Heat and Mass Transfer

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23

HEAT TRANSFER WITH TURBULENT FLOW

Heat Transfer with turbulent flow

- Forced-convection heat transfer to a fluid flowing in turbulent motion in a pipe may be the commonest heat transfer system in industry.

- Although forced convection may be associated with laminar flow and natural convection with turbulent flow, these are cases of secondary importance.

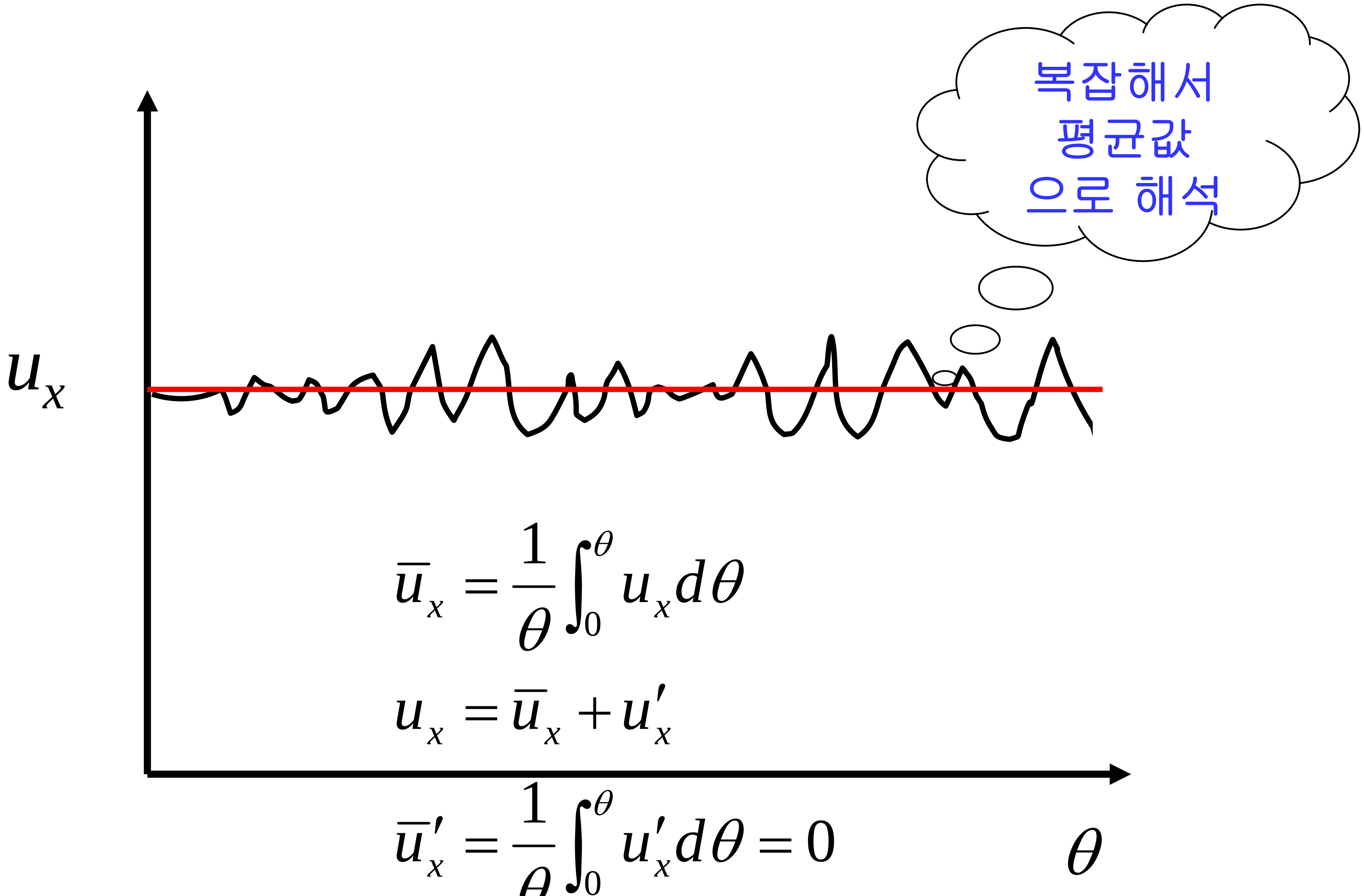
- Heat-transfer coefficients are higher with turbulent flow than with laminar flow, and heat-transfer equipment is usually designed to take advantage of this fact.

Heat Transfer with turbulent flow

- The use of the Navier-Stokes equation in the analysis of isothermal turbulent flow is complicated because of the fluctuations in the velocity motion. The use of the differential energy equation balance in the analysis of nonisothermal turbulent flow is difficult for the same reason.

- Heat is transferred in most turbulent streams by the movement of numerous macroscopic elements of fluid (eddies) between regions at different temperatures.

The fluctuations in velocity components of turbulent flow



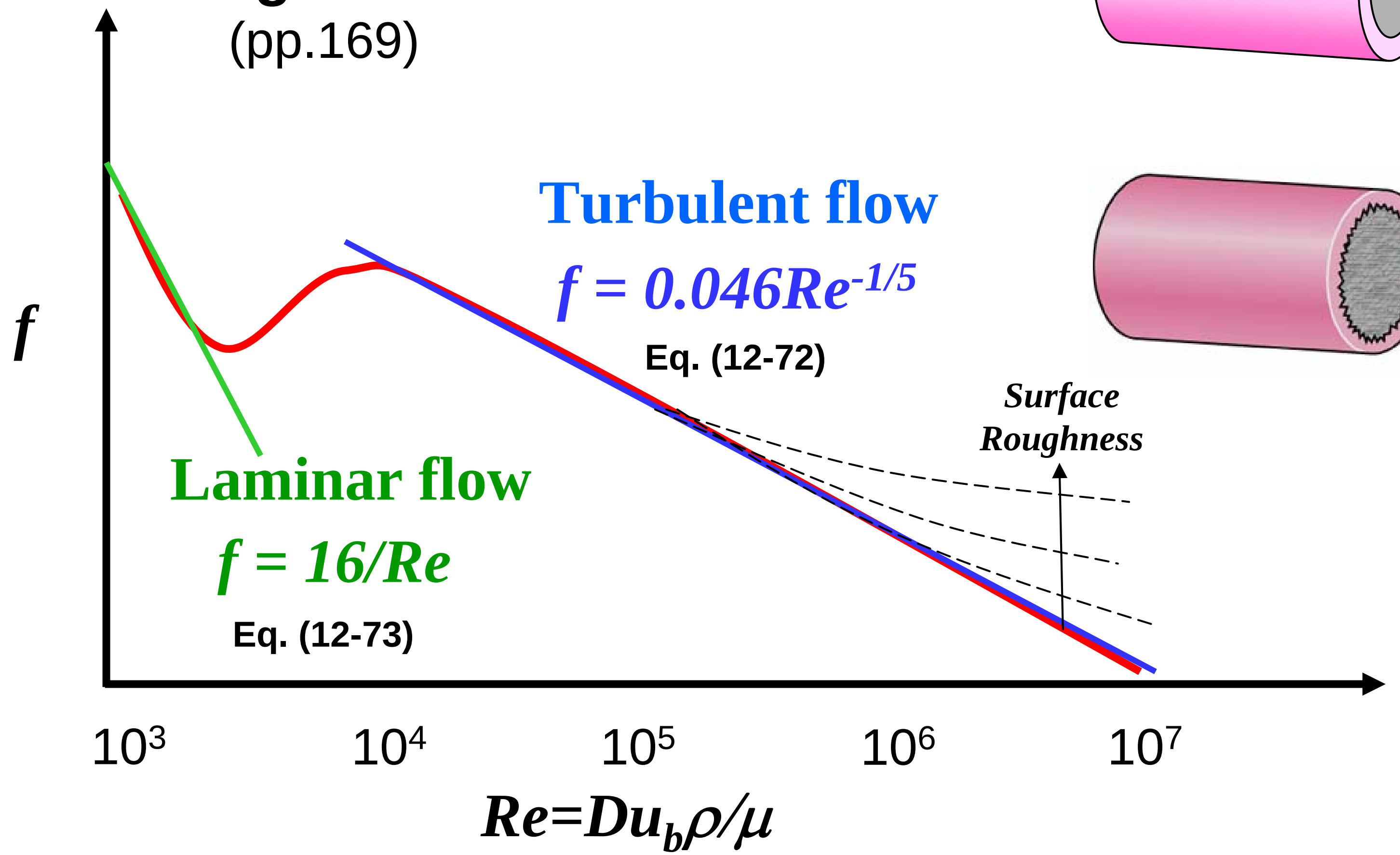
$$\bar{u}_x = \frac{1}{\theta}\int_0^{\theta} u_x \, d\theta$$

