heat balance on the thermometer bulb

\[ hA(400 - t) = \rho C_p V \frac{dt}{d\theta} \]

Rearrange and integrate

\[ \frac{dt}{400 - t} = \frac{hA}{\rho C_p V} d\theta \]

\[ \ln \frac{400 - 279}{400 - 70} = -\frac{hA \theta}{\rho C_p V} \]

For a cylinder,

\[ \frac{A}{V} = \frac{\pi DL}{\pi D^2 L / 4} = \frac{4}{D} \]
Example 21-1: time required for a mercury thermometer

\[ \theta = \frac{\rho C_p}{h(A/V)} \ln \frac{400 - 279}{400 - 70} \]

\[ \rho = 849 \ lb/ft^3 \]
\[ C_p = 0.033 \ Btu/(lb)(^\circ F) \]

\[ \theta = \frac{(849)(0.033)}{(2)(4/12)} \ln \frac{330}{121} = 0.0707 \ln(2.72) = 0.0707 \ h \]
\[ = 4.24 \ min \]

For an infinite cylinder, \( A/V = (\pi DL)/(\pi D^2/4) = 4/D \)
Example 21-1: time required for a mercury thermometer

For an infinite cylinder, $L=V/A=D/4$

$$Y = e^{-BiFo}$$

$$\ln \frac{400 - 279}{400 - 70} = -\frac{hA\theta}{\rho C_p V}$$

$$\frac{400 - 279}{400 - 70} = \exp \left( -\frac{hA\theta}{\rho C_p V} \right) = \exp \left[ -\left( \frac{hL}{k} \right) \left( \frac{k\theta}{\rho C_p L^2} \right) \right]$$

$L$ : characteristic length scale

$\tau = \frac{\rho C_p V}{hA}$ : time constant (time required for the temperature difference driving force to fall to $e^{-1}$ or 0.368 of its initial value.)

$\tau = 4.24 \text{ min}$

Biot number

Fourier number
Time constant

As \( \tau = \frac{\rho C_p V}{hA} \) decrease

\[
\frac{t_\infty - t}{t_\infty - t_0} = \exp\left(-\frac{\theta}{\tau}\right)
\]

For fast response on change in temperature

\[
\tau = \frac{\rho C_p V}{hA}
\]
Temperature response, $t$  

$\tau =$ time constant

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Temperature response, \( t \), with time

\[
\theta, \text{min} \quad \tau = 50
\]

\( \tau = 0.5, 1, 5, 10, 20 \)
Biot Number (Bi)

\[ Bi = \frac{hL}{k_s} \]

- \( Bi \) represents the internal thermal resistance of a solid divided by the boundary layer thermal resistance.
Fourier Number ($F_o$)

$$F_o = \frac{\alpha \theta}{L^2}$$

$F_o$ is equal to the rate of heat conduction over the rate of thermal energy storage in a solid.
Reynolds Number \((Re)\)

\[
Re = \frac{u_\infty L}{\nu}
\]

\(Re = \frac{\text{the inertia forces}}{\text{the viscous forces}}\)
**Prandtl Number (Pr)**

\[
Pr = \frac{C_p \mu}{k} = \frac{\nu}{\alpha}
\]

\[
Pr = \frac{\text{the molecular momentum}}{\text{the thermal diffusivity}}
\]
Individual heat transfer coefficient, $h$
Overall heat transfer coefficient, $U$

$$q = hA \Delta t_{individual}$$

$$q = UA \Delta t_{overall}$$
Overall Heat-Transfer Coefficient

Heat is transfer by a series of conduction and convection mechanisms

\[ q = h_i A_i (t_1 - t_2) = k_b A_{b,lm} \frac{t_2 - t_3}{\Delta r_b} = k_c A_{c,lm} \frac{t_3 - t_4}{\Delta r_c} = h_0 A_0 (t_4 - t_5) \]

(21-12)

