Lecture 5:

Comb Resonator Design (1)

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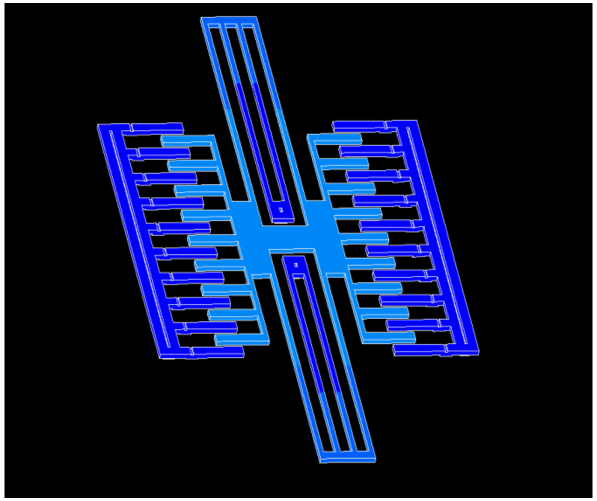
Why is Comb Drive Type ?

- ***Principle :***

  Interlacing comb fingers create large capacitor area; electrostatically actuated suspended microstructures

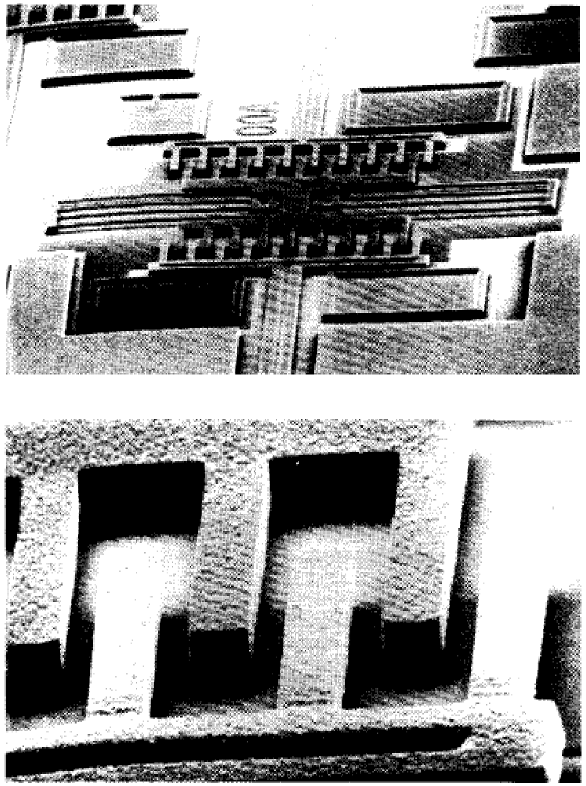
- ***Features :***
  - Fairly linear relationship between capacitance and displacement
  - Higher surface area/ capacitance than parallel plate capacitor
  - Electrostatic actuation: low power (no DC current)



Electrostatic Comb Drive Type

What is Comb Drive Resonator ?

- ***Comb drives combine mechanical and electrostatic issues :***
  - Mechanical issues
    - Elasticity
    - Stress and strain
    - Resonance (natural frequency)
    - Damping
  - Electrical issues
    - Capacitance
    - Electrostatic forces
    - Electrostatic work and energy



Tang, Nguyen and Howe JMEMS 1989.

Normal Stress and Strain

- ***Stress:*** force applied to surface

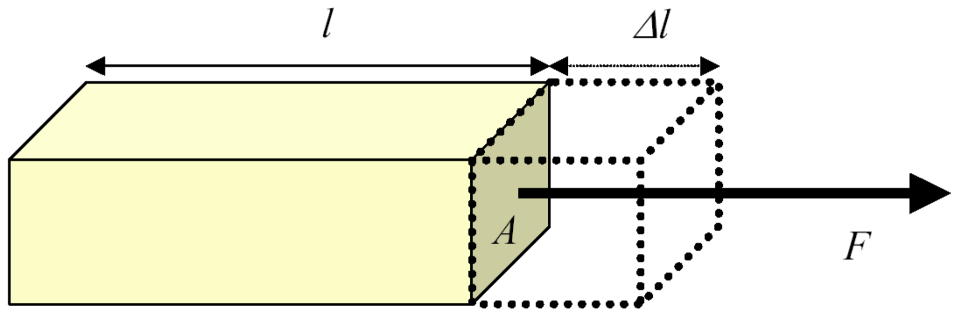
$$\sigma = F / A$$

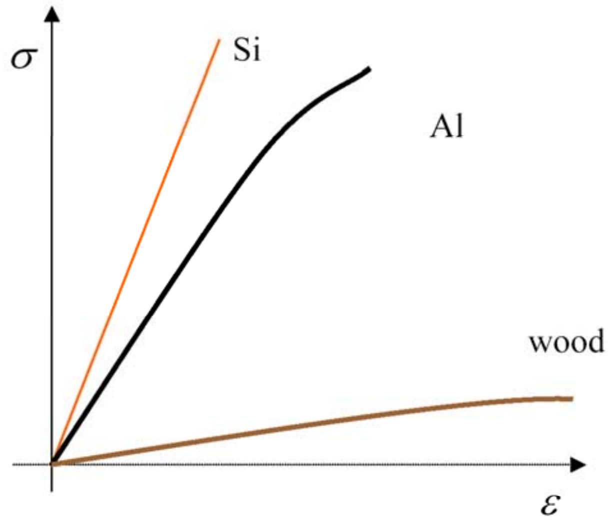
measured in N/m^2 or Pa,
compressive or tensile

- ***Strain:*** ratio of deformation to length

$$\varepsilon = \Delta l / l$$

measured in %, ppm,
or microstrain





Young's Modulus:

$$E = \sigma / \varepsilon$$

Hooke's Law:

$$K = F / \Delta l = EA / l$$

Shear Stress and Strain

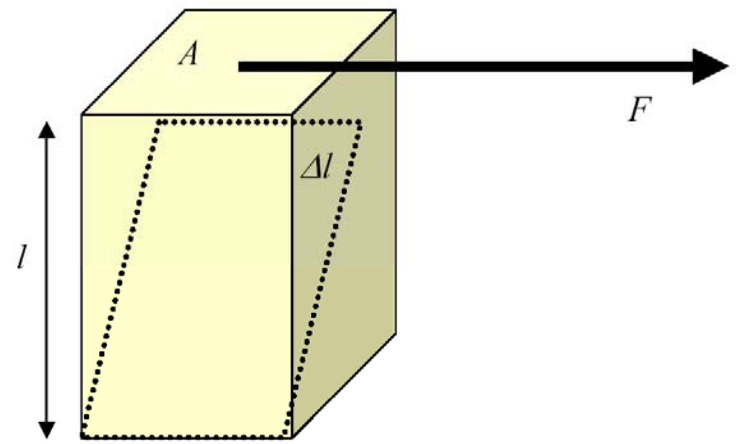
- ***Shear Stress:*** force applied parallel to surface

$$\tau = F / A$$

measured in N/m^2 or Pa,

- ***Share Strain:*** ratio of deformation to length

$$\gamma = \Delta l / l$$



Shear Modulus:

$$G = \tau / \gamma$$

Poisson's Ratio

Tensile stress in x direction results in compressive stress in y and z direction (object becomes longer and thinner)

- ***Poisson's Ratio:***

$$\nu = \left| -\frac{\varepsilon_y}{\varepsilon_x} \right| = \left| -\frac{\text{transverse strain}}{\text{longitudinal strain}} \right|$$