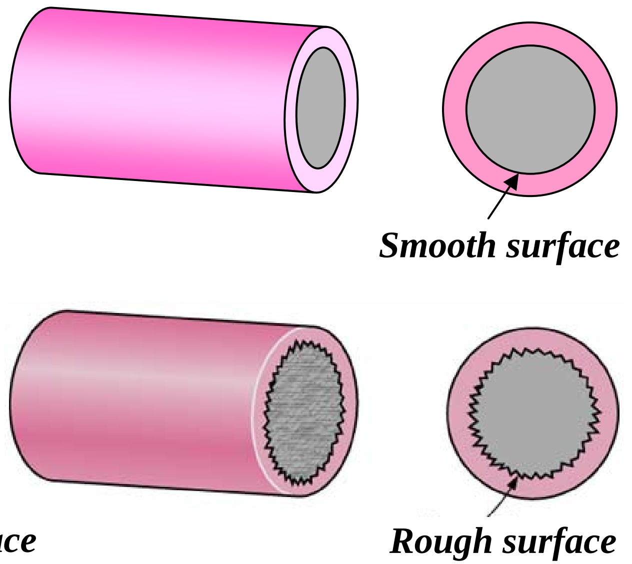
$$u_x = \bar{u}_x + u'_x$$

$$\bar{u}'_x = \frac{1}{\theta}\int_0^{\theta} u'_x \, d\theta = 0$$

Friction factors for pipe flow

Fig. 12-8
(pp.169)



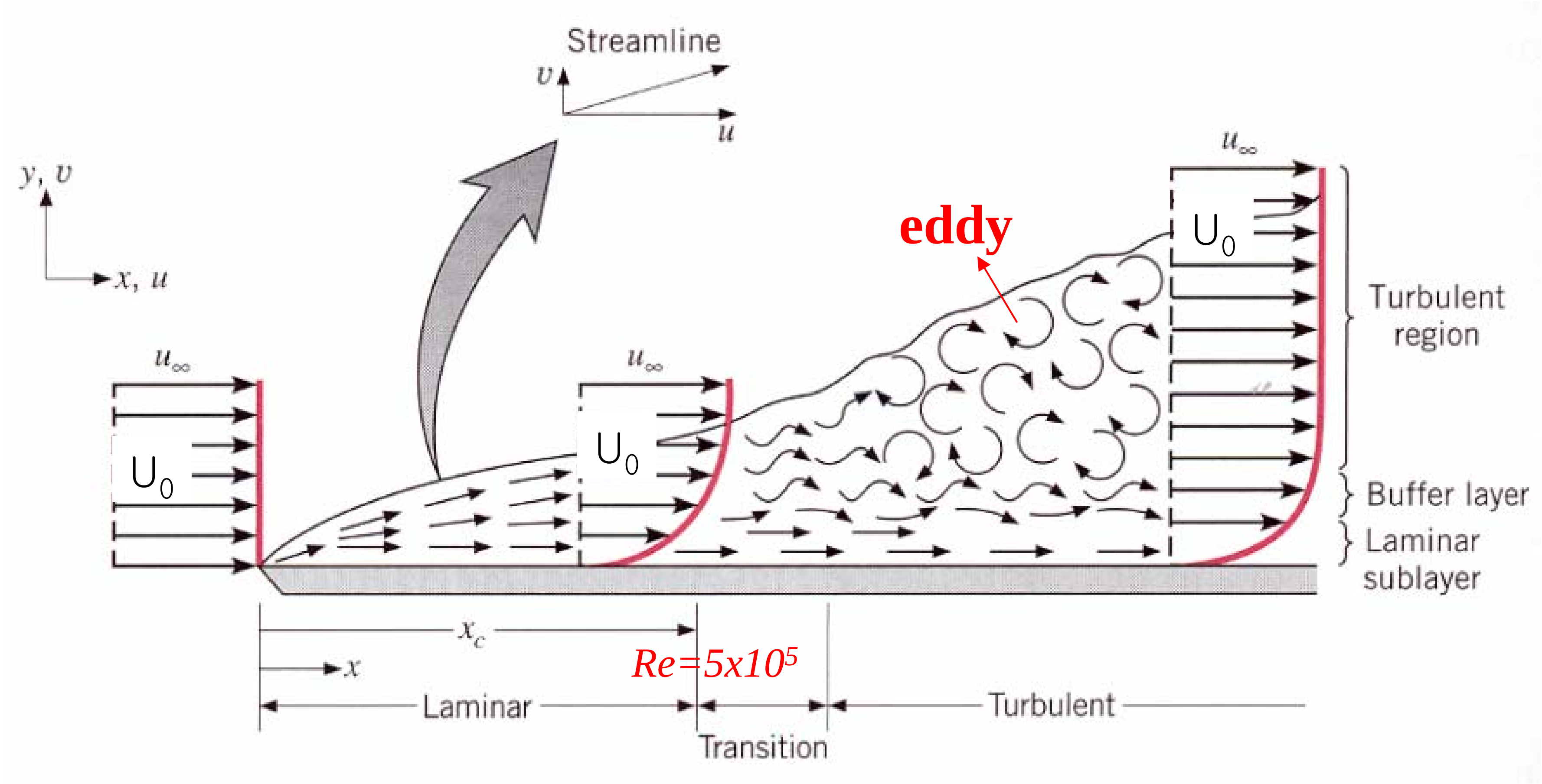


Heat Transfer with turbulent flow

- We cannot predict the behavior of these eddies with time, but even if we could, the expressions describing this behavior of these eddies would probably be so complicated that the combined solution of the equations of energy and momentum would not be possible. Nevertheless, answers must be found to these problems.

- In this chapter (Chap 23), we consider some theoretical developments in turbulent heat transfer which are use to the engineer, and in the next chapter (Chap 24) we shall examine some design equations. The theory tells us why the design equations work and what some of their limitations are.

Heat Transfer with turbulent flow



Heat transfer coefficients are higher with turbulent flow than with laminar flow, and heat-transfer equipment is usually designed to take advantage of this fact.

Entrance Effects

- Entrance effect in turbulent flow is significant at L/D<60.

- Once this entrance region has been passed, the heat transfer coefficients in developed turbulent flow remain essentially constant.

Entrance effects in a pipe

- Numerous possible combinations of thermal and hydrodynamic entrance conditions exist. We assume that the fluid enters with a uniform temperature and that the pipe wall is at some uniform temperature higher than that of the entering fluid.

- Certain flow conditions at the entrance will be considered, and their effects on the local heat-transfer coefficients deduced qualitatively from the knowledge we have gained thus far of hydrodynamic and heat-transfer theory.

Entrance effects in a pipe

1. The fluid enters the pipe with a uniform velocity profile at a rate such that **Re**<2100. Under these conditions a laminar boundary layer will build up, starting from the leading edge, until it fills the pipe at some distance down-stream.

The heat transfer coefficient will be infinite at the inlet but will continue to diminish even after the point of developed flow has been reached.

Entrance effects in a pipe

2. The fluid enters the pipe in laminar flow with a uniform velocity profile at a rate such that **Re**>2100. This condition can be achieved with a rounded entrance. At the leading edge a laminar boundary layer in a manner already described in Chap. 11 with reference to flow past a flat plate.

This turbulent boundary layer increases in thickness with increasing downstream distance until it fills the pipe with a turbulent core and a laminar sublayer at the wall. Downstream from this point the system is identical in all respects with the system that would develop if the flow had been turbulent from the entrance.

Entrance effects in a pipe

This flow behavior is reflected in the values of the local heat-transfer coefficient, which decrease from infinity at the inlet to some minimum value at the critical point where the laminar boundary layer changes into a turbulent boundary layer.

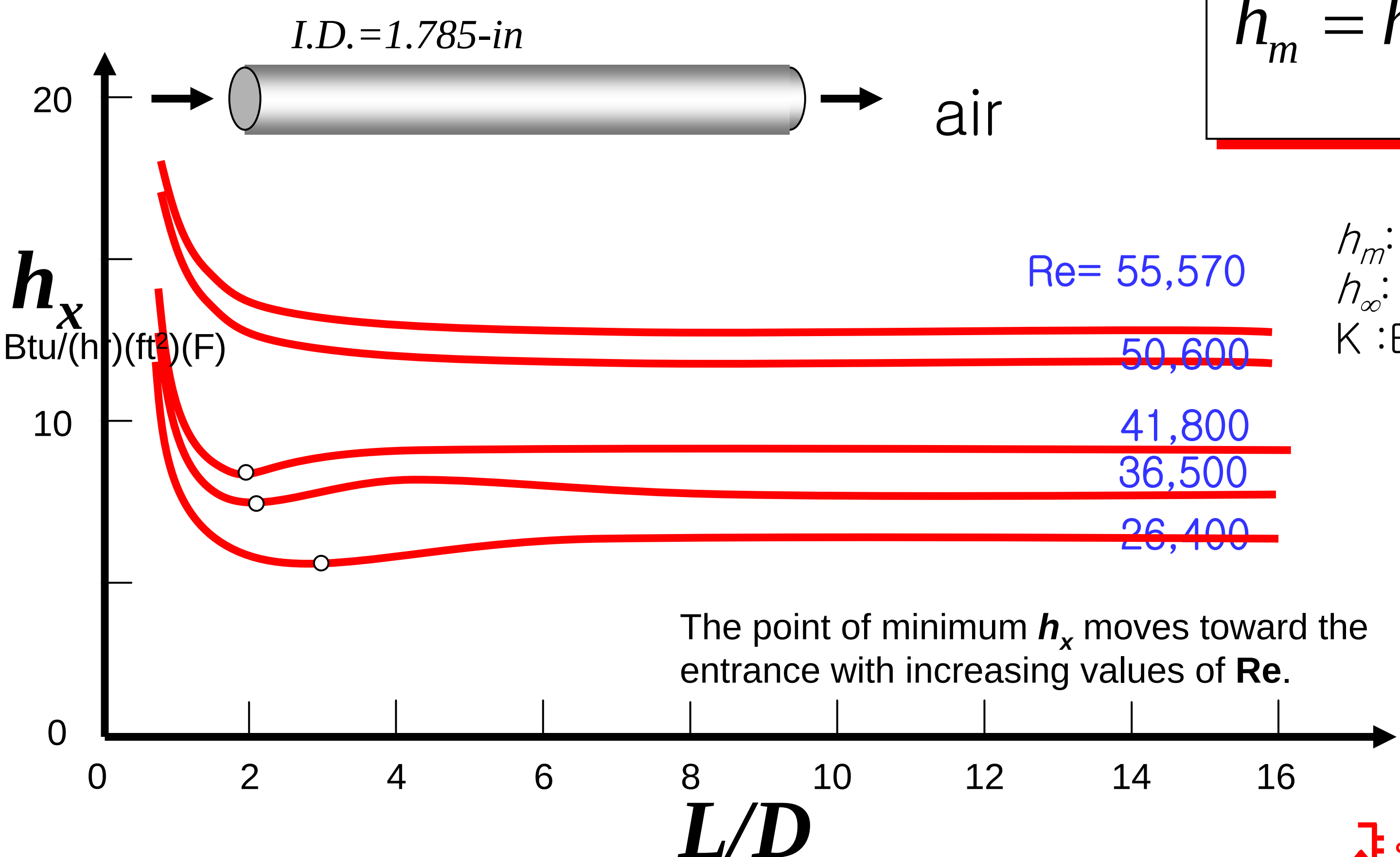
Near this point the heat-transfer coefficient increases in magnitude for a short distance, but then continues to decrease downstream until the turbulent boundary layers meet at the center of the pipe.