Thermal resistance

\[
\begin{align*}
\text{Thermal resistance} & \quad \frac{1}{h_i A_i} & \quad \frac{\Delta r_b}{k_b A_{b,lm}} & \quad \frac{\Delta r_c}{k_c A_{c,lm}} & \quad \frac{1}{h_0 A_0} \\
\end{align*}
\]
Individual temperature drops

\[ t_1 - t_2 = q \frac{1}{h_i A_i} \]

\[ t_2 - t_3 = q \frac{\Delta r_b}{k_b A_{b,lm}} \]

\[ t_3 - t_4 = q \frac{\Delta r_c}{k_c A_{c,lm}} \]

\[ t_4 - t_5 = q \frac{1}{h_0 A_0} \]
Overall temperature drops

\[ \Delta t_{\text{overall}} = t_1 - t_5 \]

\[ = q \left( \frac{1}{h_i A_i} + q \frac{\Delta r_b}{k_b A_{b,lm}} + q \frac{\Delta r_c}{k_c A_{c,lm}} + q \frac{1}{h_0 A_0} \right) \]

\[ q = \frac{t_1 - t_5}{\left( \frac{1}{h_i A_i} + \frac{\Delta r_b}{k_b A_{b,lm}} + \frac{\Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0 A_0} \right)} \]

\[ q = \frac{\Delta t_{\text{overall}}}{\Sigma R} \]
Overall coefficients of heat transfer based on the outside area $U_0$

$$q = \frac{A_0 \Delta t_{overall}}{\left( \frac{A_0}{h_i A_i} + \frac{A_0 \Delta r_b}{k_b A_{b,lm}} + \frac{A_0 \Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0} \right)}$$

$$\frac{1}{U_0} = \frac{A_0}{h_i A_i} + \frac{A_0 \Delta r_b}{k_b A_{b,lm}} + \frac{A_0 \Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0}$$

$$q = U_0 A_0 \Delta t_{overall}$$
Overall coefficients of heat transfer based on the inside area $U_0$

$$q = \frac{A_i \Delta t_{overall}}{\left( \frac{1}{h_i} + \frac{A_i \Delta r_b}{k_b A_{b,lm}} + \frac{A_i \Delta r_c}{k_c A_{c,lm}} + \frac{A_i}{h_0 A_0} \right)}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i \Delta r_b}{k_b A_{b,lm}} + \frac{A_i \Delta r_c}{k_c A_{c,lm}} + \frac{A_i}{h_0 A_i}$$

$$q = U_i A_i \Delta t_{overall}$$
Overall coefficients of heat transfer

\[ q = U_0 A_0 \Delta t_{\text{overall}} \]

\[ = U_i A_i \Delta t_{\text{overall}} \]
Overall coefficients of heat transfer

\[ q = \frac{\Delta t_{overall}}{\Sigma R} \]

\[ \Sigma R = \frac{1}{U_0 A_0} = \frac{1}{U_i A_i} \]

\[ U_0 A_0 = U_i A_i \]
Approximation of overall coefficients of heat transfer based on the inside area \( U_i \)

\[
\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i \Delta r_b}{k_b A_{b,lm}} + \frac{A_i \Delta r_c}{k_c A_{c,lm}} + \frac{A_i}{h_0 A_i}
\]

\[
\frac{1}{h_i} \gg \frac{\Delta r_b}{k_b}, \frac{\Delta r_c}{k_c}, \frac{1}{h_0}
\]

Approximation

\[
\frac{1}{U_i} = \frac{1}{h_i} + \frac{\Delta r_b}{k_b} + \frac{\Delta r_c}{k_c} + \frac{1}{h_0}
\]
Overall coefficients of heat transfer of the flat parallel walls

\[
\frac{1}{U_i} = \frac{1}{U_0} = \frac{1}{h_i} + \frac{\Delta x_b}{k_b} + \frac{\Delta x_c}{k_c} + \frac{1}{h_0}
\]
## Overall coefficients of heat transfer

### Table 21-2

<table>
<thead>
<tr>
<th>Equipment Description</th>
<th>U, Btu/(h)(ft²)(°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stabilizer reflux condenser</td>
<td>94</td>
</tr>
<tr>
<td>Oil pre-heater</td>
<td>108</td>
</tr>
<tr>
<td>Reboiler (condensing steam to re-boiling water)</td>
<td>300-800</td>
</tr>
<tr>
<td>Air heater (molten salt to air)</td>
<td>6</td>
</tr>
<tr>
<td>Steam-jacketed vessel evaporating milk</td>
<td>500</td>
</tr>
</tbody>
</table>
Fouling in heat exchanger
Fouling in heat exchanger
Fouling Coefficients

Fouling: deposits on heat transfer surface

- hard scale
  (boiler or evaporator)
- coke
  (oil heater in a refinery)
- porous deposits
  (mud, soot, vegetable)

\[
\text{Heat Transfer Coefficient} = \frac{\text{thermal conductivity}}{\text{thickness of scale}}
\]

Sandblasting, pneumatic cleaning tools, chemical cleaning

Thermal conductivity itself may be high
But effective thermal conductivity may be almost as low as that of the fluid.