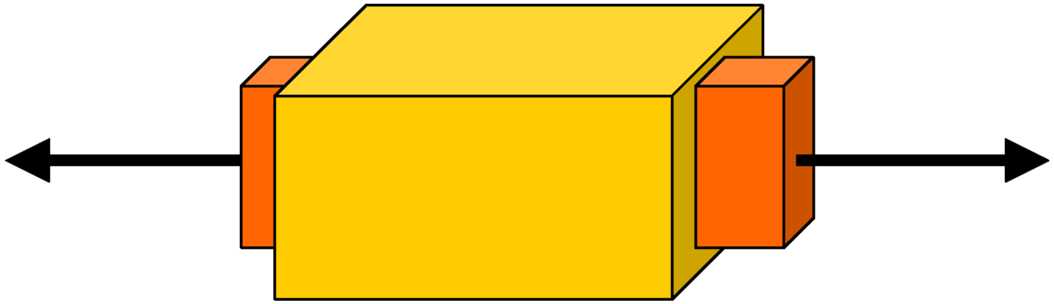
< Materials >

- **Homogeneous**
  - **Isotropic**
  - **Anisotropic**
- **Heterogeneous**

Metals : $\nu \approx 0.3$

Rubbers : $\nu \approx 0.5$

Cork : $\nu \approx 0$



Mechanical Property for Poly Crystalline Silicon

- ***Young's modulus for different deposition processes***

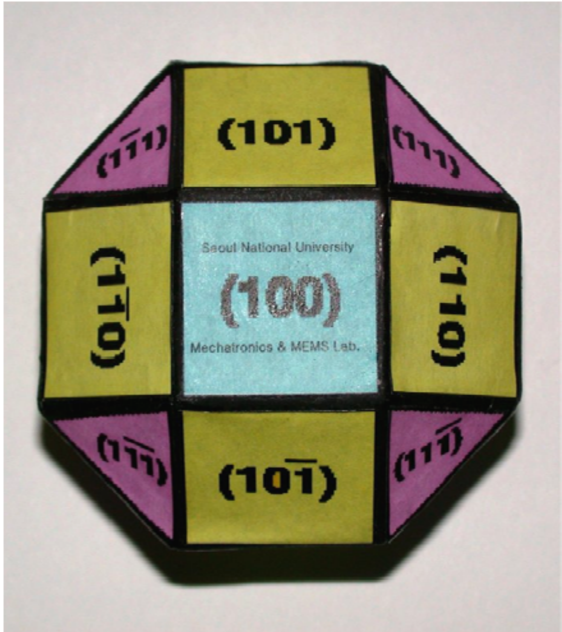
Doping Conditions / Deposition temp		PSG diffusion doping			$POCl_3$ diffusion doping							
		1000℃ 60min	1050℃ 30min	1000 ℃ 120min	850℃ 180min	850℃ 210min	850 ℃ 240min	950℃ 60min	950℃ 90min	950 ℃ 120min	1000℃ 60min	1000 ℃ 90min
Young's modulus (GPa)	625 ℃	154.3 ±2.3	155.1 ±1.8	151.1 ±3	149.6 ±2.6	150.7 ±2.5	153.1 ±2.1	149 ±1.7	151.1 ±1.1	150.4 ±1.1	152.6 ±1.4	148.6 ±2
	605 ℃	159.3 ±3.4	160.5 ±1.7	161.8 ±1.4	162.6 ±0.6	161.9 ±2.3	163.1 ±0.4	161.2 ±2	155.6 ±5.6	161.8 ±1.4	157.1 ±3.7	156 ±5.9
	585 ℃	170.1 ±2.2	169.4 ±6.2	162.4 ±1.6	168.5 ±1.9	162.2 ±2.6	163.5 ±3.1	159 ±3.3	165.6 ±1.4	160.6 ±1	159.8 ±1	156.6 ±2

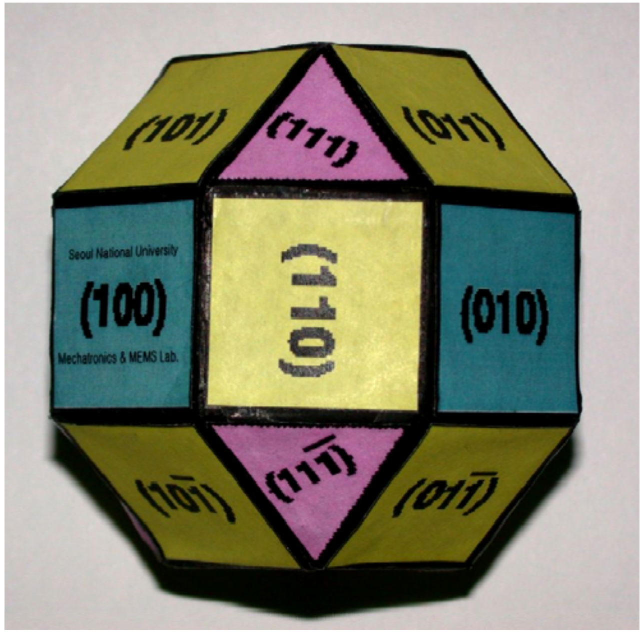
[Ref] S. Lee et al., IOP Journal of Micromechanics and Microengineering, vol. 8, no. 4, pp. 330-337, Dec. 1998.

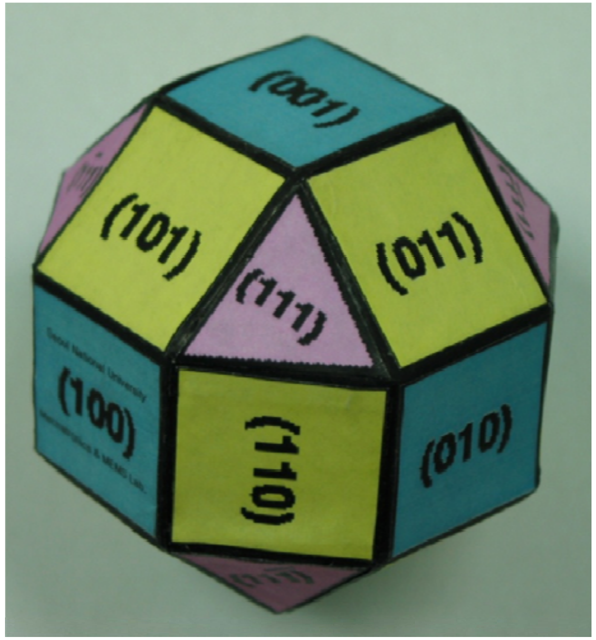
Mechanical Property for Single Crystal Silicon

- ***Silicon Crystallography***

Plane	( )	{ }
Direction	< >	[ ]



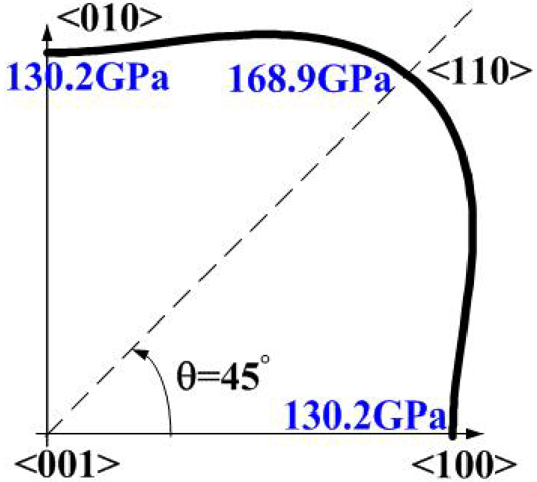




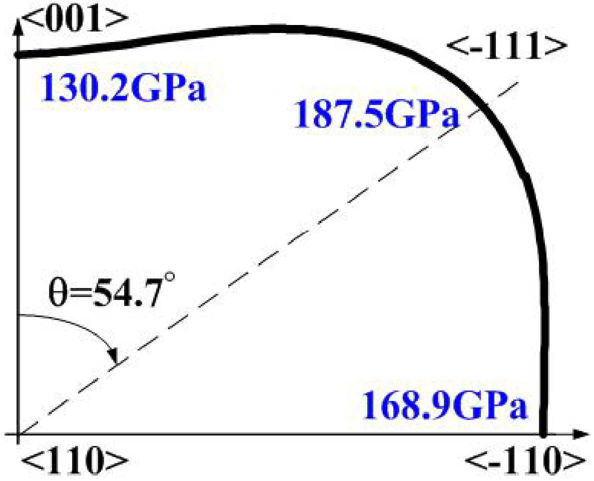
Silicon (100) **Silicon (110)** **Silicon (111)**

Mechanical Property for Single Crystal Silicon (cont'd)

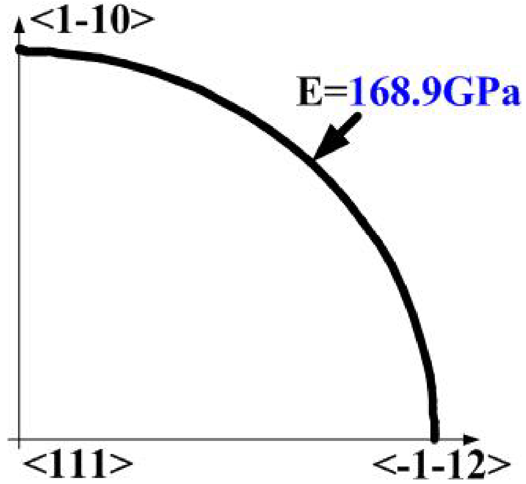
- ***Young's modulus***



Silicon (100)



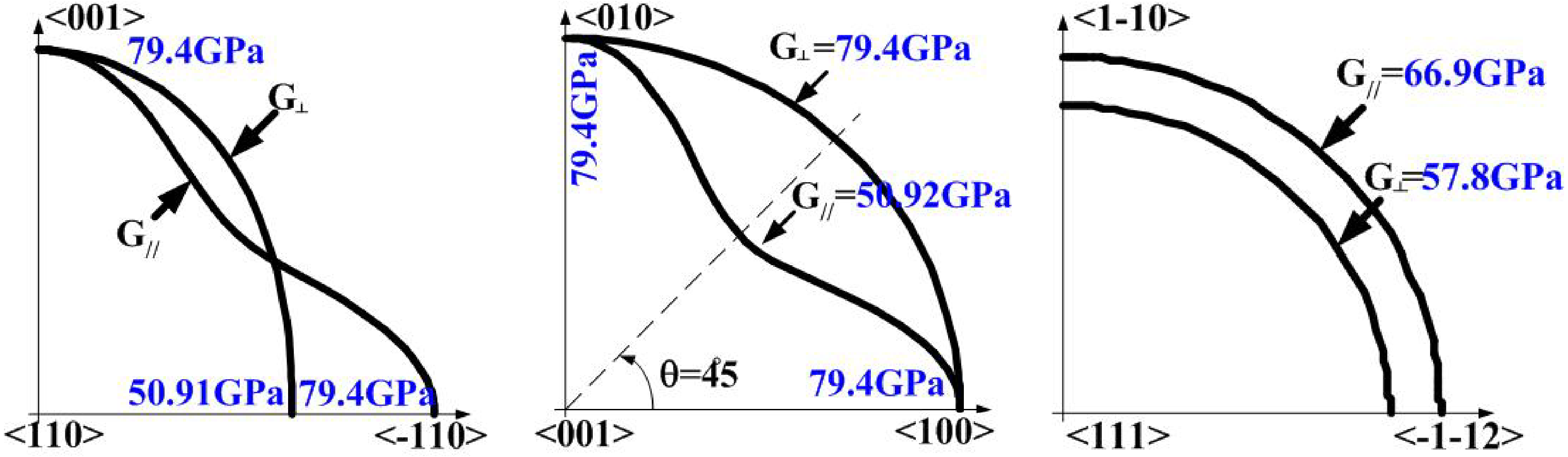
Silicon (110)



Silicon (111)