Experimental values of h_x are shown in Fig. 23-1 for air flowing into a tube with a bellmouth entrance. The point of minimum h_x moves toward the entrance with increasing values of **Re**.

Local coefficients of heat transfer near the inlet with a bellmouth entrance

L.M.K. Boelter, G. Young, and H.W. Iverson (1948)

Fig. 23-1
(pp.357)



Experimental correlation

$$h_m = h_\infty\left(1 + \frac{K}{L/D}\right)$$

h_m: Average coefficient
h_∞: Average coefficient
K :Entrance condition

The point of minimum h_x moves toward the entrance with increasing values of **Re**.

Entrance effects in a pipe

3. The fluid enters the pipe in a state of turbulence with a nonuniform velocity profile at a rate such that **Re**>2100. The velocity profile at the entrance may be caused by the presence of a sudden contration or a pipe bend immediately upstream. A thermal boundary layer will build up, starting from the beginning of the heated length, and will fill the pipe at some downstream point.

Entrance effects in a pipe

However, this entrance length for nonuniform turbulent flow usually does not have any significant effect on the local heat-transfer coefficients beyond 10 pipe diameters, whereas entrance effects in laminar flow often persist for 50 or more pipe diameters.

The local heat-transfer coefficient at the beginning of the heated length is infinite in turbulent flow because of the temperature discontinuity, just as it is for laminar flow. However, it quickly falls to some constant value, as described above.

Local coefficients of heat transfer near the inlet with a bellmouth entrance

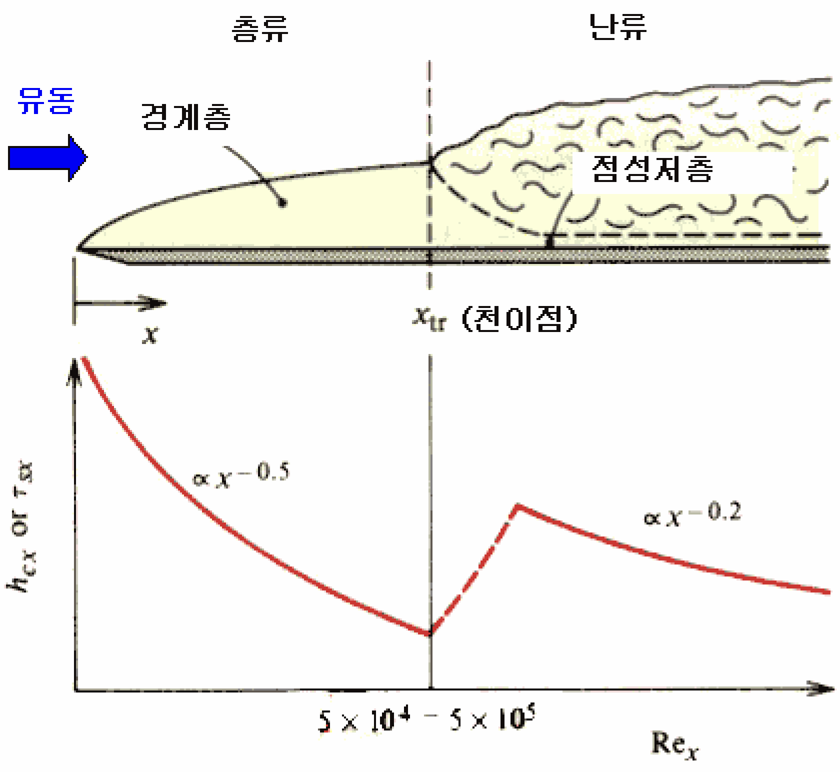
$$h_m = h_\infty \left(1 + \frac{K}{L/D}\right)$$

For L/D>5

Type of entrance	K
Bellmouth	0.7
Bellmouth with one screen	1.2
Short calming section (L/D=2.8) with sharp-edged entrance	~3
Long calming section (L/D=11.2) with sharp-edged entrance	1.4
45° angle-bend entrance	~5
90° angle-bend entrance	~7
1-in square-edge orifice, located 1 in upstream from entrance	~16
1.4-in square-edge orifice, located 1 in upstream from entrance	~7

Boelter, Young, and Iverson (1948)

Analogy between Momentum Transfer and Heat Transfer



평판 위를 지나는 강제대류 유동에서의 열전달계수 및 벽마찰계수 분포

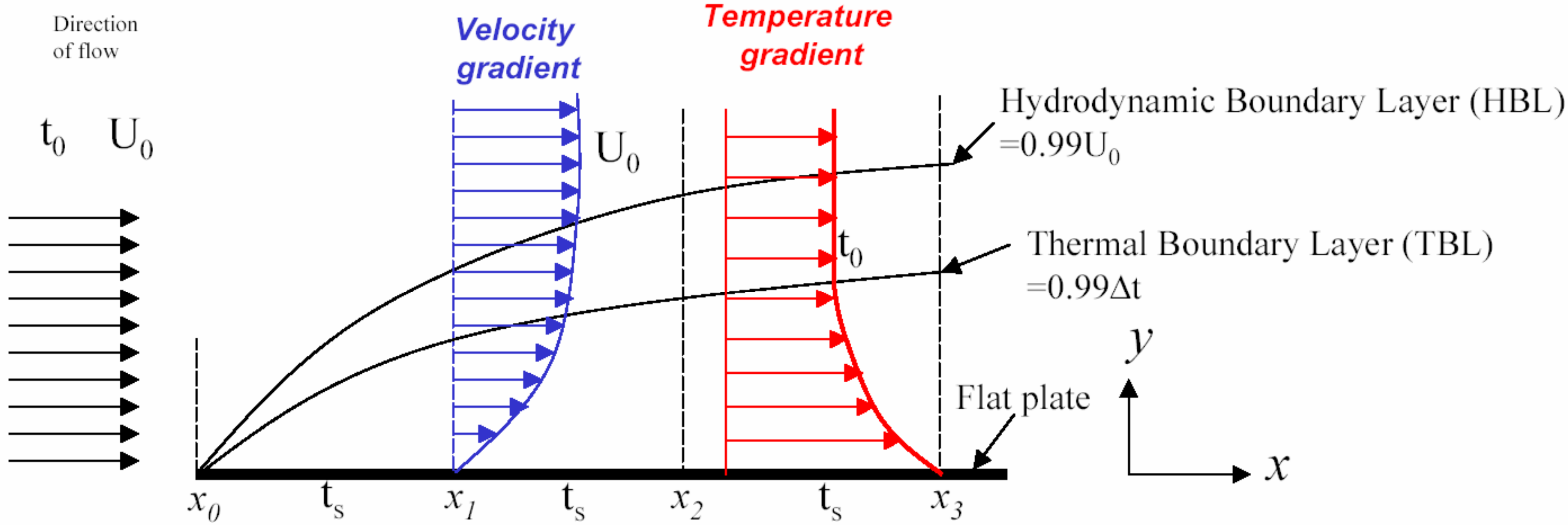
왼편 그림과 같이 유체가 평판을 만나면 경계층(boundary layer)이 발달하기 시작하여 일정한 거리를 지나면 층류경계층이 난류경계층으로 전이(transition)하게 된다. 이에 따라 **대류열전달계수(h_c)와 벽마찰(τ_s)은 왼편 그림과 같이 유사한 경향을 가지게 된다.** 평판의 시작점에서 매우 크게 나타나는 열전달계수와 벽마찰은 층류경계층이 발달함에 따라 점차 감소하다가, 경계층의 불안정성은 유동의 천이를 야기하게 된다. 천이점에서 대류열전달과 벽마찰은 갑자기 증가했다가 난류경계층으로 접어들면서 다시 유동방향으로 점차 감소하게 된다. 이 때 난류경계층의 난동성분과 혼합의 증진으로 인해 층류경계층에서 보다는 대류열전달과 벽마찰이 완만하게 감소하게 된다.

