Naval Architecture & Ocean Engineering

[2009] [11]

Innovative ship design

-Elasticity –

Sign convention, Stress, Strain

May 2009

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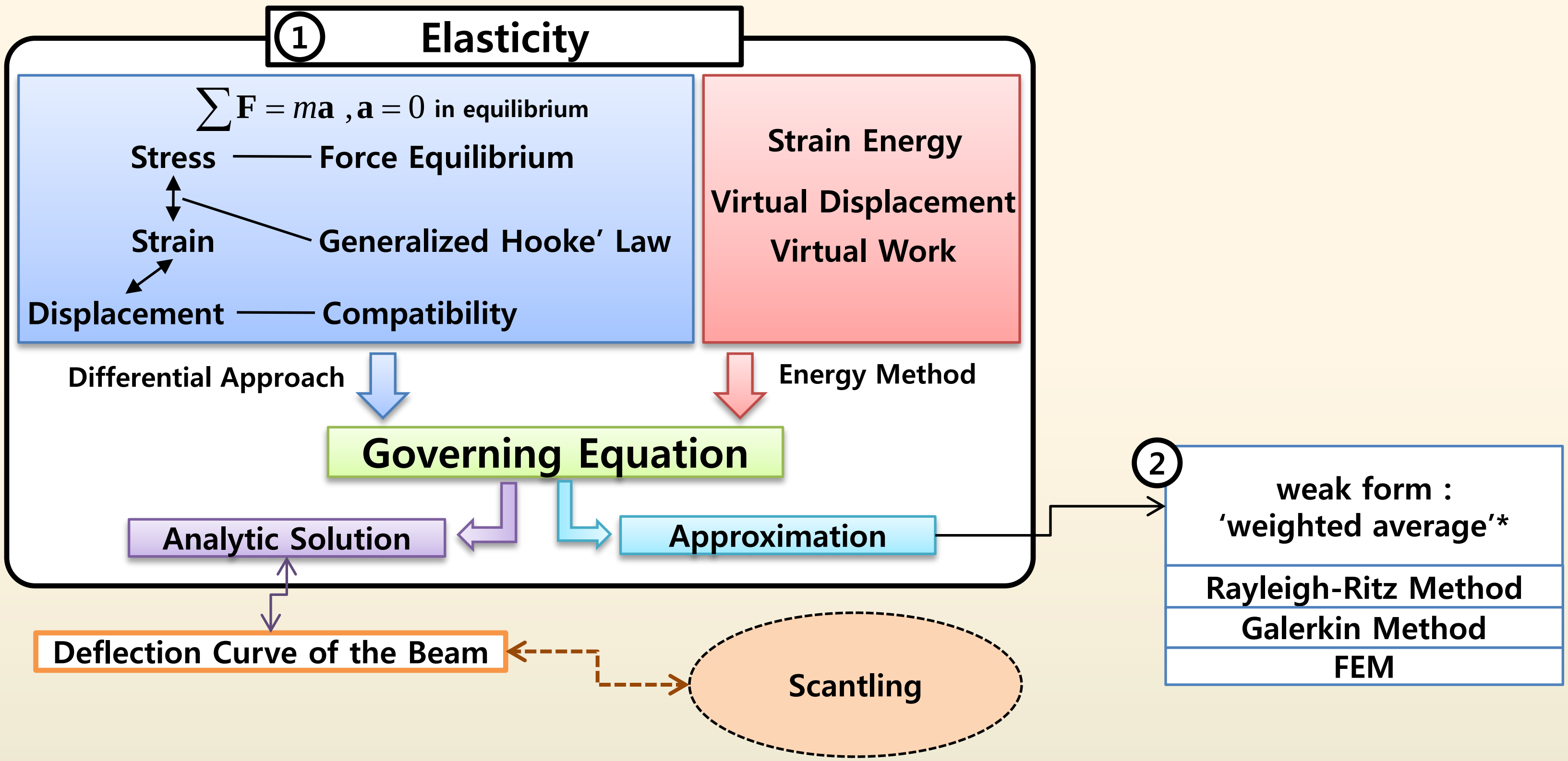

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Introduction to Ship Structural Design

E : young's modulus

I : mass moment of inertia

- **What we have studied with Beam theory**

$Load: \ f(x)$

cause

Shear Force: $V(x)$

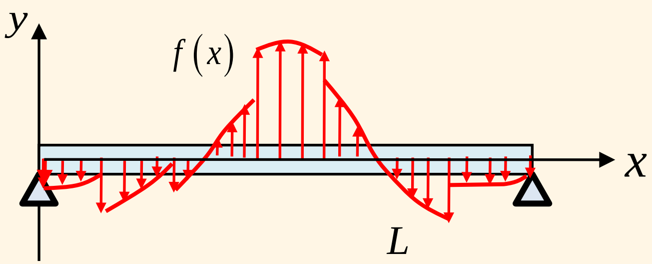
Bending Moment: $M(x)$

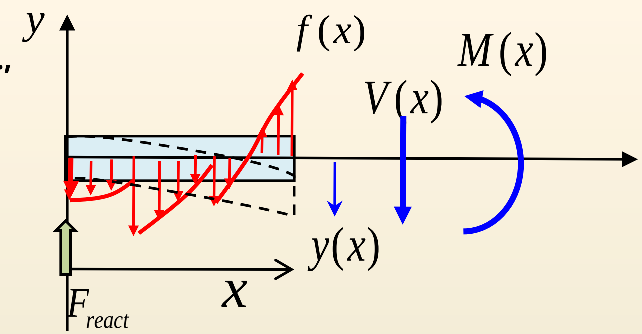
Deflection : $y(x)$

'relations' of load, S.F., B.M., and deflection

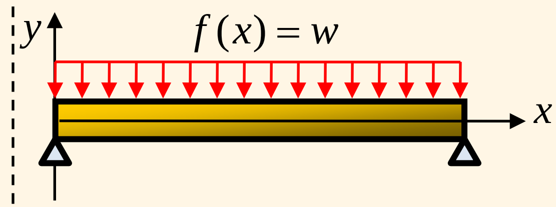
$$\frac{dV(x)}{dx} = -f(x) \ , \frac{dM(x)}{dx} = V(x)$$

$$, EI \frac{d^2y(x)}{dx^2} = M(x)$$





for example ,



$$V(x) = \frac{wL}{2} - wx$$

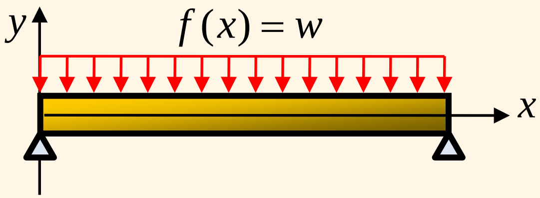
$$M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$y(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006

Simple Integration

$V(x)=\frac{wL}{2}-wx$ $\quad M(x)=\frac{wLx}{2}-\frac{wx^2}{2}$ $\quad y(x)=-\frac{wx}{24EI}(L^3-2Lx^2+x^3)$



$$EI\frac{d^4y(x)}{dx^4}=-w \qquad y(0)=0 \qquad y(L)=0 \qquad y''(0)=0 \qquad y''(L)=0$$

After integrate four times, $y(x)=c_1+c_2x+c_3x^2+c_4x^3-\frac{w}{24EI}x^4$

$$y''(x)=2c_3+6c_4x-\frac{w}{2EI}x^2$$

$y(0)=0$, $y(0)=c_1$ ⇨ $\therefore c_1=0$

$y''(0)=0$, $y''(0)=2c_3$ ⇨ $\therefore c_3=0$

$y''(L)=0$, $y''(x)=6c_4L-\frac{w}{2EI}L^2$ ⇨ $\therefore c_4=\frac{w}{12EI}L$

$y(L)=0$, $y(L)=c_2L+c_4L^3-\frac{w}{24EI}L^4$ ⇨ $\therefore c_2L+c_4L^3-\frac{w}{24EI}L^4=0$

$$c_2=-\frac{w}{24EI}L^3$$

$$\begin{aligned}c_2&=-c_4L^2+\frac{w_0}{24EI}L^3\\&=-(\frac{w}{12EI}L)L^2+\frac{w}{24EI}L^3\\&=-\frac{w}{12EI}L^3+\frac{w}{24EI}L^3\\&=-\frac{w}{24EI}L^3\end{aligned}$$

$$\begin{aligned}y(x)&=-\frac{w}{24EI}L^3x+\frac{w}{12EI}Lx^3-\frac{w}{24EI}x^4\\&=-\frac{wx}{24EI}\left(L^3-2Lx^2+x^3\right)\end{aligned}$$

E : young's modulus

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● What we have studied with Beam theory

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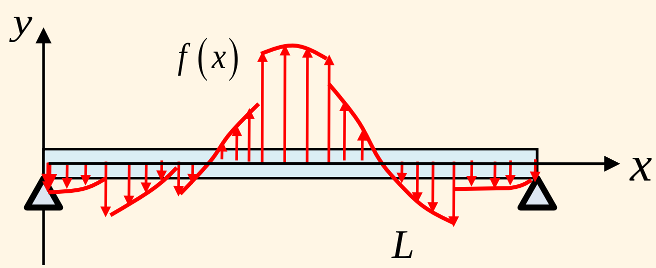

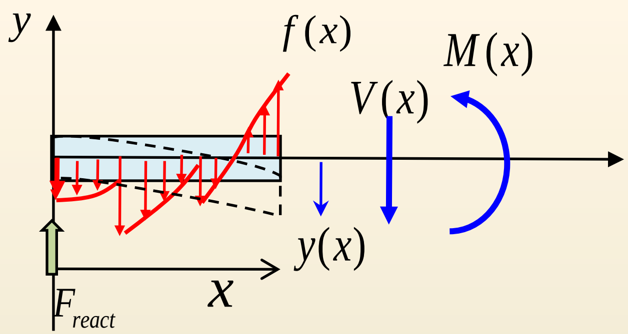

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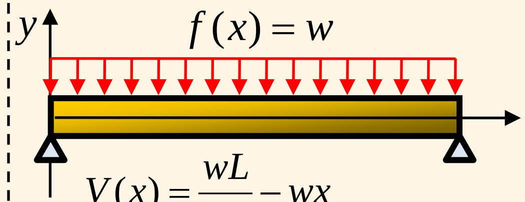

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what is our interest?

● **Safety :**
Won't it fail under the load?

Stress should meet :

$$\sigma_{act} \le \sigma_{allow}$$

< stress on beam section>

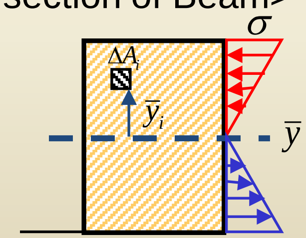
, *where* $\sigma_{act} = \dfrac{M}{I_{\bar{y}}/\bar{y}_i} = \dfrac{M}{Z}$

● **Geometry :**
How much it would be bent under the load?

Differential equations of the deflection curve

$$EI\frac{d^4y(x)}{dx^4} = -f(x)$$

< section of Beam>



$\bar{y}$: neutral axis

$\bar{y}_i$: distance from neutral axis

, ($I_{\bar{y}}$: moment of inertia from $\bar{y}$)

, (z : section modulus)

*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006

Ship Structural Design

- **Ship Structural Design**

what is designer's major interest?

- **Safety :**

Won't 'IT' fail under the load?

a ship	global
a stiffener	local
a plate	local

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cause

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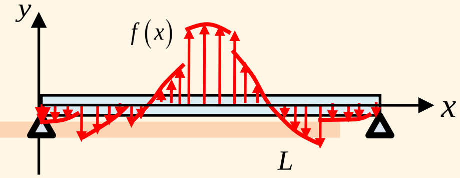

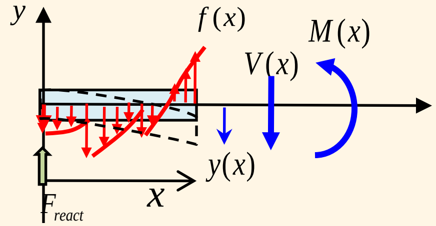

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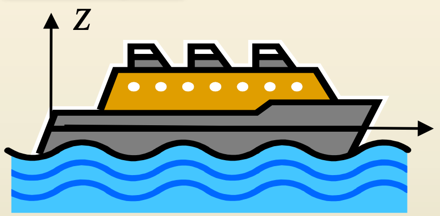

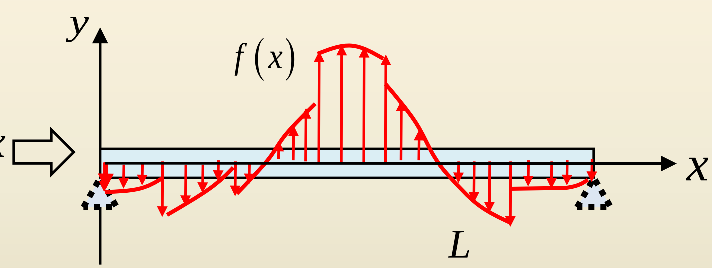

Actual stress on midship section should be less than allowable stress

$$\boxed{\sigma_{act.} \le \sigma_{allow}} \quad , \sigma_{act.} = \frac{M_{mid}}{Z_{mid}}$$

Allowable stress by Rule : (for example)

$$, \sigma_{allow} = 175 f_1 \; [N/mm^2]$$

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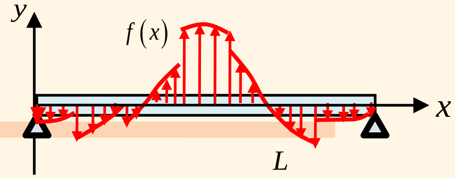

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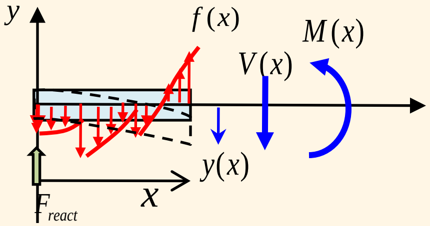

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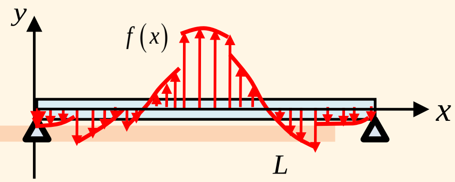

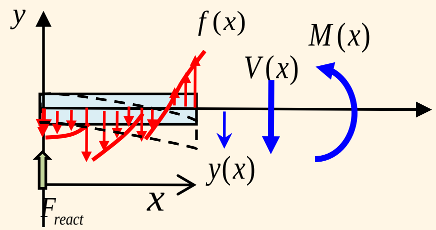

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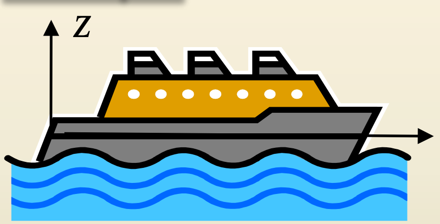

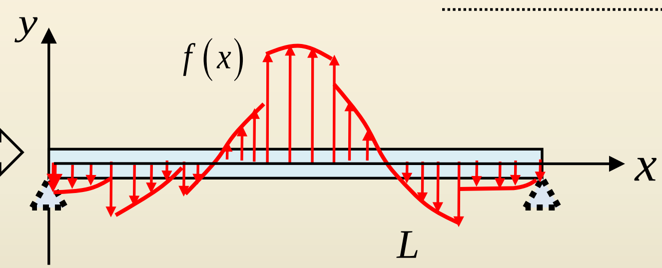

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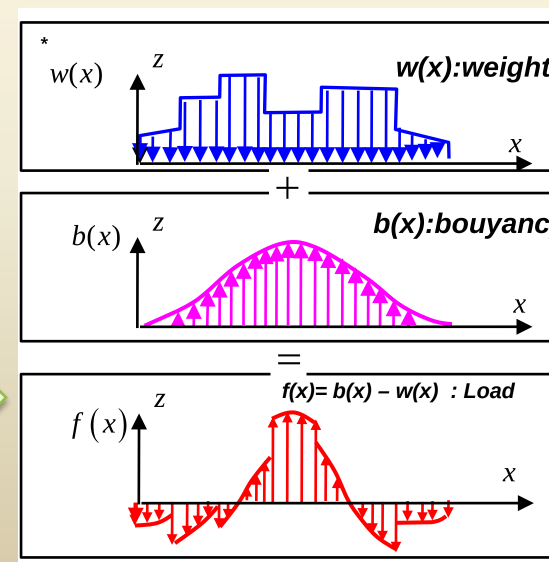
what kinds of load f cause M_{mid} ?

Hydrostatics



*대한조선학회, 선박해양공학개론, 1993, 동명사, p138

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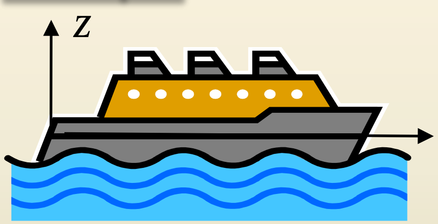

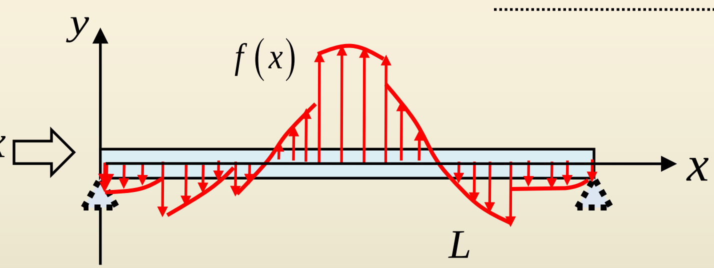

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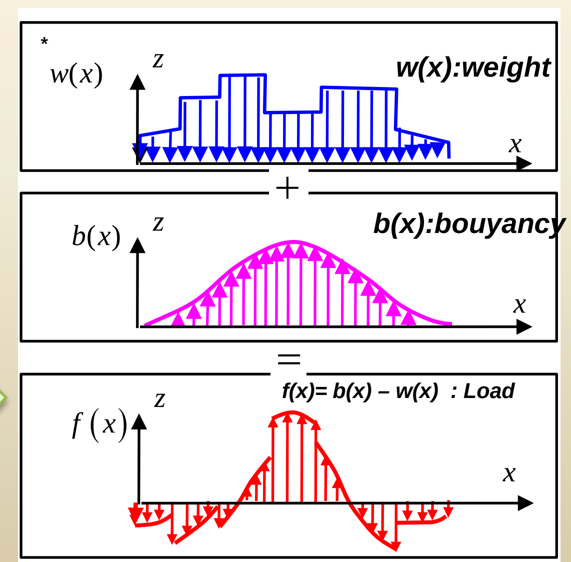
anything else?

Hydrodynamics



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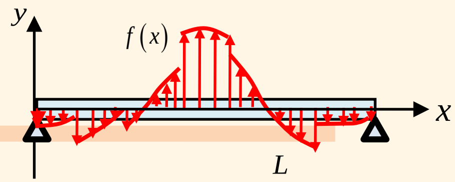

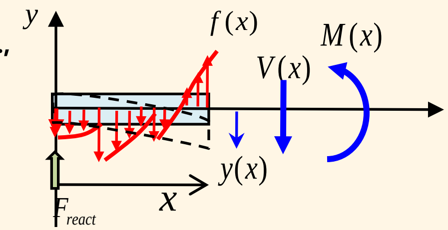

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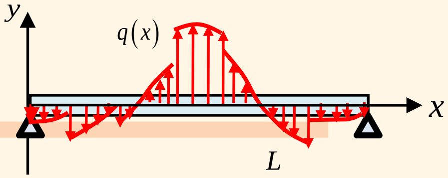

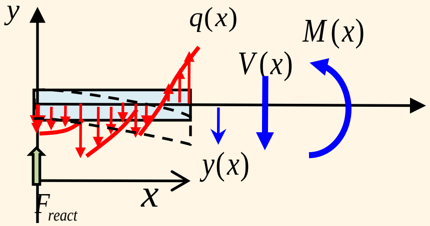

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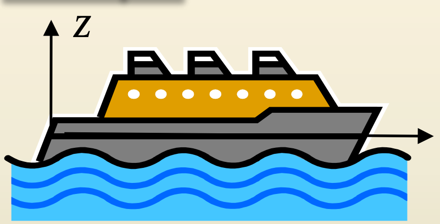
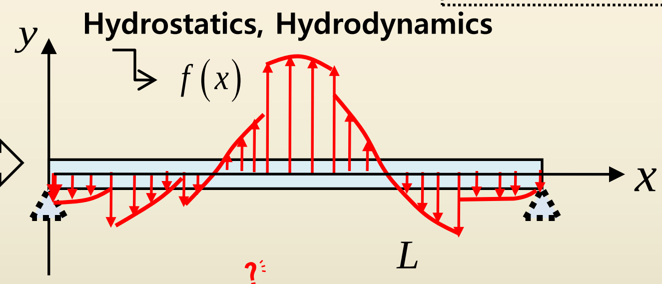
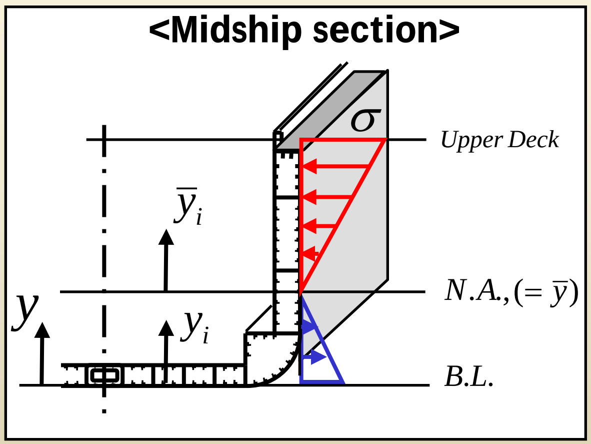






Actual stress on midship section should be less than allowable stress

$$\sigma_{act.} \le \sigma_{allow}, \quad \sigma_{act.} = \frac{M_{mid}}{Z_{mid}} = \frac{M_s + M_w}{I_{ship,N.A}/\bar{y}_i}$$

Allowable stress by Rule : (for example)

$$\sigma_{allow} = 175 f_1 \ [N/mm^2]$$

how we can meet the rule?

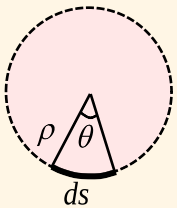
'Midship Design' is to arrange the structural members and fix the thickness of them to secure enough section modulus to the rule.

M_w : vertical wave bending moment
M_s : still water bending moment
$I_{ship,N.A.}$: moment of inertia from N.A. of Midship section

Differential Equation for Deflection of Beam in vector notation

Deflection of Beam with Vector

$\boldsymbol{\sigma}\mathbf{i}_x \boldsymbol{\varepsilon}\mathbf{i} = \sigma \;,\quad _x = \varepsilon \;,\quad \boldsymbol{\theta}\mathbf{k} = \theta \mathbf{y} \;,\quad \mathbf{j} = y \qquad \sigma = E\varepsilon$

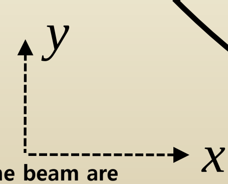


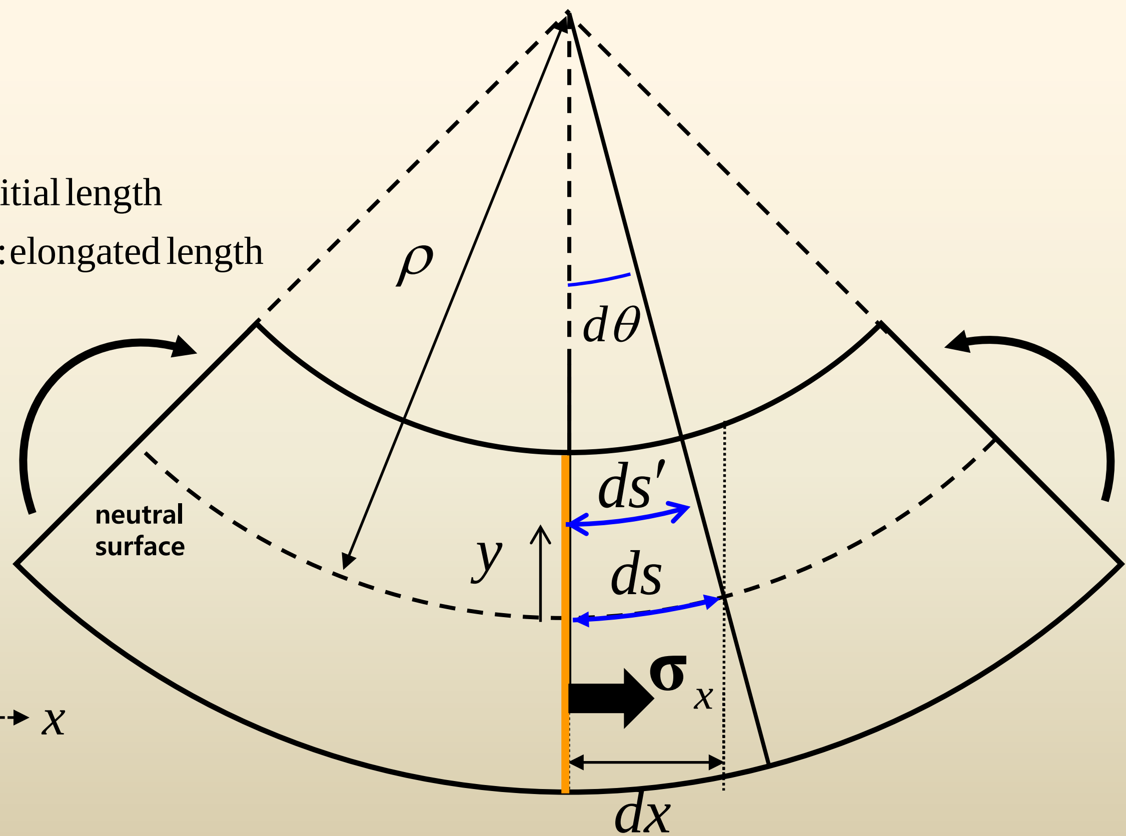
$$\rho \cdot d\theta = ds \;\Rightarrow\; \frac{d\theta}{ds} = \frac{1}{\rho}$$

① **strain at** y **in x-direction :**

$$\boldsymbol{\varepsilon}\mathbf{i}_x = \varepsilon$$

$$\varepsilon = \frac{(\rho - y)\cdot d\theta - \rho d\theta}{ds} = -y\frac{d\theta}{ds} = -\frac{y}{\rho} \quad \begin{array}{l} , ds : \text{initial length} \\ , y\cdot d\theta : \text{elongated length} \end{array}$$

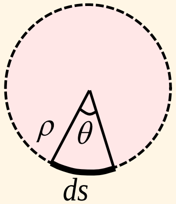




* neutral surface : Longitudinal lines on the lower part of the beam are elongated, whereas those on the upper part are shortened. Thus the lower part of the beam is in tension and the upper part is in compression. Somewhere between the top and bottom of the beam is a surface in which longitudinal lines do not change in length. This surface is called neutral surface

Deflection of Beam with Vector

$\boldsymbol{\sigma} = \sigma_x \mathbf{i}$, $\boldsymbol{\varepsilon} = \varepsilon_x \mathbf{i}$, $\boldsymbol{\theta} = \theta \mathbf{k}$, $\mathbf{y} = y\mathbf{j}$ $\sigma = E\varepsilon$



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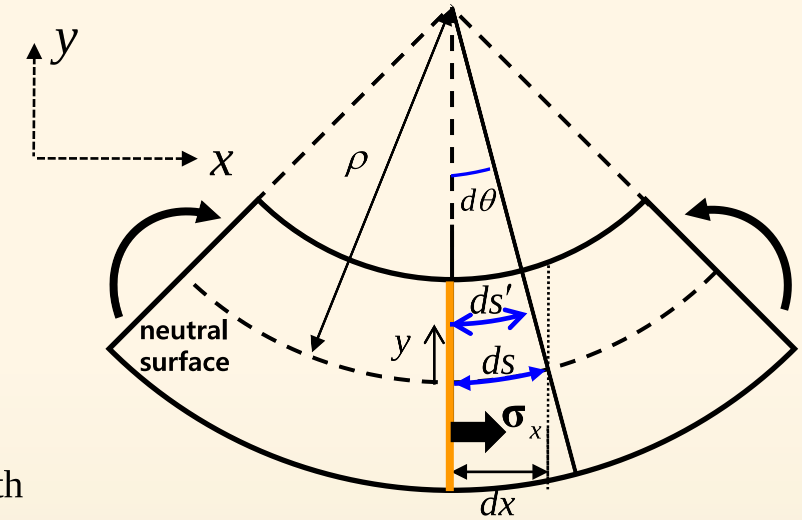


① **strain at y in x-direction :**

$$\boldsymbol{\varepsilon} = \varepsilon_x \mathbf{i}$$

$$\varepsilon = \frac{(\rho - y)\cdot d\theta - \rho d\theta}{ds} = -y\frac{d\theta}{ds} = -\frac{y}{\rho}$$

$, ds$: initial length

$, y \cdot d\theta$: elongated length

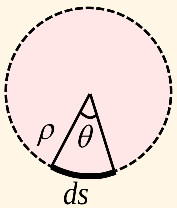
② **stress at y in x-direction :** $\boldsymbol{\sigma} = \sigma_x \mathbf{i} = E \cdot \varepsilon \mathbf{i}$, where $\varepsilon = -\dfrac{y}{\rho}$ $\quad \therefore \boldsymbol{\sigma} = \sigma_x \mathbf{i} = -E\dfrac{y}{\rho}\mathbf{i}$