Steam (air, hot water) blowing
Fouling Coefficients

\[ q = h_d A \Delta t_{\text{scale}} \]

\[ U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{\Delta r_b}{k_b A_{b,lm}} + \frac{\Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_o}} \]
Fouling Coefficients

\[ q = h_d A \Delta t_{scale} \]

\[ U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{\Delta r_b}{k_b} A_o \frac{A_o}{k_b} A_{b,lm} + \frac{\Delta r_c}{k_c} A_o \frac{A_o}{k_c} A_{c,lm} + \frac{1}{h_{do}} + \frac{1}{h_o}} \]
### Fouling coefficients

Table 21-3

<table>
<thead>
<tr>
<th>h&lt;sub&gt;d&lt;/sub&gt;, Btu/(h)(ft&lt;sup&gt;2&lt;/sup&gt;)(°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overhead vapors from crude-oil distillation</td>
</tr>
<tr>
<td>Dry crude oil (300-100°F):</td>
</tr>
<tr>
<td>Velocity under 2 ft/sec</td>
</tr>
<tr>
<td>Velocity 2-4 ft/sec</td>
</tr>
<tr>
<td>Velocity over 4 ft/sec</td>
</tr>
<tr>
<td>Air</td>
</tr>
<tr>
<td>Steam (non-oil-bearing)</td>
</tr>
<tr>
<td>Water, Great Lakes, over 125°F</td>
</tr>
</tbody>
</table>

Tubular Exchanger Manufacturers Association, New York, 1949
Example 21-2

A reflux condenser contains ¾ in. 16-gauge copper tubes in which cooling water circulates. Hydrocarbon vapors condense on the exterior surfaces of the tubes. Find the overall heat-transfer coefficient $U_o$. The inside convective coefficient can be taken as 4500 W/m²·K, and the outside coefficient as 1500 W/m²·K.

Approximately fouling coefficients from Table 21-3 are

\[ h_{d0} = 5700 \text{ W/m}^2\cdot\text{K} \]
\[ h_{di} = 2840 \text{ W/m}^2\cdot\text{K} \]

<table>
<thead>
<tr>
<th></th>
<th>Btu/(h)(ft²)(°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overhead vapors from crude-oil distillation</td>
<td>1000</td>
</tr>
<tr>
<td>Water, Great Lakes, over 125°F</td>
<td>500</td>
</tr>
</tbody>
</table>
Example 21-2   Find $U_o$

$\frac{3}{4}$" 16-gauge copper tube
$r_i=0.0157$
$r_o=0.0191$

Outside fluid

$\begin{align*}
\text{Condensing vapor} & \quad h_{do} = 5700 \\
\text{Cooling water} & \quad h_o = 1500 \\
\text{Outside fluid} & \quad h_{di} = 2840 \\
\text{Inside fluid} & \quad h_i = 4500
\end{align*}$

$k$

unit: $W/m^2\cdot K$
Fouling Coefficients

\[ U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{\Delta r_b}{k_b A_{b,lm}} + \frac{\Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_{do}} + \frac{1}{h_o}} \]
Example 21-2

\[ h_i = 4500 W / m^2 \cdot K, \quad h_0 = 1500 W / m^2 \cdot K \]

\[ h_{d_0} = 1000 \times 5.678 = 5700 W / m^2 \cdot K \]

\[ h_{d_i} = 500 \times 5.678 = 2840 W / m^2 \cdot K \]

\[ k = 380 W / m \cdot K, \quad r_i = 0.0157 m, \quad r_0 = 0.0191 m \]

\[ r_{l_m} = \frac{r_0 - r_i}{\ln \frac{r_0}{r_i}} = \frac{0.0191 - 0.0157}{\ln \frac{0.0191}{0.0157}} = 0.0175 m \]

\[ r_0 = \frac{0.0191}{(4500)(0.0157)} = 0.00027 \]

\[ r_0 = \frac{0.0191}{(2840)(0.0157)} = 0.00043 \]