$$\boxed{\frac{\delta_H}{\delta_{th}} = \mathrm{Pr}^{1/3}}$$

valid for ***Pr > 0.06***

Blasius flow (laminar flow)

$$\mathrm{Re}_x = \frac{\rho u_0 x}{\mu} \le 5\times 10^5 \text{ (Laminar flow)}$$



$$\mathrm{Pr} = \frac{\nu}{\alpha} > 1 \cdots \mathrm{HBL} > \mathrm{TBL}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha} < 1 \cdots \mathrm{HBL} < \mathrm{TBL}$$

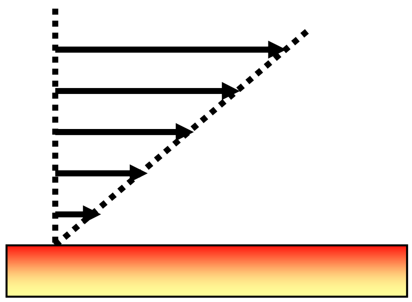
$$\mathrm{Pr} : \text{water} = 6.5,\ \text{air} = 0.7,\ \text{Hg} = 0.025$$

$$\text{water } (350^{\circ}\,\mathrm{F}) = 1$$

Analogy between Momentum Transfer and Heat Transfer

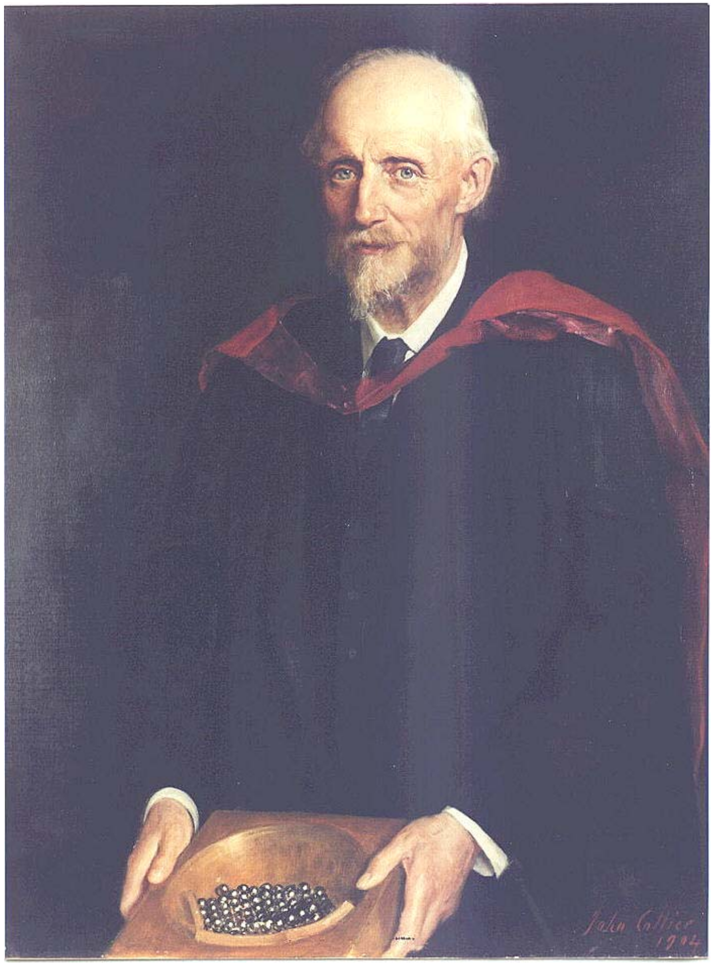
$$\tau_{yx} = g_c \mu \frac{\partial u_x}{\partial y}$$

$$q = -k \frac{\partial t}{\partial x}$$



The heat transfer coefficient (***h***) can be correlated by friction factor (***f***)

The similarity of heat and momentum transport



Osborne Reynolds

In 1874, Osborne Reynolds noted that the similarity of heat, momentum, and mass transport.

His work has led to useful, simple equations relating the friction factor, the heat-transfer coefficient, and the mass-transfer coefficient.

History of the Analogy theory

Osborne Reynolds(1874)

-similarity of heat and momentum transport

Prandtl (1910) and Taylor (1916)

-improvement (Prandtl-Taylor equation)

Murphree (1932) and von Kármán(1939)

- extended further

Reichardt (1940)
Boelter, Martinelli, and Jonassen (1941)
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Deissler (1954)- variation of viscosity with temperature

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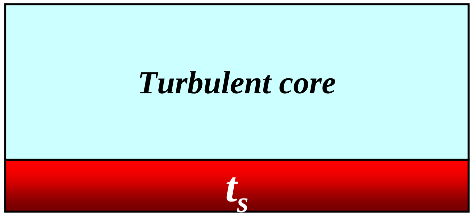
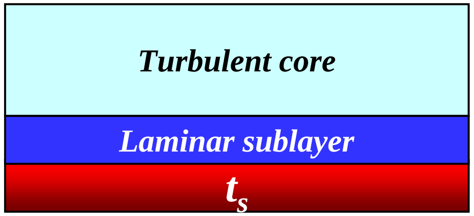
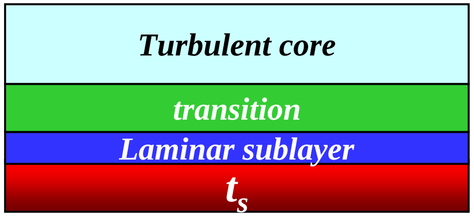
Reynolds analogy



Prandtl-Taylor



von Kármán



Reynolds analogy

Osborne Reynolds stated that in the transport of heat or momentum between a fluid and a solid surface, two mechanisms contributed to the transport process:

1. The natural internal diffusion of the fluid when at rest
2. The eddies caused by visible motion which mixes the fluid up and continually brings fresh particles into contact with the surface.

The first cause is dependent on the nature of the fluid. The second cause is a function of the velocity of the fluid past the surface.

Reynolds analogy

Combination of two causes led to a heat transfer equation

$$H = At + B\rho vt$$

where t = different in temperature between surface and fluid

ρ = density

υ = bulk velocity

A and B = constants

H = heat transmitted

per unit of area of surface per unit of time

The resistance R to motion offered by friction in the fluid

$$R = A'v + B'\rho v^2$$

A' and *B'* = constants

Reynolds analogy

$$H = At + B\rho vt$$

$$R = A'v + B'\rho v^2$$

Various considerations, which Reynolds did not name specifically, led to the supposition that **A** and **B** were proportional to **A'** and **B'**.

Reynolds wrote as ***Bρv*** is equivalent to ***h*** and ***B'*** is proportional to ***f***.

h was proportional to ***f***.

Reynolds analogy

The ***proportionality*** of heat and momentum transfer can be stated in terms of four quantities, which we shall define with reference to a fluid at a bulk temperature t_b flowing through a pipe and losing heat to the wall of the pipe, which is at temperature t_s. The four quantities are:

(a) Heat flux (fluid →wall): $h(t_b - t_s)$ Btu/(h)(ft^2)
(b) Momentum flux at wall: $\tau_s g_c$ (lb)(ft/h)/(h)(ft^2)
(c) Heat transfer parallel to wall: $wC_p(t_b - t_s)$ Btu/(h)
(d) Momentum transfer parallel to wall: wu_p (lb)(ft/h)/(h)

proportionality ⇨ $$\frac{(a)}{(c)} = \frac{(b)}{(d)}$$

Reynolds analogy

$$\frac{\text{The heat flux from the fluid to the pipe wall}}{\text{The heat trans fer rate parallel to the pipe wall}}$$

$$= \frac{\text{The momentum flux at the wall}}{\text{The momentum transport paralle to the pipe wall}}$$

$$\frac{h(t_b - t_s)}{wC_p(t_b - t_s)} = \frac{\tau_s g_c}{wu_b}$$

$$\tau_s g_c = \frac{fu_b^2 \rho}{2} \qquad \textbf{(12-67)}$$

$$h = \frac{(\tau_s g_c)C_p}{u_b} = \frac{\left(fu_b^2 \rho / 2\right)C_p}{u_b} = \frac{fu_b \rho C_p}{2} \qquad \textbf{(23-4)}$$