③ **force acting on dA in x-direction :** $d\mathbf{F} = \boldsymbol{\sigma} dA = (\sigma_x \mathbf{i})dA = \boldsymbol{\sigma} dA$ $\quad \therefore d\mathbf{F} = -E\dfrac{y}{\rho}dA\mathbf{i}$

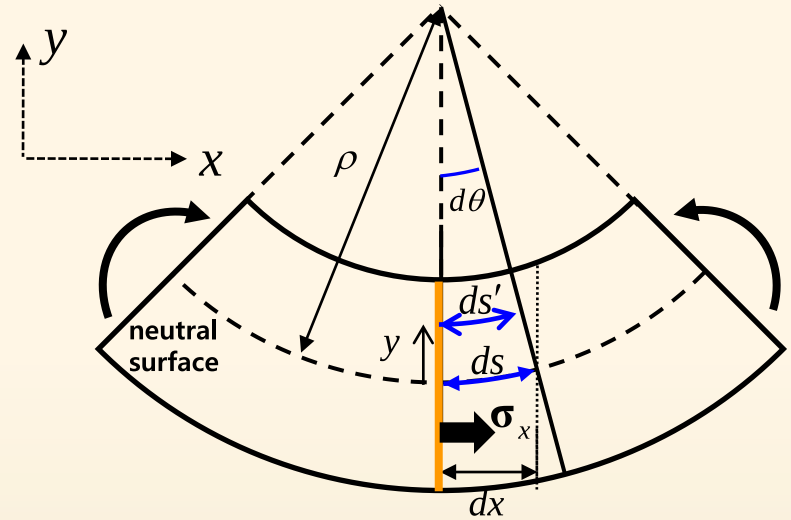
④ **moment about z-axis :** $d\mathbf{M} = \mathbf{y} \times d\mathbf{F} = (y\mathbf{j}) \times (-E\dfrac{y}{\rho}dA\mathbf{i}) = E\dfrac{y^2}{\rho}dA\mathbf{k}$

Deflection of Beam with Vector

$\boldsymbol{\sigma} = \sigma_x \mathbf{i} = \sigma$, $\boldsymbol{\varepsilon} = \varepsilon_x \mathbf{i} = \varepsilon$, $\boldsymbol{\theta} = \theta\mathbf{k} = \theta$, $\mathbf{y} = y\mathbf{j} = y$ $\qquad \sigma = E\varepsilon$



$$\rho \cdot d\theta = ds \Rightarrow \frac{d\theta}{ds} = \frac{1}{\rho}$$



① **strain at** y **in x-direction :** $\varepsilon = \dfrac{(\rho - y)\cdot d\theta - \rho \cdot d\theta}{ds} = -y\dfrac{d\theta}{ds}$

$\boldsymbol{\varepsilon} = \varepsilon_x \mathbf{i} = \varepsilon$ $\quad , d\theta$: initial length, $\; y \cdot d\theta$: elongated length

② **stress at** y **in x-direction :** $\boldsymbol{\sigma} = \sigma_x \mathbf{i} = \sigma = -E\dfrac{y}{\rho}\mathbf{i}$

③ **force acting on** dA **in x-direction** $: d\mathbf{F} = -E\dfrac{y}{\rho}dA\mathbf{i}$

④ **moment about z-axis :** $d\mathbf{M} = \mathbf{y} \times d\mathbf{F} = (y\mathbf{j}) \times (-E\dfrac{y}{\rho}dA\mathbf{i}) = E\dfrac{y^2}{\rho}dA\mathbf{k} \quad \therefore \mathbf{M} = \int_A d\mathbf{M} = \int_A E\dfrac{y^2}{\rho}dA\mathbf{k}$

Define $I = \int_A y^2 dA$ $\quad$ then, $\mathbf{M} = \dfrac{EI}{\rho}\mathbf{k}, \quad M = \dfrac{EI}{\rho}$

$$\Rightarrow \mathbf{M} = \frac{EI}{\rho}\mathbf{k}$$

$$\Downarrow$$

$$\mathbf{M} = EI\frac{d\theta}{ds}\mathbf{k}$$

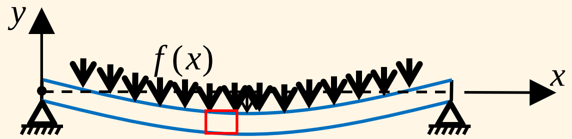
$$\Downarrow$$

⑤ **assume** $ds \approx dx, \; \theta \approx \tan(\theta) = \dfrac{dy}{dx}$

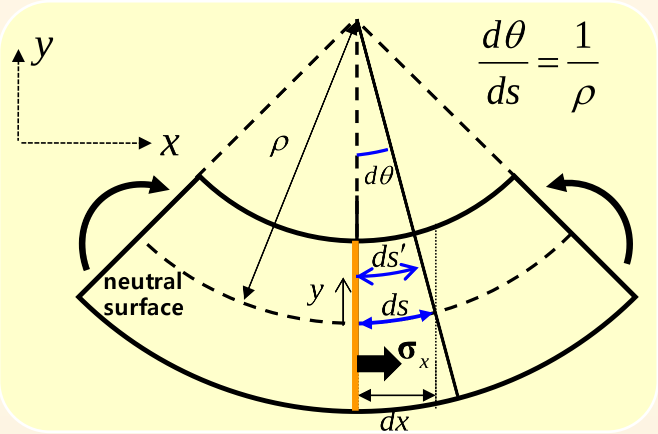
$$\frac{d\theta}{ds} = \frac{d}{ds}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \Rightarrow \frac{d\theta}{ds} = \frac{d^2y}{dx^2}$$

$$\mathbf{M} = EI\frac{d^2y}{dx^2}\mathbf{k} \qquad , M = EI\frac{d^2y}{dx^2}$$

Deflection of Beam with Vector



$\boldsymbol{\sigma}_x = \sigma\mathbf{i}$, $\boldsymbol{\varepsilon}_x = \varepsilon\mathbf{i}$, $\boldsymbol{\theta} = \theta\mathbf{k}$, $\mathbf{y} = y\mathbf{j}$ $\quad \sigma = E\varepsilon$



$$\frac{d\theta}{ds} = \frac{1}{\rho}$$

① **strain at y in x-direction :** $\varepsilon = \dfrac{(\rho - y)\cdot d\theta - \rho\cdot d\theta}{ds} = -y\dfrac{d\theta}{ds}$

$\boldsymbol{\varepsilon}_x = \varepsilon\mathbf{i}$ $\quad$, $d\theta$: initial length, $\; y\cdot d\theta$: elongated length

② **stress at y in x-direction :** $\boldsymbol{\sigma}_x = \sigma\mathbf{i} = -E\dfrac{y}{\rho}\mathbf{i}$

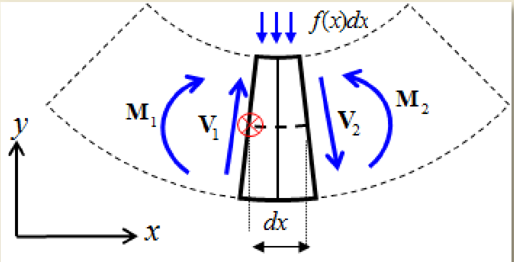
③ **force acting on dA in x-direction :** $d\mathbf{F} = -E\dfrac{y}{\rho}dA\mathbf{i}$

④ **moment about z-axis :**

$$d\mathbf{M} = \mathbf{y}\times d\mathbf{F} = (y\mathbf{j})\times\left(-E\frac{y}{\rho}dA\mathbf{i}\right) = E\frac{y^2}{\rho}dA\mathbf{k} \quad \therefore \mathbf{M} = \int_A d\mathbf{M} = \int_A E\frac{y^2}{\rho}dA\mathbf{k} = \frac{EI}{\rho}\mathbf{k} \;,\; I = \int_A y^2 dA$$

⑤ **assume** $ds \approx dx,\ \theta \approx \tan(\theta) = \dfrac{dy}{dx}$ ⇨ $\dfrac{d\theta}{ds} = \dfrac{d^2y}{dx^2}$ ⇨ $\mathbf{M} = \dfrac{EI}{\rho}\mathbf{k} = EI\dfrac{d\theta}{ds}\mathbf{k}$ ⇨ $\mathbf{M} = EI\dfrac{d^2y}{dx^2}\mathbf{k}$, $M = EI\dfrac{d^2y}{dx^2}$

⑥ **relationships between loads, shear forces, and bending moments**



$$\mathbf{V}_1 = V\mathbf{j},\ \ \mathbf{V}_2 = -\left(V + \frac{\partial V}{\partial x}dx\right)\mathbf{j},\ \ \mathbf{M}_1 = -M\mathbf{k},\ \ \mathbf{M}_2 = \left(M + \frac{\partial M}{\partial x}dx\right)\mathbf{k}$$

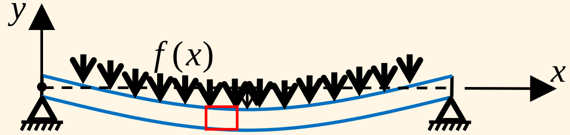
•**force equilibrium** $\quad \sum \mathbf{F}_y = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{f}(x) = 0$

$$(V_1\mathbf{j}) + \left(-\left(V_1 + \frac{\partial V_1}{\partial x}dx\right)\mathbf{j}\right) + (-f(x)\mathbf{j}) = 0$$

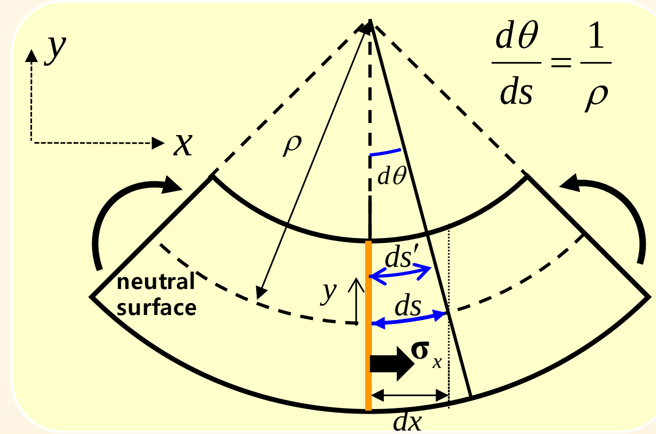
$$\left(V_1 - V_1 - \frac{\partial V_1}{\partial x}dx - f(x)\right)\mathbf{j} = 0$$

$$\therefore \frac{dV}{dx} = -f(x)$$

Deflection of Beam with Vector



$\boldsymbol{\sigma}_x = \sigma\mathbf{i}$, $\boldsymbol{\varepsilon}_x = \varepsilon\mathbf{i}$, $\boldsymbol{\theta} = \theta\mathbf{k}$, $\mathbf{y} = y\mathbf{j}$ $\qquad \sigma = E\varepsilon$



$$\frac{d\theta}{ds} = \frac{1}{\rho}$$

① **strain at y in x-direction :** $\varepsilon = \dfrac{(\rho - y)\cdot d\theta - \rho\cdot d\theta}{ds} = -y\dfrac{d\theta}{ds}$

$\boldsymbol{\varepsilon}_x = \varepsilon\mathbf{i}$, $d\theta$: initial length, $y\cdot d\theta$: elongated length

② **stress at y in x-direction :** $\boldsymbol{\sigma}_x = \sigma\mathbf{i} = -E\dfrac{y}{\rho}\mathbf{i}$

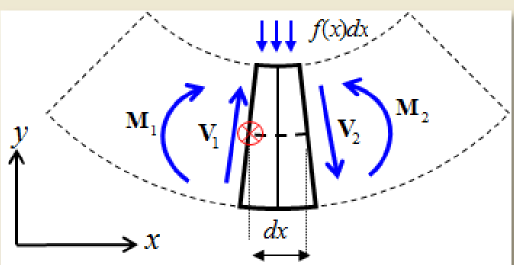
③ **force acting on dA in x-direction** : $d\mathbf{F} = -E\dfrac{y}{\rho}dA\mathbf{i}$

④ **moment about z-axis :**

$$d\mathbf{M} = \mathbf{y}\times d\mathbf{F} = (y\mathbf{j})\times\left(-E\frac{y}{\rho}dA\mathbf{i}\right) = E\frac{y^2}{\rho}dA\mathbf{k} \quad \therefore \mathbf{M} = \int_A d\mathbf{M} = \int_A E\frac{y^2}{\rho}dA\mathbf{k} = \frac{EI}{\rho}\mathbf{k} \ , \ I = \int_A y^2 dA$$

⑤ **assume** $ds \approx dx,\ \theta \approx \tan(\theta) = \dfrac{dy}{dx}$ ⇨ $\dfrac{d\theta}{ds} = \dfrac{d^2y}{dx^2}$ ⇨ $\mathbf{M} = \dfrac{EI}{\rho}\mathbf{k} = EI\dfrac{d\theta}{ds}\mathbf{k}$ ⇨ $\mathbf{M} = EI\dfrac{d^2y}{dx^2}\mathbf{k}$, $M = EI\dfrac{d^2y}{dx^2}$

⑥ **relationships between loads, shear forces, and bending moments**



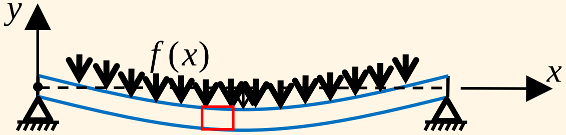
$$\mathbf{V}_1 = V\mathbf{j},\ \ \mathbf{V}_2 = -\left(V + \frac{\partial V}{\partial x}dx\right)\mathbf{j},\ \ \mathbf{M}_1 = -M\mathbf{k},\ \ \mathbf{M}_2 = \left(M + \frac{\partial M}{\partial x}dx\right)\mathbf{k}$$

- **force equilibrium** $\dfrac{dV}{dx} = -f(x)$
- **moment equilibrium** $\sum \mathbf{M}_z = \mathbf{M}_1 + \mathbf{M}_2 + d\mathbf{x}\times\mathbf{V}_2 + \dfrac{1}{2}d\mathbf{x}\times(\mathbf{f}(x)\cdot dx) = 0$

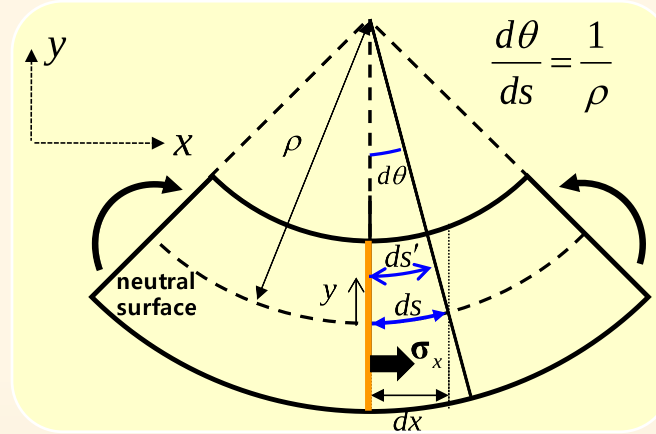
$$-M\mathbf{k} + \left(M + \frac{\partial M}{\partial x}dx\right)\mathbf{k} + (dx\,\mathbf{i})\times\left(-\left(V + \frac{\partial V}{\partial x}dx\right)\mathbf{j}\right) + \left(\frac{1}{2}dx\,\mathbf{i}\right)\times(-f(x)\mathbf{j}) = 0$$

$$\therefore \frac{dM}{dx} = V(x)$$

Deflection of Beam with Vector



$\boldsymbol{\sigma} = \sigma_x \mathbf{i}$, $\boldsymbol{\varepsilon} = \varepsilon_x \mathbf{i}$, $\boldsymbol{\theta} = \theta \mathbf{k}$, $\mathbf{y} = y \mathbf{j}$ $\quad \sigma = E\varepsilon$



① **strain at** y **in x-direction :** $\varepsilon = \dfrac{(\rho - y)\cdot d\theta - \rho \cdot d\theta}{ds} = -y\dfrac{d\theta}{ds}$

$\boldsymbol{\varepsilon} = \varepsilon_x \mathbf{i}$ $\quad , d\theta$: initial length, $y \cdot d\theta$: elongated length

② **stress at** y **in x-direction :** $\boldsymbol{\sigma} = \sigma_x \mathbf{i} = -E\dfrac{y}{\rho}\mathbf{i}$

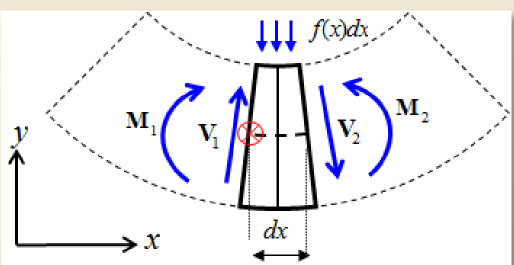
③ **force acting on** dA **in x-direction** : $d\mathbf{F} = -E\dfrac{y}{\rho}dA\mathbf{i}$

④ **moment about z-axis :**

$$d\mathbf{M} = \mathbf{y} \times d\mathbf{F} = (y\mathbf{j}) \times \left(-E\frac{y}{\rho}dA\mathbf{i}\right) = E\frac{y^2}{\rho}dA\mathbf{k} \quad \therefore \mathbf{M} = \int_A d\mathbf{M} = \int_A E\frac{y^2}{\rho}dA\mathbf{k} = \frac{EI}{\rho}\mathbf{k} \quad , I = \int_A y^2 dA$$

⑤ **assume** $ds \approx dx,\ \theta \approx \tan(\theta) = \dfrac{dy}{dx}$ ⇨ $\dfrac{d\theta}{ds} = \dfrac{d^2y}{dx^2}$ ⇨ $\mathbf{M} = \dfrac{EI}{\rho}\mathbf{k} = EI\dfrac{d\theta}{ds}\mathbf{k}$ ⇨ $\mathbf{M} = EI\dfrac{d^2y}{dx^2}\mathbf{k}$, $M = EI\dfrac{d^2y}{dx^2}$

⑥ **relationships between loads, shear forces, and bending moments**



$$\mathbf{V}_1 = V\mathbf{j},\ \ \mathbf{V}_2 = -\left(V + \frac{\partial V}{\partial x}dx\right)\mathbf{j},\ \ \mathbf{M}_1 = -M\mathbf{k},\ \ \mathbf{M}_2 = \left(M + \frac{\partial M}{\partial x}dx\right)\mathbf{k}$$

•**force equilibrium** $\dfrac{dV}{dx} = -f(x)$ •**moment equilibrium** $\dfrac{dM}{dx} = V(x)$

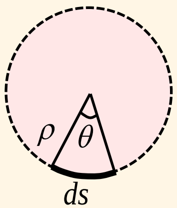
$$\frac{d^2y}{dx^2} = \frac{M}{EI} \rightarrow \frac{d^3y}{dx^3} = \frac{1}{EI}\cdot\frac{dM}{dx} = \frac{1}{EI}\cdot V(x) \rightarrow \frac{d^4y}{dx^4} = \frac{1}{EI}\cdot\frac{dV}{dx} = -\frac{1}{EI}\cdot f(x)$$

$$\therefore EI\frac{d^4y}{dx^4} = -f(x)$$

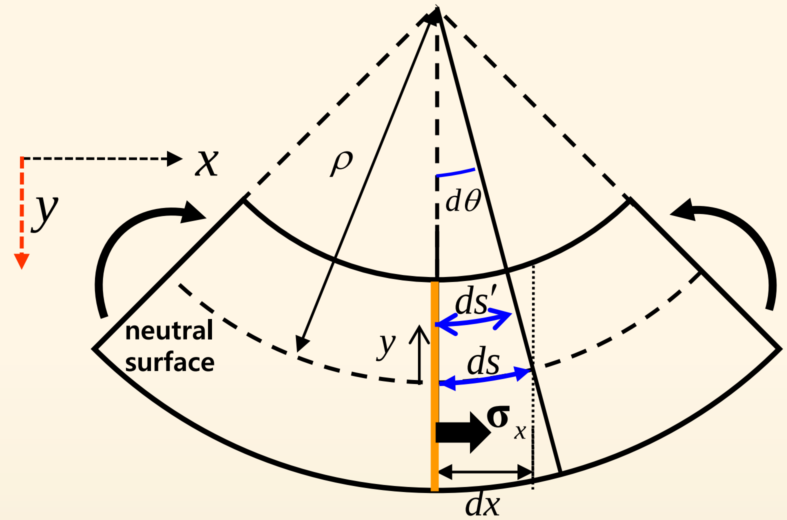
Deflection of Beam with Vector

what happen
if we take the direction of
y axis reversed?

$\boldsymbol{\sigma}_x = \sigma\mathbf{i}$, $\boldsymbol{\varepsilon}_x = \varepsilon\mathbf{i}$, $\boldsymbol{\theta} = \theta\mathbf{k}$, $\mathbf{y} = y\mathbf{j}$ $\qquad \sigma = E\varepsilon$



$$\rho\cdot d\theta = ds \;\Rightarrow\; \frac{d\theta}{ds} = \frac{1}{\rho}$$



① **strain at** y **in x-direction :** $\varepsilon = \dfrac{(\rho+y)\cdot d\theta - \rho\cdot d\theta}{ds} = y\dfrac{d\theta}{ds}$

$\boldsymbol{\varepsilon}_x = \varepsilon\mathbf{i}$ $\qquad , d\theta$: initial length, $y\cdot d\theta$: elongated length

② **stress at** y **in x-direction :** $\boldsymbol{\sigma}_x = \sigma\mathbf{i} = E\dfrac{y}{\rho}\mathbf{i}$

③ **force acting on** dA **in x-direction :** $d\mathbf{F} = E\dfrac{y}{\rho}dA\mathbf{i}$

④ **moment about z-axis :** $d\mathbf{M} = \mathbf{y}\times d\mathbf{F} = (y\mathbf{j})\times(E\dfrac{y}{\rho}dA\mathbf{i}) = -E\dfrac{y^2}{\rho}dA\mathbf{k} \;\therefore \mathbf{M} = \int_A d\mathbf{M} = -\int_A E\dfrac{y^2}{\rho}dA\mathbf{k}$

Define $I = \int_A y^2 dA$ then, $\mathbf{M} = -\dfrac{EI}{\rho}\mathbf{k}$, $M = -\dfrac{EI}{\rho}$

$$\Rightarrow \mathbf{M} = -\frac{EI}{\rho}\mathbf{k}$$

$$\mathbf{M} = -EI\frac{d\theta}{ds}\mathbf{k}$$

⑤ **assume** $ds \approx dx,\ \theta \approx \tan(\theta) = \dfrac{dy}{dx}$

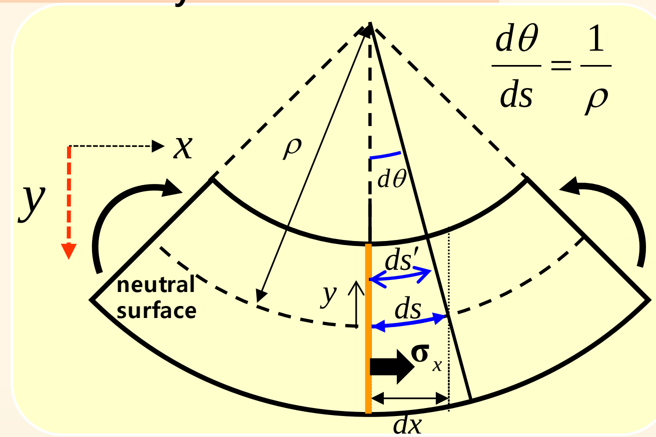
$$\frac{d\theta}{ds} = \frac{d}{ds}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \;\Rightarrow\; \frac{d\theta}{ds} = \frac{d^2y}{dx^2}$$

$$\mathbf{M} = -EI\frac{d^2y}{dx^2}\mathbf{k} \quad , M = -EI\frac{d^2y}{dx^2}$$

Deflection of Beam with Vector

what happen
if we take the direction of
y axis reversed?

$\boldsymbol{\sigma}_x \mathbf{i} = \sigma$, $\boldsymbol{\varepsilon}_x \mathbf{i} = \varepsilon$, $\boldsymbol{\theta}\mathbf{k} = \theta$, $\mathbf{y}\mathbf{j} = y$ $\sigma = E\varepsilon$



① strain at y in x-direction : $\varepsilon = \frac{(\rho + y)\cdot d\theta - \rho\cdot d\theta}{ds} = y\frac{d\theta}{ds}$

$\boldsymbol{\varepsilon}_x \mathbf{i} = \varepsilon$, $d\theta$: initial length, $y\cdot d\theta$: elongated length

② stress at y in x-direction : $\boldsymbol{\sigma}_x \mathbf{i} = \sigma \mathbf{i} = E\frac{y}{\rho}$

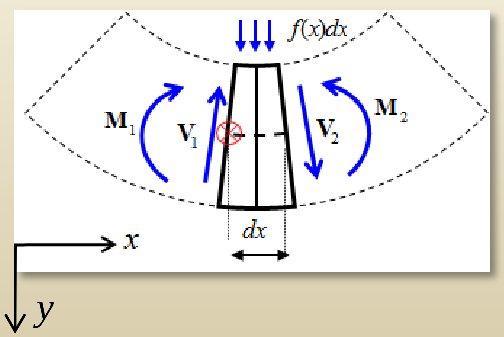
③ force acting on dA in x-direction : $d\mathbf{F} = E\frac{y}{\rho}dA\mathbf{i}$

④ moment about z-axis :

$$d\mathbf{M} = \mathbf{y}\times d\mathbf{F} = (y\mathbf{j})\times(E\frac{y}{\rho}dA\mathbf{i}) = -E\frac{y^2}{\rho}dA\mathbf{k} \quad \therefore \mathbf{M} = \int_A d\mathbf{M} = -\int_A E\frac{y^2}{\rho}dA\mathbf{k} = \frac{EI}{\rho}\mathbf{k} \quad , I = \int_A y^2 dA$$

⑤ assume $ds \approx dx,\ \theta \approx \tan(\theta) = \frac{dy}{dx}$ ⇨ $\frac{d\theta}{ds} = \frac{d^2y}{dx^2}$ ⇨ $\mathbf{M} = -\frac{EI}{\rho}\mathbf{k} = -EI\frac{d\theta}{ds}\mathbf{k}$ ⇨ $\mathbf{M} = -EI\frac{d^2y}{dx^2}\mathbf{k}$, $M = -EI\frac{d^2y}{dx^2}$

⑥ relationships between loads, shear forces, and bending moments



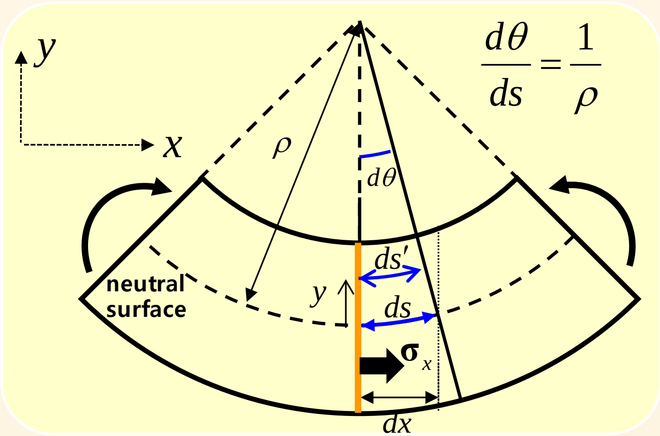
$$\mathbf{V}_1 = V\mathbf{j},\ \mathbf{V}_2 = -\left(V + \frac{\partial V}{\partial x}dx\right)\mathbf{j},\ \mathbf{M}_1 = -M\mathbf{k},\ \mathbf{M}_2 = \left(M + \frac{\partial M}{\partial x}dx\right)\mathbf{k}$$