Mechanical Property for Single Crystal Silicon (cont'd)

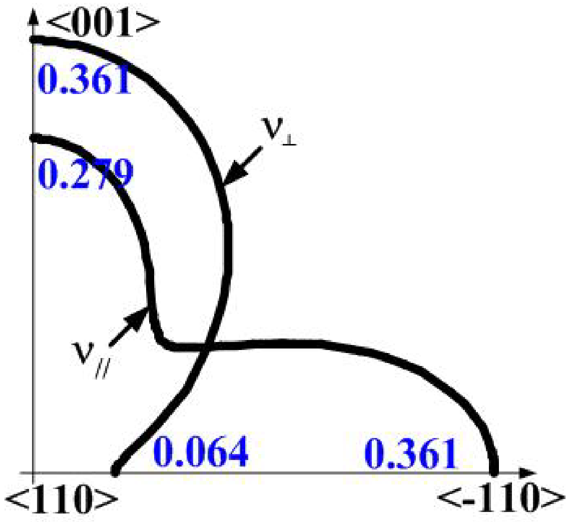
- ***Shear modulus***



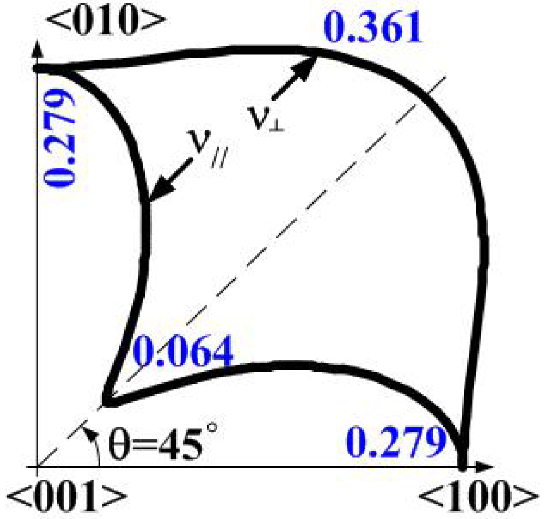
Silicon (100) **Silicon (110)** **Silicon (111)**

Mechanical Property for Single Crystal Silicon (cont'd)

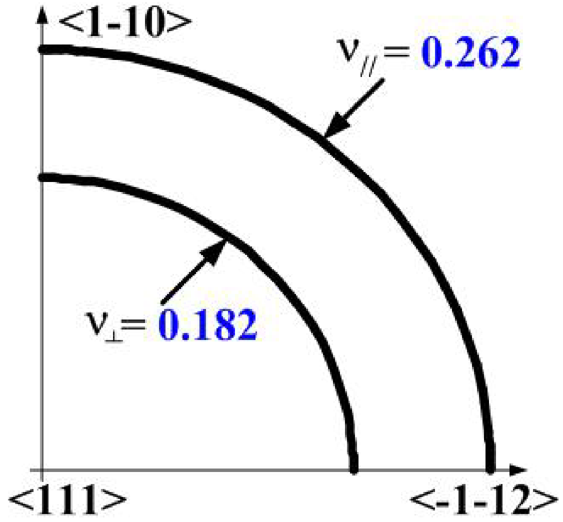
- ***Poisson's ratio***



Silicon (100)



Silicon (110)

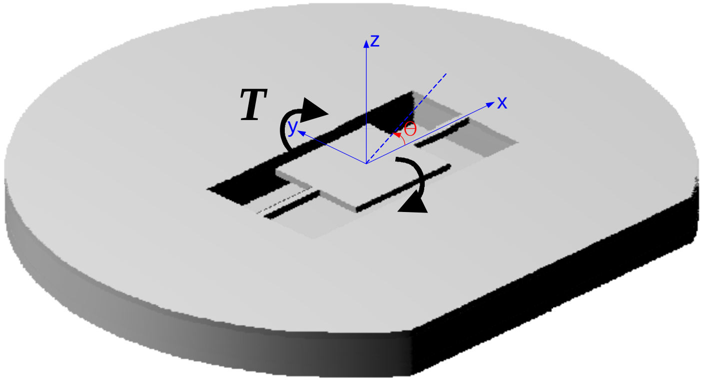


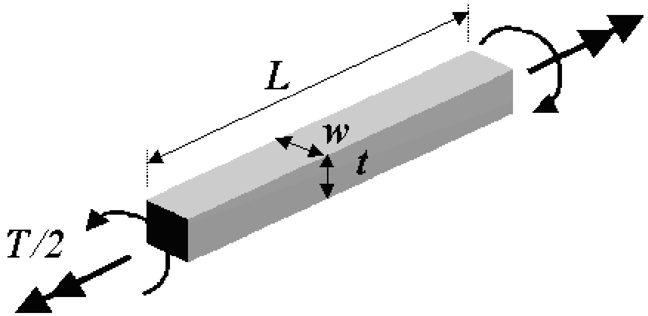
Silicon (111)

[Ref] J. Kim et al., Proceedings of Transducers 2001, pp. 662-665, Munich, Germany, June 2001.

Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (1)***
  – Schematic of torsional spring





✓ **The dimension splits of torsional springs**

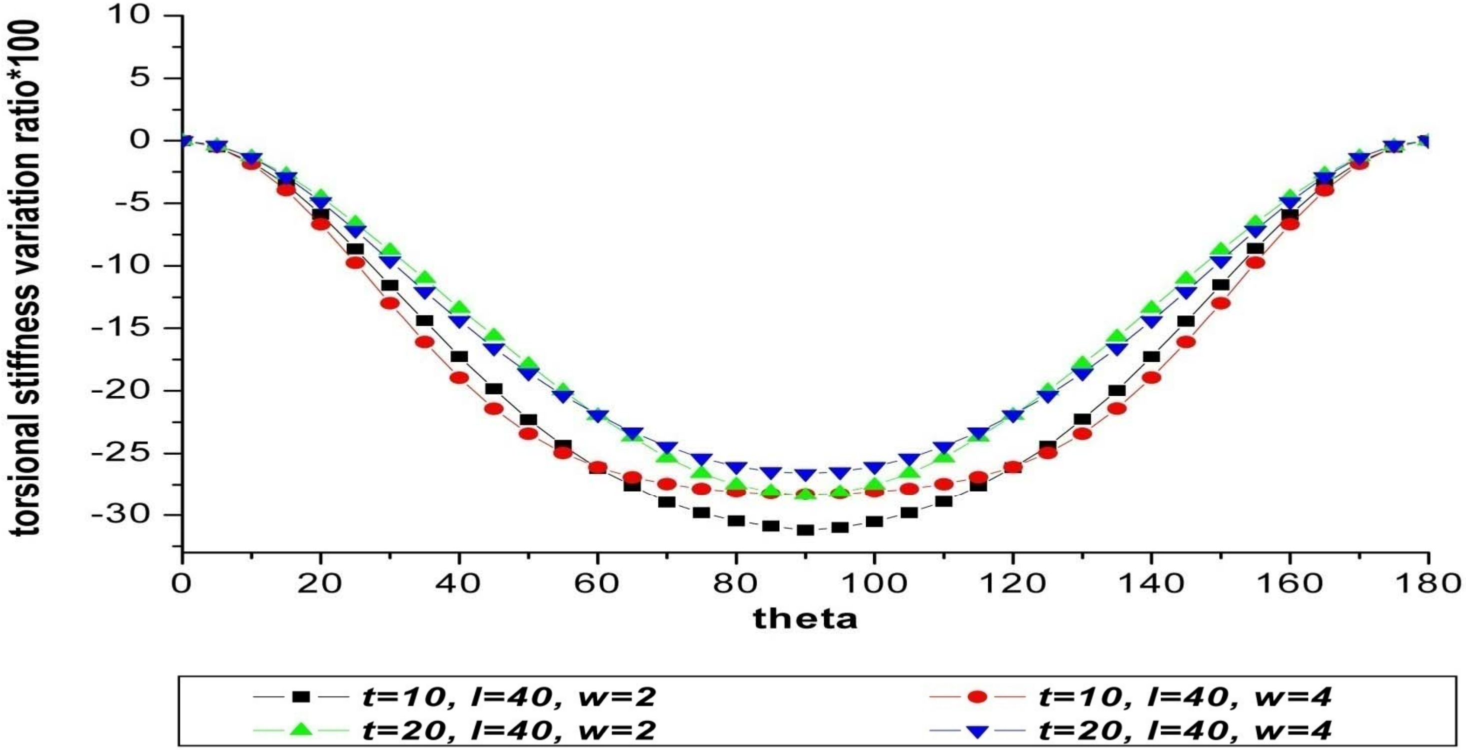
- Thickness(t) : 10 μm, 20 μm

- Length(l) : 40 μm

- Width(w) : 2 μm, 4 μm

Mechanical Property for Single Crystal Silicon (cont'd)