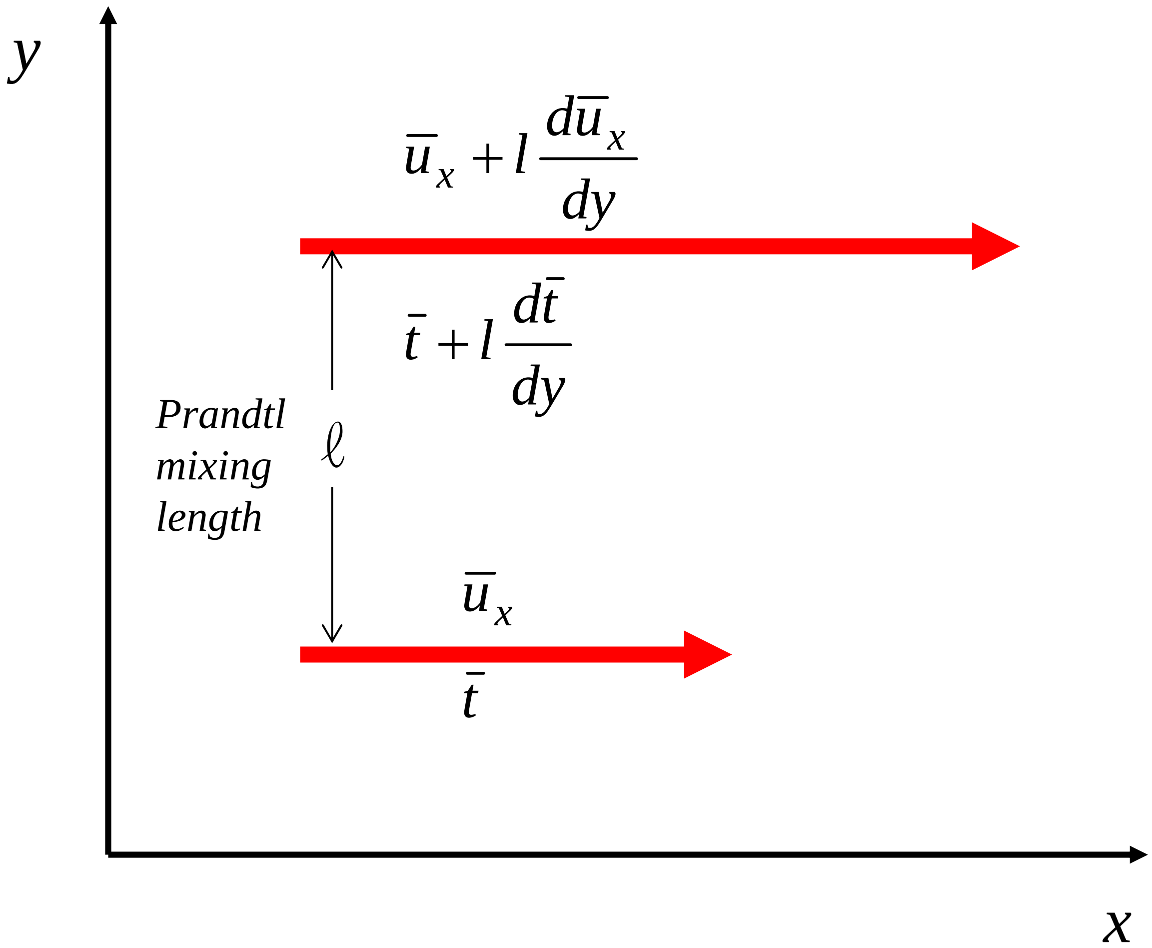
Reynolds analogy

Based on intuition

$$\boxed{h = \frac{f u_b \rho C_p}{2}} \qquad \textbf{(23-4)}$$

Though not actually derived by Reynolds,
the above equation is known as the Reynolds analogy

Eddy thermal diffusivity and mixing length



Velocities and temperatures are shown at two planes separated by a distance equal to the Prandtl mixing length ℓ.

Fluid is assumed to be transported between the planes with a velocity equal to the time-average magnitude of the fluctuating velocity component $\left|u'_y\right|$.

Energy is transported with the packets of fluid at a rate per unit area which equals the mass flux times the product of specific heat and temperature difference.

$$\left|u'_y\right| \rho C_p \left[\ell\left(d\bar{t}\,/\,dy\right)\right]$$

Eddy thermal diffusivity and mixing length

On turbulent flow, total shear stress $\bar{\tau}_{xy}{}^{t}$ is related to $\dfrac{d\bar{u}_x}{dy}$

$$\bar{\tau}_{xy}{}^{t} = \frac{\rho}{g_c}(\nu + \nu_e)\frac{d\bar{u}_x}{dy}$$

$$\nu_e = l^2 \left| \frac{d\bar{u}_x}{dy} \right|$$

Eddy kinematic viscosity

Prandtl mixing length

Eddy thermal diffusivity and mixing length

The fluctuating velocity component at a point u'_x and u'_y had the same time-average magnitude, and a relation was given for these quantities in terms of the Prandtl mixing length:

$$\left|\overline{u'_y}\right| = \left|\overline{u'_x}\right| = \ell \left|\frac{d\,\overline{u}_x}{dy}\right| \qquad (12\text{-}40)$$

Therefore the heat flux due to the turbulent motion of the fluid becomes

$$\left|u'_y\right| \rho C_p \left[\ell\left(d\bar{t}/dy\right)\right] = \rho C_p \underbrace{\ell^2 \left|d\overline{u}_x/dy\right|}_{\nu_e =} d\bar{t}/dy$$

Eddy
kinematic
viscosity

Eddy thermal diffusivity and mixing length

On turbulent flow, total shear stress $\bar{\tau}_{xy}^{\,t}$ is related to $\dfrac{d\bar{u}_x}{dy}$

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$$\nu_e = l^2\left|\frac{d\bar{u}_x}{dy}\right|$$

Eddy kinematic viscosity

Prandtl mixing length

$$\frac{q}{A} = -\rho C_p(\alpha + \alpha_e)\frac{d\bar{t}}{dy}$$

$$\alpha_e = l^2\left|\frac{d\bar{u}_x}{dy}\right|$$

Eddy thermal diffusivity

$$\alpha_e = \nu_e$$

Eddy thermal diffusivity and mixing length

The fluid is cooled in a pipe with heat being transferred radially. The distance y is measured from the wall,

$$\frac{q}{A} = \rho C_p (\alpha + \alpha_e) \frac{d\bar{t}}{dy}$$

$$\tau g_c = \rho(\nu + \nu_e) \frac{du}{dy}$$

If the molecular transport coefficients α and υ are assumed to be negligible when compared with the turbulent transport coefficients, dy can be expressed as followings:

$$dy = \frac{\rho C_p \alpha_e dt}{q / A} = \frac{\rho \nu_e du}{\tau g_c}$$

Eddy thermal diffusivity and mixing length

Assume that $\alpha_e = \nu_e$

$$du = \frac{C_p \tau g_c}{q/A} dt \qquad \textbf{(23-10)}$$

$$\frac{\tau}{\tau_s} = 1 - \frac{y}{r_i}; \qquad \frac{q/A}{(q/A)_s} = 1 - \frac{y}{r_i} \qquad \textbf{(23-11)}$$

From (23-10) and (23-11),

$$\frac{\tau g_c}{q/A} = const$$ For all radial positions.

Eddy thermal diffusivity and mixing length

Integration yields

$$du = \frac{C_p \tau g_c}{q/A} dt$$

$$u_b - \cancelto{0}{u_s} = \frac{C_p \tau_s g_c}{(q/A)_s}(t_b - t_s)$$

$$\tau_s g_c = \frac{f u_b^2 \rho}{2}$$

$$\left(\frac{q}{A}\right)_s = h(t_b - t_s)$$

$$u_b = C_p \frac{f u_b^2 \rho}{2} \frac{(t_b - t_s)}{h(t_b - t_s)} = C_p \frac{f u_b^2 \rho}{2} \qquad \textbf{(23-12)}$$

Reynolds analogy

$$h = \frac{f u_b \rho C_p}{2} \qquad \textbf{(23-4)}$$

One of the major assumption was α and υ are negligible compared with α_e and υ_e. The ratio υ/α is equal to the Prandtl number, so for fluid with **Pr**=1. Therefore, in this special case, $\upsilon + \upsilon_e = \alpha + \alpha_e$, and (23-4) can be obtained without disregarding the molecular transport terms. It is good for gases, **Pr**~1

$$Nu = \frac{f}{2} \mathrm{Re}\,\mathrm{Pr} \qquad \textbf{(23-28)}$$

History of the Analogy theory

Osborne Reynolds(1874)
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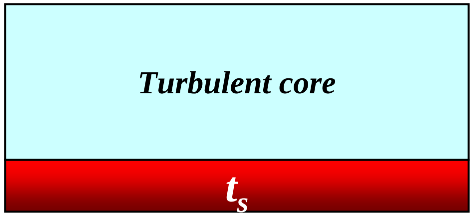
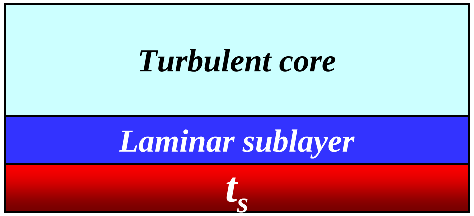
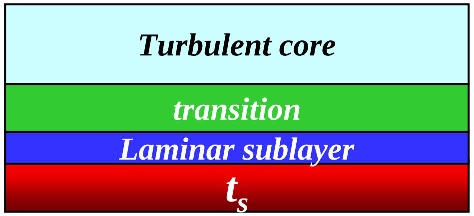
Reynolds analogy



Prandtl-Taylor



von Kármán



Analogy between Momentum Transfer and Heat Transfer

$$St = \frac{f}{2}$$

$$St = \frac{f/2}{1 + 5\sqrt{f/2} \cdot (\Pr - 1)}$$

$$St = \frac{f/2}{1 + 5\sqrt{f/2} \cdot \left\{ \Pr - 1 + \ln[1 + \frac{5}{6}(\Pr - 1)] \right\}}$$

$$St = \frac{f}{2}\Pr^{-2/3}$$

Reynolds Analogy: Pr=1

$$\frac{\text{The heat flux from the fluid to the pipe wall}}{\text{The heat transfer rate parallel to the pipe wall}}$$

$$= \frac{\text{The momentum flux at the wall}}{\text{The momentum transport paralle to the pipe wall}}$$

$$\frac{h(t_b - t_s)}{wC_p(t_b - t_s)} = \frac{\tau_s g_c}{wu_b}$$

$$h = \frac{(\tau_s g_c)C_p}{u_b} = \frac{\left(fu_b^2\rho/2\right)C_p}{u_b} = \frac{fu_b\rho C_p}{2}$$