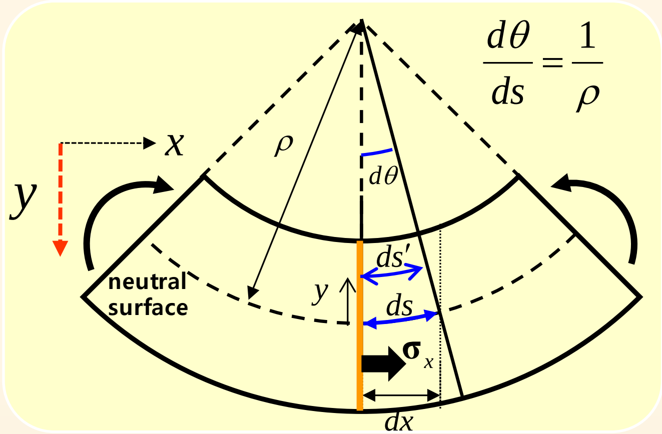
•force equilibrium $\frac{dV}{dx} = -f(x)$ •moment equilibrium $\frac{dM}{dx} = V(x)$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \rightarrow \frac{d^3y}{dx^3} = -\frac{1}{EI}\cdot\frac{dM}{dx} = -\frac{1}{EI}\cdot V(x) \rightarrow \frac{d^4y}{dx^4} = -\frac{1}{EI}\cdot\frac{dV}{dx} = \frac{1}{EI}\cdot f(x)$$

$$\therefore EI\frac{d^4y}{dx^4} = f(x)$$

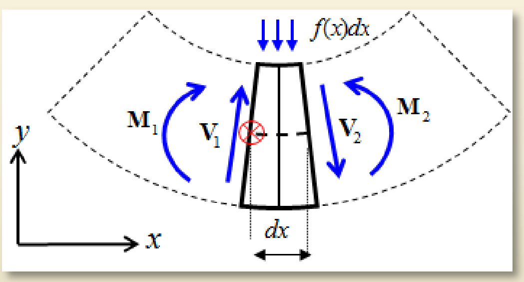
Deflection of Beam with Vector



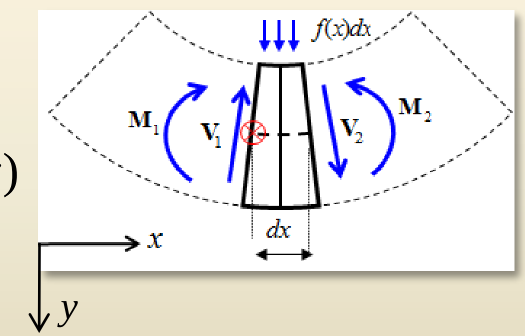


$$M = EI\frac{d^2y}{dx^2}$$

$$M = -EI\frac{d^2y}{dx^2}$$



$$\frac{dV}{dx} = -f(x) \quad , \frac{dM}{dx} = V(x)$$



$$\therefore EI\frac{d^4y}{dx^4} = -f(x)$$

$$\therefore EI\frac{d^4y}{dx^4} = f(x)$$

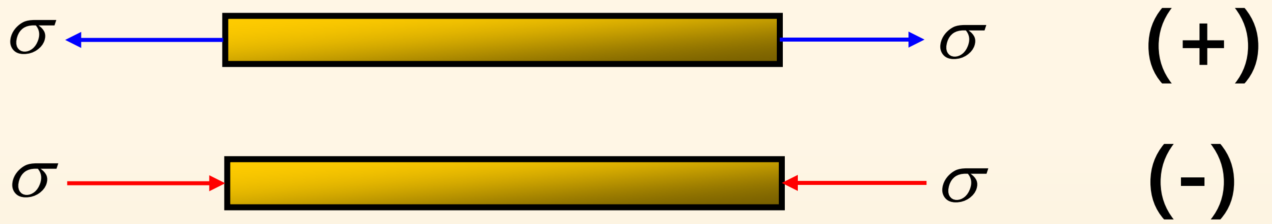
Differential Equation for Deflection of Beam in conventional notation

References : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006	
Sign Convention for Normal Stress	Sec. 1.2 p4
Deformation Sign Convention and Static Sign Convention	Sec. 4.3, p270~p271
Curvature Sign Convention	Sec. 5.3, p303
Differential Equation of the Deflection Curve	Sec. 9.2, p594~p599

Sign Convections and Deflection of Beams

Sign Convention for Normal Stress*

Definition
Coordinates independent



When a sign convention for normal stresses is required, it is customary to define...

tensile stress as positive and

compressive stresses as negative

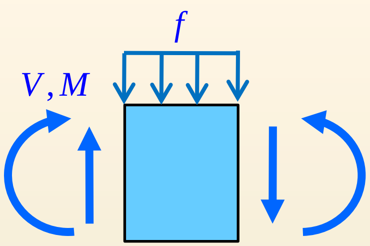
*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, Sec.1.2 p4

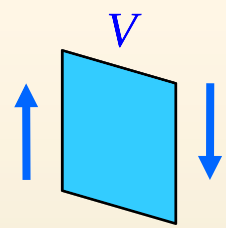
Deformation Sign Conventions*

Definition
Coordinates independent

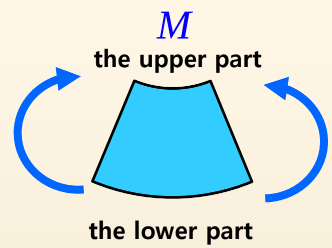
-"the algebraic sign of a stress resultant is determined by <u>how it deforms</u> the material on which it acts rather than by its direction in space"

Positive
-Shear,
-Bending Moment,
-Intensity of distributed load

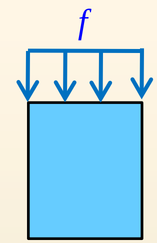




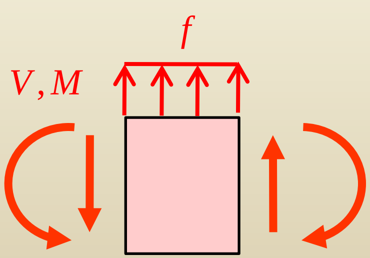
a positive shear force acts clockwise against the material

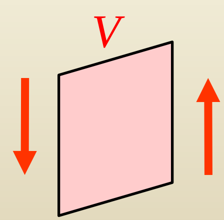


a positive bending <u>compresses</u> the upper part of the beam

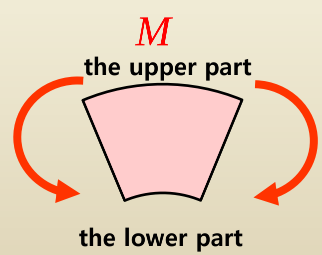


Negative
-Shear,
-Bending Moment,
-Intensity of distributed load

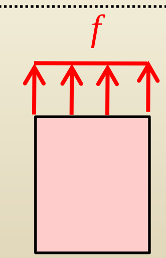




a negative shear force acts counterclockwise against the material



a negative bending <u>compresses</u> the lower part of the beam



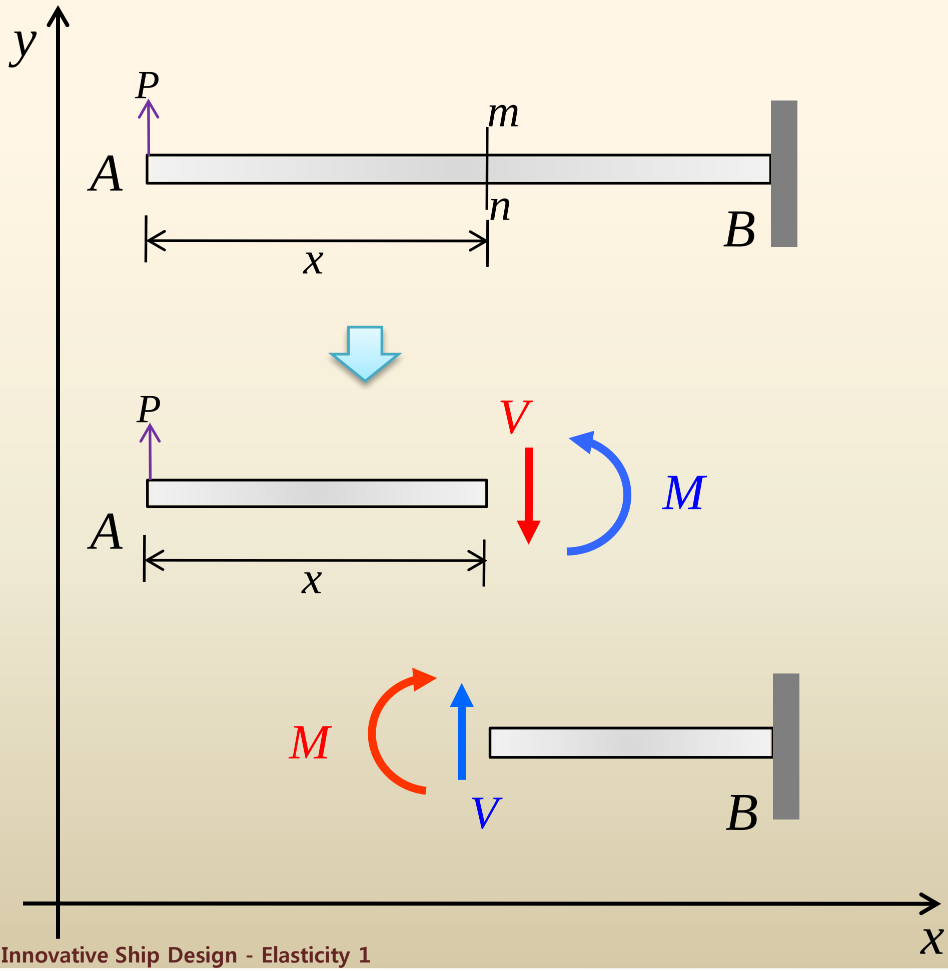
*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, Sec.4.3 p271
*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, Sec.9.2 p598 (figure9-4)

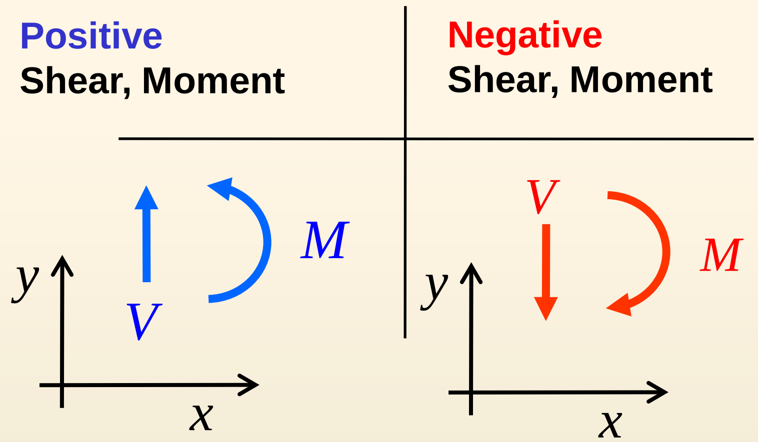
Static Sign Conventions*

Definition
Coordinates dependent

-when writing <u>equations of equilibrium</u>

-forces are positive or negative <u>according to their direction along the coordinate axes</u>





*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, Sec.4.3 p271

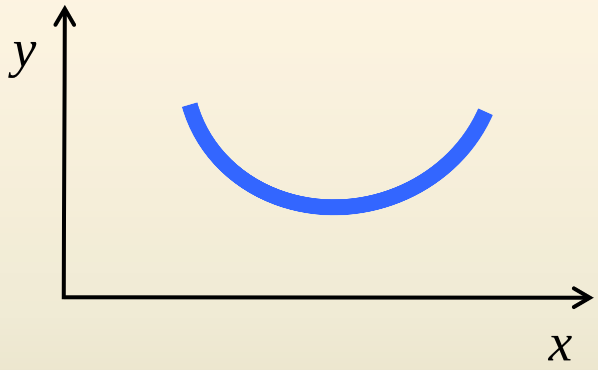
Curvature Sign Conventions

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \cong \frac{d\theta}{dx}$$

Definition
Coordinates dependent

-The sign convention for curvature depends upon the orientation of the coordinate axes*
-Curvature is positive when the angle of rotation increase as moving along the beam in the positive x-direction

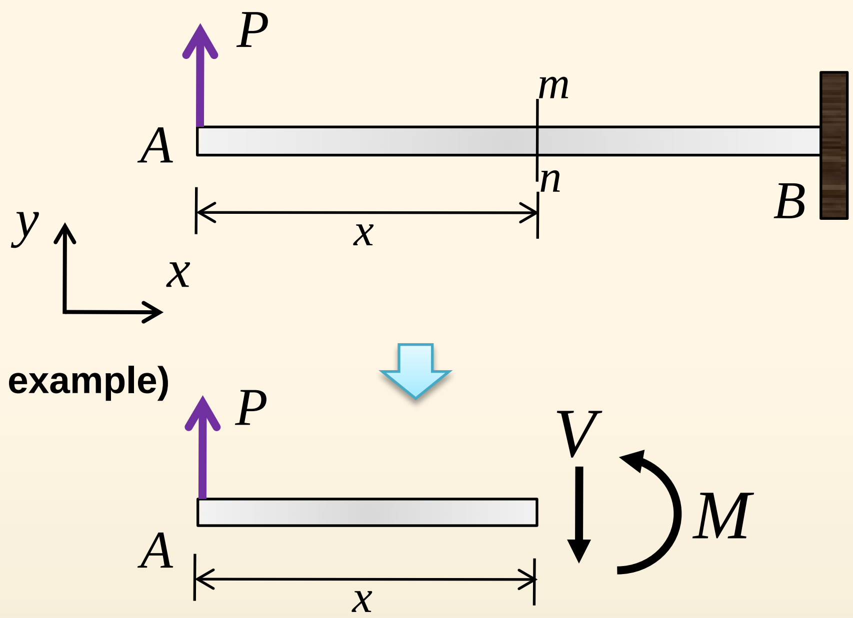
Positive Curvature

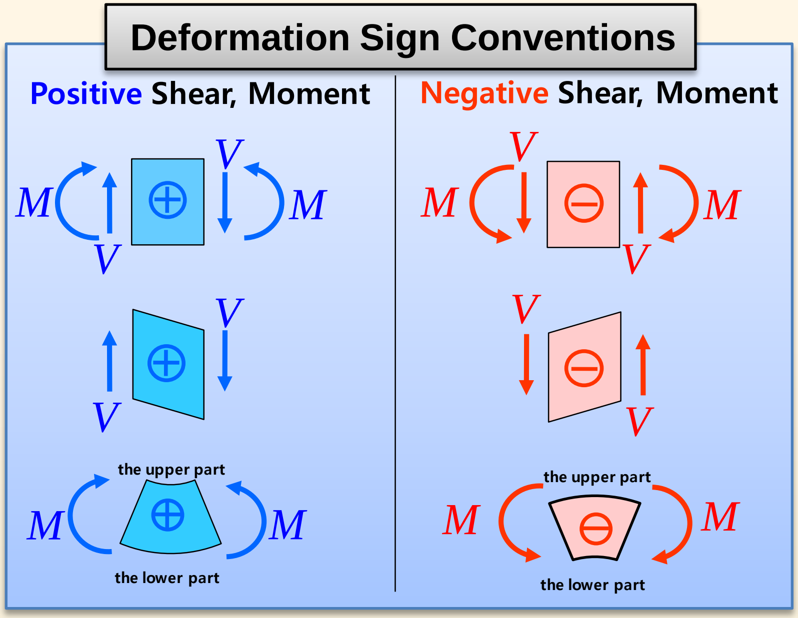


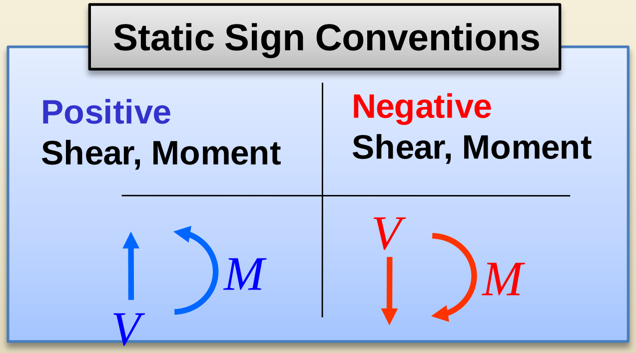
Negative Curvature

*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, Sec.5.3 p303

Free-Body Diagram & Convention



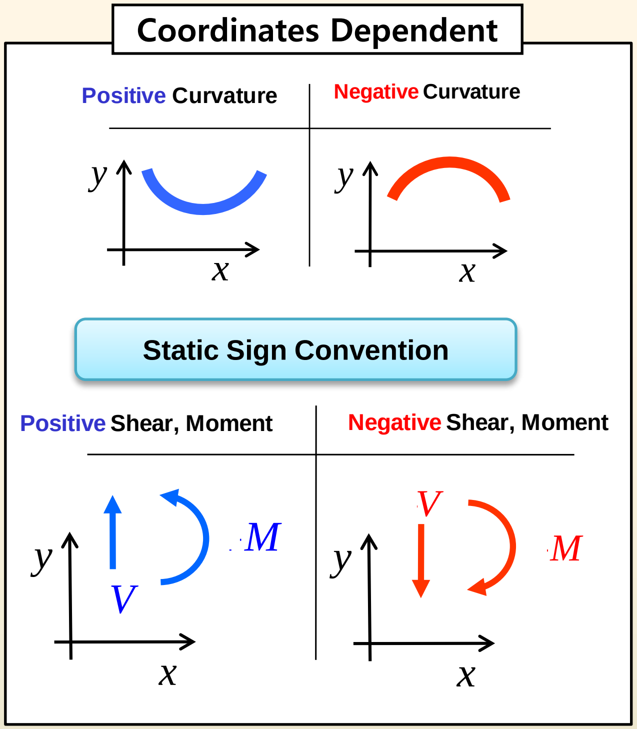


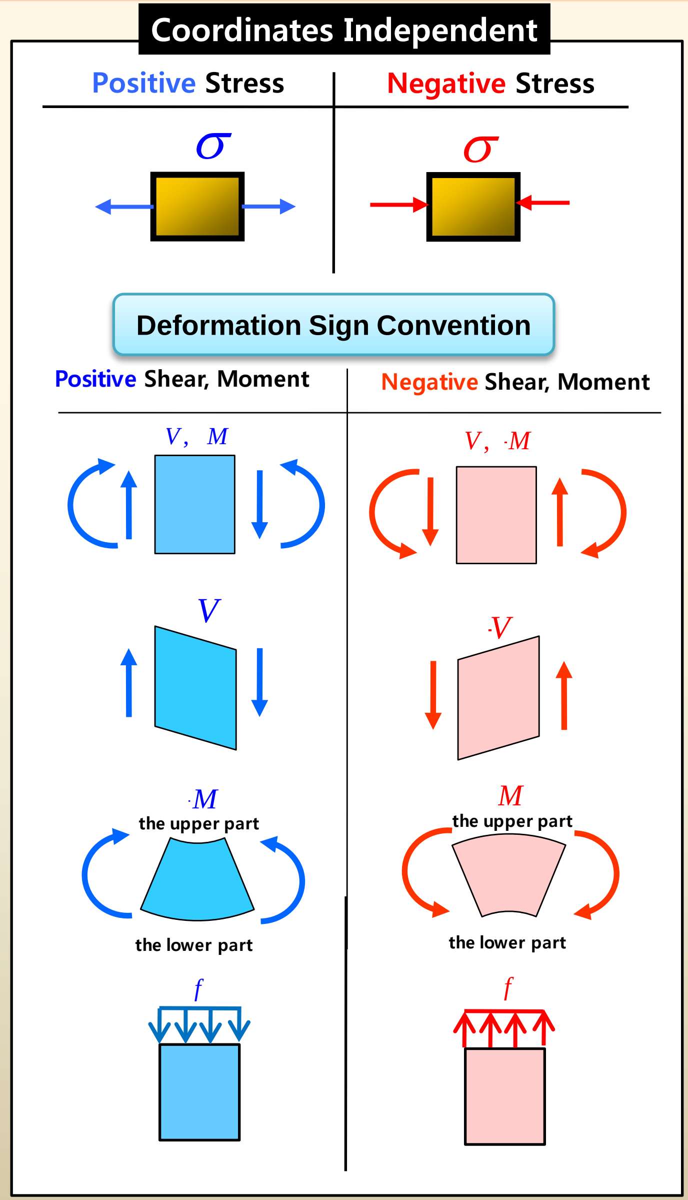


The shear force V
(which is a positive shear force) is given
a negative sign because it acts downward

This example shows the distinction between
the deformation sign convention used for shear force and
the static sign convention used in the equation of equilibrium

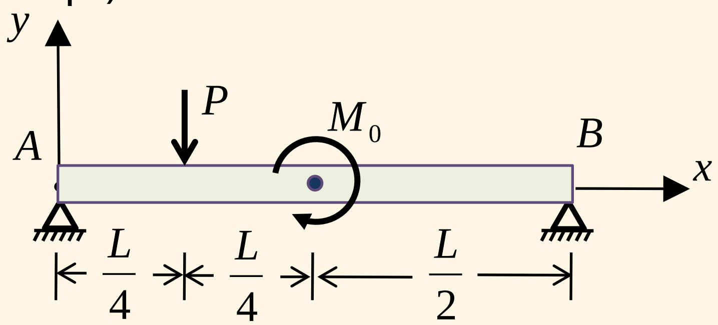
Comparison of Sign Conventions



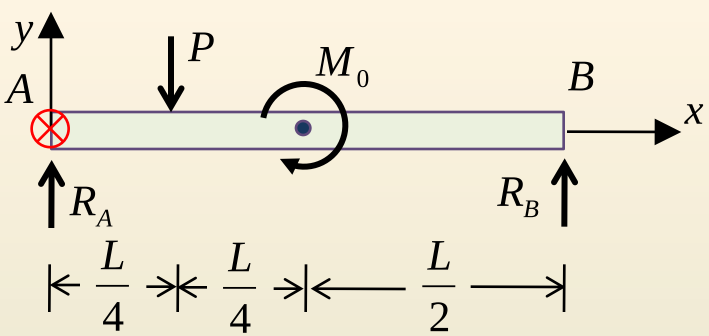


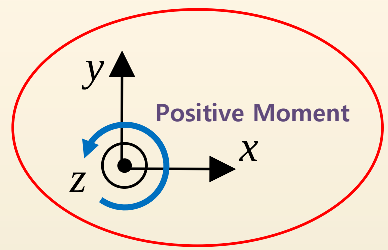
Comparison of Sign Conventions

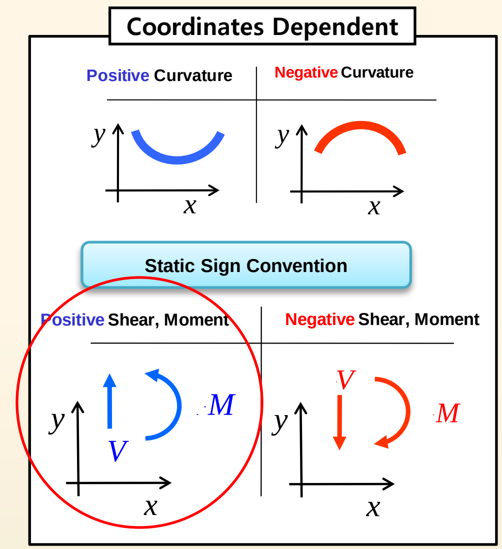
Example)



1) Reaction (Free-body diagram)





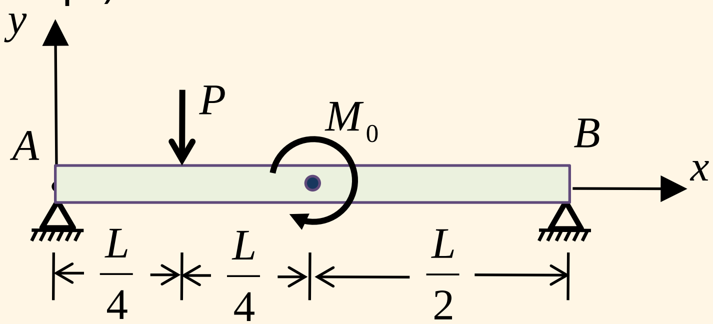


Moment Equilibrium at A

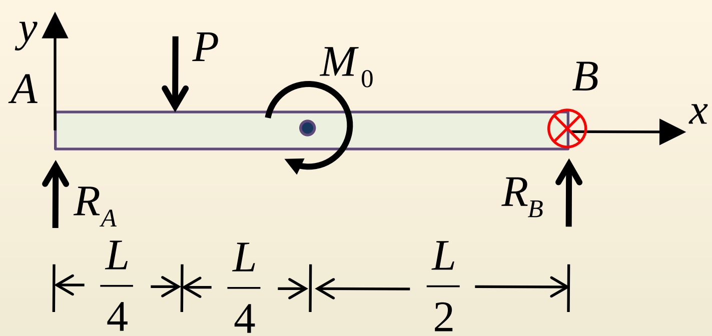
$$\sum M_{z\ \text{at A}} = -\ P\cdot\frac{L}{4} - M_0 + R_B\cdot L = 0 \qquad \therefore R_B = \frac{P}{4} + \frac{M_0}{L}$$

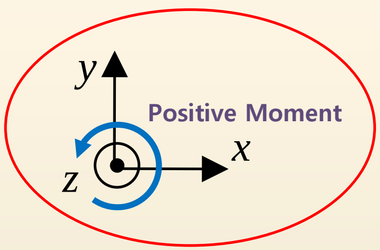
Comparison of Sign Conventions

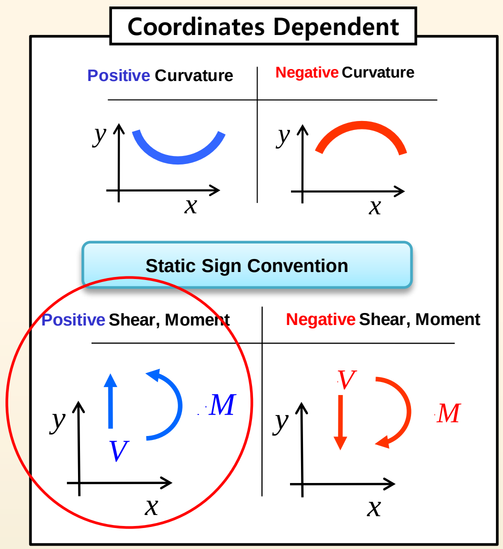
Example)



1) Reaction (Free-body diagram)







Moment Equilibrium at A

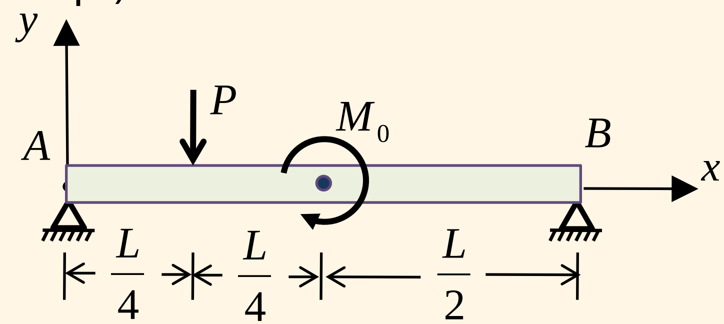
$$\sum M_{z \text{ at A}} = - P\cdot\frac{L}{4} - M_0 + R_B\cdot L = 0 \qquad \therefore R_B = \frac{P}{4}+\frac{M_0}{L}$$

Moment Equilibrium at B

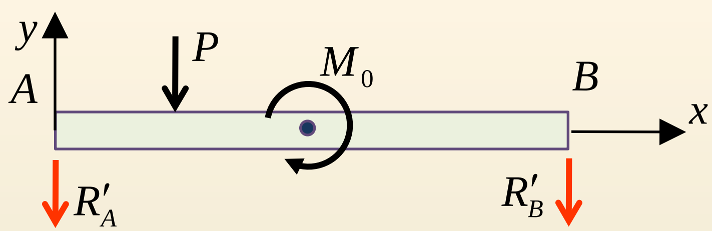
$$\sum M_{z \text{ at B}} = + P\cdot\frac{3L}{4} - M_0 - R_A\cdot L = 0 \qquad \therefore R_A = \frac{3P}{4}-\frac{M_0}{L}$$

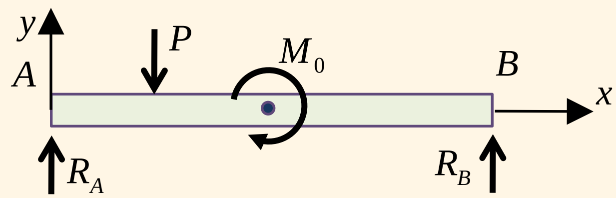
Comparison of Sign Conventions

Example)



1) Reaction (Free-body diagram)

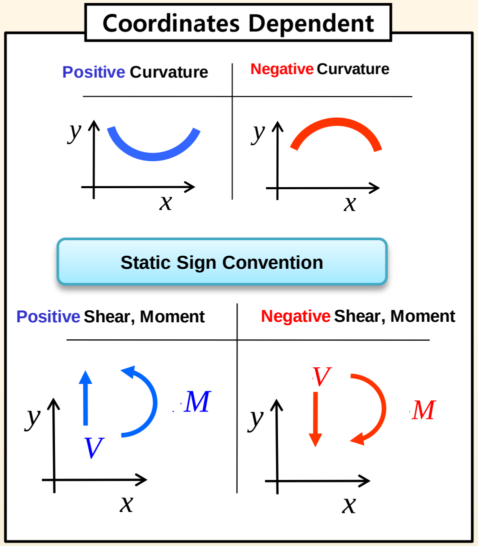




$$\sum M_{z\ \text{at A}} = -P\cdot\frac{L}{4} - M_0 + R_B\cdot L \quad \therefore R_B = \frac{P}{4} + \frac{M_0}{L}$$

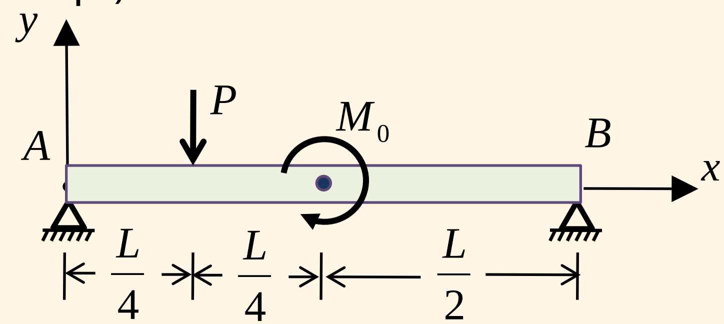
$$\sum M_{z\ \text{at B}} = +P\cdot\frac{3L}{4} - M_0 - R_A\cdot L \quad \therefore R_A = \frac{3P}{4} - \frac{M_0}{L}$$

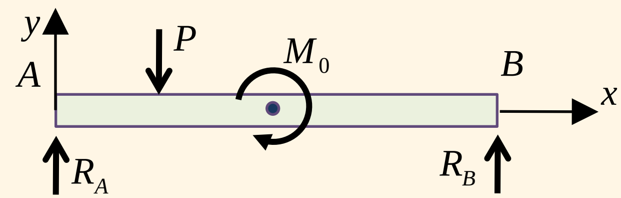
What happen if the direction is assumed to be opposite?