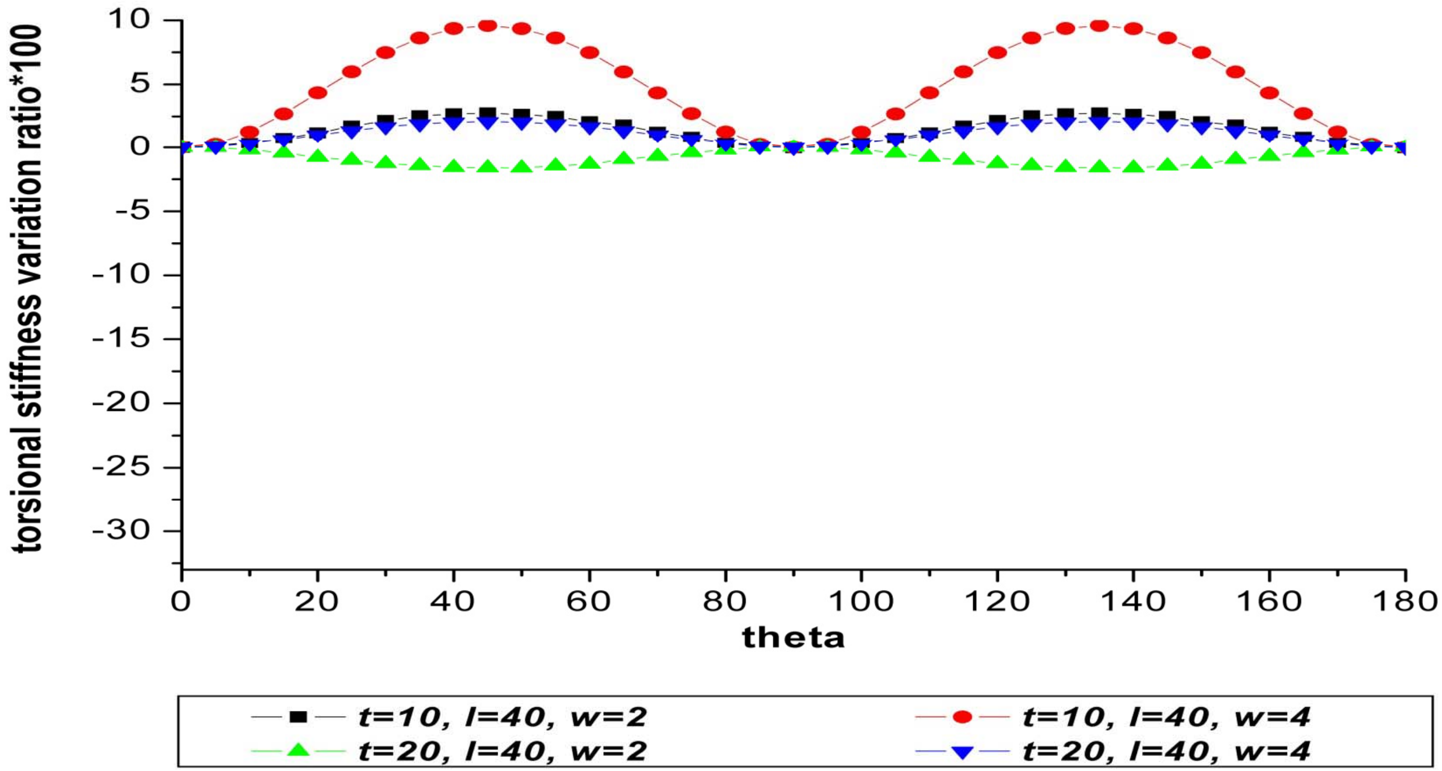
- ***Torsional stiffness (2)***
  - Silicon (110)



The torsional stiffness on silicon (110) varies from -31.2 % to -26.6 %

Mechanical Property for Single Crystal Silicon (cont'd)

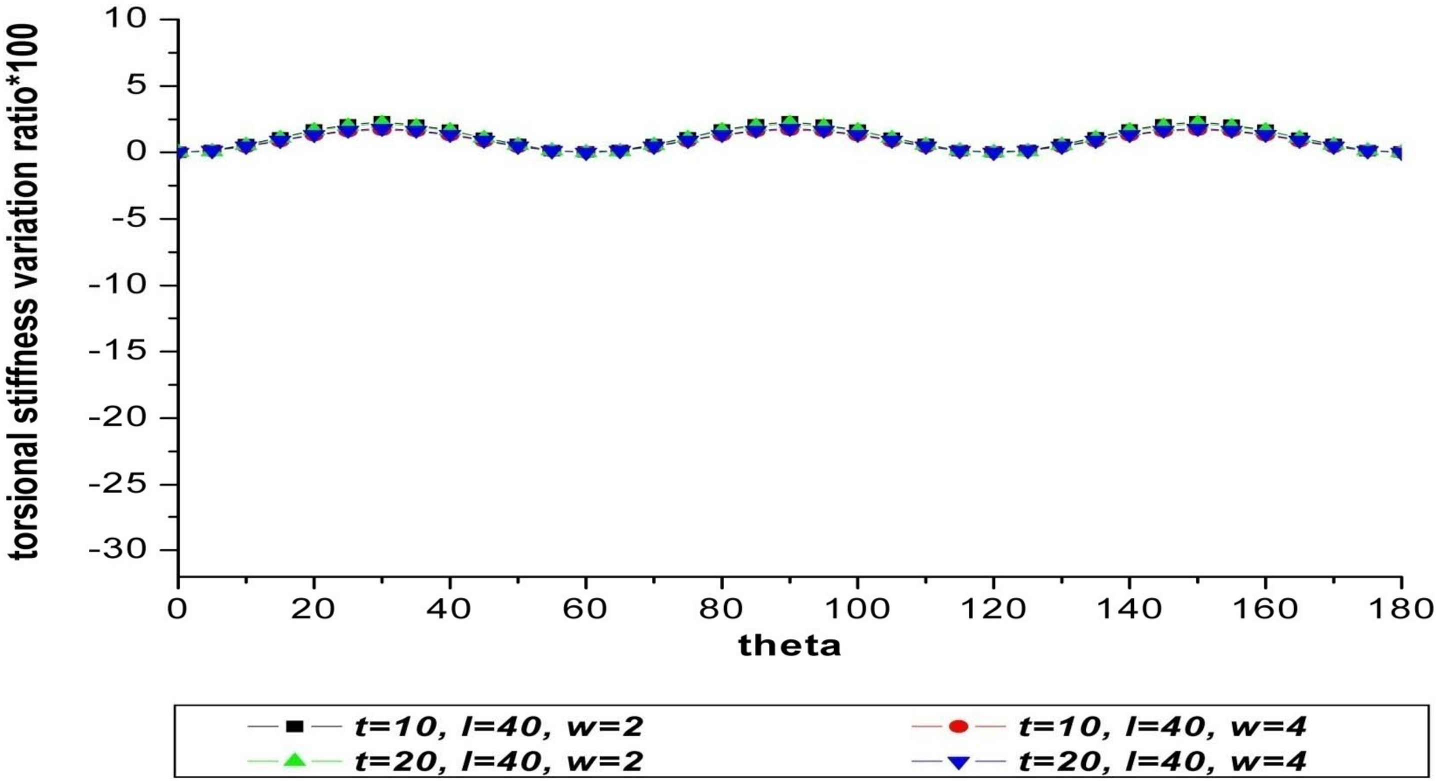
- ***Torsional stiffness (3)***
  - Silicon (100)



The torsional stiffness on silicon (100) varies from -1.6 % to 9.6 %.

Mechanical Property for Single Crystal Silicon (cont'd)

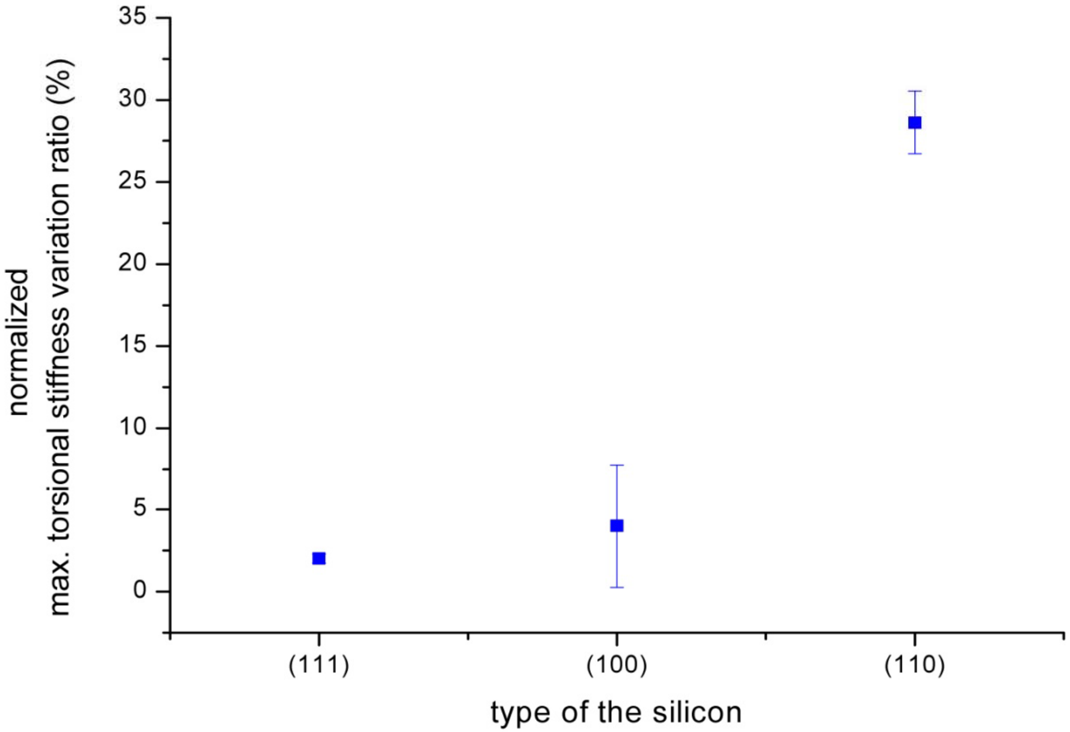
- ***Torsional stiffness (4)***
  - Silicon (111)



The torsional stiffness on silicon (111) varies from 1.7 % to 2.3 %.

Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (5)***
  - Summary for results



The average for the normalized maximum torsional stiffness variation ratio (%)

[Ref] D. Kwak et al., Proceedings of IMECE 2003, Washington D.C, USA, November 2003.