Reynolds Analogy: Pr=1

$$\frac{d}{dy}\frac{u_x}{u_\infty}\bigg|_{y=0} = \frac{d}{dy}\left(\frac{t-t_s}{t_\infty - t_s}\right)\bigg|_{y=0} \quad \Leftarrow \quad \mathrm{Pr} = \frac{\mu C_p}{k} = 1, \quad \mu C_p = k$$

$$q = h(t_s - t_\infty) = -k\frac{\partial}{\partial x}(t - t_s)\big|_{y=0}$$

$$\mu C_p \frac{d}{dy}\left(\frac{u_x}{u_\infty}\right)\bigg|_{y=0} = k\frac{d}{dy}\left(\frac{t-t_s}{t_\infty - t_s}\right)\bigg|_{y=0}$$

$$\frac{\mu C_p}{u_\infty}\frac{du_x}{dy}\bigg|_{y=0} = \frac{k}{t_\infty - t_s}\frac{d}{dy}(t - t_s)\big|_{y=0} = h$$

$$f \equiv \frac{\tau_0}{\rho u_\infty^2/2} = \frac{2\mu}{\rho u_\infty^2}\frac{du_x}{dy}\bigg|_{y=0}$$

$$\frac{h}{\rho u_\infty C_p} \equiv St = \frac{f}{2}$$

For gases, Pr~1

Prandtl-Taylor Analogy: Pr≠1

- Pr=1이 아닌 경우 Reynolds analogy은 잘 맞지 않음

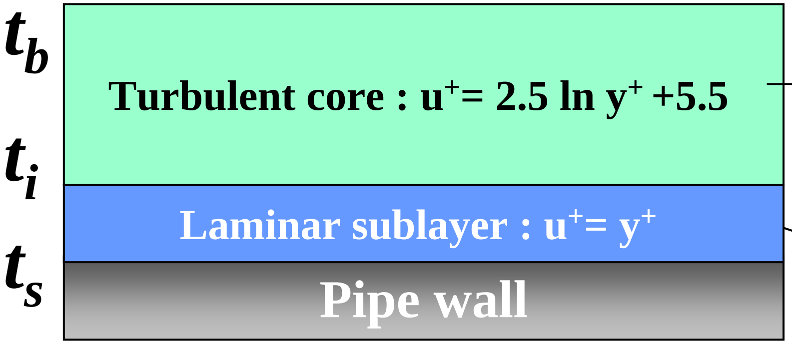
- 파이프 내의 turbulent core에서는 분자확산에 의한 열확산계수와 동점도계수가 무시됨.

- 파이프 내의 laminar sublayer는 열전달에 있어서 중요한 영역이며, 이 영역에서는 eddy coefficient는 무시되고, 분자확산에 의한 열확산계수(α)와 동점도계수(υ)는 무시할 수 없음.

- Prandtle-Taylor analogy에서는 이러한 단점을 극복하고 laminar sublayer에서는 열전도식을 사용하고 turbulent core에서는 Reynolds-analogy식을 사용함.

- 두 영역에서의 열전달 저항을 합하는 방식으로 총괄저항에 대한 단일식으로 표현함.

Prandtl-Taylor Analogy: Pr≠1



$$q/A = \frac{f u_b^2 \rho C_p}{2(u_b - u_i)}(t_b - t_i)$$

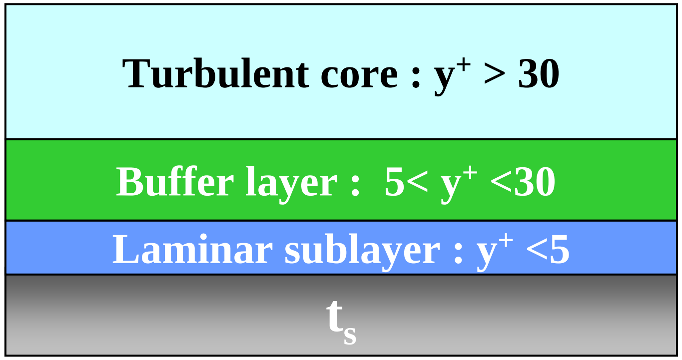
$$q/A = \frac{k(t_i - t_s)}{L} = \frac{k f u_b^2 \rho}{2\mu u_i}(t_i - t_s)$$

$$u_i = 5 u_b \sqrt{f/2}$$

$$h = \frac{1}{\left(\frac{k f u_b^2 \rho}{2\mu u_i}\right)^{-1} + \left(\frac{f u_b^2 \rho C_p}{2(u_b - u_i)}\right)^{-1}}$$

$$\boxed{St = \frac{f/2}{\underbrace{1}_{\text{Turbulent core}} + \underbrace{5\sqrt{f/2}\cdot(\mathrm{Pr}-1)}_{\text{Laminar sublayer}}}}$$

von Kármán Analogy



eddy

conduction + eddy $u^+ = 5 + 5\ln(y^+/5)$

conduction only

$$St = \frac{f/2}{1 + 5\sqrt{f/2}\cdot\{\Pr - 1 + \ln[(1+5\Pr)/6]\}}$$

Laminar sublayer

Buffer layer

Turbulent core

Analogy between Momentum Transfer and Heat Transfer

$$St = \frac{h}{u_b \rho C_p} = \frac{\dfrac{hx}{k}}{\dfrac{\rho u_b x}{\mu} \cdot \dfrac{C_p \mu}{k}} = \frac{Nu}{\mathrm{Re} \cdot \mathrm{Pr}}$$

Stanton number

Reynolds : $$St = \frac{f}{2}$$ **(23-4)**

Prandtl-Taylor : $$St = \frac{f/2}{1 + 5\sqrt{f/2} \cdot (\mathrm{Pr} - 1)}$$ **(23-25)**

von Kármán : $$St = \frac{f/2}{1 + 5\sqrt{f/2} \cdot \left\{\mathrm{Pr} - 1 + \ln[(1 + 5\,\mathrm{Pr})/6]\right\}}$$ **(23-26)**

Colburn : $$St = \frac{f}{2}\mathrm{Pr}^{-2/3}$$ **(23-27)**

Pr → 1, all equation approaches to Reynolds analogy equation

Prandtl-Taylor :

$$St = \frac{f/2}{1+5\sqrt{f/2}\cdot(\Pr-1)}$$

von Kármán :

$$St = \frac{f/2}{1+5\sqrt{f/2}\cdot\left\{\Pr-1+\ln[1+\frac{5}{6}(\Pr-1)]\right\}}$$

Colburn :

$$St = \frac{f}{2}\Pr^{-2/3}$$

Pr=1

Reynolds analogy equation

$$St = \frac{f}{2}$$

Colburn Analogy for smooth pipe

Empirical *j*-factor relation (1933)

$$St = \frac{f}{2}\mathrm{Pr}^{-2/3}$$

Colburn analogy
or Chilton-Colburn analogy

$$f = 0.046\,\mathrm{Re}^{-0.2} \qquad (\mathrm{Re} > 10^5) \qquad \textbf{(12-72)}$$

$$St = \frac{Nu}{\mathrm{Re}\cdot\mathrm{Pr}} = \frac{0.046\,\mathrm{Re}^{-0.2}}{2}\mathrm{Pr}^{-2/3}$$

$$\boxed{Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}} \qquad \textbf{(23-32)}$$

Dittus-Boelter equation for smooth pipe Pr=1

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

Colburn analogy

Pr=1

$$\boxed{Nu = 0.023\,\mathrm{Re}^{0.8}} \qquad \textbf{(23-30)}$$

Dittus-Boelter equation

Turbulent flow parallel to a flat plate
von Kármán integral method

Viscous dissipation & conduction in the x and z-direction are neglected.

energy balance
- **Heat in by fluid: A_1, A_3**
- **Heat out by fluid: A_2**
- **Heat in by conduction: A_4**

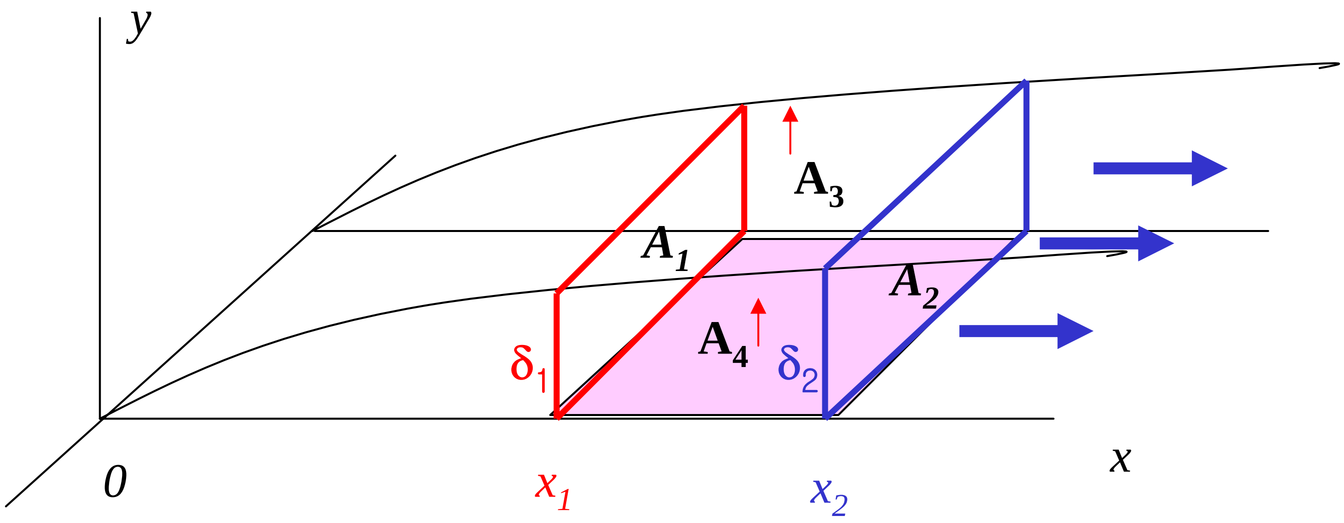


Fig. 23-5

Control volume for analysis of heat transfer in turbulent boundary layer

Turbulent flow parallel to a flat plate
von Kármán integral method

Heat-transfer coefficients in the region of the laminar boundary layer on a flat plate were analyzed in the previous chapter, starting with the basic differential balances. The problem could also have been solved in a semi-empirical fashion by what is known as the **von Kármán integral method**.