Comparison of Sign Conventions

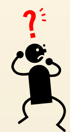
Example)



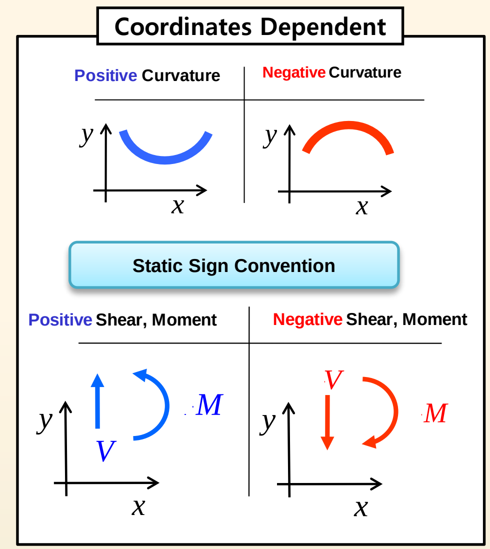


$$\sum M_{z\ \text{at A}} = -P\cdot\frac{L}{4} - M_0 + R_B\cdot L \quad \therefore R_B = \frac{P}{4} + \frac{M_0}{L}$$

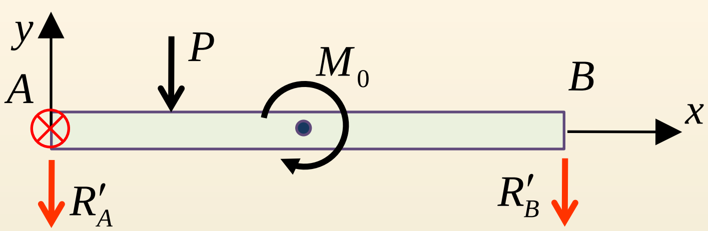
$$\sum M_{z\ \text{at B}} = +P\cdot\frac{3L}{4} - M_0 - R_A\cdot L \quad \therefore R_A = \frac{3P}{4} - \frac{M_0}{L}$$



What happen if the direction is assumed to be opposite?



1) Reaction (Free-body diagram)

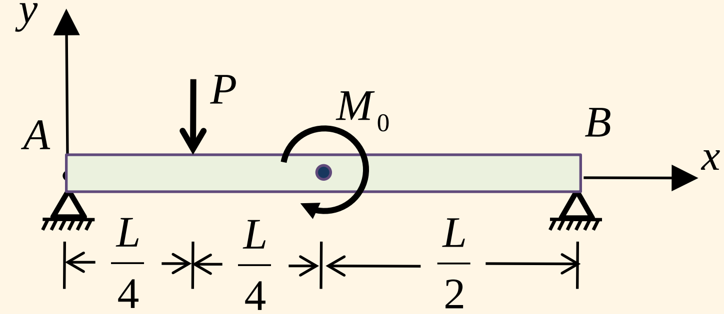


Moment Equilibrium at A

$$\sum M_{z\ \text{at A}} = -\ P\cdot\frac{L}{4} - M_0 \boxed{-}\ R'_B\cdot L = 0 \qquad \therefore R'_B = \boxed{-}\left(\frac{P}{4} + \frac{M_0}{L}\right)$$

Comparison of Sign Conventions

Example)



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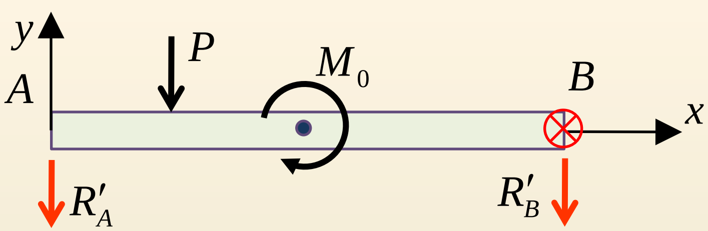
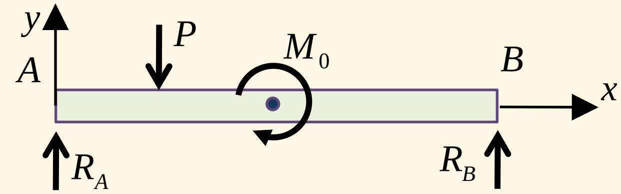
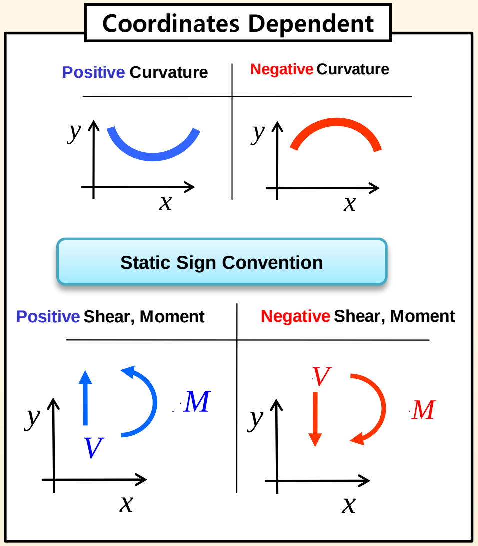




$$\sum M_{z \text{ at A}} = -P\cdot\frac{L}{4} - M_0 + R_B\cdot L \quad \therefore R_B = \frac{P}{4} + \frac{M_0}{L}$$

$$\sum M_{z \text{ at B}} = +P\cdot\frac{3L}{4} - M_0 - R_A\cdot L \quad \therefore R_A = \frac{3P}{4} - \frac{M_0}{L}$$

What happen if the direction is assumed to be opposite?



Moment Equilibrium at A

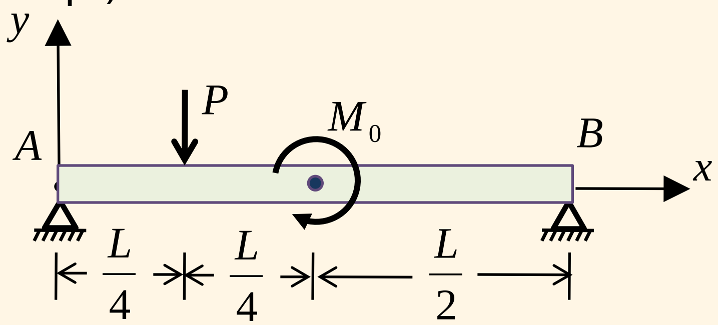
$$\sum M_{z \text{ at A}} = -\,P\cdot\frac{L}{4} - M_0 \boxed{-} R'_B\cdot L = 0 \qquad \therefore R'_B = \boxed{-}\left(\frac{P}{4} + \frac{M_0}{L}\right)$$

Moment Equilibrium at B

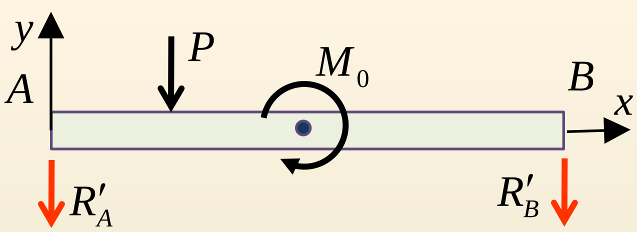
$$\sum M_{z \text{ at B}} = +P\cdot\frac{3L}{4} - M_0 \boxed{+} R'_A\cdot L = 0 \qquad \therefore R'_A = \boxed{-}\left(\frac{3P}{4} - \frac{M_0}{L}\right)$$

Comparison of Sign Conventions

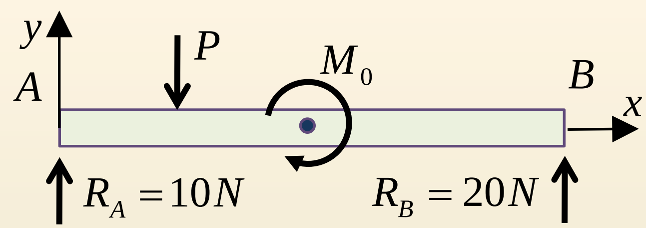
Example)

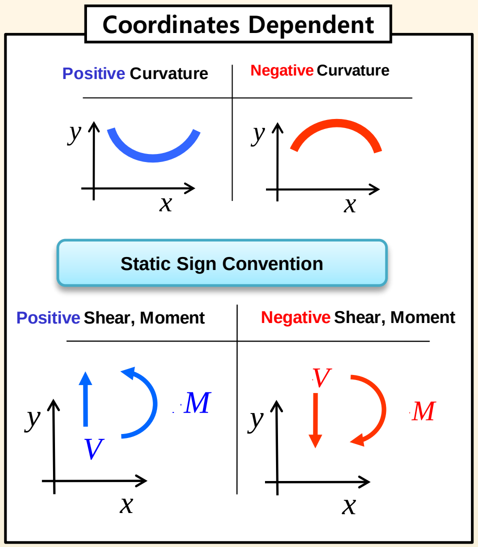


1) Reaction (Free-body diagram)



What happen if the direction is assumed to be opposite?





$$R'_B = -\left(\frac{P}{4} + \frac{M_0}{L}\right)$$

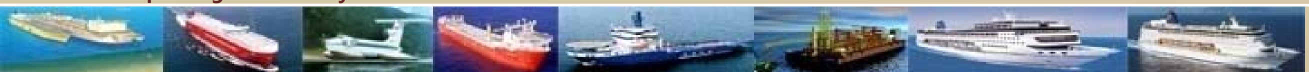
$$R'_A = -\left(\frac{3P}{4} - \frac{M_0}{L}\right)$$

$$R_B = \frac{P}{4} + \frac{M_0}{L}$$

$$R_A = \frac{3P}{4} - \frac{M_0}{L}$$

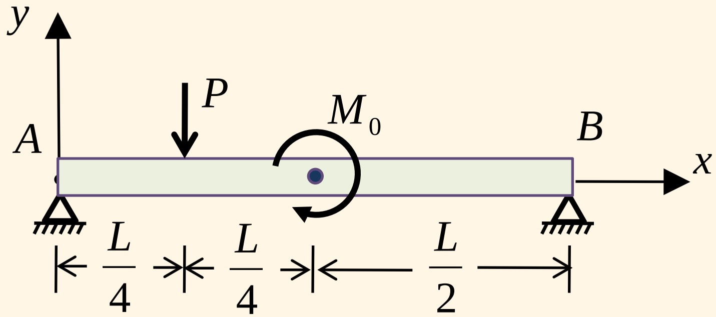
$$R'_B = -R_B$$

$$R'_A = -R_A$$



Comparison of Sign Conventions

Example)



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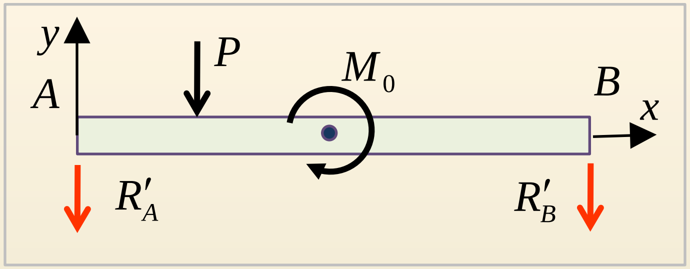
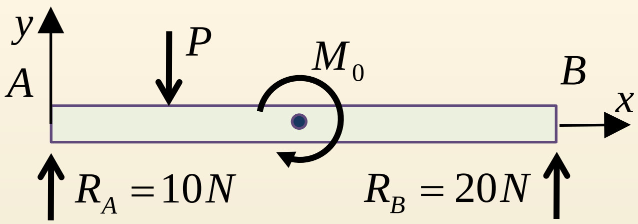


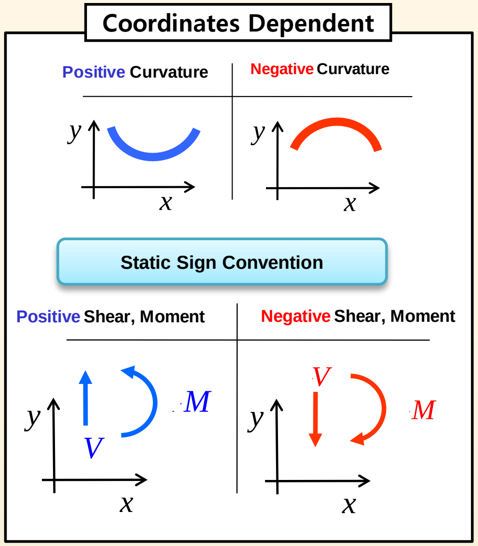
What happen if the direction is assumed to be opposite?



for instance, $R_A = 10N, \; R_B = 20N$

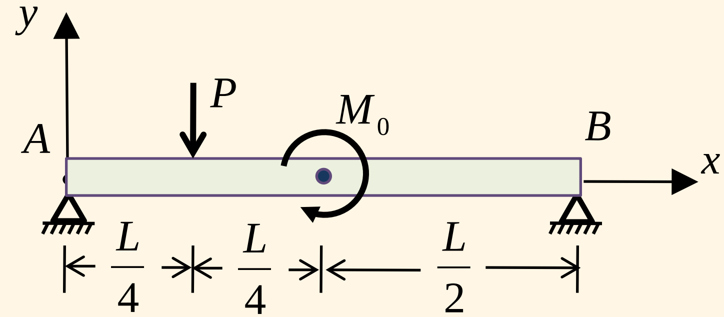
$$R'_B = -20N, \; R'_A = -10N$$

$$R'_A = -R_A, \; R'_B = -R_B$$

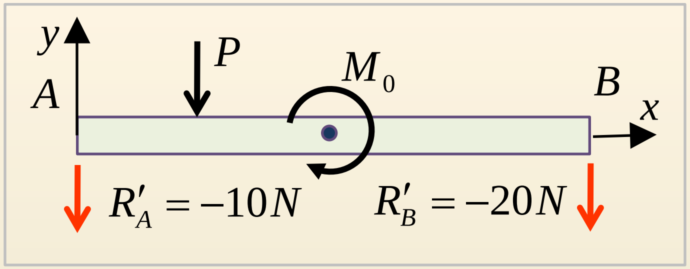


Comparison of Sign Conventions

Example)

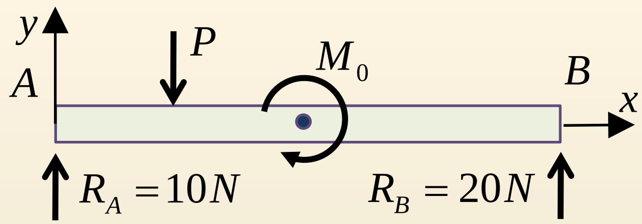


1) Reaction (Free-body diagram)

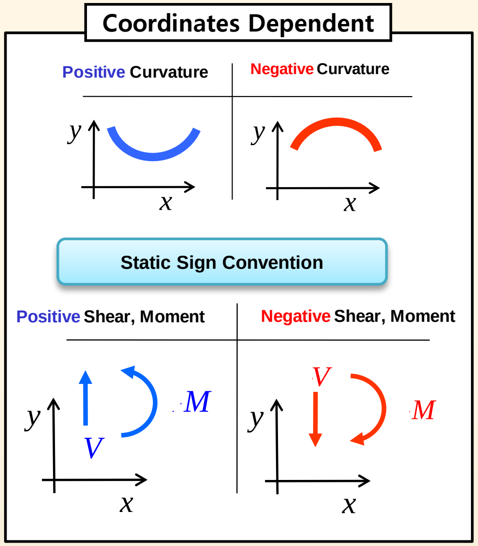


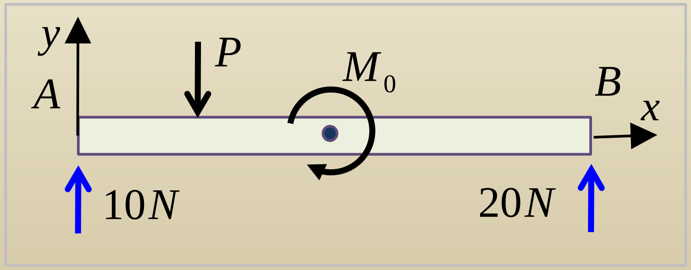
$$R'_A = -R_A,\ R'_B = -R_B$$

What happen if the direction is assumed to be opposite?



for instance, $R_A = 10N,\ R_B = 20N$

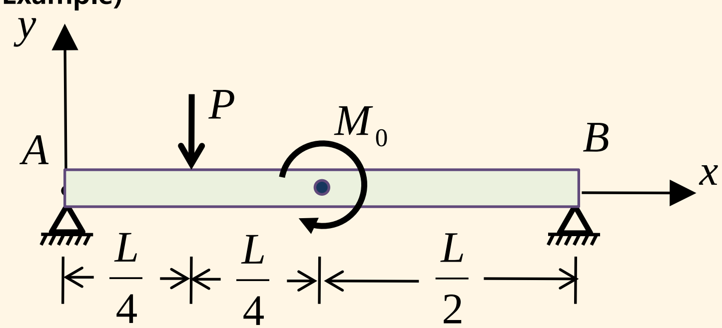




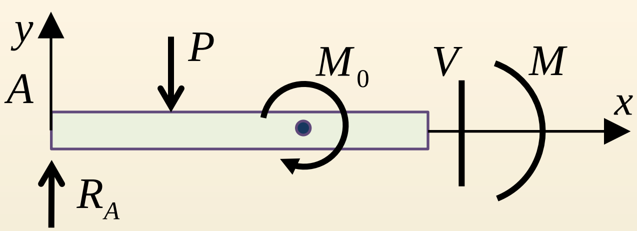
the same

Comparison of Sign Conventions

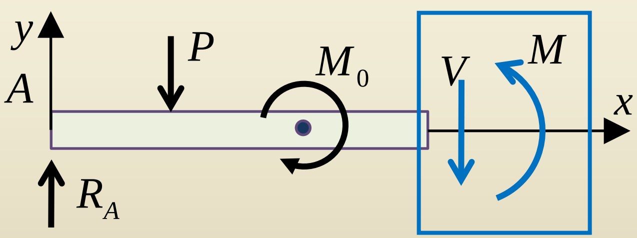
Example)



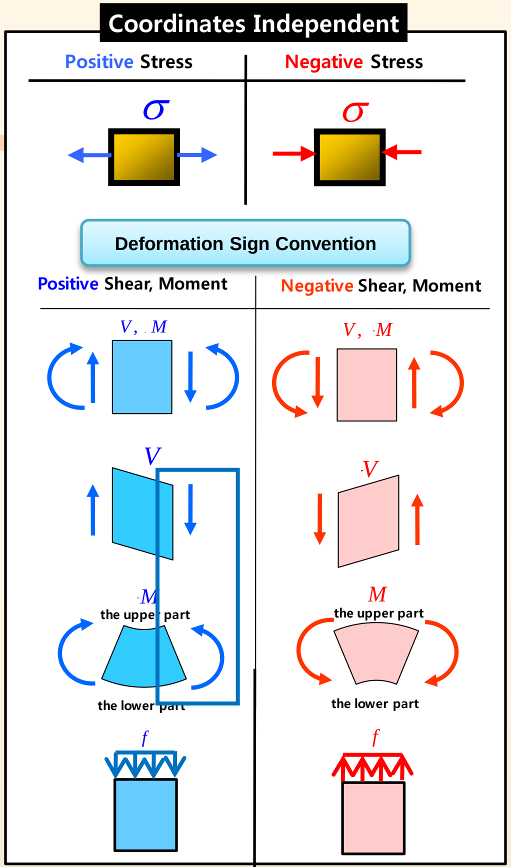
2) Shear force and bending moment at x (Free-body diagram)

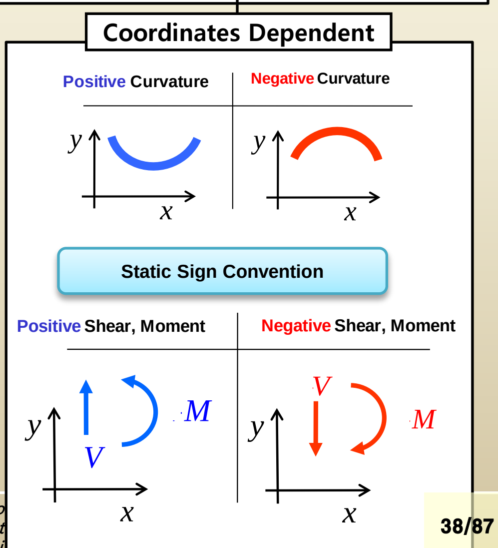


Which direction can be assumed for shear force and bending moment?



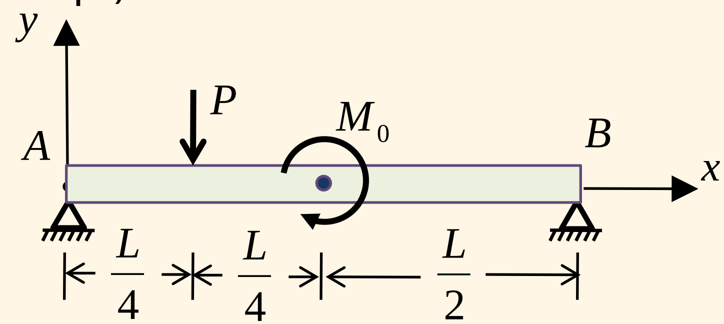
Let us assume the direction with which they have positive deform





Comparison of Sign Conventions

Example)



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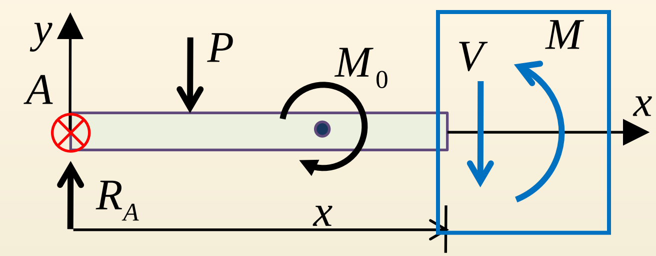
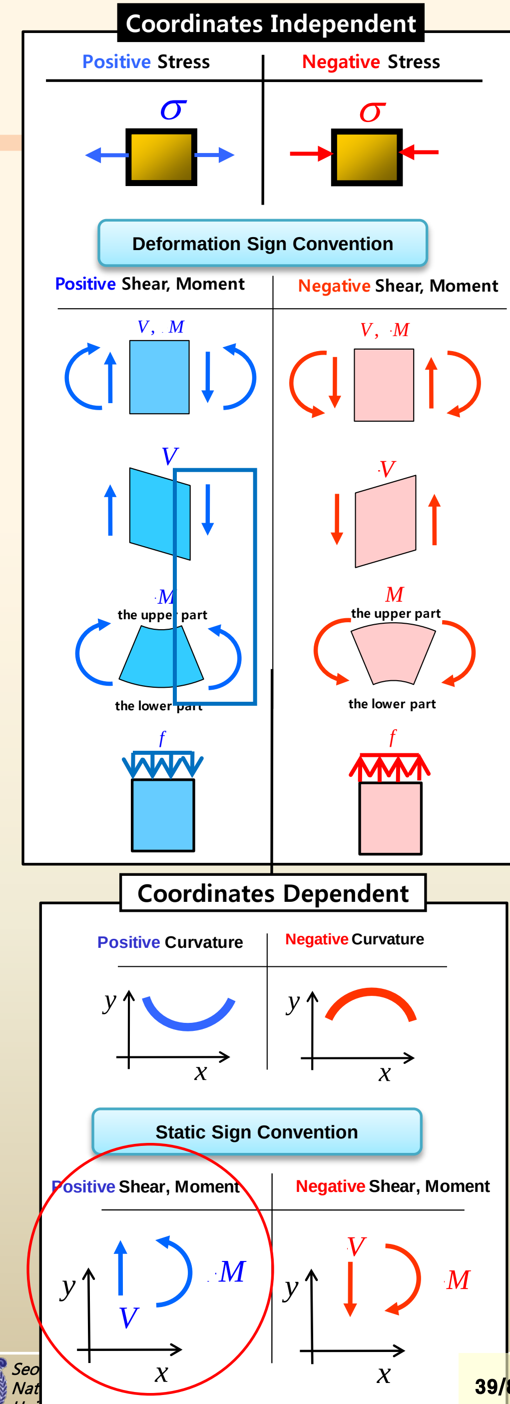
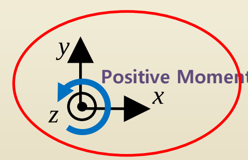


Let us assume the direction with which they have positive deform



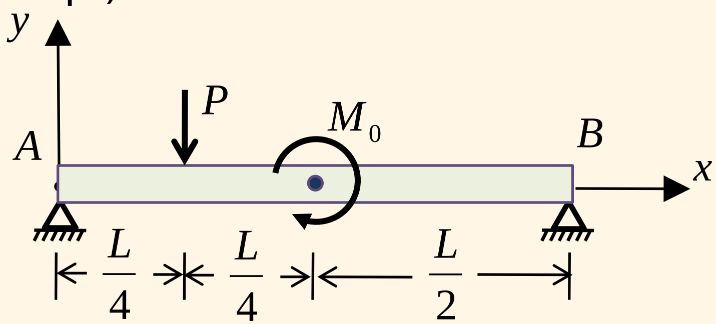
Force Equilibrium

$$\sum F_y = +R_A - P - V = 0 \qquad \therefore V = R_A - P$$

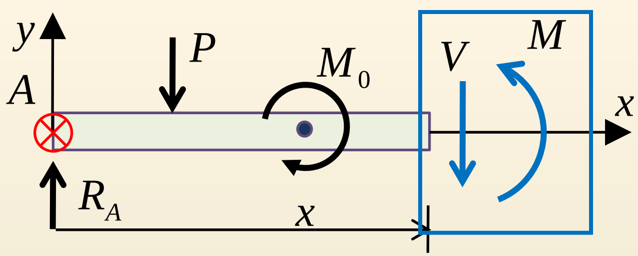


Comparison of Sign Conventions

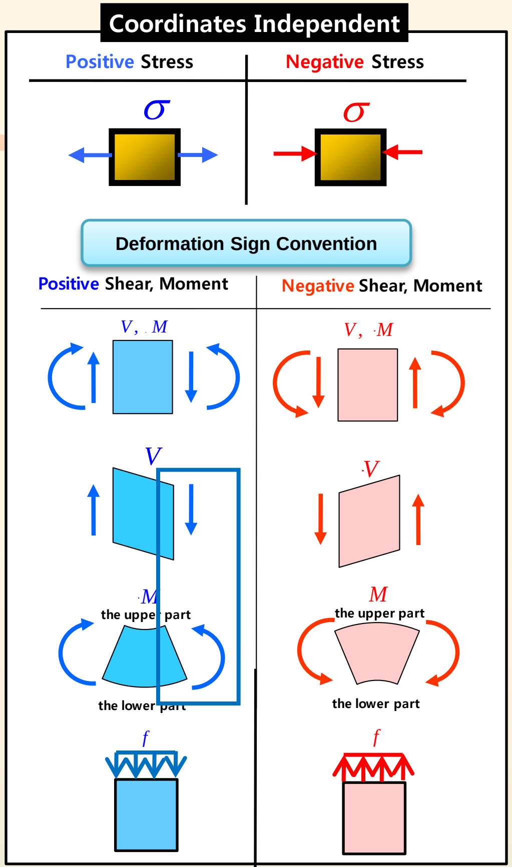
Example)



2) Shear force and bending moment at x (Free-body diagram)

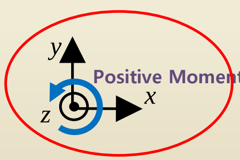


Let us assume the direction with which they have positive deform



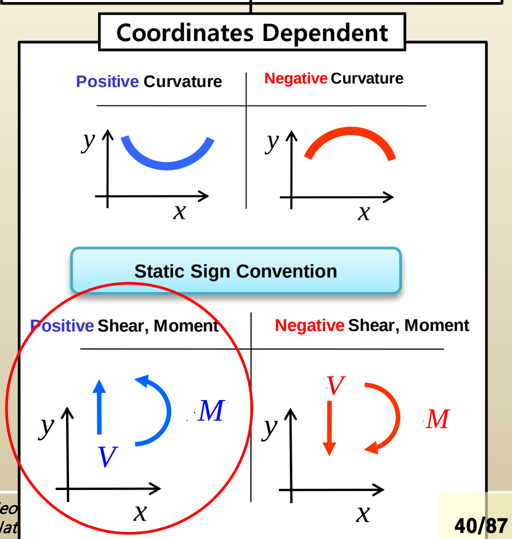
Force Equilibrium

$$\sum F_y = +R_A - P - V = 0 \qquad \therefore V = R_A - P$$



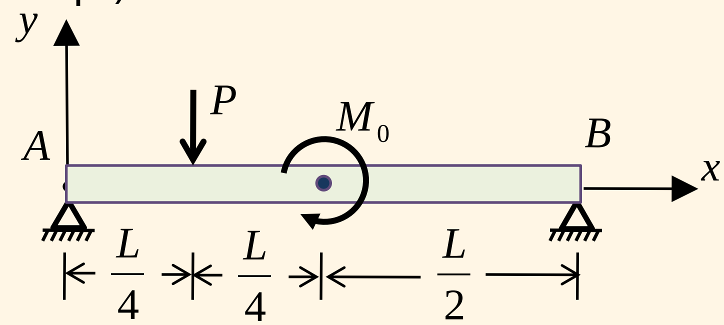
Moment Equilibrium at A

$$\sum M_{z\ \text{at A}} = -P\cdot\frac{L}{4} - M_0 - V\cdot x + M = 0 \qquad \therefore M = \frac{PL}{4} + M_0 + V\cdot x$$

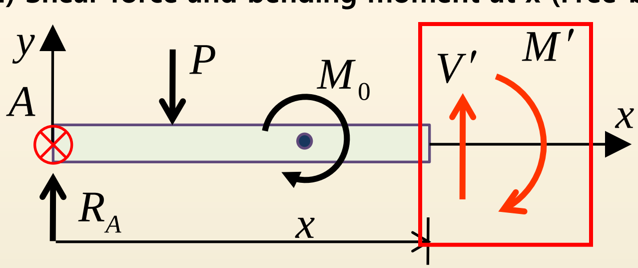


Comparison of Sign Conventions

Example)

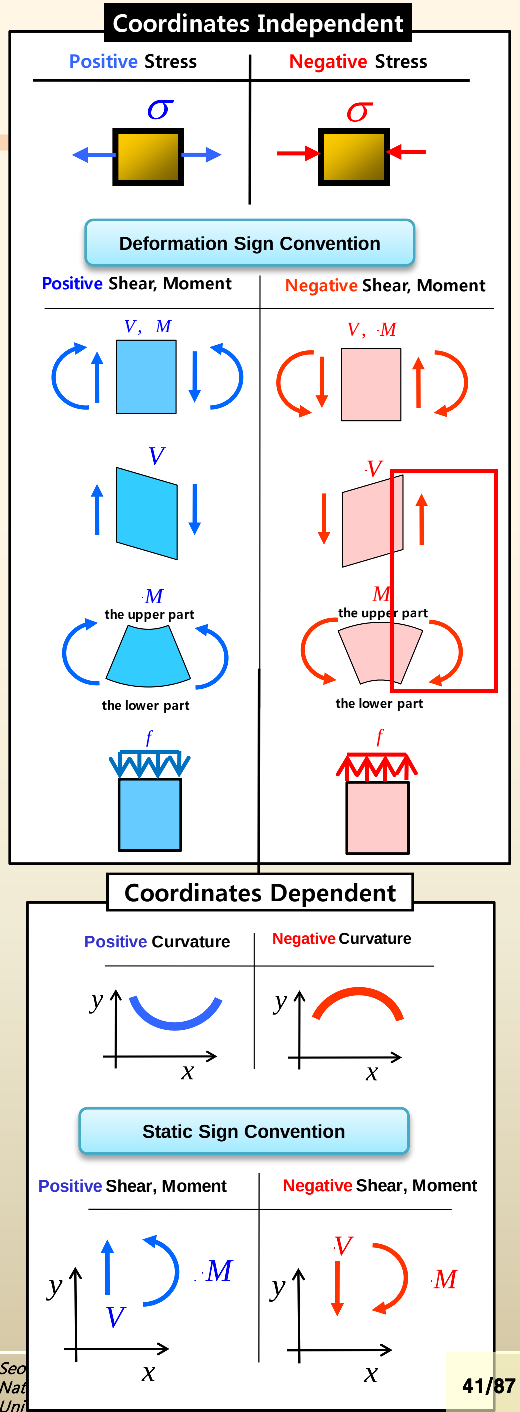


2) Shear force and bending moment at x (Free-body diagram)



What happen if the direction is assumed to be opposite?