Electrostatic Forces

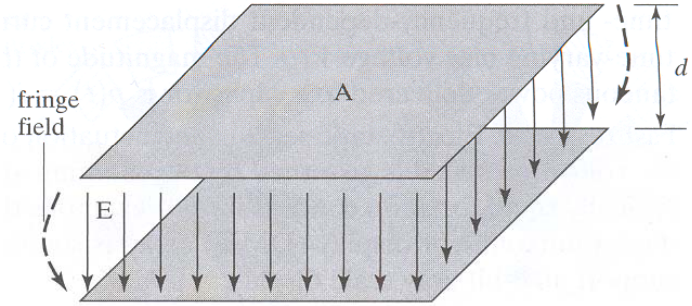
- ***Parallel Plate Capacitor:***
  - **Capacitance:**

$$C = Q/V = \varepsilon_0 \varepsilon_r A/d$$

ε_0; dielectric constant of free space

$(\approx 8.854\times10^{-12} F/m)$

ε_r; dielectric permittivity



  - **Stored energy:**

$$W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}\left(\rightarrow C = \frac{\varepsilon A}{d}\right)$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

  - **Electrostatic force between plates:**

$$F_x = -\frac{\partial W}{\partial x} = -\frac{1}{2}\frac{\varepsilon A}{d^2}V^2 = -\frac{1}{2}\frac{CV^2}{d}$$

Electrostatic Actuation

- ***Positioning of capacitor plate:***

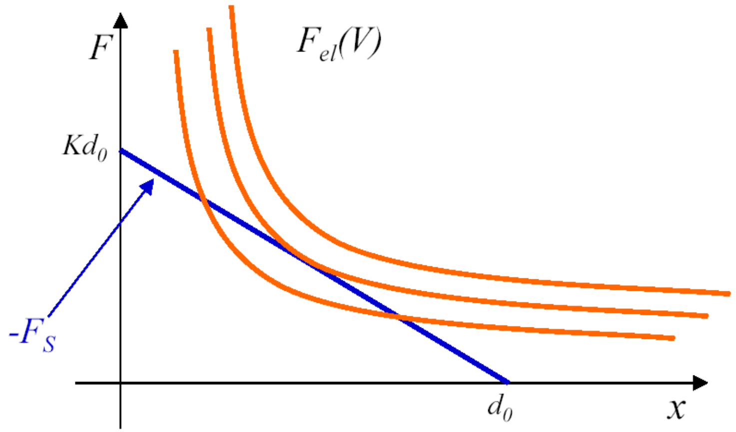
$$F_{el} = -\frac{1}{2}\varepsilon_0\varepsilon_r A V^2 / x^2 < 0$$

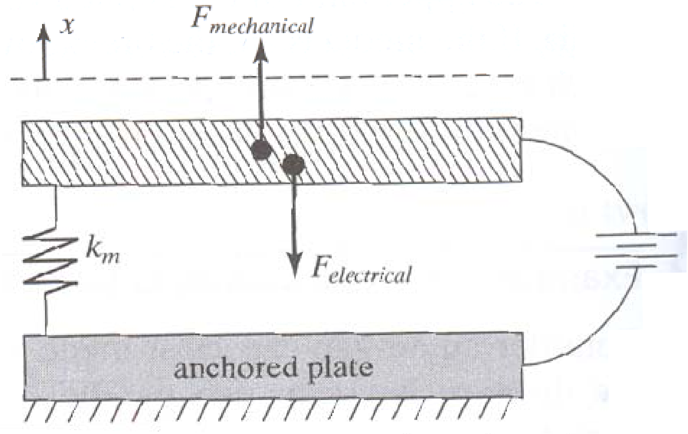
$$F_S = K(x - d_0) < 0$$

where d_0:distance at rest

(no applied voltage)

- ***Stable equilibrium when*** $F_{el} = F_S$





Pull-In Point

The higher *V*, the closer the plate is pulled in. $F_{el} \rightarrow \infty$ when *d* → 0. What is the closest stable distance x_{min}?

- ***F_{el} and F_S must be tangential:***

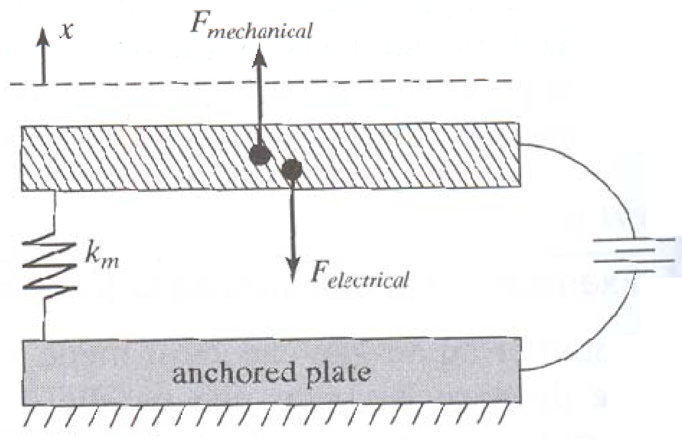
  $\varepsilon_0 \varepsilon_r A / x^3 V^2 = K$, so

  $V^2 = K x^3 / \varepsilon_0 \varepsilon_r A$

- ***Substitute into $F_{el}=F_S$ to get***

  $x_{\min} = 2/3 d_0$

  can control x only from $2/3d_0$ to d_0



Electrostatic Comb Drive

- ***Capacitance is approximately:***

$$C = \varepsilon_0 \varepsilon_r A / d$$
$$= 2n\varepsilon_0 \varepsilon_r lh / d$$

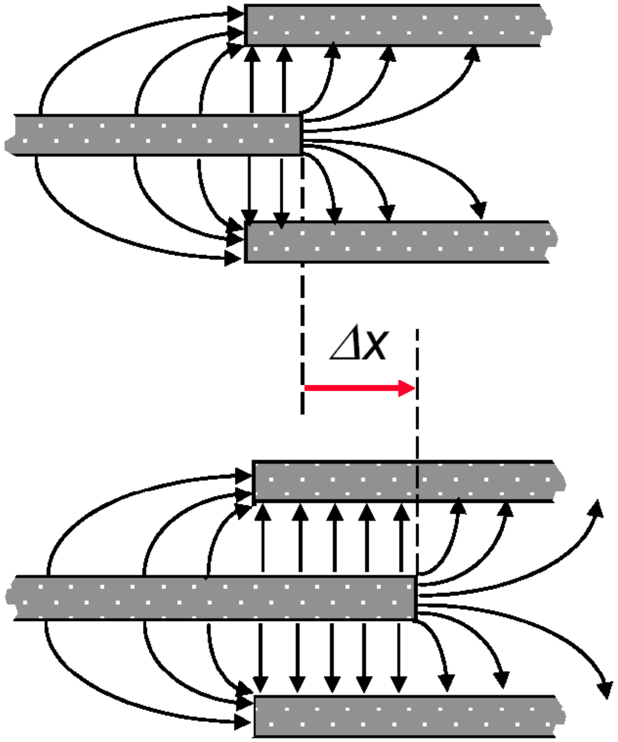
(L>>d)
(ignore fringing field effect)

- ***Change in capacitance when moving by △x :***

$$\Delta C = \varepsilon_0 \varepsilon_r \Delta A / d$$
$$= 2n\varepsilon_0 \varepsilon_r \Delta x h / d$$

- ***Electrostatic force :***

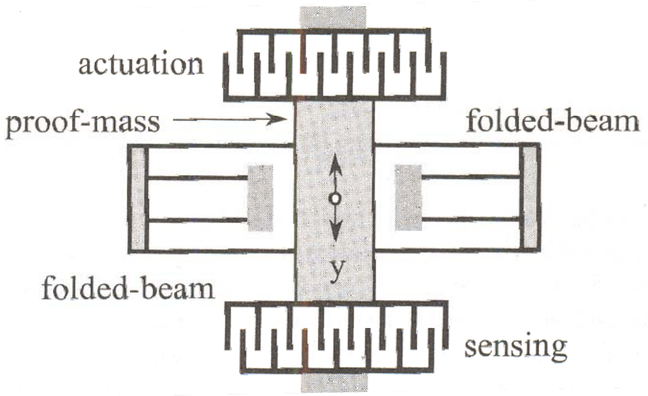
$$F_{el} = 1/2V^2 dC/dx = n\varepsilon_0 \varepsilon_r h / dV^2$$



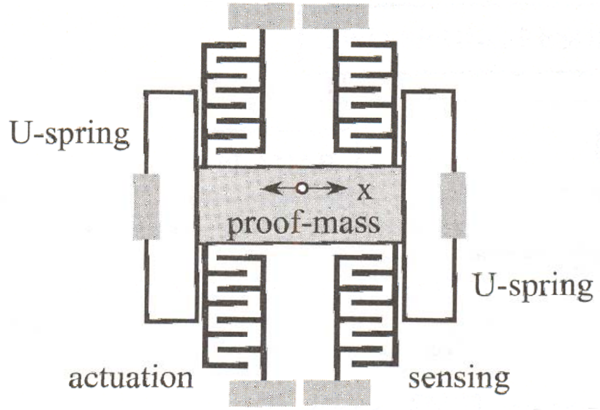
Electric field distribution in com-finger gaps

Note: F_{el} is independent of △x over wide range (fringing field), and quadratic in *V*.