In this chapter we are dealing turbulent flow and are unable to use the Navier-Stokes equations, so we shall use the integral method to obtain an approximate solution to the problem.

The use of the integral method for isothermal flow has already been illustrated in Chapter 12. An empirical equation was written for the velocity distribution in the boundary layer, and force-momentum balance made on a segment of the layer.

Turbulent flow parallel to a flat plate
von Kármán integral method

Laminar flow

$$\frac{\delta}{\delta_{th}} = \mathrm{Pr}^{1/3}, \qquad \delta_{th} = 5x\,\mathrm{Re}^{-1/2}, \qquad h_x = 0.332\frac{k}{x}\left(\mathrm{Re}_x\right)^{1/2}\mathrm{Pr}^{1/3}$$

Turbulent flow (Re>10^5)

$$\frac{u_x}{u_0} = \left(\frac{y}{\delta}\right)^{1/7}, \qquad \frac{t_s - t}{t_s - t_0} = \left(\frac{y}{\delta_{th}}\right)^{1/7}, \qquad \delta_{th} = 0.376x\,\mathrm{Re}^{-1/5}\ \left(\mathrm{Pr} = 1\right)$$

(12-96) & (23-33)

von Kármán's idea: $\frac{u_x}{u_b} = 1 - \left(\frac{r}{R}\right)^{1/7}$

Overall energy balance

$$\boxed{\iint_{A_1} \rho C_p t u \cos\alpha dA + \iint_{A_2} \rho C_p t u \cos\alpha dA + \iint_{A_3} \rho C_p t u \cos\alpha dA = \iint_{A_4} h_x (t_s - t_0) dA}$$

(23-34)

$$\boxed{h_x = \rho C_p \frac{d}{dx}\left(\int_0^\delta \frac{t - t_0}{t_s - t_0} u_x dy \right)}$$

(23-39)

$$\frac{t - t_0}{t_s - t_0} = 1 - \frac{t_s - t}{t_s - t_0} = 1 - \left(\frac{y}{\delta}\right)^{1/7} ; u_x = u_0 \left(\frac{y}{\delta}\right)^{1/7}$$

(23-40) & (23-41)

$$h_x = \rho C_p u_0 \frac{d}{dx}\left\{ \delta \int_0^1 \left[1 - \left(\frac{y}{\delta}\right)^{1/7} \right] \left(\frac{y}{\delta}\right)^{1/7} d\frac{y}{\delta} \right\} = \rho C_p u_0 \frac{7}{72} \frac{d\delta}{dx}$$

(23-43)

Local heat transfer coefficient

$$\boxed{h_x = \rho C_p u_0 \frac{7}{72}\frac{d\delta}{dx}} \qquad \textbf{(23-43)}$$

Boundary Layer thickness

$$\frac{\delta}{x} = 0.376(\mathrm{Re}_x)^{-1/5} \qquad \textbf{(12-102)}$$

$$\frac{d\delta}{dx} = 0.376\left(\frac{\mu}{u_0\rho}\right)^{1/5}\left(\frac{4}{5}\right)x^{-1/5} = 0.301(\mathrm{Re}_x)^{-1/5} \qquad \textbf{(23-44)}$$

$$\frac{h_x}{\rho C_p u_0} = St = 0.0292(\mathrm{Re}_x)^{-1/5} \qquad \textbf{(23-45)}$$

Local heat transfer coefficient (Pr=1)

1

$$Nu_x = 0.0292 \left(\mathrm{Re}_x \right)^{4/5} \cancel{\mathrm{Pr}}$$

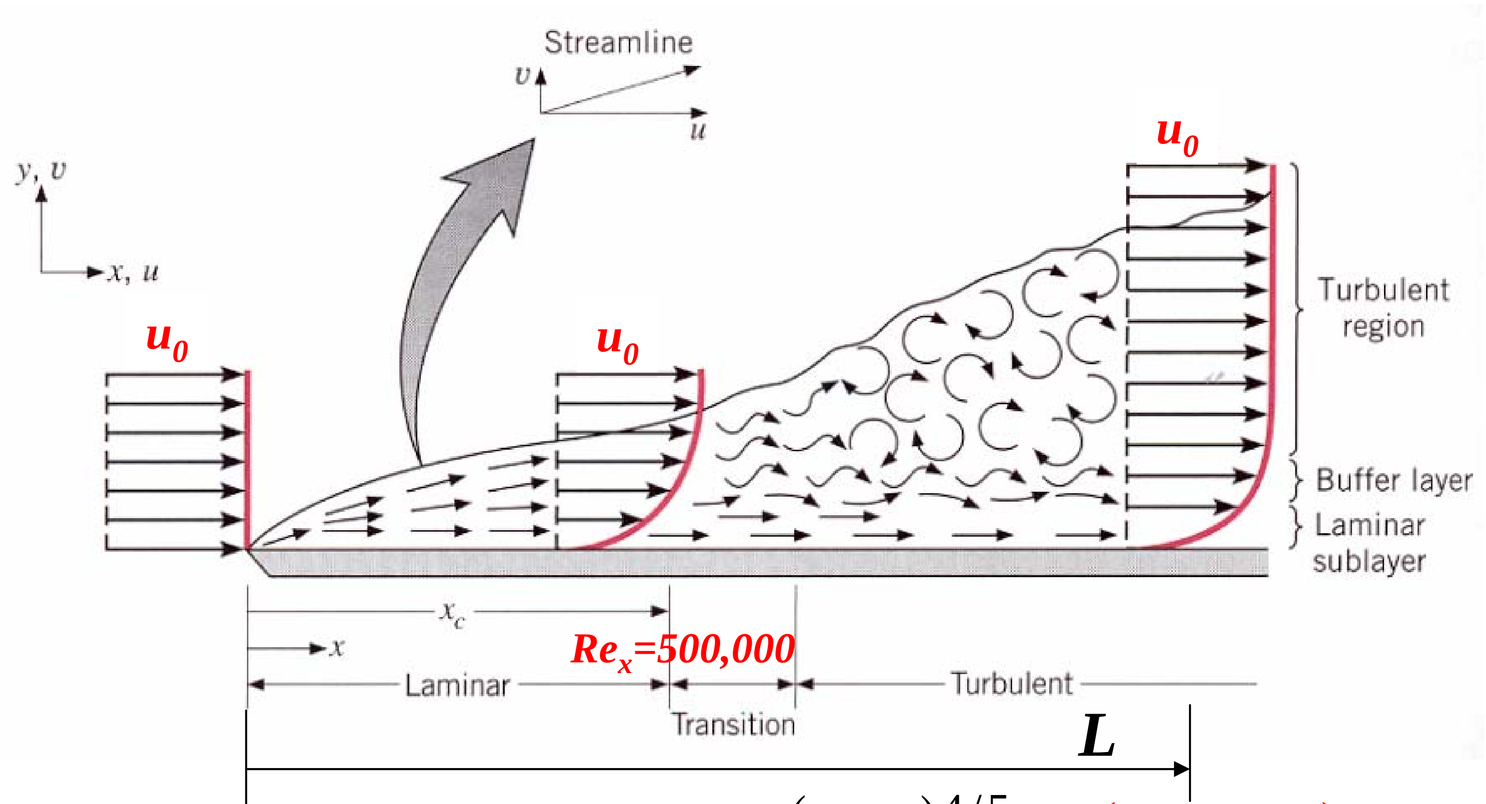
Local Nusselt number

$$\boxed{Nu_x = 0.0292 \left(\mathrm{Re}_x \right)^{4/5}} \qquad \textbf{(23-46)}$$

Mean Nusselt number

$$\boxed{Nu_m = 0.0365 \left(\mathrm{Re}_L \right)^{4/5}} \qquad \textbf{(23-47)}$$