\[ \Delta r \cdot r_0 = \frac{(0.00165)(0.0191)}{(380)(0.0175)} = 0.0000047 \]

\[ \frac{1}{h_{d_0}} = \frac{1}{5700} = 0.00018, \quad \frac{1}{h_0} = \frac{1}{1500} = 0.00067 \]
Example 21-2

\[
U_0 = \frac{1}{\frac{r_0}{h_i r_i} + \frac{r_0}{h_d r_i} + \frac{\Delta r \cdot r_0}{k_b r_{lm}} + \frac{1}{h_{d_0}} + \frac{1}{h_0}}
\]

\[
= \frac{1}{0.00027 + 0.00043 + 0.0000047 + 0.00018 + 0.00067}
\]

\[
= \frac{1}{0.00155} = 645 \text{ } W / m^2 \cdot K
\]

Resistance of metal wall is negligible

Fouling Resistance is significant~39%

\[
U_i A_i = U_o A_o
\]

\[
U_i = (645)(0.0191)/((0.0157) = 785 \text{ } W / m^2 \cdot K
\]
Heat Exchangers

Fig. 403. Cutaway view of a single-pass shell and tube exchanger. (Ross Heater and Mfg. Co.)
Fig. 410. Cross-sectional drawing of a typical four-pass tube side, single-pass shell side, floating head heat exchanger. (Tubular Exchanger Manufacturers Association.)
Heat Exchangers

Fig. 412. Component parts of a feed water heater. (*American Locomotive Co.*) 1, shell; 2, assembled tube bundle; 3, clamp ring for floating head; 4, channel with integral tube sheet; 5, clamp ring for channel cover; 6, shell cover; 7, floating head tube sheet; 8, floating head cover; 9, channel cover.
Heat Exchangers
Heat Exchangers

Figure 15.3. Heat exchanger with four tube passes and one shell pass. (Courtesy The Whitlock Manufacturing Co.)
Heat Exchangers

(c) Detail of floating head.

Figure 15.4. Two-tube-pass, one-shell-pass, floating-head heat exchangers. (Courtesy National U.S. Radiator Corp., Heat Transfer Division.)
Shell–and–Tube Heat Exchangers

Figure 11.3  Shell-and-tube heat exchanger with one shell pass and one tube pass (cross-counterflow mode of operation).
Shell–and–Tube Heat Exchangers

**Figure 11.4** Shell-and-tube heat exchangers. (a) One shell pass and two tube passes. (b) Two shell passes and four tube passes.
Heat Exchangers
Crossflow heat exchangers

(a) Finned with both fluid unmixed
(b) Unfinned with one fluid mixed and the other unmixed
Compact heat exchangers

**Figure 11.5** Compact heat exchanger cores. (a) Fin–tube (flat tubes, continuous plate fins). (b) Fin–tube (circular tubes, continuous plate fins). (c) Fin–tube (circular tubes, circular fins). (d) Plate–fin (single pass). (e) Plate–fin (multipass).
Double-pipe heat exchanger
The choice of path for each fluid depends on considerations such as corrosion, plugging, fluid pressure, and permissible pressure drop.
Double-pipe heat exchanger

\[ \Delta t = \Delta t_1 = \Delta t_2 = \text{constant} \]
Double-pipe heat exchanger

(a) Condenser ($C_h \rightarrow \infty$)

(b) Boiler ($C_c \rightarrow \infty$)
Double-pipe heat exchanger

The determination of the required heat transfer area is one of the principal objective in the design of heat exchangers. For a differential segment of the exchanger for which the outside tube area is $dA_0$,

$$q = U_0 A_0 \Delta t_{overall}$$

Generally,

$$dq = U_0 \Delta t_{overall} dA_0 \quad \leftarrow \quad U_0 \text{ and } \Delta t \text{ are varying}$$

$$= U_i \Delta t dA_i$$

\[ (21-23) \]
Double-pipe heat exchanger

Under steady conditions, the mixing-cup temperature of the hot and cold fluids in a heat exchanger are assumed to be fixed at any cross section normal to the flow. The overall temperature difference is $\Delta t = t_h - t_c$. 

$$\int_0^q \frac{dq}{U_0 \Delta t_{overall}} = \int_0^{A_0} dA_0$$

The overall coefficient of heat transfer $U_0$

The difference between the mixing-cup temperatures of the hot and cold fluids, $\Delta t$