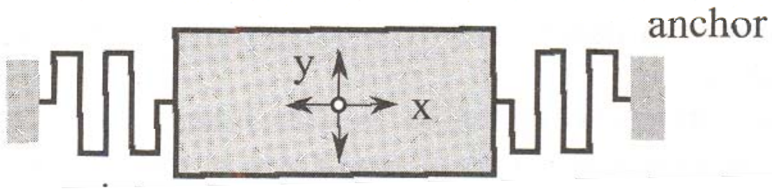
Comb Drive Design



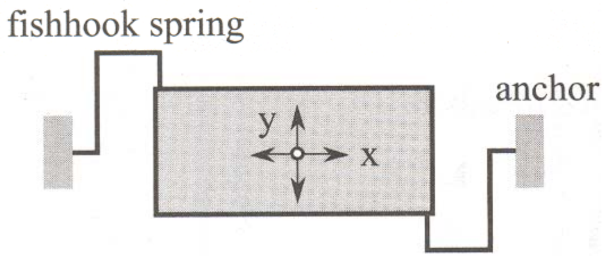
[Folded suspension type]



[U spring suspension type]

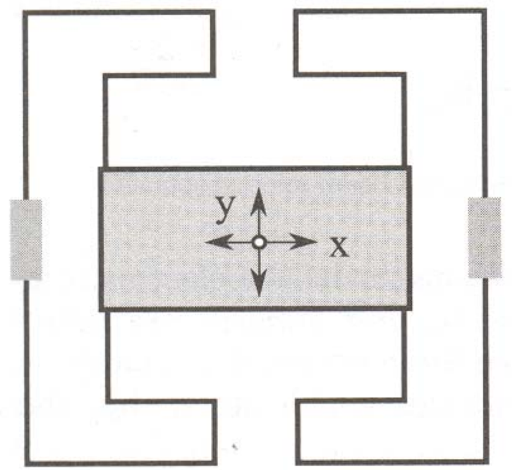


[Serpentine suspension type]

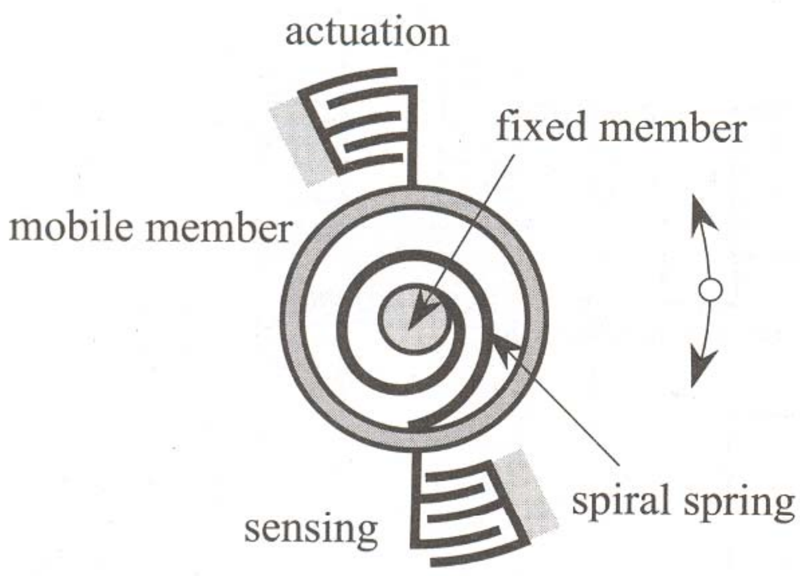


[Fishhook suspension type]

Comb Drive Design (cont'd)



[Bent beam serpentine suspension type]

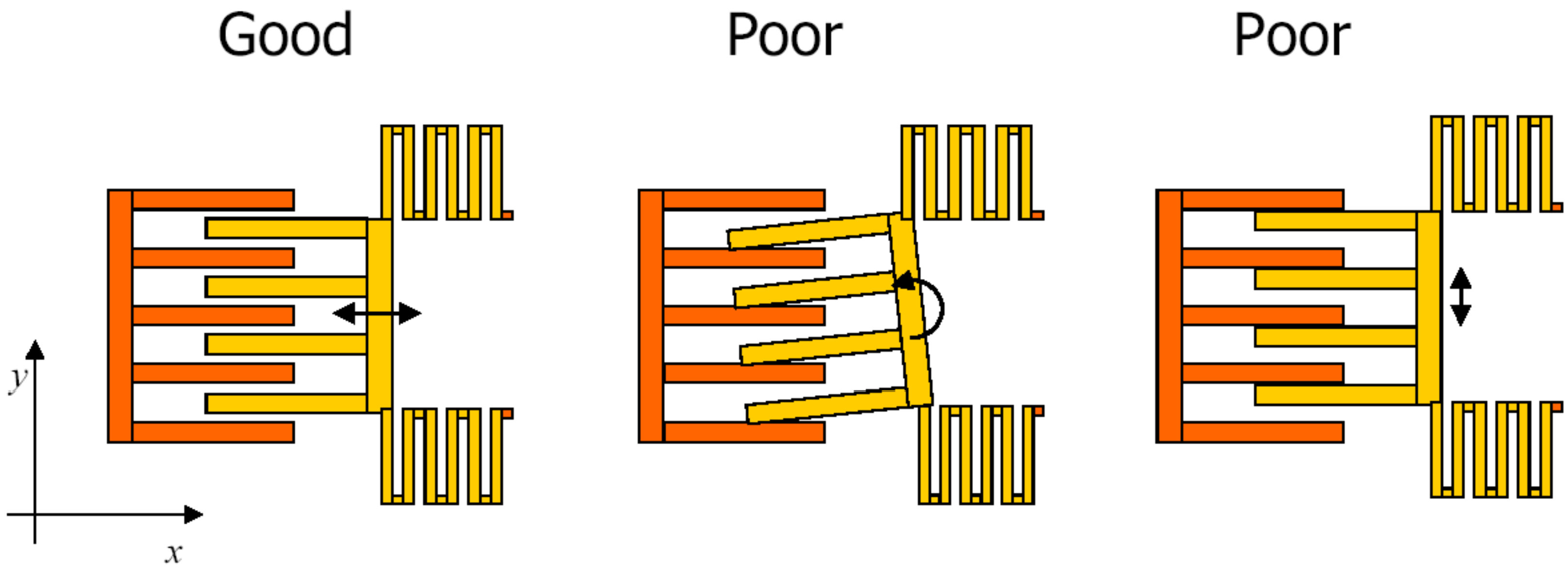


[Spiral spring suspension type]

Comb Drive Failure Modes

- ***Comb drives require low stiffness in x direction but high stiffness in y, z direction as well as rotations.***

Note: comb fingers are in unstable equilibrium with respect to the y direction.



Summary of Translation Motion

- ***Acceleration, velocity, distance***

$$a = \dot{v} = \ddot{x}$$

- ***Force, momentum***

$$F$$

$$p = mv = Ft$$

- ***Kinetic energy***

$$E = \frac{1}{2} mv^2$$

- ***Dynamic (spring, damper, mass)***

$$F = Kx + b\dot{x} + m\ddot{x}$$

- ***Oscillation (assume b=0)***

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad (Hz) \qquad \omega = \sqrt{\frac{K}{m}}$$

Summary of Rotation Motion

- ***Angular acceleration, angular velocity, angle***

$$\alpha = \dot{\omega} = \ddot{\phi}$$

- ***Torque, angular momentum***

$$\vec{T} = \vec{r} \times \vec{F}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

- ***Kinetic energy***

$$E = \frac{1}{2} I\omega^2$$

- ***Dynamic (moment of inertia)***

$$T = K\phi + \beta\dot{\phi} + I\ddot{\phi}$$

- ***Oscillation (assume b=0)***

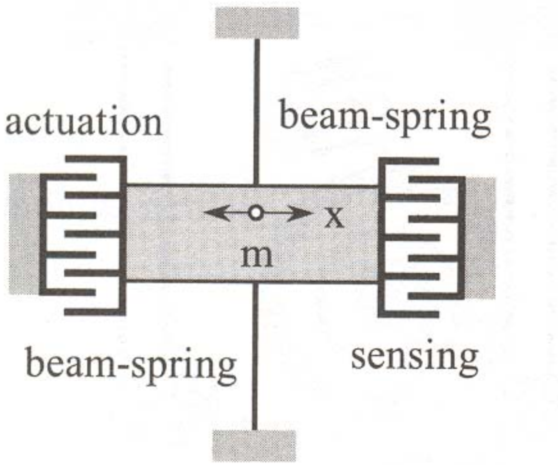
$$f = \frac{1}{2\pi}\sqrt{\frac{K}{I}}$$

Lumped-Parameter Model

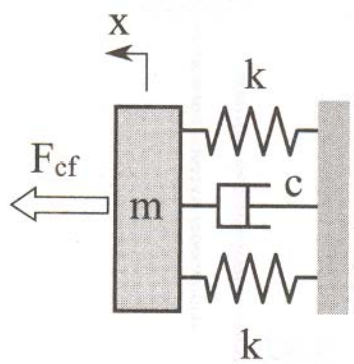
- The dynamic of motion is based on the lumped-parameter model
  - ***Input: external fore F, output: displacement x***

$$m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = F_0 \sin(wt)$$

$$where\ \ m : \text{mass},\ \ b : \text{damping},\ \ K : \text{stiffness}$$



Schematic microdevice

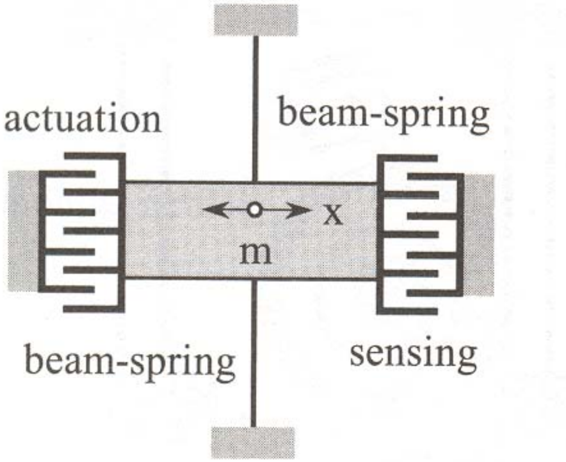


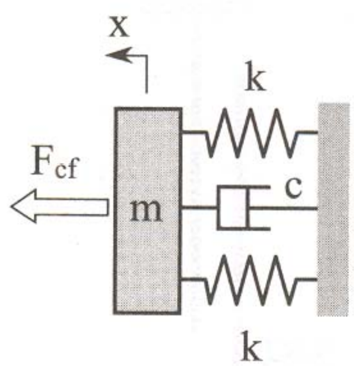
Lumped-parameter model

Lumped-Parameter Model (cont'd)