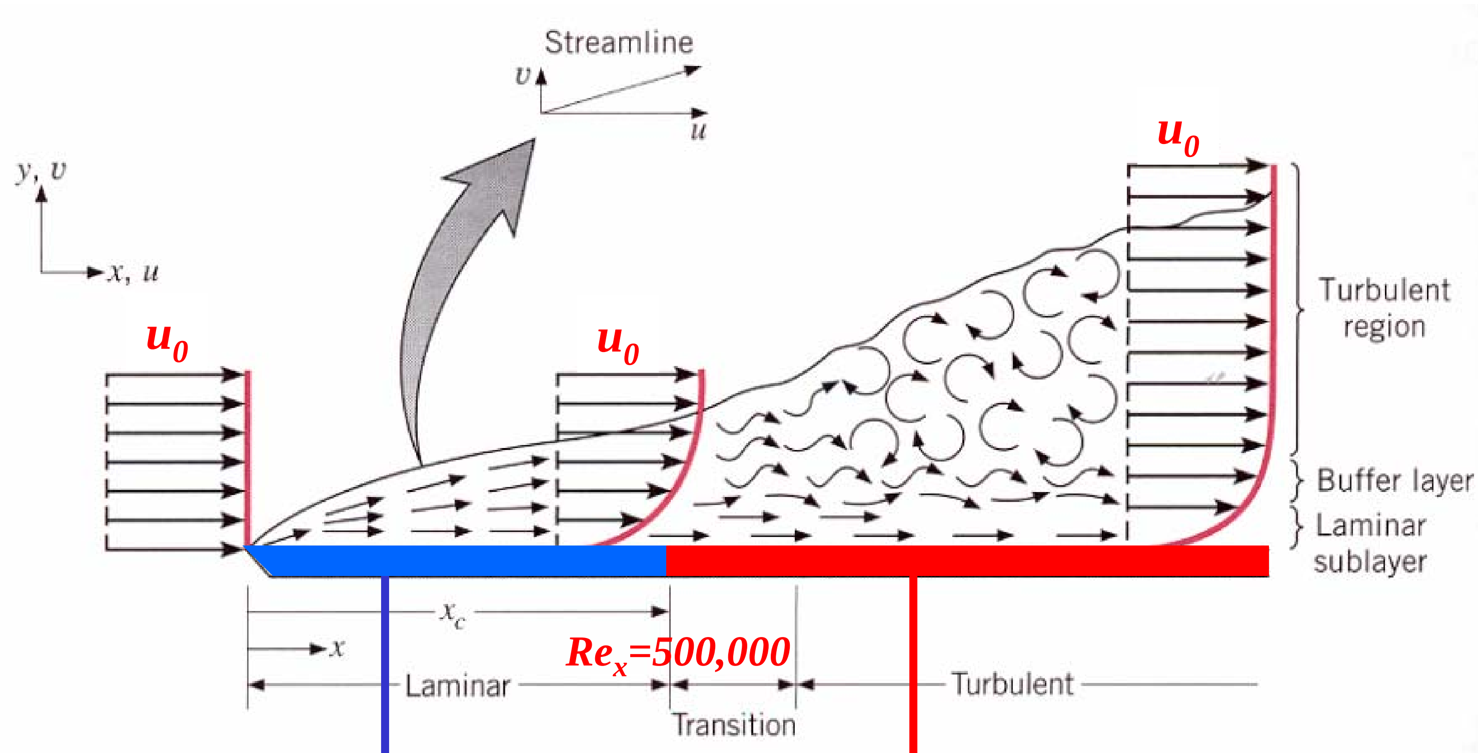
Mean Nusselt number (Pr=1)



$$Nu_m = 0.0365(\mathrm{Re}_L)^{4/5}$$ **(wrong)**

여기에서 평균 Nusselt수는 평판의 시작점에서 부터 난류가 형성되었다고 했을 때를 가정하여 유도되었기 때문에 이 식을 사용하면 안 된다.

Local Nusselt number (Pr=1)



Re_x<500,000

Re_x>500,000

(22-19) $$Nu_x = 0.332(\mathrm{Re}_x)^{1/2}$$

$$Nu_x = 0.0292(\mathrm{Re}_x)^{4/5}$$ (22-46)

Laminar flow

Turbulent flow

x must always be the distance from the leading edge of the plate, Not the distance from the point where turbulence starts.

Local heat transfer coefficient (Pr≠1)

$$\frac{\delta}{\delta_{th}} = \mathrm{Pr}^{n}$$

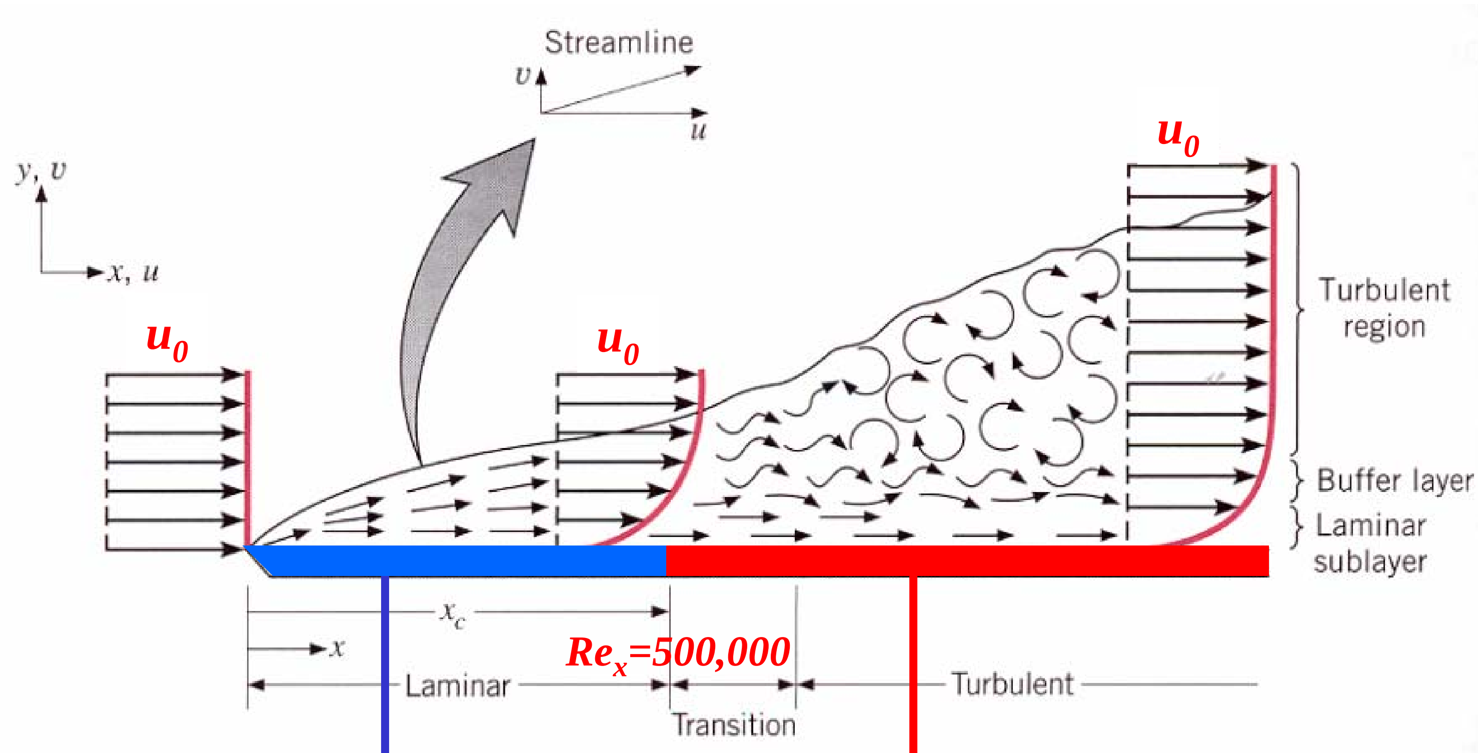
$$\frac{h_x}{\rho C_p u_0} = 0.0292\left(\mathrm{Re}_x\right)^{-1/5}\mathrm{Pr}^{-8n/7}$$

$$n = 1/3$$

$$Nu_x = 0.0292\left(\mathrm{Re}_x\right)^{4/5}\mathrm{Pr}^{0.62} \qquad \textbf{(23-50)}$$

$$Nu_m = 0.0365\left(\mathrm{Re}_L\right)^{4/5}\mathrm{Pr}^{0.62} \qquad \textbf{(23-51)}$$

Local Nusselt number (Pr≠1)



Re_x<500,000

Re_x>500,000

(22-19) $$Nu_x = 0.332(\mathrm{Re}_x)^{1/2}\,\mathrm{Pr}^{1/3}$$

$$Nu_x = 0.0292(\mathrm{Re}_x)^{4/5}\,\mathrm{Pr}^{0.62}$$ (22-50)

Laminar flow

Turbulent flow

x must always be the distance from the leading edge of the plate,
Not the distance from the point where turbulence starts.

Other study on local heat transfer coefficient (Pr≠1)

Turbulent flow over flat plate

Hanna & Myers

$$Nu_x = 0.0292(\mathrm{Re}_x)^{4/5}\,\mathrm{Pr}^{0.5}$$

$$Nu_m = 0.0365(\mathrm{Re}_L)^{4/5}\,\mathrm{Pr}^{0.5}$$

Colburn j-factor Analogy

$$Nu_x = 0.0292(\mathrm{Re}_x)^{4/5}\,\mathrm{Pr}^{0.33}$$

$$Nu_m = 0.0365(\mathrm{Re}_L)^{4/5}\,\mathrm{Pr}^{0.33}$$

Zhukauskas & Ambrazyavichus

$$Nu_x = 0.0292(\mathrm{Re}_x)^{4/5}\,\mathrm{Pr}^{0.43}$$

$$Nu_m = 0.0365(\mathrm{Re}_L)^{4/5}\,\mathrm{Pr}^{0.43}$$

$0.7 < \mathrm{Pr} < 380$

Summary

How to obtain equations for predicting heat-transfer coefficients

1. Combination of momentum, energy, and continuity equations for laminar flow

2. von Karman integral method for turbulent flow

3. The analogy between heat and momentum transfer for turbulent flow

4. The dimensional analysis

Homework

23-1

23-3