Let us assume the direction with which they have negative deform



Comparison of Sign Conventions

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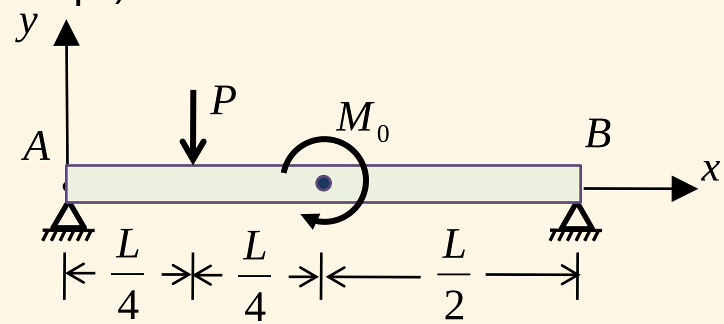

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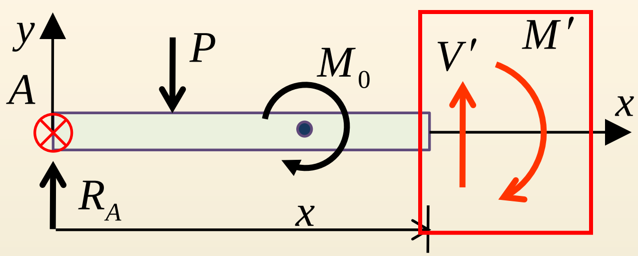
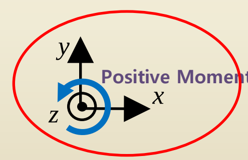


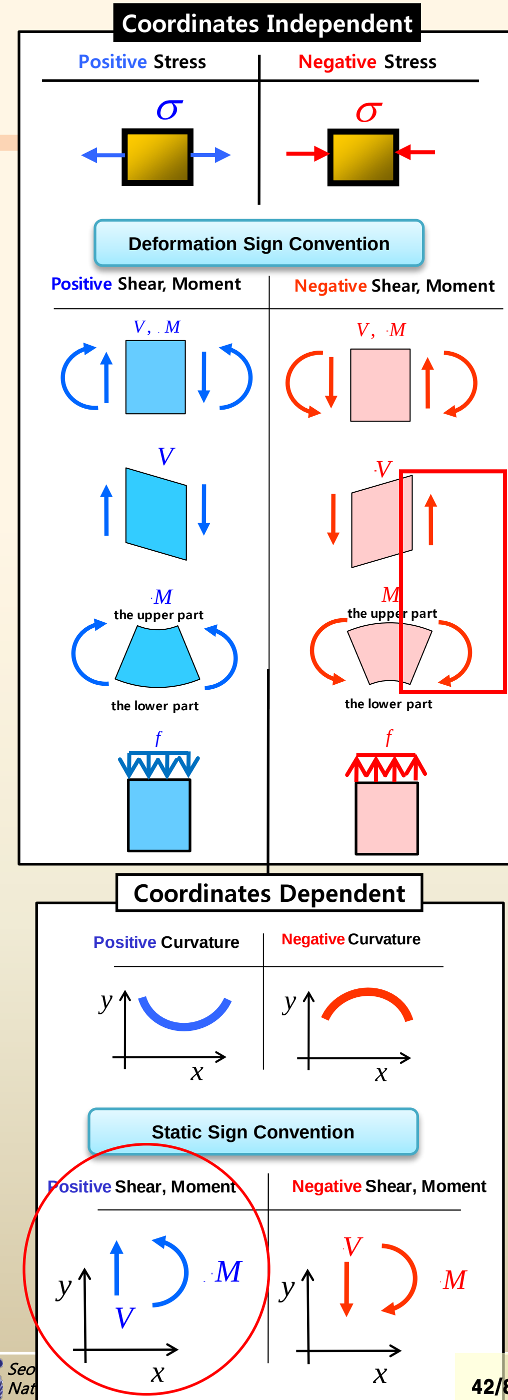
What happen if the direction is assumed to be opposite?

Let us assume the direction with which they have <u>negative</u> deform

Force Equilibrium

$$\sum F_y = +R_A - P + V' = 0 \qquad \therefore V' = -(R_A - P)$$





Comparison of Sign Conventions

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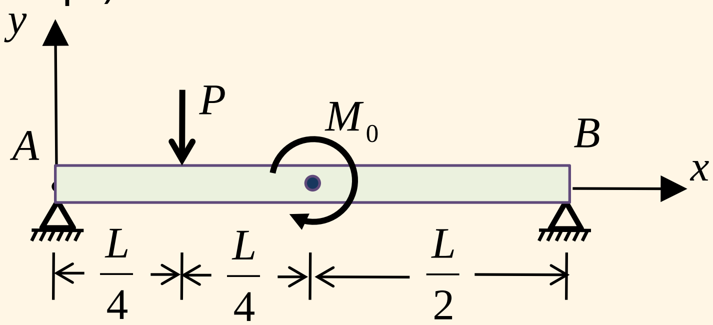

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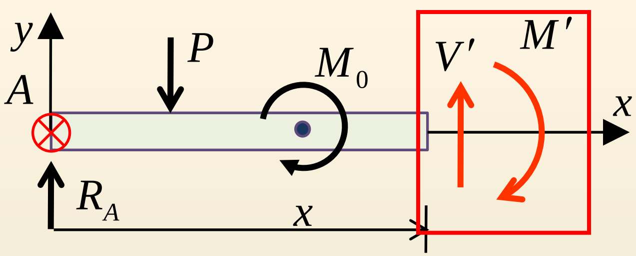

What happen if the direction is assumed to be opposite?

Let us assume the direction with which they have negative deform

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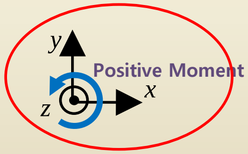
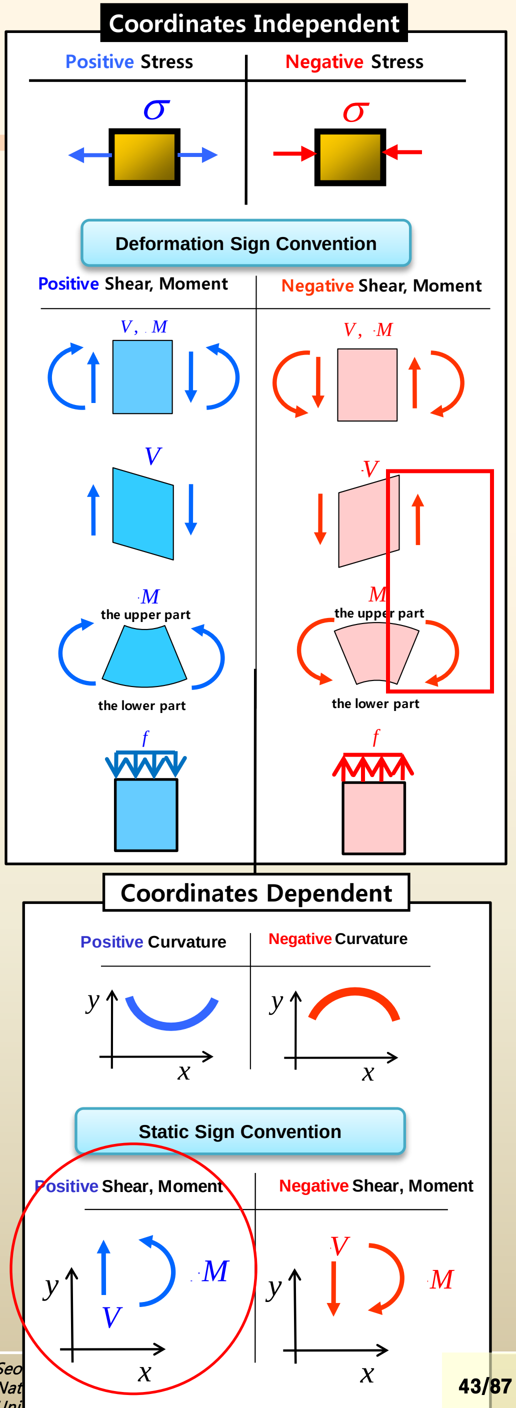
$$\sum F_y = +R_A - P + V' = 0 \qquad \therefore V' = -(R_A - P)$$



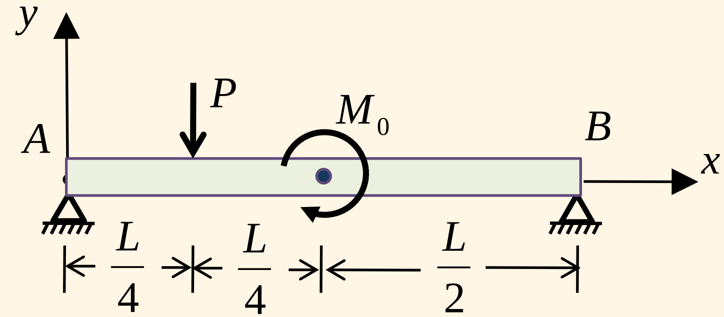
Moment Equilibrium at A

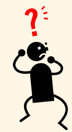
$$\sum M_{z \text{ at A}} = -P \cdot \frac{L}{4} - M_0 + V' \cdot x - M' = 0 \quad \therefore M' = -\frac{PL}{4} - M_0 + V' \cdot x$$



Comparison of Sign Conventions

Example)





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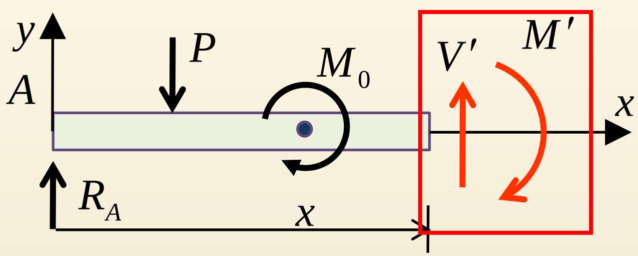
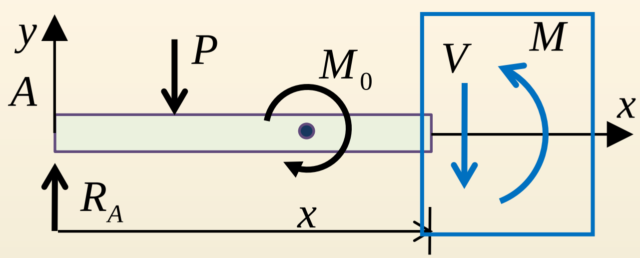
2) Shear force and bending moment at x (Free-body diagram)





$$V' = -(R_A - P)$$

$$M' = -\frac{PL}{4} - M_0 + V' \cdot x$$

$$M' = -\frac{PL}{4} - M_0 - (R_A - P) \cdot x$$

$$= -\left(\frac{PL}{4} + M_0 + (R_A - P) \cdot x\right)$$

$$= -M$$

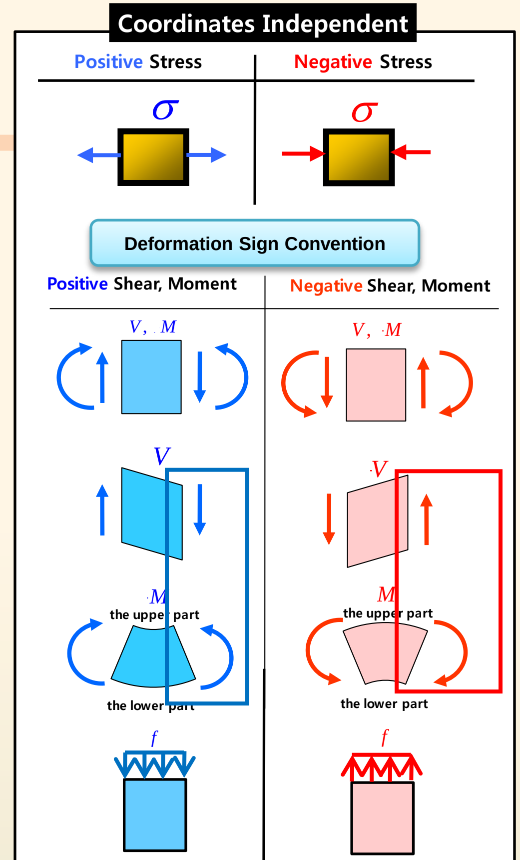
$$V = R_A - P$$

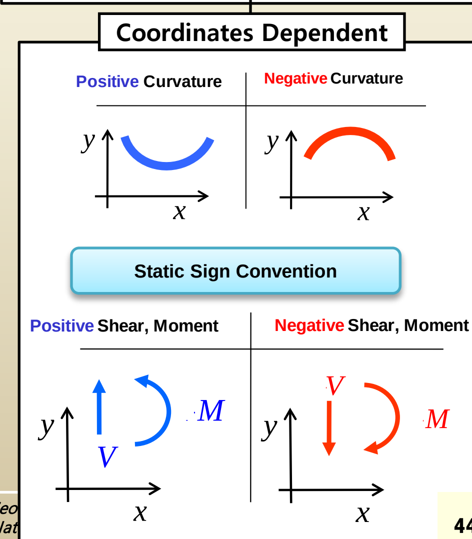
$$M = \frac{PL}{4} + M_0 + V \cdot x$$

$$M = \frac{PL}{4} + M_0 + (R_A - P) \cdot x$$

$$V' = -V$$

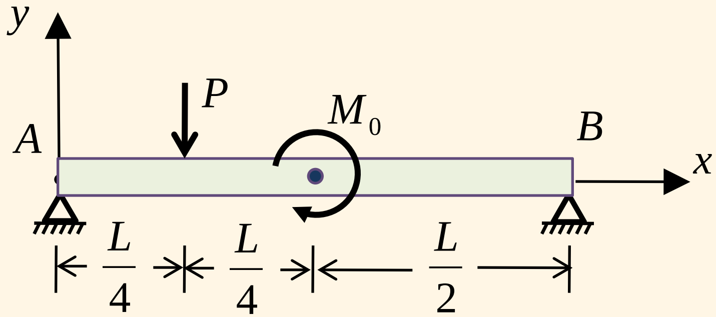
$$M' = -M$$





Comparison of Sign Conventions

Example)



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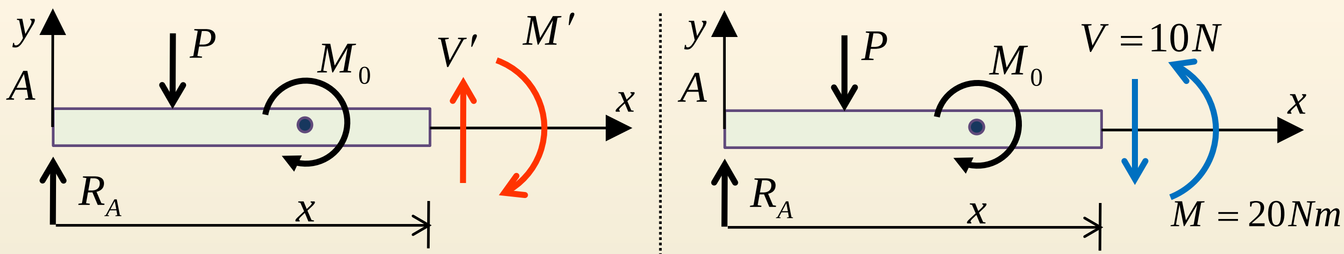
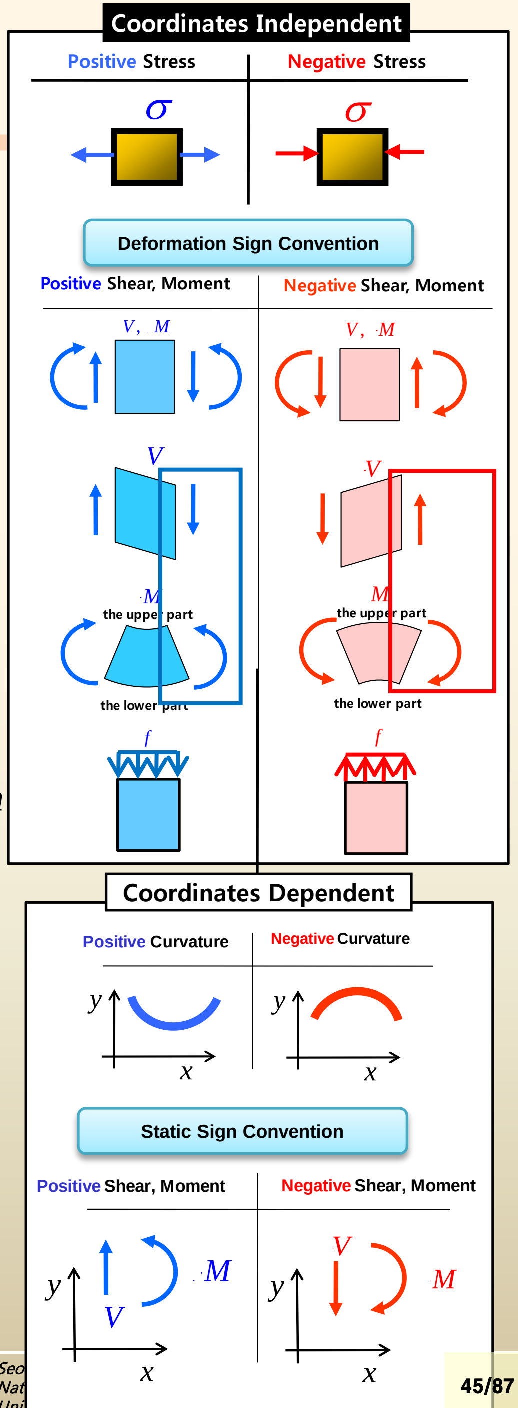


for instance $V = 10N, M = 20Nm$

$$V' = -10N,\ M' = -20Nm$$

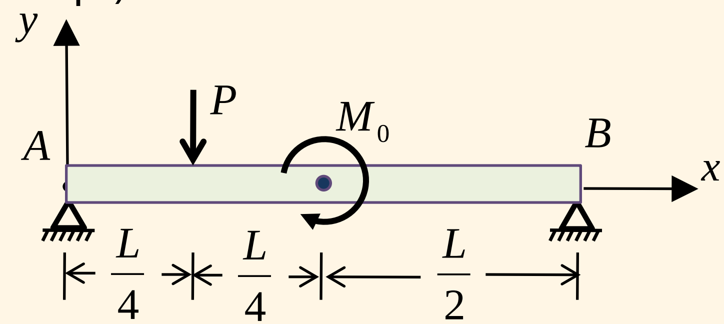
$$V' = -V$$

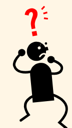
$$M' = -M$$



Comparison of Sign Conventions

Example)





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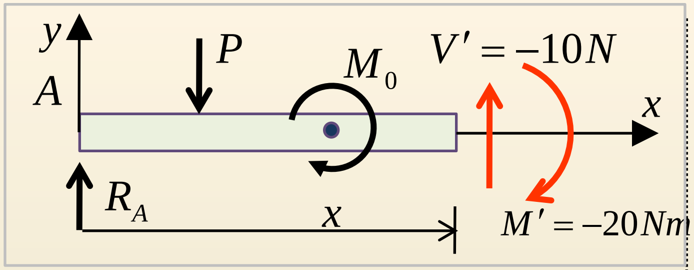

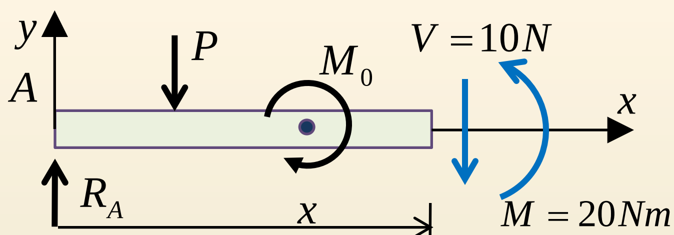

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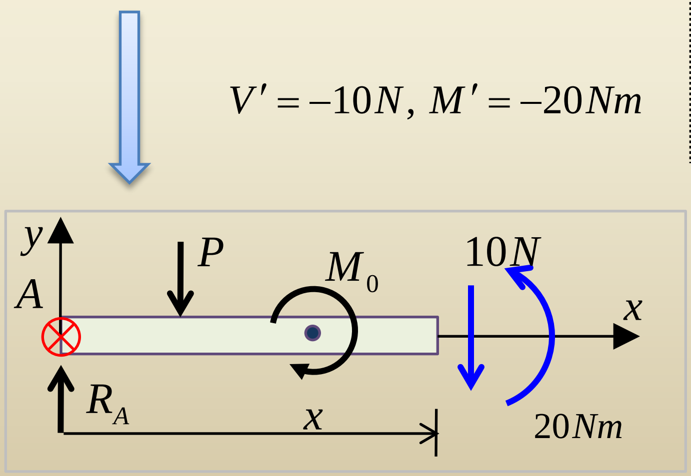
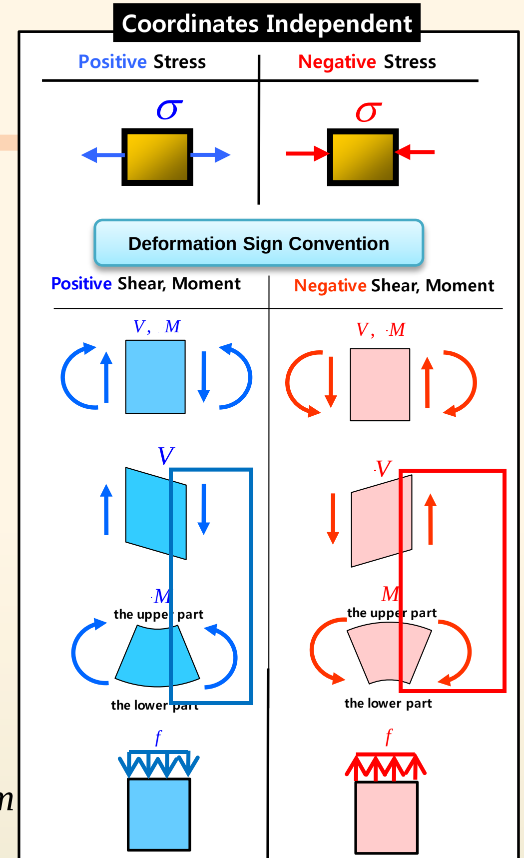
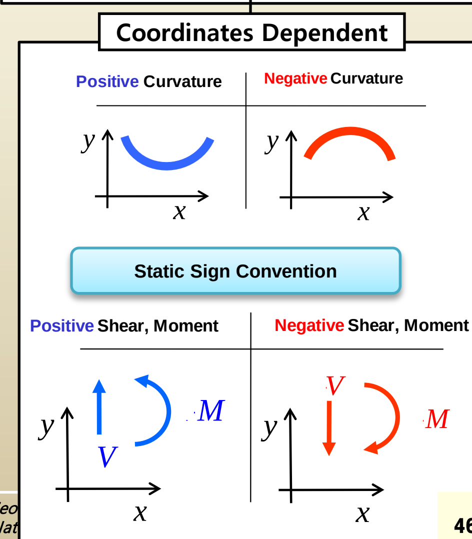


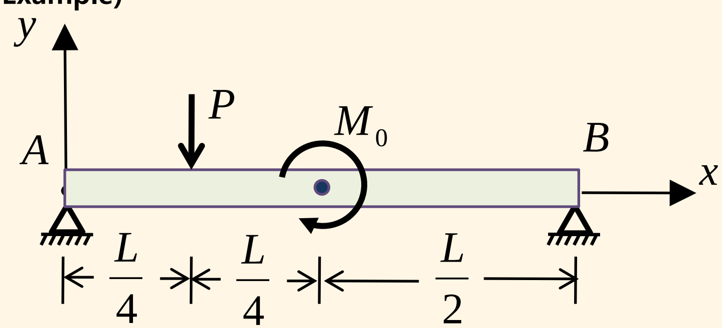
the same





Comparison of Sign Conventions

Example)



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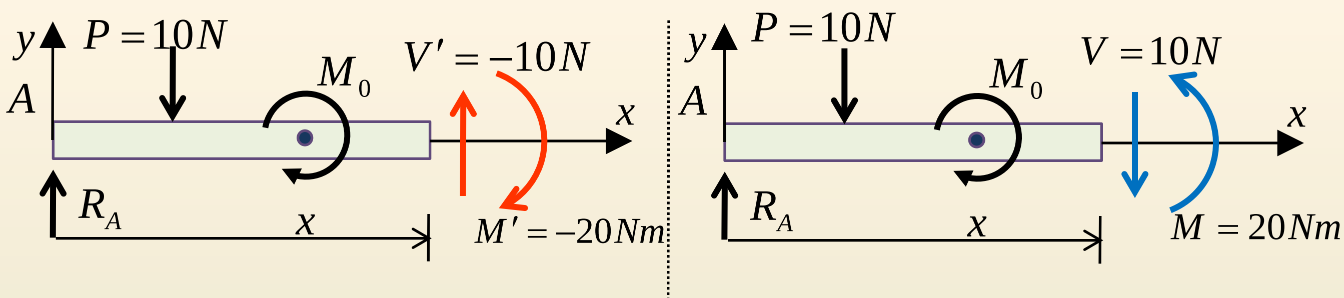
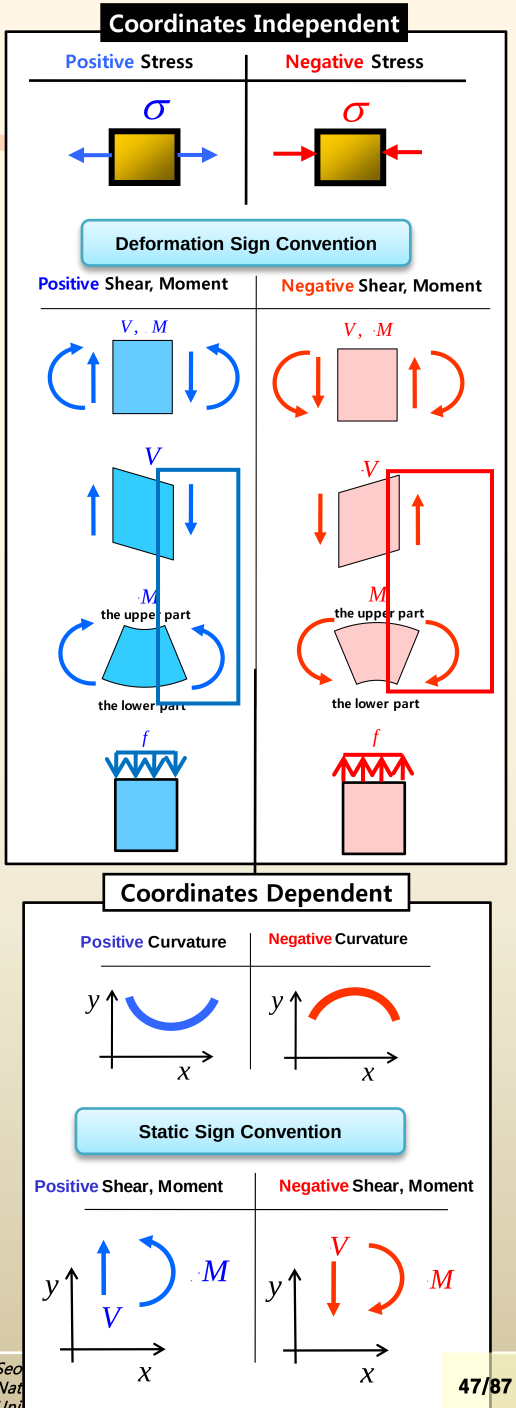
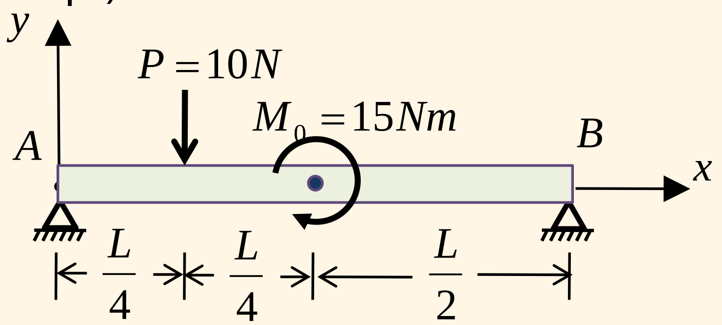




if we describe them in vector notation?