Outside heat transfer area, $A_0$
Countercurrent vs. Concurrent

Countercurrent

Cold Fluid in $W_c$ lb/hr

Hot Fluid Out $W_h$ lb/hr

“1”

Cold Fluid out $W_c$ lb/hr

Hot Fluid In $W_h$ lb/hr

“2”

Concurrent (Parallel-flow)

Cold Fluid in $W_c$ lb/hr

Hot Fluid In $W_h$ lb/hr

“1”

Cold Fluid out $W_c$ lb/hr

Hot Fluid Out $W_h$ lb/hr

“2”
Double-pipe heat exchanger
Countercurrent

If \( C_p = \text{constant} \)

\[
\Delta t = t_h - t_c
\]

\[
\frac{d(\Delta t)}{dq}
\]

Components:
- Hot fluid, \( t_h \)
- Cold fluid, \( t_c \)
Heat balance for the differential section

\[ dq = w_c C_{pc} dt_c = w_h C_{ph} dt_h = U_0 (t_h - t_c) dA_0 \]
If $C_p$ = constant

The difference between the mixing cup temperatures, $\Delta t$, is also linear w.r.t. $q$.

$$\frac{d(\Delta t)}{dq} = \frac{\Delta t_2 - \Delta t_1}{q}$$
Heat Balance in the differential section of the exchanger

\[ dq = w_c C_{pc} dt_c = w_h C_{ph} dt_h = U_0 (t_h - t_c) dA_0 = U_0 \Delta t dA_0 \] (21-25)

\[ \frac{d(\Delta t)}{dq} = \frac{\Delta t_2 - \Delta t_1}{q} \] (21-26)
Heat Balance in the differential section of the exchanger

\[ dq = U_o \Delta t dA_o \]

\[ \frac{d(\Delta t)}{dq} = \frac{d(\Delta t)}{U_o \Delta t dA_o} = \frac{\Delta t_2 - \Delta t_1}{q} \]  

(21-27)

\[ \int_{\Delta t_1}^{\Delta t_2} \frac{d(\Delta t)}{U_o \Delta t/ q} = \frac{\Delta t_2 - \Delta t_1}{q} \int_0^{A_o} dA_o \]  

(21-28)

If \( U_o = \text{constant} \)

\[ q = U_o A_o \left[ \frac{\Delta t_2 - \Delta t_1}{\ln(\Delta t_2 / \Delta t_1)} \right] \]  

LMTD
Logarithmic Mean Temperature Difference

(21-29)

Supercritical Fluid Process Lab
Heat Exchange in double pipe heat exchanger

\[
q = U_o A_o \left[ \frac{\Delta t_2 - \Delta t_1}{\ln(\Delta t_2 / \Delta t_1)} \right] = w_c C_{pc} \Delta t_c = w_h C_{ph} \Delta t_h
\]
Example 21-3

If the temperature of approach (minimum temperature difference between fluids) is 20°F, determine the $A_o$, $w_h$ for (a) concurrent flow and (b) countercurrent flow.

Overall coefficient $U_o = 80 \text{ Btu/(h)(ft}^2)(\text{oF})$
Specific heat of crude oil = 0.56 Btu/(lb)(\text{oF})
Specific heat of kerosene = 0.60 Btu/(lb)(\text{oF})
Example 21-3 (a) Concurrent flow

Total heat load:

\[ q = w_c C_{pc} \Delta t_c = (2000)(0.56)(200 - 90) = 123,000 \text{ Btu/h} \]

\[ w_h = \frac{q}{C_{ph} \Delta t_h} = \frac{123,000}{(0.60)(450 - 220)} = 891 \text{ lb/h} \]

\[ A_o = \frac{q}{U_o \left[ \frac{\Delta t_2 - \Delta t_1}{\ln(\Delta t_2 / \Delta t_1)} \right]} = \frac{123,000}{80 \left[ \frac{360 - 20}{\ln(360 / 20)} \right]} = 13.1 \text{ ft}^2 \]

The temperature distribution

\( \Delta t_1 = 360 \)