- ***Transfer function:***

$$H(s) = \frac{x}{F} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$





Schematic microdevice **Lumped-parameter model**

Resonators

- ***Analogy between mechanical and electrical system:***
  - Mass *m* – Inductivity *L*
  - Spring *K* – Capacitance *1/C*
  - Damping *b* – Resistance R (depending where R is placed in circuit)
- ***Solution to 2nd order differential equation:***

$$H(s) = \frac{\omega_0^{\,2}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^{\,2}} \quad \text{or} \quad \frac{\omega_0^{\,2}}{s^2 + 2\xi\omega_0 s + \omega_0^{\,2}}$$

where $\omega_0 = 2\pi f_o$: natural frequency

$$\omega_0 = \sqrt{\frac{K}{m}} \text{ : mechanical system,}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \text{ : electrical system}$$

Q: quality factor ($Q = 1/2\xi$, ξ : *damping ratio*)

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Mechanical Resonators

- ***Frequency and phase shift under damping:***

$$x(t) = Ae^{-t/2\tau}\cos(\omega_1 t + \varphi)$$

$where\ \ \tau = \dfrac{m}{b}$: damping time

$$\omega_1 = \omega_0\sqrt{1-\frac{1}{4\omega_0^{\,2}\tau^2}} = \omega_0\sqrt{1-\frac{b^2}{4Km}} = \omega_0\sqrt{1-\xi^2} = \omega_d$$

φ: phase shift

- ***Energy dissipation:*** $E(t) = E_0 e^{-t/\tau}$

Quality Factor

- ***Definition: Quality factor (Q factor)***
  - Ratio of stored energy and lost energy:

$$Q = 2\pi \frac{E}{|\Delta E|} = 2\pi \frac{\tau}{T} = \omega_0 \tau$$

  - Mechanical system: $Q = \omega_0 \dfrac{m}{b} = \dfrac{\sqrt{Km}}{b}$

  - Similar for electric systems: (a) $Q = \omega_0 \dfrac{L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

(b) $Q = \omega_0 RC = R\sqrt{\dfrac{C}{L}}$

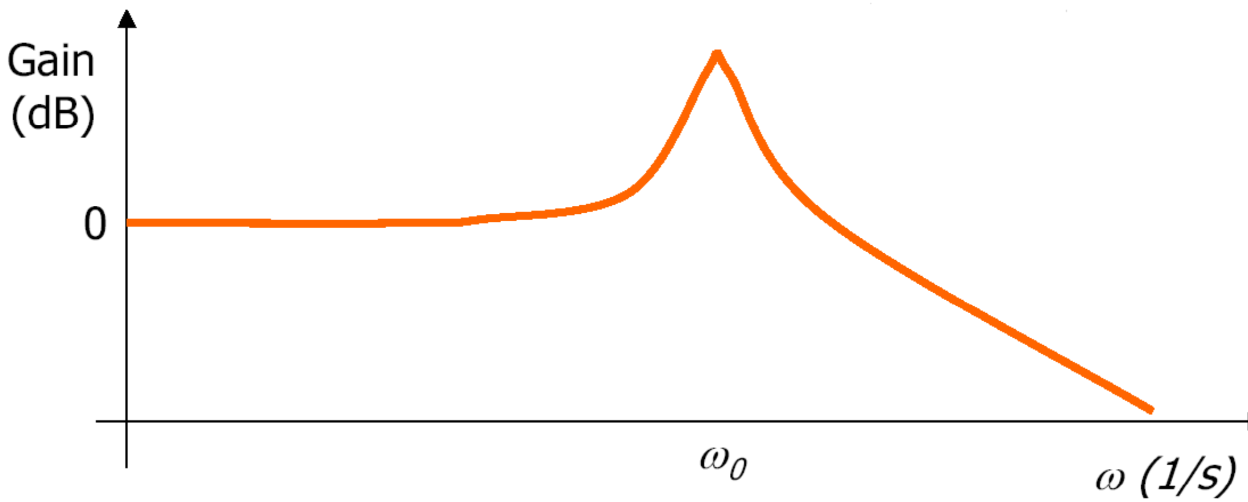
Quality Factor (cont'd)

- **How fast does energy dissipate?**

$$\tau = \frac{Q}{\omega_0} = \frac{m}{b} \quad \text{(mechanical)}$$

- **What is the maximum amplitude for a given frequency?**

  : At resonance, amplitude is Q times the DC response



Summary: Mechanical/Electrical Resonator

- **Mechanical resonator :**

  $m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = 0$

  m : mass, b: damping, K: stiffness

  natural frequency $\omega_0 = \sqrt{K/m}$ (for small b)

- **Torsional resonator :**

  $I\ddot{\theta}(t) + b\dot{\theta}(t) + k\theta(t) = 0$

  I : moment of inertia, b: damping, k: stiffness

  natural frequency $\omega_0 = \sqrt{k/I}$

- **Electrical resonator :**

  $L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = 0$

  L : inductivity, R: resistance, C: capacitance

  natural frequency $\omega_0 = \sqrt{1/LC}$

Measurement

V_P : DC bias

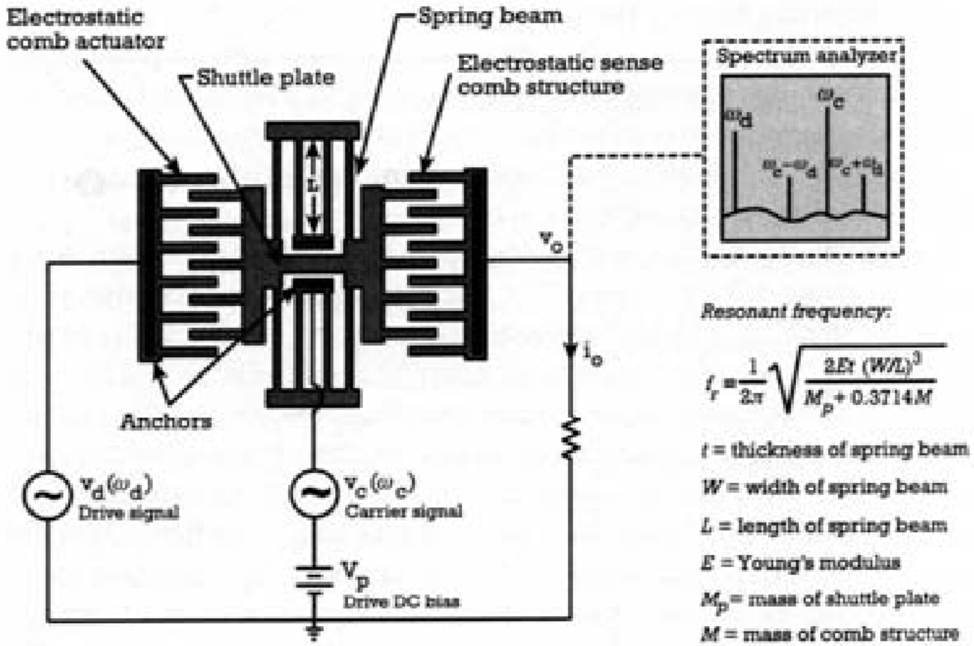
V_d : drive signal at ω_d

I_0 : output current $\propto V_P dC / dt$

Feedback

via transimpedance amplifier

V_c carrier signal at ω_c

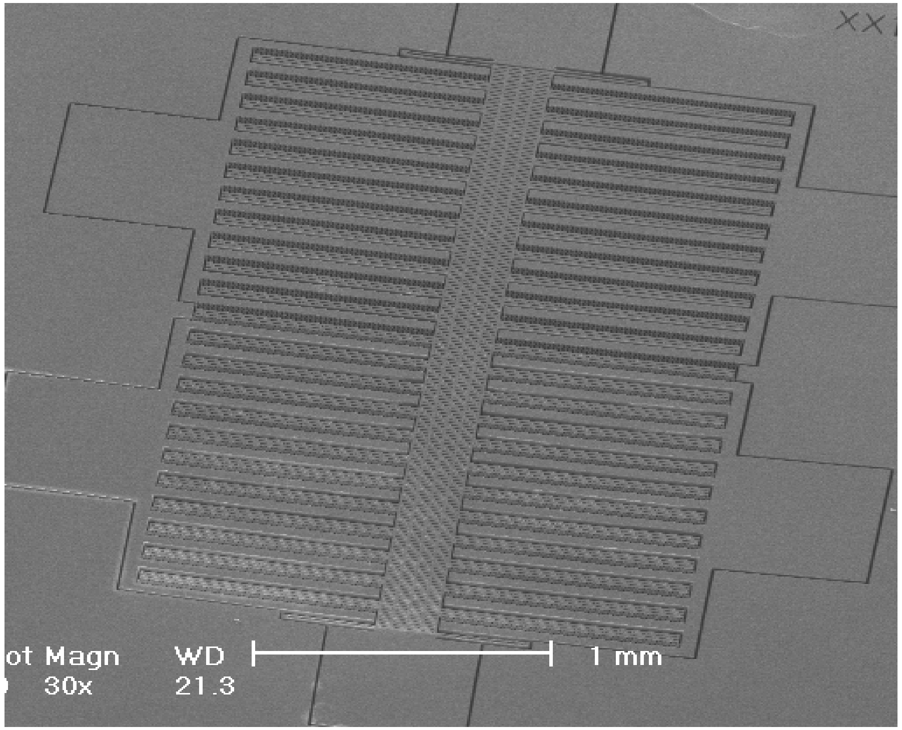


Output signal:

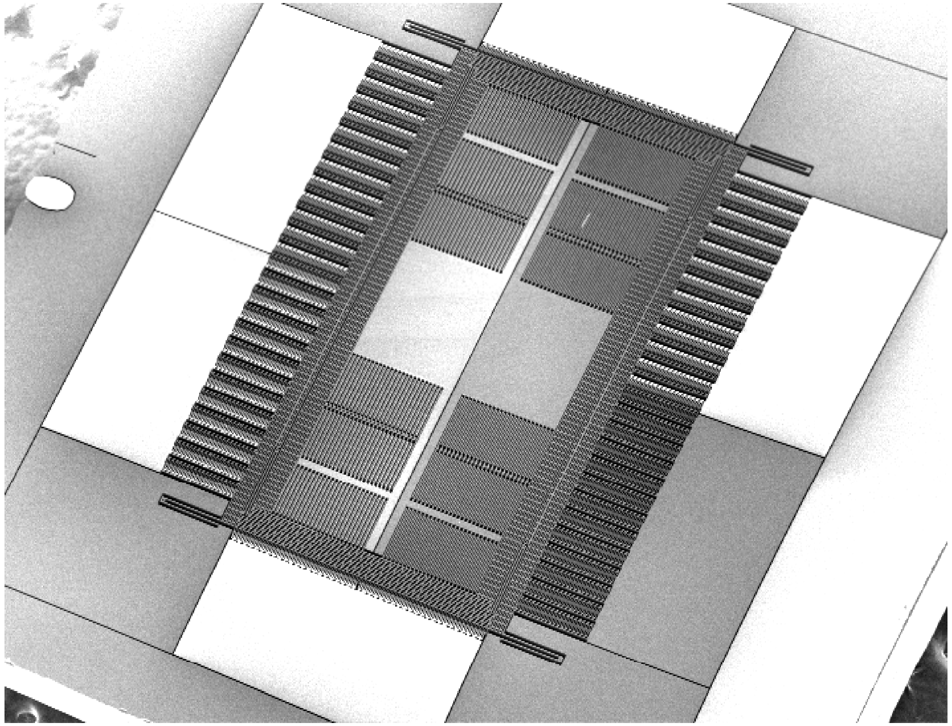
frequency spectrum includes ω_d, ω_c, but also $\omega_c \pm \omega_d$

basis for frequency transfer

Application of Comb Drive

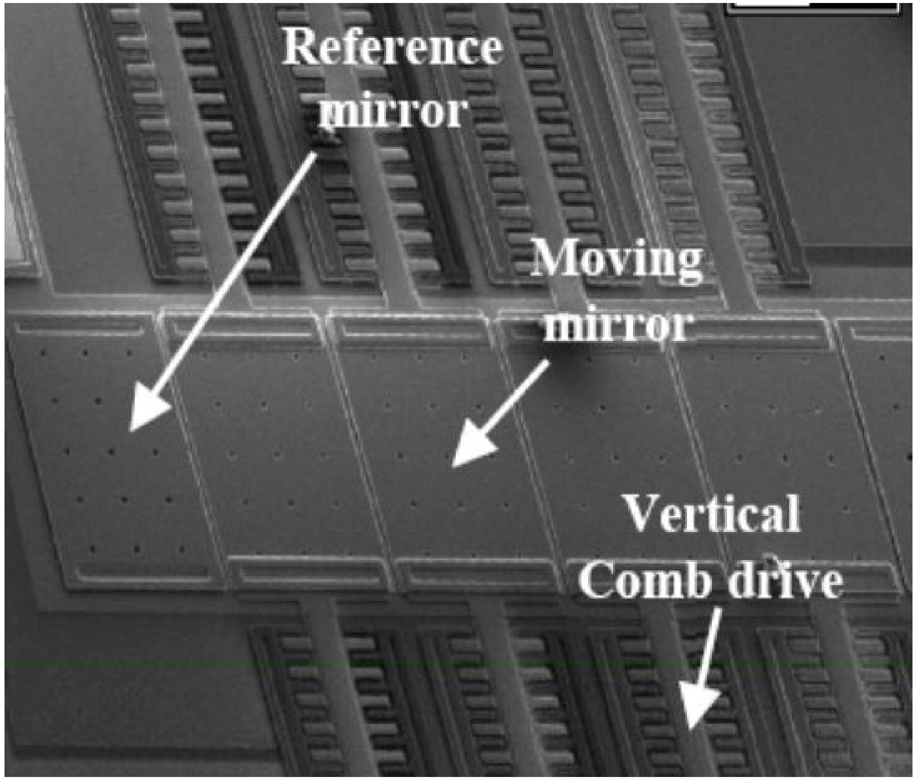


Accelerometer (NML SNU)

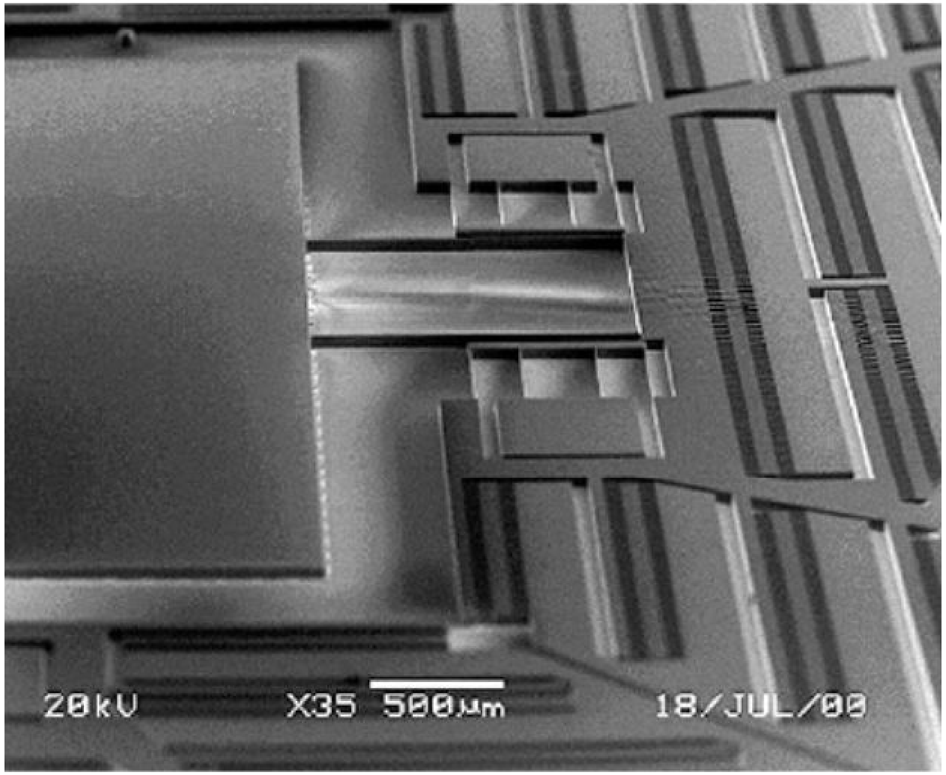


Gyroscope (NML SNU)

Application of Comb Drive (cont'd)



Micromirror (UC Davis & Stanford)



x/y stage (MiSA SNU)

Reference

- Lee, S. W., Cho, C. H., Kim, J. P., Park, S. J., Yi, S. W., Kim, J. J., and Cho, D. I., "The Effects of Post-deposition Processes on Polysilicon Young's Modulus, IOP Journal of Micromechanics and Microengineering, vol. 8, no. 4, pp. 330-337, Dec. 1998.
- Kim, J. P., Cho, D. I., and Muller, R. S., "Why is (111) Silicon a Better Mechanical Material for MEMS," Proceedings of Transducers 2001: 11th International Conference on Solid State Sensors and Actuators, pp. 662-665, Munich, Germany, June 10-14, 2001.
- Kwak, D. H., Kim, J. P., Park, S. J., Ko, H. H., and Cho, D. I., "Why is (111) Silicon a better Mechanical Material for MEMS: Torsion Case," 2003 ASME International Mechanical Engineering Congress (IMECE 2003), Washington D.C., USA, Nov. 15-21, 2003.
- Lee, S. W., Park, S. J., Yi, S. W., Lee, S. C., Cho, D. I., Ha, B. J., Oh, Y. S., and Song, C. M., "Electrostatic Actuation of Surface/Bulk Micromachined Single-crystal Silicon Microresonators," Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, vol. 2, pp. 1057-1062, Kyeongju, Korea, Oct. 17-21, 1999.
- Ko, H. H., Kim, J. P., Park, S. J., Kwak, D. H., Song, T. Y., Setiadi, D., Carr, W., Buss, J., and Cho, D. I., "A High-performance X/Y-axis Microaccelerometer Fabricated on SOI Wafer without Footing Using the Sacrificial Bulk Micromachining (SBM) Process," 2003 International Conference on Control, Automation and Systems (ICCAS 2003), pp. 2187-2191, Gyeongju, Korea, Oct. 22-25, 2003.
- Kim, C. H., Jeong, H. M., Jeon, J. U., Kim, Y. K., “Silicon micro XY-stage with a large area shuttle and no-etching holes for SPM-based data storage”, Journal of Microelectromechanical Systems, vol. 12 , issue 4 , pp. 470 - 478, Aug 2003.
- Chang Liu, “Foundations of MEMS”, Pearson, 2006.
- Nicolae O. Lobontiu, “Mechanical design of microresonators”, McGraw-Hill, 2006.