Comparison of Sign Conventions

Example)



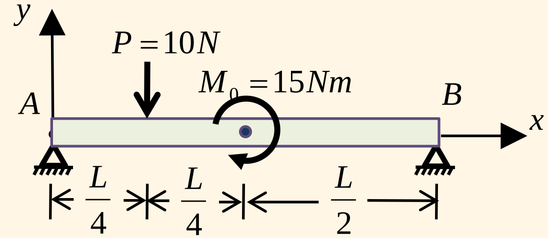
What if we describe them in <u>vector</u> notation?

$$\mathbf{P} = -10\mathbf{j},\ P = -10$$

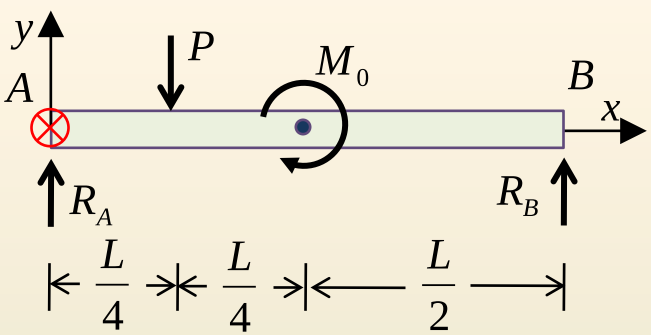
$$\mathbf{M_0} = -15\mathbf{k},\ M_0 = -15$$

Comparison of Sign Conventions

Example)



1) Reaction (Free-body diagram)

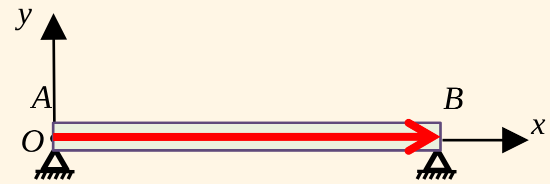


if we describe them in <u>vector</u> notation?

$\mathbf{P} = -10\mathbf{j},\ P = -10$

$\mathbf{M_0} = -15\mathbf{k},\ M_0 = -15$

$\mathbf{L} = 30\mathbf{i},\ L = 30$



$\mathbf{L} = \overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 30\mathbf{i} - 0\mathbf{i} = 30\mathbf{i}$

$\overrightarrow{OB},\ \overrightarrow{OA}$: position vector

Moment Equilibrium at A

$$\sum \mathbf{M}_{z\ \text{at A}} = \frac{\mathbf{L}}{4} \times \mathbf{P} + \mathbf{M_0} + \mathbf{L} \times \mathbf{R}_B = \frac{L}{4}\mathbf{i} \times P\mathbf{j} + M_0\mathbf{k} + L\mathbf{i} \times R_B\mathbf{j}$$

$$\therefore \frac{L}{4} \cdot P\mathbf{k} + M_0\mathbf{k} + L \cdot R_B\mathbf{k} = 0$$

$$\left(\frac{L}{4} \cdot P + M_0 + L \cdot R_B\right)\mathbf{k} = 0$$

for instance,

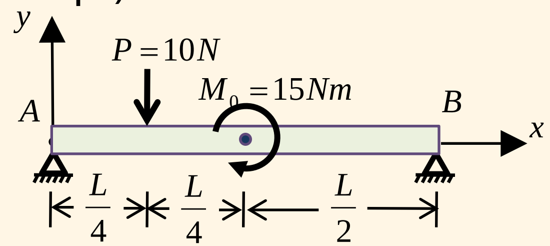
$$\left(\frac{30}{4} \cdot (-10) + (-15) + 30 \cdot R_B\right)\mathbf{k} = 0 \Rightarrow 30 \cdot R_B = \frac{30}{4} \cdot 10 + 15$$

$$\therefore R_B = 3 \qquad R_B = \frac{10}{4} + \frac{15}{30} = \frac{5}{2} + \frac{1}{2} = 3$$

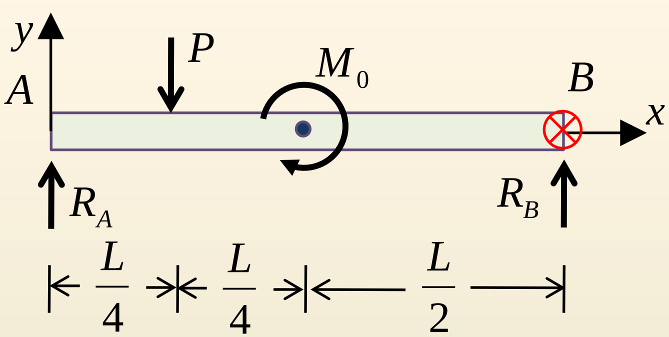
means, $\mathbf{R}_B = R_B\mathbf{j} = 3\mathbf{j}$

Comparison of Sign Conventions

Example)



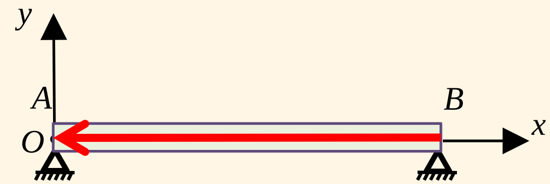
1) Reaction (Free-body diagram)



if we describe them in <u>vector</u> notation?

$\mathbf{P} = -10\mathbf{j},\ P = -10$

$\mathbf{M_0} = -15\mathbf{k},\ M_0 = -15$



$\mathbf{L} = \overrightarrow{BA}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 0\mathbf{i} - 30\mathbf{i} = -30\mathbf{i}$

$\overrightarrow{OA},\ \overrightarrow{OB}$: position vector

$\mathbf{L} = -30\mathbf{i},\ L = -30$

Moment Equilibrium at B

$$\sum \mathbf{M}_{z\ \text{at B}} = \frac{3\mathbf{L}}{4} \times \mathbf{P} + \mathbf{M_0} + \mathbf{L} \times \mathbf{R}_A \quad = \frac{3L}{4}\mathbf{i} \times P\mathbf{j} + M_0\mathbf{k} + L\mathbf{i} \times R_A\mathbf{j}$$

$$\therefore \frac{3L}{4} \cdot P\mathbf{k} + M_0\mathbf{k} + L \cdot R_A\mathbf{k} = 0$$

$$\left(\frac{3L}{4} \cdot P + M_0 + L \cdot R_A\right)\mathbf{k} = 0$$

for instance,

$$\left(\frac{3\cdot(-30)}{4} \cdot (-10) + (-15) + (-30) \cdot R_A\right)\mathbf{k} = 0 \Rightarrow 30 \cdot R_A = \frac{3\cdot(-30)}{4} \cdot (-10) + (-15)$$

$$R_A = \frac{3\cdot 10}{4} - \frac{15}{30}$$

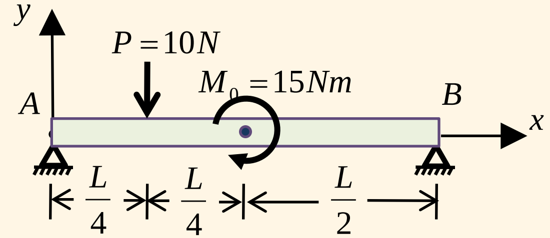
$$\therefore R_A = 7$$

$$R_A = \frac{15}{2} - \frac{1}{2} = 7$$

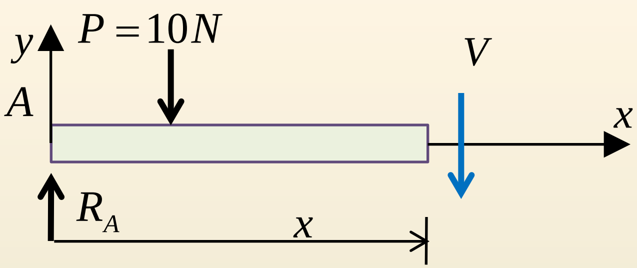
means, $\mathbf{R}_A = R_A\mathbf{j} = 7\mathbf{j}$

Comparison of Sign Conventions

Example)



2) Shear force and bending moment at x (Free-body diagram)



if we describe them in <u>vector</u> notation?

$$\mathbf{P} = -10\mathbf{j},\ P = -10$$

$$\mathbf{M_0} = -15\mathbf{k},\ M_0 = -15$$

Force Equilibrium

$$\sum \mathbf{F}_y = \mathbf{R}_A + \mathbf{P} + \mathbf{V}$$

$$= R_A\mathbf{j} + P\mathbf{j} + V\mathbf{j} = 0$$

$$(R_A + P + V)\mathbf{j} = 0$$

$$\mathbf{R}_A = R_A\mathbf{j} = 7\mathbf{j}$$

for instance,

$$(7 + (-10) + V)\mathbf{j} = 0$$

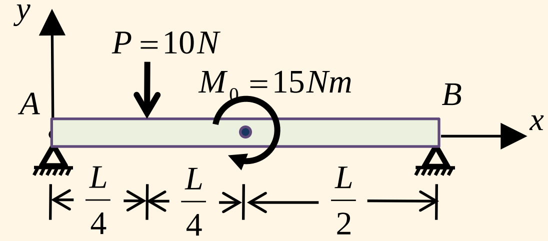
$$V = 3$$

means,

$$\mathbf{V} = V\mathbf{j} = 3\mathbf{j}$$

Comparison of Sign Conventions

Example)



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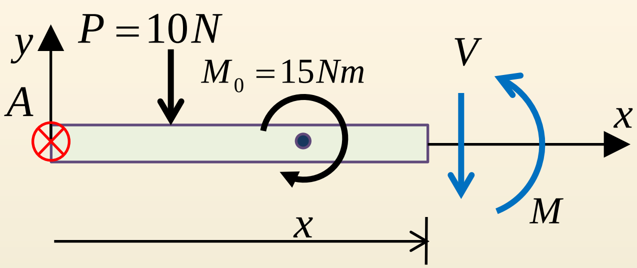
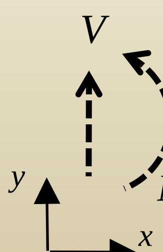


$$\mathbf{V} = V\mathbf{j} = 3\mathbf{j}$$

$$\mathbf{M} = M\mathbf{k} = 25.5\mathbf{k}$$

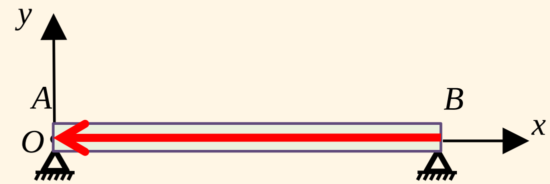


if we describe them in <u>vector</u> notation?

$$\mathbf{P} = -10\mathbf{j},\ P = -10$$

$$\mathbf{M_0} = -15\mathbf{k},\ M_0 = -15$$

$$\mathbf{L} = 30\mathbf{i},\ L = 30$$



$\mathbf{L} = \overrightarrow{AB}$: position vector

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 0\mathbf{i} - 30\mathbf{i} = -30\mathbf{i}$$

Moment Equilibrium at A

$$\sum \mathbf{M}_{z \text{ at A}} = \frac{\mathbf{L}}{4} \times \mathbf{P} + \mathbf{M}_0 + \mathbf{x} \times \mathbf{V} + \mathbf{M} = \frac{L}{4}\mathbf{i} \times P\mathbf{j} + M_0\mathbf{k} + x\mathbf{i} \times V\mathbf{j} + \mathbf{M}$$

$$\therefore \frac{L}{4} \cdot P\mathbf{k} + M_0\mathbf{k} + x \cdot V\mathbf{k} + M\mathbf{k} = 0$$

$$\left(\frac{L}{4} \cdot P + M_0 + x \cdot V + M\right)\mathbf{k} = 0$$

for instance, at $x = \frac{3}{4}L$

$$\left(\frac{30}{4} \cdot (-10) + (-15) + \frac{3}{4}(30) \cdot (3) + M\right)\mathbf{k} = 0$$

$$\Rightarrow M = \frac{30}{4} \cdot (10) + (15) - \frac{3}{4}(30) \cdot (3)$$

$$M = 25.5$$

means, $\mathbf{M} = M\mathbf{k} = 25.5\mathbf{k}$

D.E. for Deflection of Beam

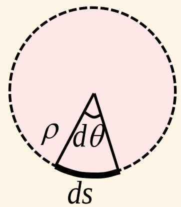
$\sigma = E\varepsilon$

① **definition** ρ: radius of curvature

$\kappa = \dfrac{1}{\rho}$: curvature

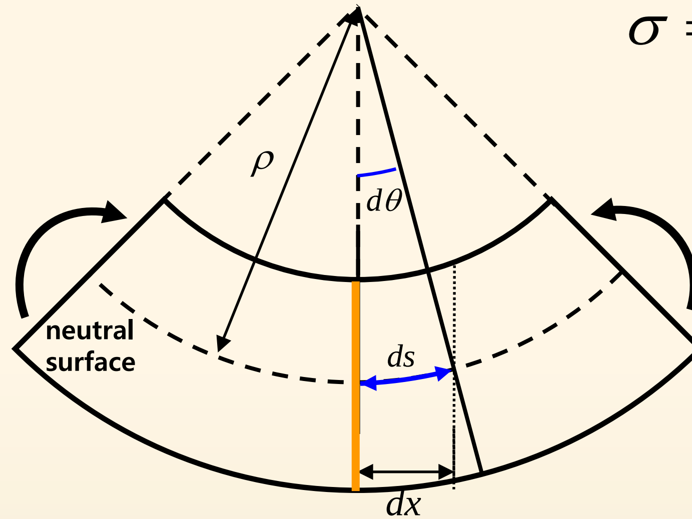
② **from the geometry**

$\rho \cdot d\theta = ds$ ⇨ $\dfrac{1}{\rho} = \dfrac{d\theta}{ds}$



linearization

$if \;\; \theta << 1, \; then \; ds \approx dx$



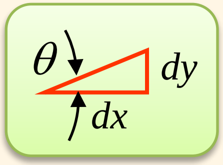
ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

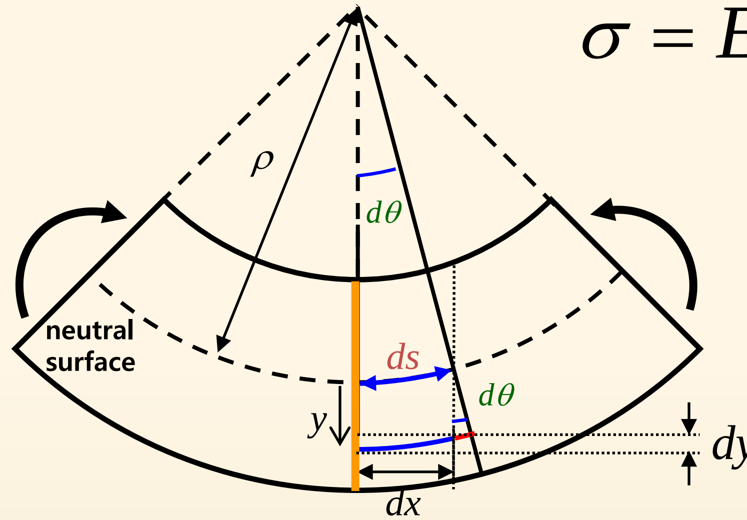
D.E. for Deflection of Beam

Linearization

$$if \quad \theta \ll 1$$

$$1)\; ds \approx dx$$





$$ds^2 = dx^2 + dy^2 \rightarrow ds = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\text{let, } z = \left(\frac{dy}{dx}\right)^2 \text{ then, } \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+z}$$

$$f(z) = \sqrt{1+z}$$

$$f(0) = 1$$

$$f'(0) = \frac{1}{2}(1+z)^{-\frac{1}{2}}\Big|_{z=0} = \frac{1}{2}$$

$$f''(0) = -\frac{1}{4}(1+z)^{-\frac{3}{2}}\Big|_{z=0} = -\frac{1}{4}$$

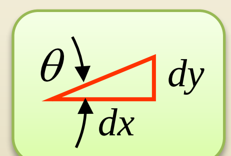
$$\therefore f(z) = 1 + \frac{1}{2}z + \frac{1}{2}\left(-\frac{1}{4}\right)z^2 + \cdots$$

$$if,\theta \ll 1$$

$$\therefore f(z) = \sqrt{1+z} \approx 1$$

$$\therefore ds \approx dx$$

$$2)\; \theta \approx \tan(\theta) = \frac{dy}{dx}$$



$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \cdots$$

D.E. for Deflection of Beam

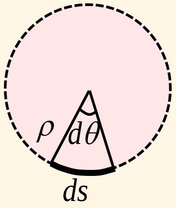
① **definition** ρ: radius of curvature

$$\kappa = \frac{1}{\rho} : \text{curvature}$$

$$\sigma = E\varepsilon$$

② **from the geometry**

$$\rho \cdot d\theta = ds \Rightarrow \frac{1}{\rho} = \frac{d\theta}{ds}$$



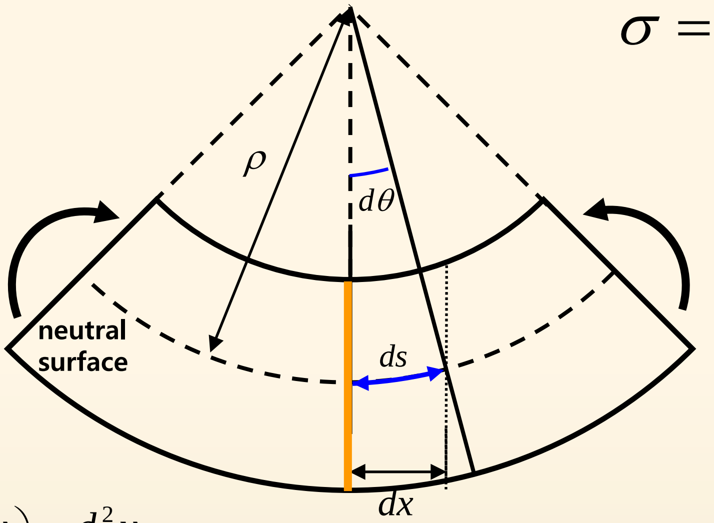
linearization

$$if \;\; \theta << 1, \; then \; ds \approx dx$$

$$\therefore \kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \cong \frac{d\theta}{dx} \Rightarrow \kappa = \frac{d\theta}{dx}$$

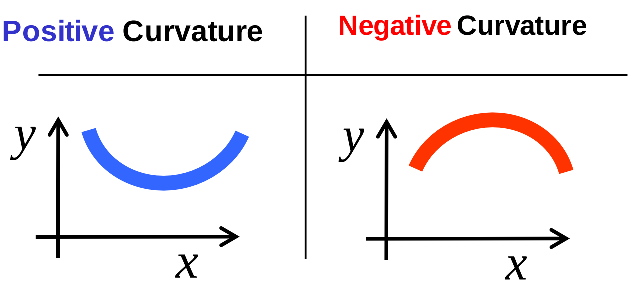
by linearlization

$$\kappa = \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2 y}{dx^2}$$



observe the deformation sign convention?

Coordinates Dependent



D.E. for Deflection of Beam

$$\sigma = E\varepsilon$$

① **definition** ρ: radius of curvature

$$\kappa = \frac{1}{\rho} : \text{curvature}$$

② **from the geometry and linearization**

$$\frac{1}{\rho} = \frac{d\theta}{dx} \quad , \kappa = \frac{d\theta}{dx}$$

③ **change in length**

length of AB : $\rho d\theta = ds \cong dx$

length of A′B′ : $(\rho - y)d\theta = ds' \cong dx'$

$$(\rho - y)d\theta = dx'$$

$$\rho d\theta - y d\theta = dx'$$

$$dx - y d\theta = dx' \qquad \text{since } \frac{dx}{\rho} = d\theta,$$

$$-y\frac{dx}{\rho} = dx' - dx$$

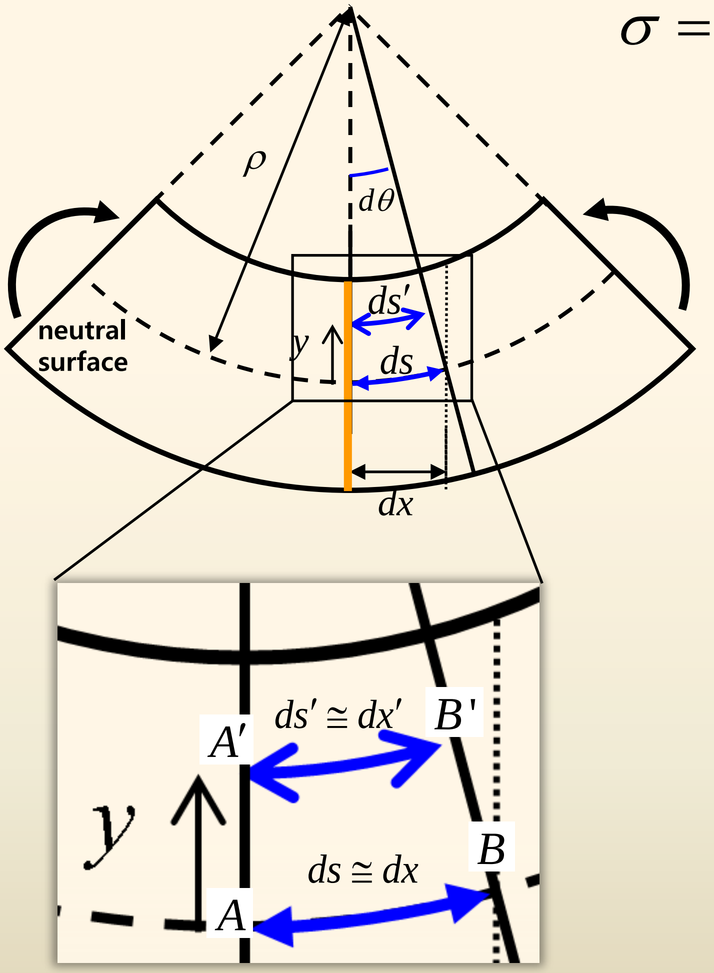
(after deformation) – (before deformation)

④ **strain**

$$-\frac{y}{\rho} = \frac{dx' - dx}{dx}$$

definition of ε_x

$$\therefore \varepsilon_x = -\frac{y}{\rho}$$



ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

D.E. for Deflection of Beam

$$\sigma = E\varepsilon$$

① **definition** ρ: radius of curvature

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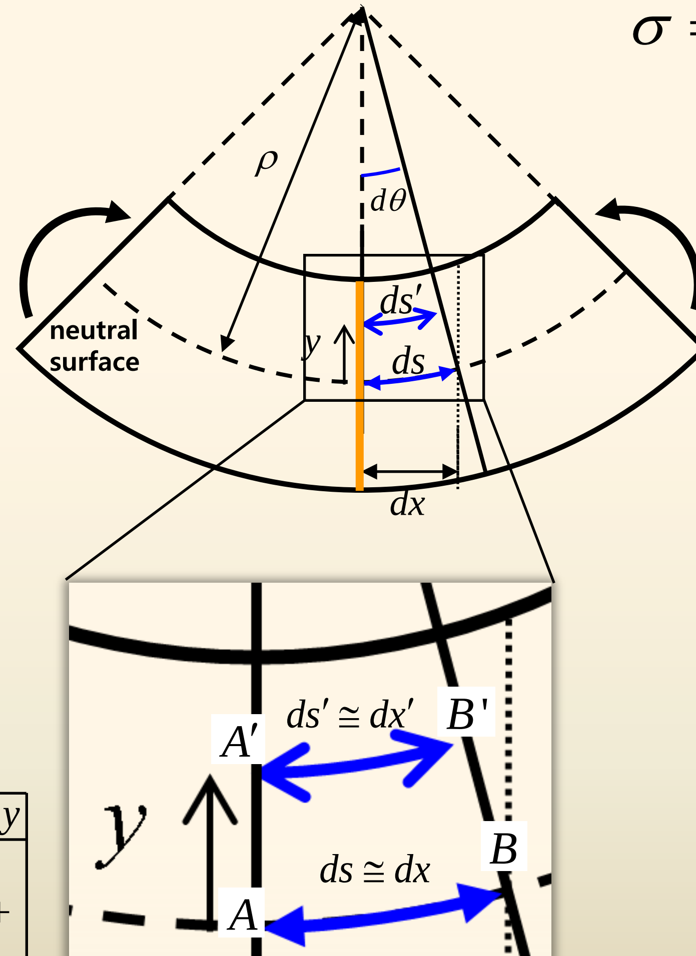
③ **change in length**

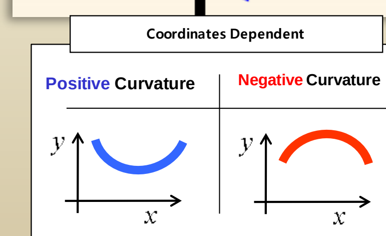
$$-\frac{y}{\rho} = \frac{dx' - dx}{dx}$$

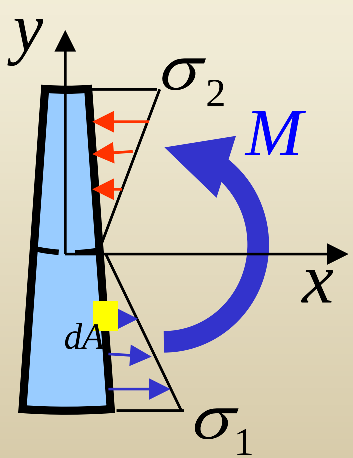
④ **strain**

$$\varepsilon_x = -\frac{y}{\rho} \quad \text{or} \quad \varepsilon_x = -\kappa y$$

observe the deformation sign convention?







	ε_x		$-\kappa y$	$-$	κ	y
compression	$-$	match!	$-$	$-$	$+$	$+$
tension	$+$		$+$	$-$	$+$	$-$

What about the left side?

ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

D.E. for Deflection of Beam

① **definition** ρ: radius of curvature

$$\kappa = \frac{1}{\rho} \text{ : curvature}$$

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$$\frac{1}{\rho} = \frac{d\theta}{dx} \quad , \kappa = \frac{d\theta}{dx}$$

③ **change in length**

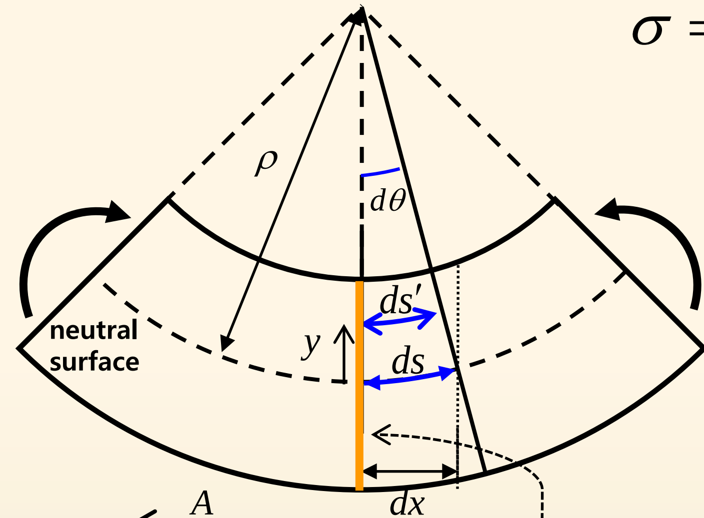
$$-\frac{y}{\rho} = \frac{dx' - dx}{dx}$$

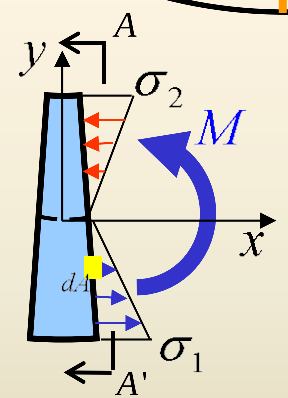
④ **strain**

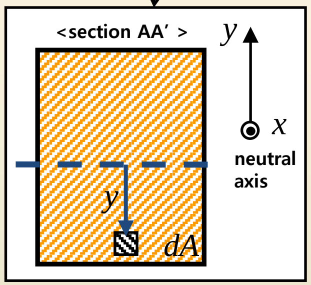
$$\varepsilon_x = -\frac{y}{\rho} \quad or \ \varepsilon_x = -\kappa y$$

⑤ **stress** $\sigma_x = E\varepsilon_x \qquad \therefore \sigma_x = -E\frac{y}{\rho} \quad or \ \ -\kappa y$

$$\sigma = E\varepsilon$$







⑥ **moment :**

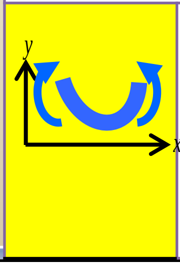
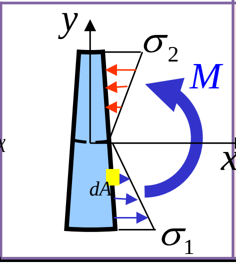
the moment <u>**about neutral axis**</u> **resultant of the normal stresses** σ_x **acting over the cross section is equal to the bending moment**

the element force acting on the element of area : $\sigma_x y dA$

$$dM = \sigma_x y dA$$

observe the deformation sign convention?

$$\therefore dM = \square \sigma_x y dA$$

deformation		κ =1/ρ	B.M	yσ	y	σ	dM (σydA)
		+	+	-	+	-	□σ ydA
				-	-	+	

D.E. for Deflection of Beam

$$\sigma = E\varepsilon$$

① **definition** ρ: radius of curvature

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② **from the geometry and linearization**

$$\frac{1}{\rho} = \frac{d\theta}{dx} \quad , \kappa = \frac{d\theta}{dx}$$

③ **change in length**

$$-\frac{y}{\rho} = \frac{dx' - dx}{dx}$$

④ **strain**

$$\varepsilon_x = -\frac{y}{\rho} \quad or \quad \varepsilon_x = -\kappa y$$

⑤ **stress**

$$\sigma_x = E\varepsilon_x \qquad \therefore \sigma_x = -E\frac{y}{\rho} \quad or \quad -\kappa y$$

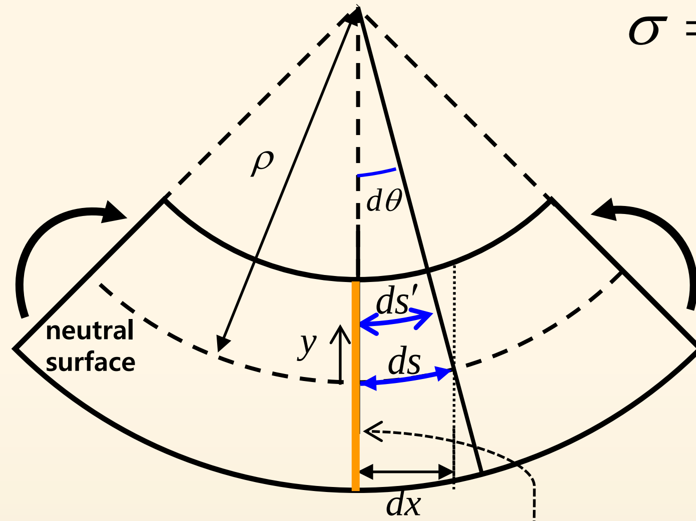
⑥ **moment**

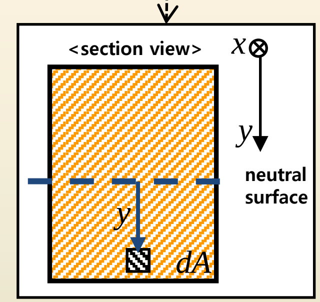
$$dM = -\sigma_x y dA \qquad M = \int_A dM$$

$$M = -\int_A \sigma_x y dA$$

$$= -\int_A (-E\frac{y}{\rho}) y \, dA$$

$$\therefore M = \frac{E}{\rho}\int_A y^2 dA \qquad \text{Define } I = \int_A y^2 dA \quad \text{then, } M = \frac{EI}{\rho}$$





observe the deformation sign convention?

$$\boxed{\frac{M}{EI} = \frac{1}{\rho}} \quad \text{or} \quad \boxed{\frac{M}{EI} = \kappa}$$

D.E. for Deflection of Beam

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④ **strain**

$$\varepsilon_x = -\frac{y}{\rho} \quad or \;\; \varepsilon_x = -\kappa y$$

⑤ **stress** $\sigma_x = E\varepsilon_x \qquad \therefore \sigma_x = -E\frac{y}{\rho} \quad or \;\; -\kappa y$

⑥ **moment** $dM = -\sigma_x y dA$

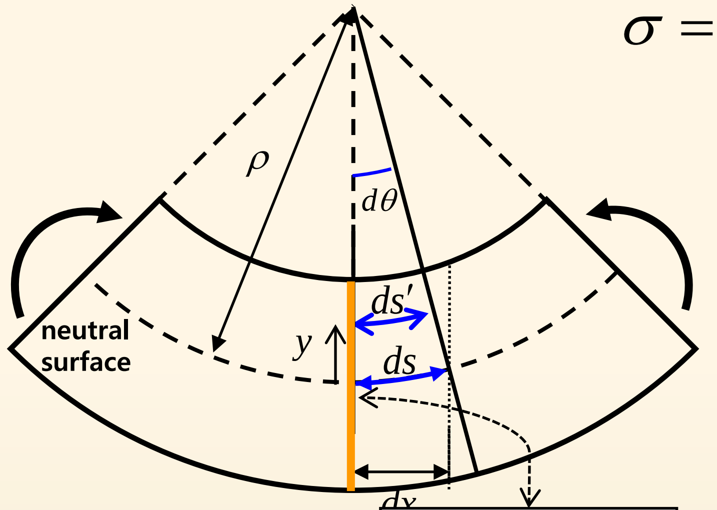
$$M = \frac{EI}{\rho} \Rightarrow \boxed{\frac{M}{EI} = \frac{1}{\rho}} \text{ or } \boxed{\frac{M}{EI} = \kappa}$$

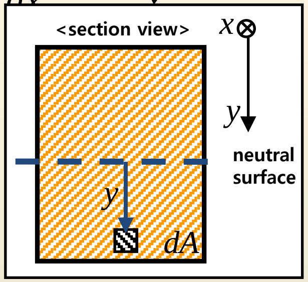
observe the deformation sign convention?

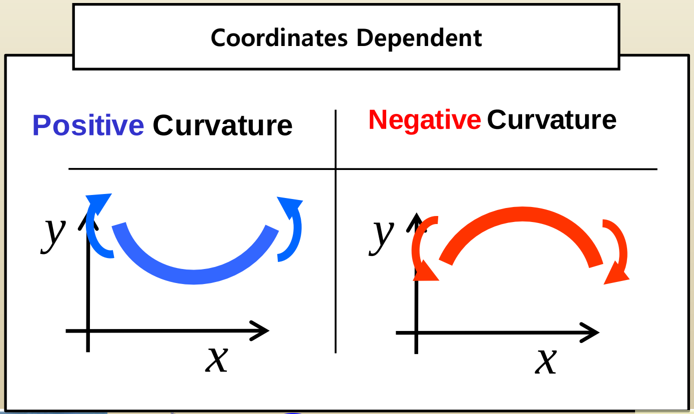
$$\left(I = \int_A y^2 dA \right)$$

"a positive bending moment produces positive curvature and a negative bending moment produces negative curvature*"

$$\sigma = E\varepsilon$$

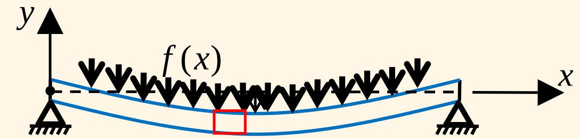






*Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p312

D.E. for Deflection of Beam



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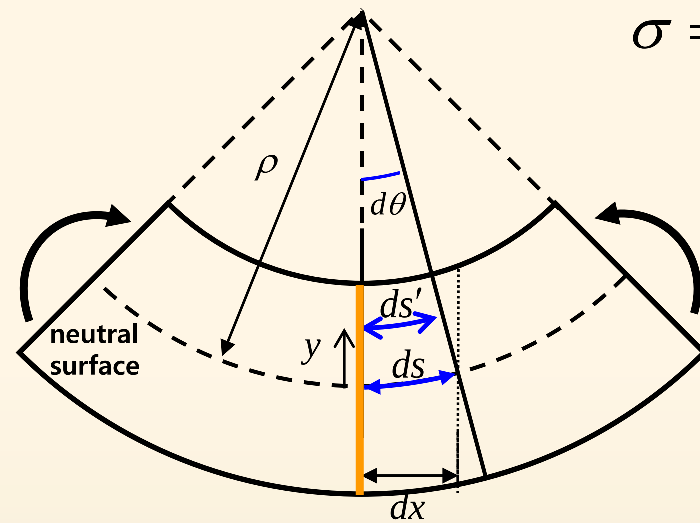

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③ **change in length** $-\frac{y}{\rho} = \frac{dx' - dx}{dx}$

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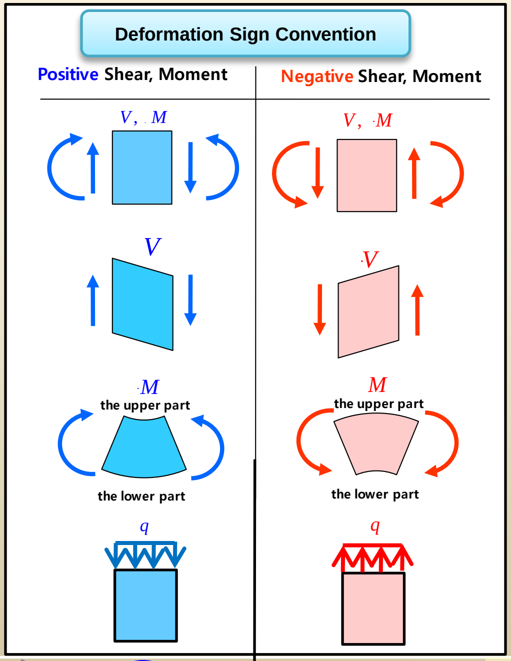
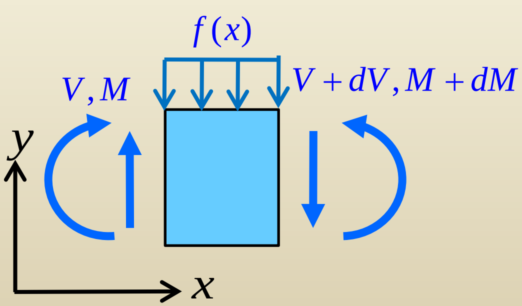


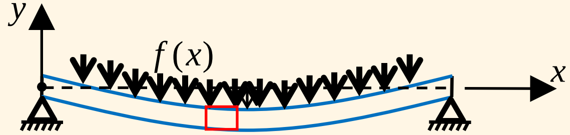
⑦ **relationships between loads, shear forces, and bending moments**



•**free-body diagram (positive deformation)**

ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

D.E. for Deflection of Beam



$\sigma = E\varepsilon$

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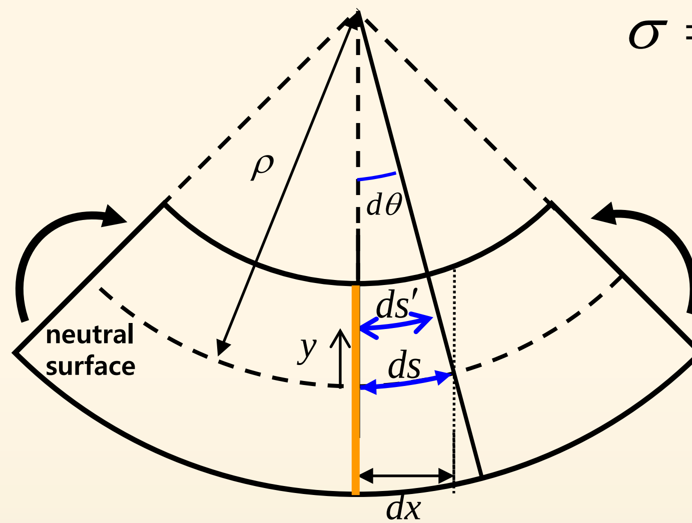
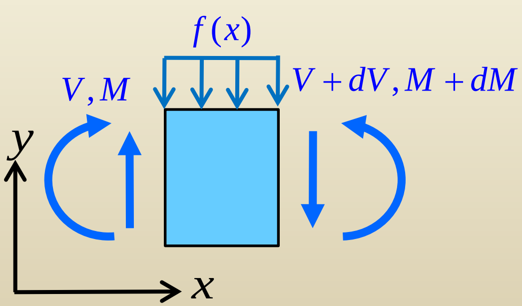


⑥ **moment** $dM = -\sigma_x y dA$ $\boxed{\dfrac{M}{EI} = \dfrac{1}{\rho}}$ **or** $\boxed{\dfrac{M}{EI} = \kappa}$ $, I = \int_A y^2 dA$

⑦ **relationships between loads, shear forces, and bending moments**

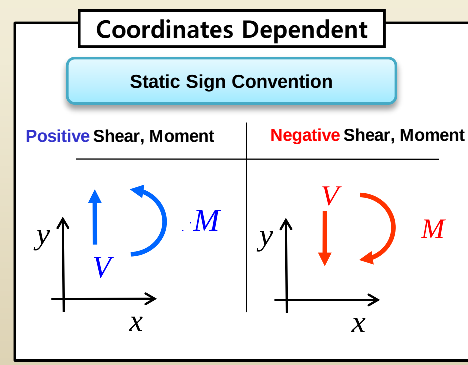


•force equilibrium

$$\sum F_y = V - f(x)dx - (V + dV) = 0$$

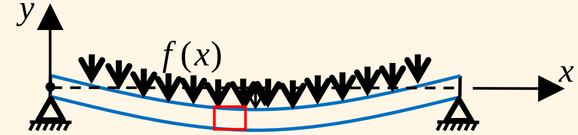
$$-dV - f(x)dx = 0$$

$$\boxed{\frac{dV}{dx} = -f(x)}$$



ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

D.E. for Deflection of Beam



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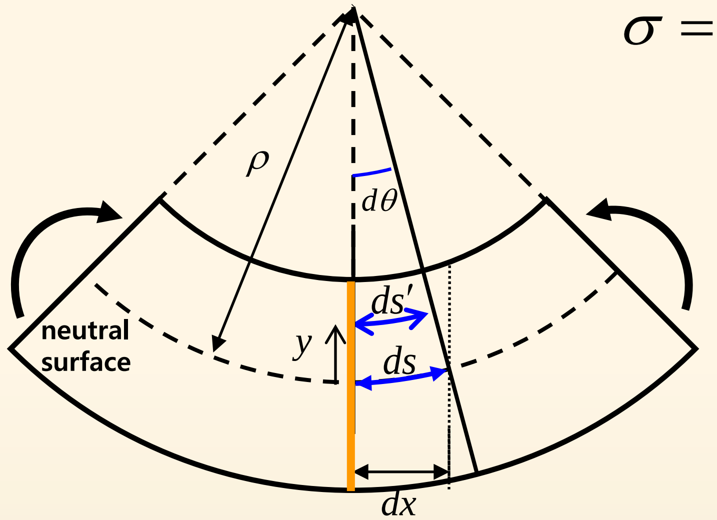

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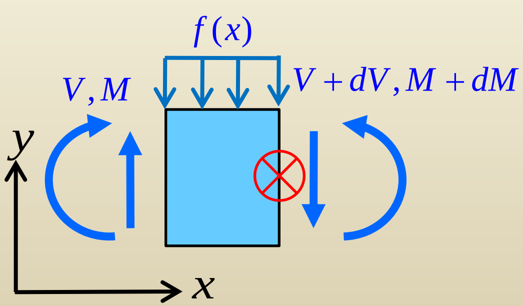
⑦ **relationships between loads, shear forces, and bending moments**

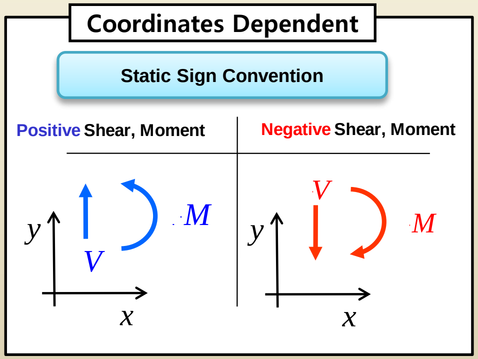


•**force equilibrium** $\boxed{\dfrac{dV}{dx} = -f(x)}$

•**moment equilibrium**

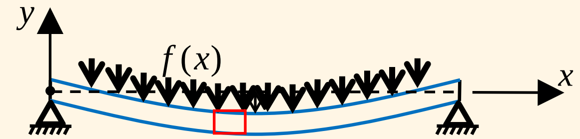
$$-M + f(x)dx \cdot \frac{1}{2}dx + (M + dM) - Vdx = 0$$

$$dM - Vdx = 0 \qquad \boxed{\frac{dM}{dx} = V(x)}$$



ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

D.E. for Deflection of Beam



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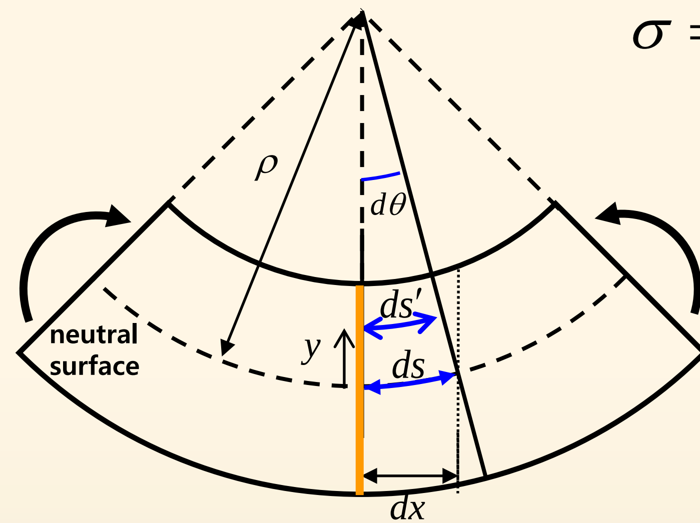

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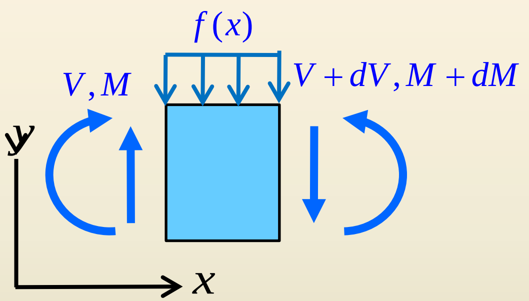


⑦ **relationships between loads, shear forces, and bending moments**

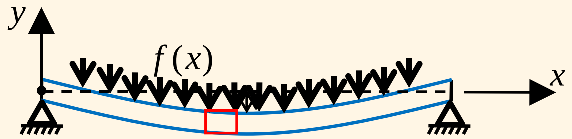
•**force equilibrium** $\boxed{\dfrac{dV}{dx} = -f(x)}$ •**moment equilibrium** $\boxed{\dfrac{dM}{dx} = V(x)}$

⑧ **by the linearization** $ds \approx dx, \ \theta \approx \tan(\theta) = \dfrac{dy}{dx}$

$$\frac{d\theta}{ds} = \frac{d}{ds}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \Rightarrow \boxed{\frac{d\theta}{ds} = \frac{d^2y}{dx^2}} \text{ or } \boxed{\kappa = \frac{d^2y}{dx^2}}$$

ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

D.E. for Deflection of Beam



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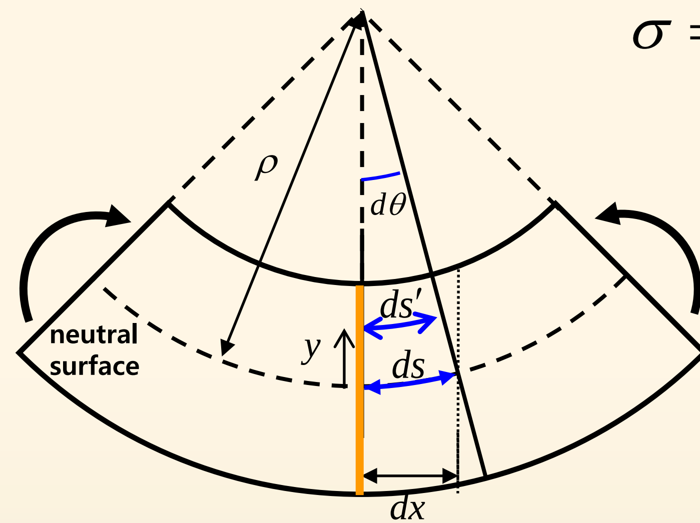
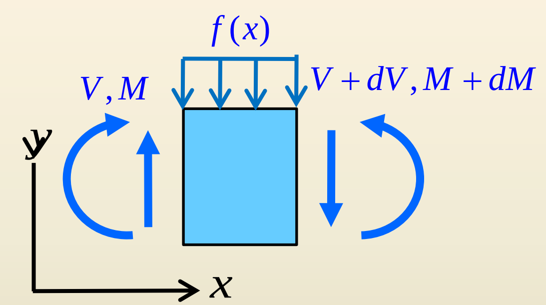


⑥ **moment** $dM = -\sigma_x y dA$ $\boxed{\frac{M}{EI} = \frac{1}{\rho}}$ or $\boxed{\frac{M}{EI} = \kappa}$ ① $, I = \int_A y^2 dA$



⑦ **relationships between loads, shear forces, and bending moments**

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⑧ **by the linearization** $ds \approx dx, \ \theta \approx \tan(\theta) = \frac{dy}{dx}$

$$\boxed{\kappa = \frac{d^2y}{dx^2}} \xrightarrow{①} \boxed{\therefore \frac{d^2y}{dx^2} = \frac{M}{EI}} \rightarrow \boxed{EI\frac{d^2y}{dx^2} = M} \xrightarrow{②} \boxed{EI\frac{d^3y}{dx^3} = V} \xrightarrow{③} \boxed{EI\frac{d^4y}{dx^4} = -f(x)}$$

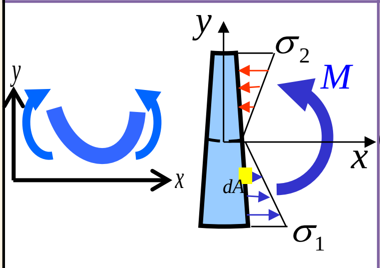
ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

$\varepsilon \Rightarrow \sigma = E \cdot \varepsilon \Rightarrow y\sigma dA \rightarrow dM$

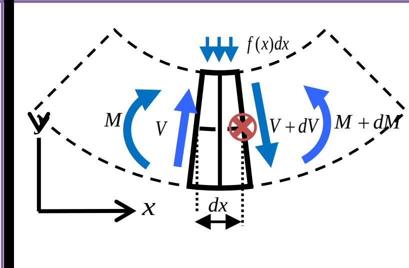
Summary : Sign Convention & Equations

convention

Derived

deformation (ref : Gere)	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M = \int_A dM$
	⊕	+	+	$-\kappa y$ or $-\frac{y}{\rho}$	-	comp.	⊖	$-\sigma y dA$ (modified)	$M = -\int_A y\sigma dA$ $= -\int_A y(-\frac{E}{\rho}y)dA$ $M = \frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI} = \frac{d^2y}{dx^2}$
			-		+	tension			

not match!

relation btw V, M, f(x)	
	$V - f(x)dx - (V + dV) = 0$ $\Rightarrow \frac{dV}{dx} = -f(x)$ $-M + f(x)dx \cdot \frac{1}{2}dx + (M + dM) - Vdx = 0$ $\Rightarrow \frac{dM}{dx} = V(x)$ $EI\frac{d^4y}{dx^4} = -f(x)$

what is the difference with the vector notation?

④ moment about z-axis : $d\mathbf{M} = \mathbf{y} \times d\mathbf{F}$

$$= (y\mathbf{j}) \times (-E\frac{y}{\rho}dA\mathbf{i})$$

$$= E\frac{y^2}{\rho}dA\mathbf{k}$$

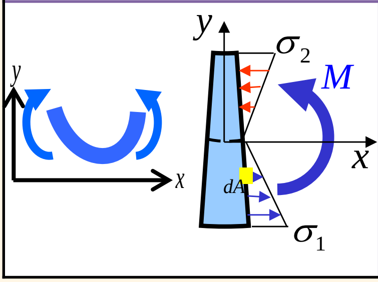
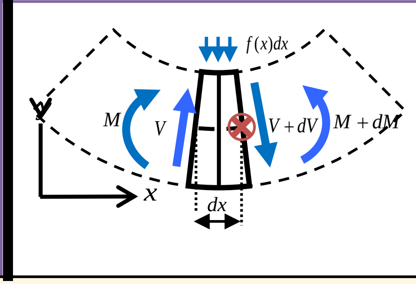
$$\therefore \mathbf{M} = \int_A d\mathbf{M} = \int_A E\frac{y^2}{\rho}dA\mathbf{k}$$

$\varepsilon \Rightarrow \sigma = E\cdot\varepsilon \Rightarrow y\sigma dA \rightarrow dM$

Summary : Sign Convention & Equations

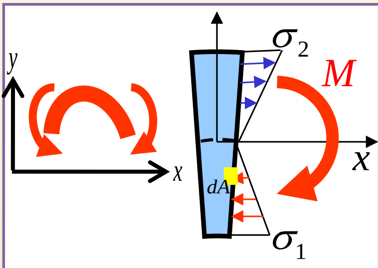
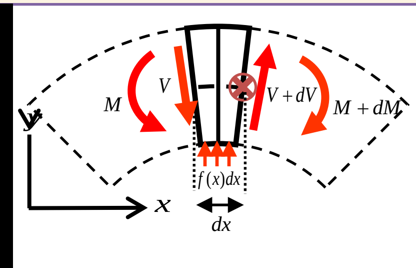
convention

Derived

deformation (ref : Gere)	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)	
	⊕	+	+ -	$-\kappa y$ or $-\frac{y}{\rho}$	- +	comp. tension	⊖	$-\sigma y dA$ (modified)	$M=-\int_A y\sigma dA$ $=-\int_A y(-\frac{E}{\rho}y)dA$ $M=\frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI}=\frac{d^2y}{dx^2}$		$V-f(x)dx-(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $-M+f(x)dx\cdot\frac{1}{2}dx+(M+dM)-Vdx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=-f(x)$

not match!

what about for 'negative bending'?

deformation	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)	
	⊖	-	+ -	$-\kappa y$ or $-\frac{y}{\rho}$	+ -	tension comp.	⊕	$-\sigma y dA$ (modified)	$M=-\int_A y\sigma dA$ $=-\int_A y(-\frac{E}{\rho}y)dA$ $M=\frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI}=\frac{d^2y}{dx^2}$		$-V+f(x)dx+(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $M-f(x)dx\cdot\frac{1}{2}dx-(M+dM)+Vdx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=-f(x)$

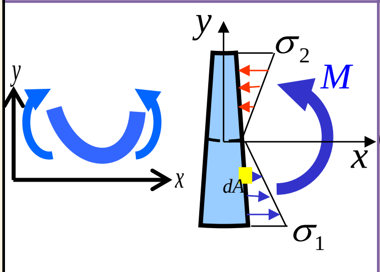
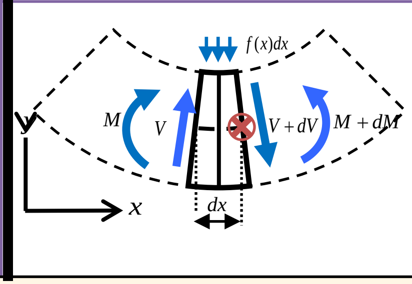
not match!