The approach=20°F

\( \Delta t_2 = 20 \)
Example 21-3 (b) Countercurrent flow

Total heat load:

\[ q = w_c C_{pc} \Delta t_c = (2000)(0.56)(200 - 90) = 123,000 \text{ Btu} / h \]

\[ w_h = \frac{q}{C_{ph} \Delta t_h} = \frac{123,000}{(0.60)(450 - 110)} = 603 \text{ lb} / h \]

\[ A_o = \frac{q}{U_o \left[ \frac{\Delta t_2 - \Delta t_1}{\ln(\Delta t_2 / \Delta t_1)} \right]} \]
\[ = \frac{123,000}{80 \left[ \frac{250 - 20}{\ln(250 / 20)} \right]} \]
\[ = 16.9 \text{ ft}^2 \]

The temperature distribution

\[ \Delta t_1 = 20 \]
\[ \Delta t_2 = 250 \]

The approach=20\degree F

kerosene
Crude oil

Supercritical Fluid Process Lab
If $U_0$ is a function of temperature

$$U_0 = a + b \Delta t$$

(21-30) \[
\int_{\Delta t_1}^{\Delta t_2} \frac{d(\Delta t)}{U_0 \Delta t} = \frac{\Delta t_2 - \Delta t_1}{q} \int_0^{A_0} dA_0 \rightarrow \int_{\Delta t_1}^{\Delta t_2} \frac{d(\Delta t)}{(a + b \Delta t) \Delta t} = \frac{\Delta t_2 - \Delta t_1}{q} A_0
\]

\[
\int_{\Delta t_1}^{\Delta t_2} \frac{d(\Delta t)}{(a + b \Delta t) \Delta t} = \int_{\Delta t_1}^{\Delta t_2} \left( \frac{1/a}{\Delta t} + \frac{-b/a}{a + b \Delta t} \right) d(\Delta t)
\]

\[
= \left[ \frac{1}{a} \ln \Delta t - \frac{1}{a} \ln(a + b \Delta t) \right]_{\Delta t_1}^{\Delta t_2}
\]

\[
= \left[ \frac{1}{a} \ln \frac{\Delta t}{a + b \Delta t} \right]_{\Delta t_1}^{\Delta t_2}
\]

\[
= \frac{1}{a} \ln \left( \frac{\Delta t_2}{a + b \Delta t_2} \right) \left( \frac{a + b \Delta t_1}{\Delta t_1} \right)
\]

\[
= \frac{1}{a} \ln \frac{U_{o1} \Delta t_2}{U_{o2} \Delta t_1}
\]  \hspace{1cm} (21-31)
If $U_0$ is a function of temperature

$$U_0 = a + b\Delta t$$

$$U_{o2} = a + b\Delta t_2$$

$$U_{o1} = a + b\Delta t_1$$

$$U_{o2} - U_{o1} = b(\Delta t_2 - \Delta t_1)$$

$$b = \frac{U_{o2} - U_{o1}}{\Delta t_2 - \Delta t_1}$$

$$a = \frac{U_{o1}\Delta t_2 - U_{o2}\Delta t_1}{\Delta t_2 - \Delta t_1} \quad (21-32)$$
If $U_o$ is a function of temperature

$$U_o = a + b\Delta t$$

\[
\frac{\Delta t_2 - \Delta t_1}{q} A_o = \frac{1}{a} \ln \frac{U_{o1}\Delta t_2}{U_{o2}\Delta t_1}
\]

\[
a = \frac{U_{o1}\Delta t_2 - U_{o2}\Delta t_1}{\Delta t_2 - \Delta t_1}
\]

\[
q = \frac{U_{o1}\Delta t_2 - U_{o2}\Delta t_1}{\ln \frac{U_{o1}\Delta t_2}{U_{o2}\Delta t_1}} A_o
\]

(21-34)
Heat transfer rate in double pipe heat exchanger

\[ U_o = \text{const} \]

\[ q = U_o A_o \frac{\Delta t_2 - \Delta t_1}{\ln\left(\frac{\Delta t_2}{\Delta t_1}\right)} \]  \hspace{1cm} (21-29)

\[ U_o = a + b\Delta t \]

\[ q = \frac{U_{o1}\Delta t_2 - U_{o2}\Delta t_1}{\ln\left(\frac{U_{o1}\Delta t_2}{U_{o2}\Delta t_1}\right)} A_o \]  \hspace{1cm} (21-34)
Homework

PROBLEMS

21-1
21-6
21-10
21-15
21-17

Due on November 2