$\varepsilon \Rightarrow \sigma = E \cdot \varepsilon \Rightarrow y\sigma dA \rightarrow dM$

Summary : Sign Convention & Equations

convention

Derived

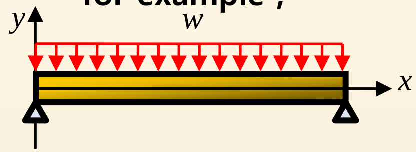
deformation (ref : Gere)	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M = \int_A dM$	relation btw V, M, f(x)
	⊕	+	+	$-\kappa y$ or $-\frac{y}{\rho}$	-	comp.	⊖	$-\sigma y dA$	$M = -\int_A y\sigma dA$ $= -\int_A y(-\frac{E}{\rho}y)dA$ $M = \frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI} = \frac{d^2y}{dx^2}$	 $V - f(x)dx - (V + dV) = 0$ $\Rightarrow \frac{dV}{dx} = -f(x)$ $-M + f(x)dx \cdot \frac{1}{2}dx + (M + dM) - Vdx = 0$ $\Rightarrow \frac{dM}{dx} = V(x)$ $EI\frac{d^4y}{dx^4} = -f(x)$
			-		+	tension				

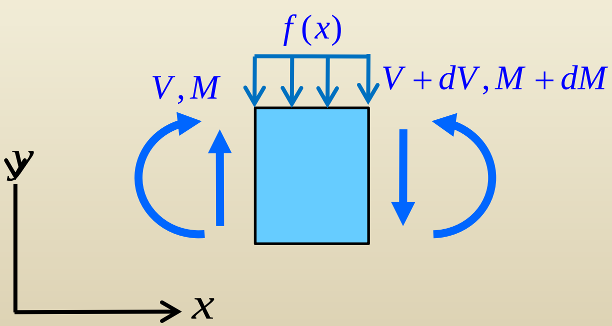
not match!

modified

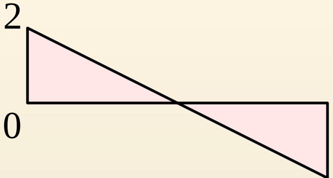
what about the sign of values?

for example ,

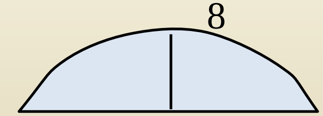




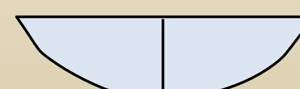
$$V(x) = \frac{wL}{2} - wx$$



$$M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$$

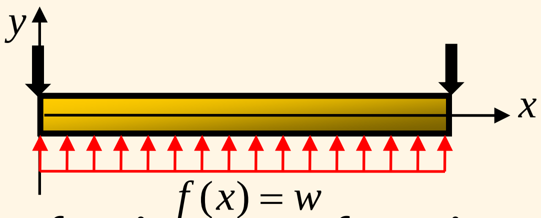


$$y(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$$



Summary : Sign Convention & Equations

$V(x) = \frac{wL}{2} - wx$ $\quad M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$ $\quad y(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$



$$EI\frac{d^4 y(x)}{dx^4} = -w \qquad y(0)=0 \qquad y(L)=0 \qquad y''(0)=0 \qquad y''(L)=0$$

After integrate four times, $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3 - \frac{w}{24EI}x^4$

$$y''(x) = 2c_3 + 6c_4 x + \frac{w}{2EI}x^2$$

$y(0)=0$, $y(0)=c_1$ ⇨ $\therefore c_1 = 0$

$y''(0)=0$, $y''(0)=2c_3$ ⇨ $\therefore c_3 = 0$

$y''(L)=0$, $y''(x) = 6c_4 L - \frac{w}{2EI}L^2$ ⇨ $\therefore c_4 = \frac{w}{12EI}L$

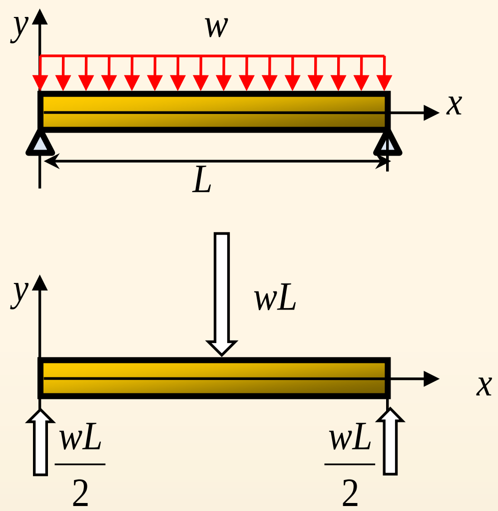
$y(L)=0$, $y(L) = c_2 L + c_4 L^3 - \frac{w}{24EI}L^4$ ⇨ $\therefore c_2 L + c_4 L^3 - \frac{w}{24EI}L^4 = 0$

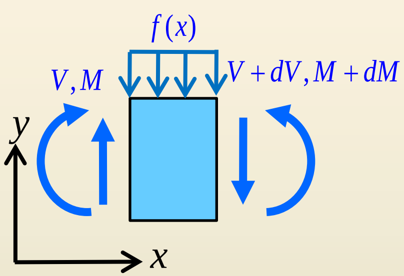
$$c_2 = -\frac{w}{24EI}L^3$$

$$\begin{aligned} c_2 &= -c_4 L^2 + \frac{w_0}{24EI}L^3 \\ &= -(\frac{w}{12EI}L)L^2 + \frac{w}{24EI}L^3 \\ &= -\frac{w}{12EI}L^3 + \frac{w}{24EI}L^3 \\ &= -\frac{w}{24EI}L^3 \end{aligned}$$

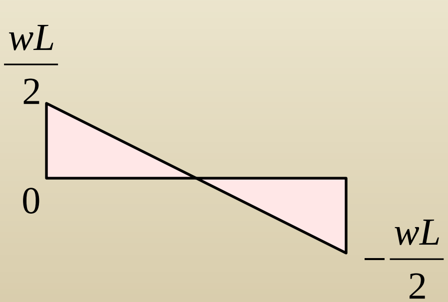
$$\begin{aligned} y(x) &= -\frac{w}{24EI}L^3 x + \frac{w}{12EI}Lx^3 - \frac{w}{24EI}x^4 \\ &= -\frac{wx}{24EI}\left(L^3 - 2Lx^2 + x^3\right) \end{aligned}$$

Summary : Sign Convention & Equations





$$V(x) = \frac{wL}{2} - wx$$

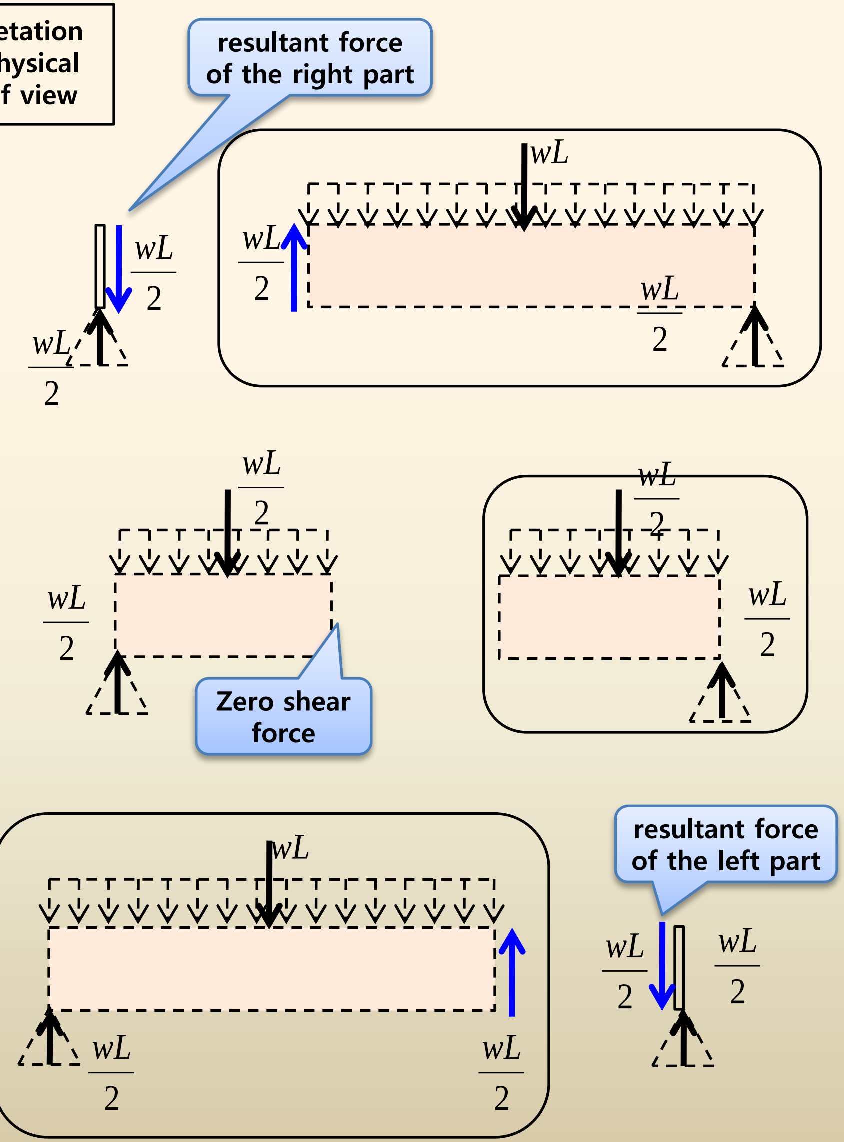


sign convention & solution ⇔ interpretation from physical point of view

$at\ \ x = 0$

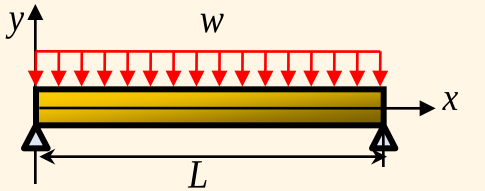
$at\ \ x = \frac{L}{2}$

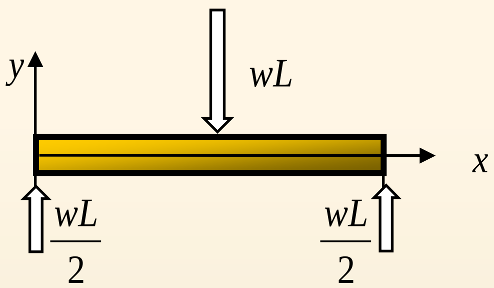
$at\ \ x = L$

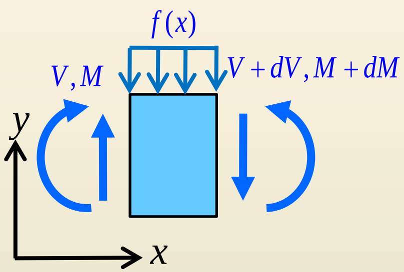


The solution should be interpreted by the sign convention used.

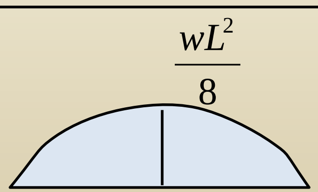
Summary : Sign Convention & Equations







$$M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$$

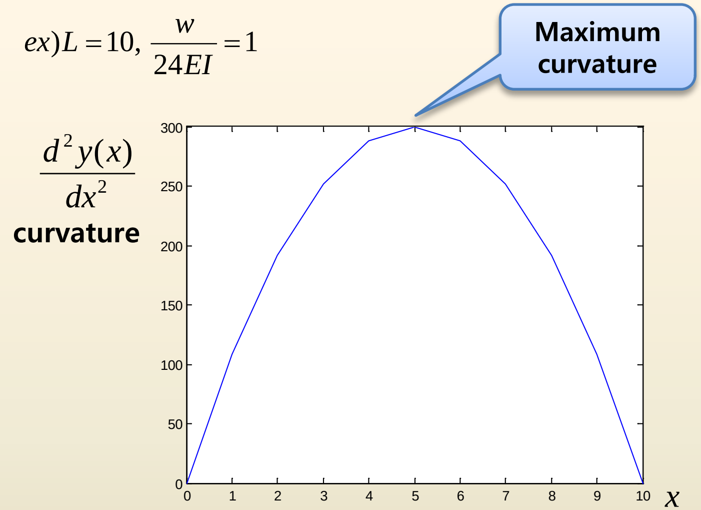


why the bending moment is maximum at x=L/2 ?

$$y(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$\frac{d^2y(x)}{dy^2} = -\frac{wx}{24EI}(12x - 6L)$$

$$ex)L = 10,\ \frac{w}{24EI} = 1$$



since, $\frac{M(x)}{EI} = \frac{d^2y(x)}{dx^2}$

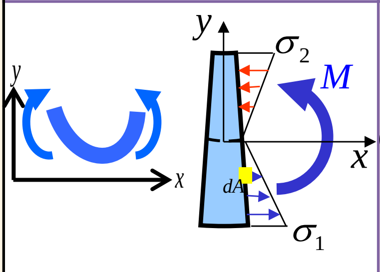
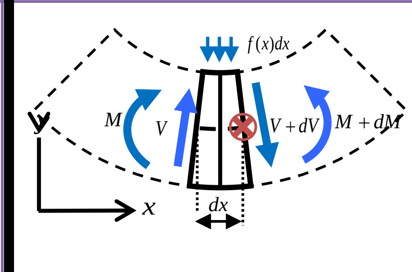
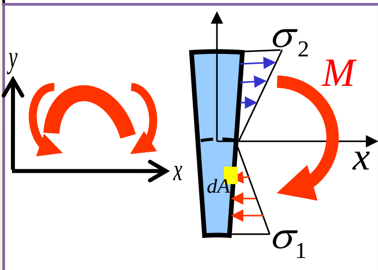
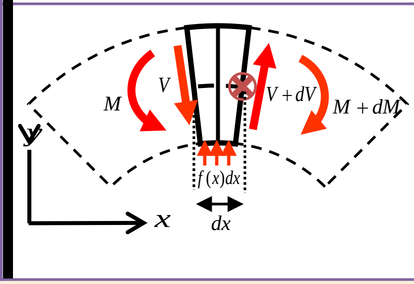
moment is maximum at the point with maximum curvature

$\varepsilon \Rightarrow \sigma = E \cdot \varepsilon \Rightarrow y\sigma dA \rightarrow dM$

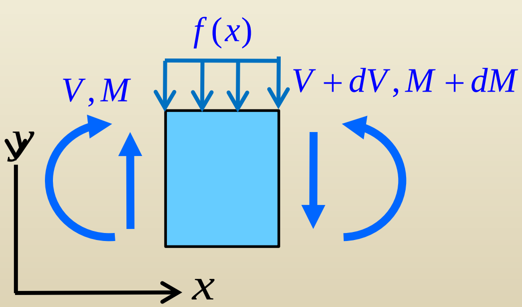
Summary : Sign Convention & Equations

convention

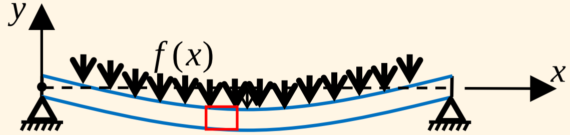
Derived

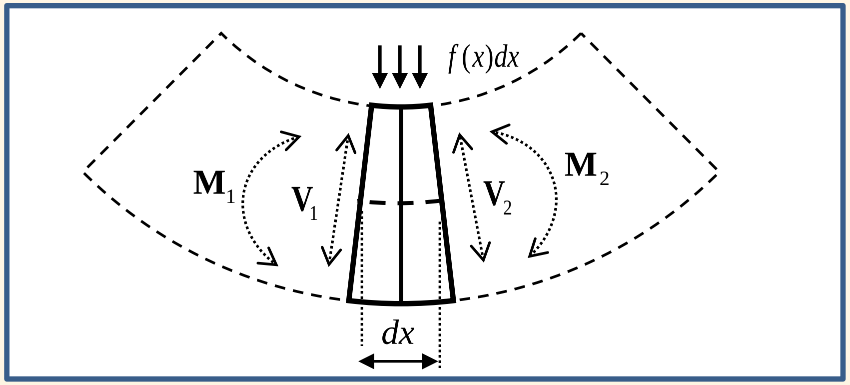
deformation (ref : Gere)	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)	
	⊕	+	+ -	$-\kappa y$ or $-\frac{y}{\rho}$	- +	comp. tension not match!	⊖	$-\sigma y dA$ modified	$M=-\int_A y\sigma dA$ $=-\int_A y(-\frac{E}{\rho}y)dA$ $M=\frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI}=\frac{d^2y}{dx^2}$		$V-f(x)dx-(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $-M+f(x)dx\cdot\frac{1}{2}dx+(M+dM)-Vdx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=-f(x)$
	⊖	-	+ -	$-\kappa y$ or $-\frac{y}{\rho}$	+ -	tension comp. not match!	⊕	$-\sigma y dA$ modified	$M=-\int_A y\sigma dA$ $=-\int_A y(-\frac{E}{\rho}y)dA$ $M=\frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI}=\frac{d^2y}{dx^2}$		$-V+f(x)dx+(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $M-f(x)dx\cdot\frac{1}{2}dx-(M+dM)+Vdx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=-f(x)$

why we should use the set of direction?



Summary : Sign Convention & Equations

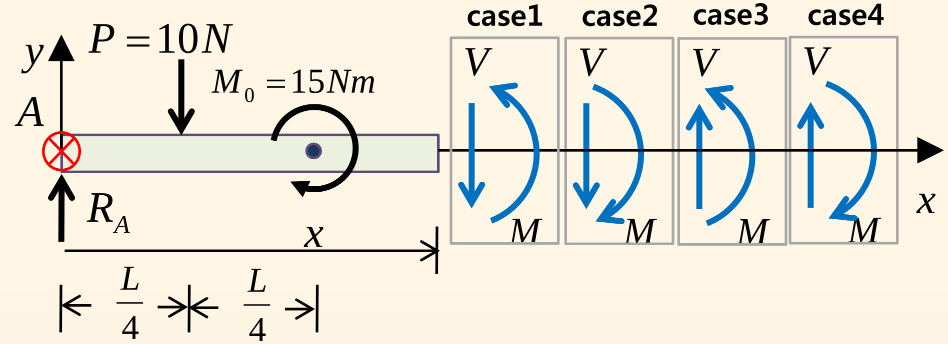




is it ok to assume arbitrary direction for V, M?

what is the difference between the two problems?

recall, and compare



given or known : $\mathbf{P}, \mathbf{M}_0, \mathbf{R}_A$

fine : $\mathbf{V}, \mathbf{M}$

$$\sum \mathbf{F}_y = \mathbf{R}_A + \mathbf{P} + \mathbf{V} \qquad \therefore R_A\mathbf{j} + P\mathbf{j} + V\mathbf{j} = 0$$

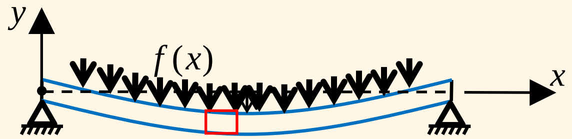
$$\sum \mathbf{M}_{z\ \text{at A}} = \frac{\mathbf{L}}{4} \times \mathbf{P} + \mathbf{M}_0 + \mathbf{x} \times \mathbf{V} + \mathbf{M}$$

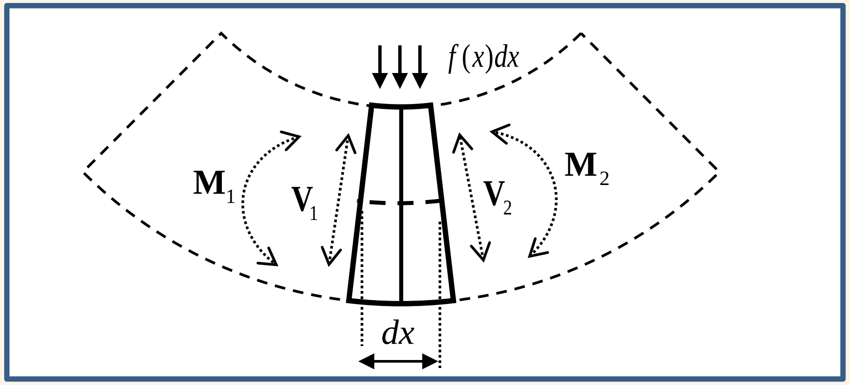
$$\therefore \frac{L}{4} \cdot P\mathbf{k} + M_0\mathbf{k} + x \cdot V\mathbf{k} + M\mathbf{k} = 0$$

can you see any difference in result with the cases of assumption?

Why?

Summary : Sign Convention & Equations





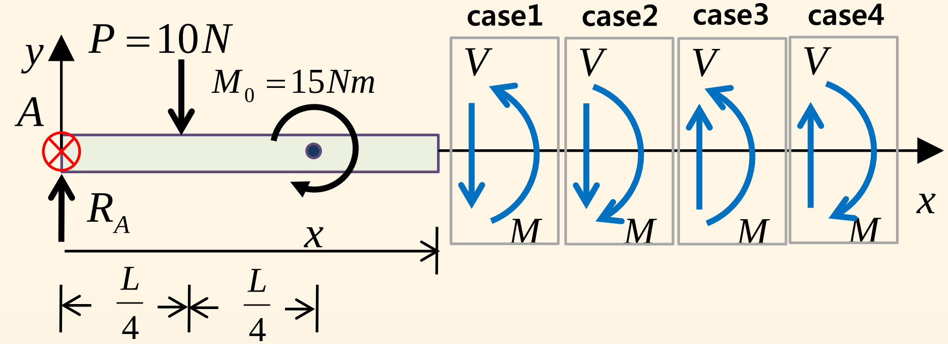
is it ok to assume arbitrary direction for V, M?

what is the difference between the two problems?

the direction of two unknowns should be assumed → 'a criteria' is required

'a criteria' : the directions which are able to explain the bending consistently with the physical law (to make the equation 'solvable')

recall, and compare



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$\sum \mathbf{F}_y = \mathbf{R}_A + \mathbf{P} + \mathbf{V} \quad \therefore R_A\mathbf{j} + P\mathbf{j} + V\mathbf{j} = 0$

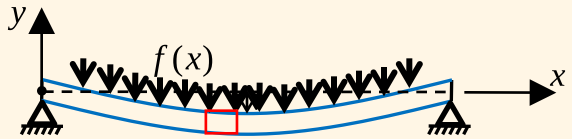
$$\sum \mathbf{M}_{z\ \text{at A}} = \frac{\mathbf{L}}{4} \times \mathbf{P} + \mathbf{M}_0 + \mathbf{x} \times \mathbf{V} + \mathbf{M}$$

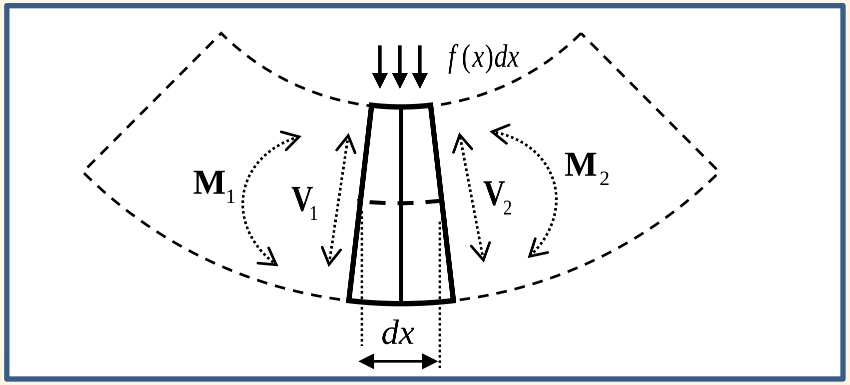
$$\therefore \frac{L}{4} \cdot P\mathbf{k} + M_0\mathbf{k} + x \cdot V\mathbf{k} + M\mathbf{k} = 0$$

can you see any difference in result with the cases of assumption?

Why?

Summary : Sign Convention & Equations





is it ok to assume arbitrary direction for V, M?

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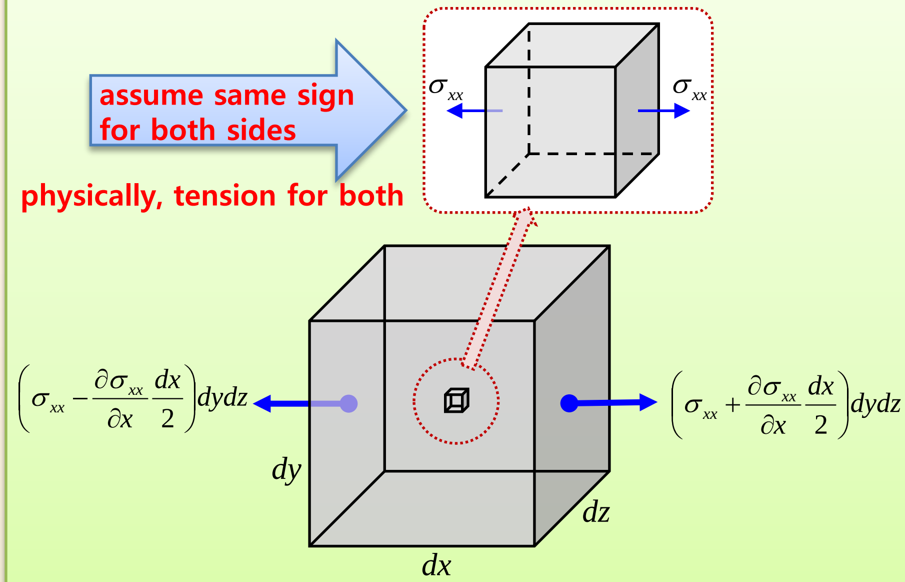
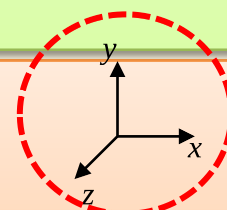
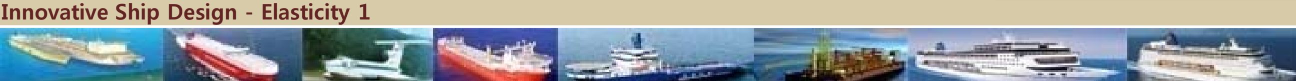




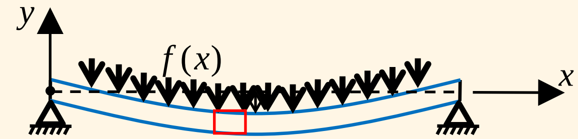
then, consider the direction of coordinates

$$F_{xx} = \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz - \left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz$$

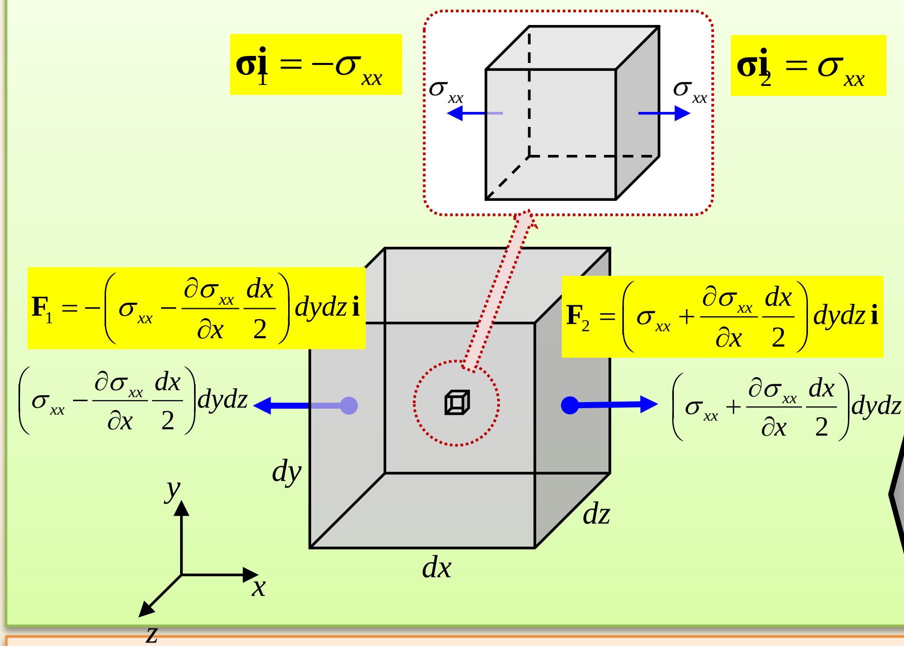
$$= \frac{\partial \sigma_{xx}}{\partial x} dxdydz$$



Summary : Sign Convention & Equations



if we use vector



$\boldsymbol{\sigma}\mathbf{i}_1 = -\sigma_{xx}$ $\quad$ $\boldsymbol{\sigma}\mathbf{i}_2 = \sigma_{xx}$

$$\mathbf{F}_1 = -\left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz\,\mathbf{i} \qquad \mathbf{F}_2 = \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz\,\mathbf{i}$$

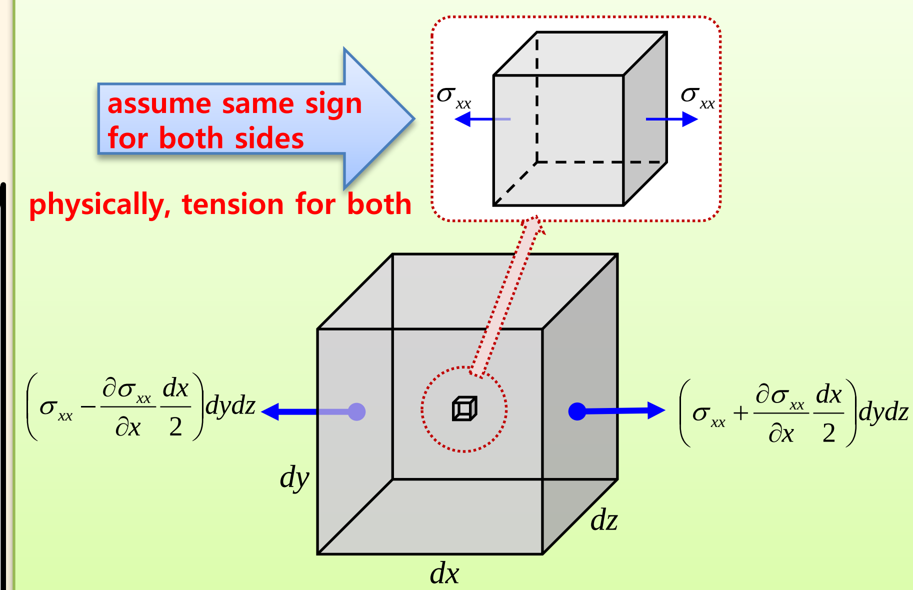
$$\left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz \qquad \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz$$

$$\sum \mathbf{F}_x = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -\left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz\,\mathbf{i} + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz\,\mathbf{i}$$

$$= \frac{\partial \sigma_{xx}}{\partial x}dxdydz\,\mathbf{i}$$

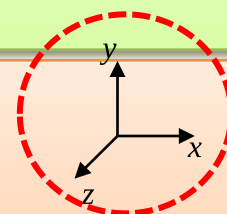
recall,



assume same sign for both sides

physically, tension for both

$$\left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz \qquad \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz$$



then, consider the direction of coordinates

$$F_{xx} = \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz - \left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz$$

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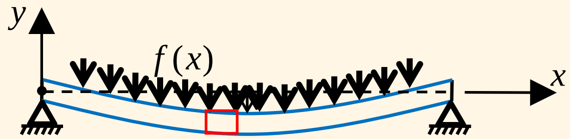
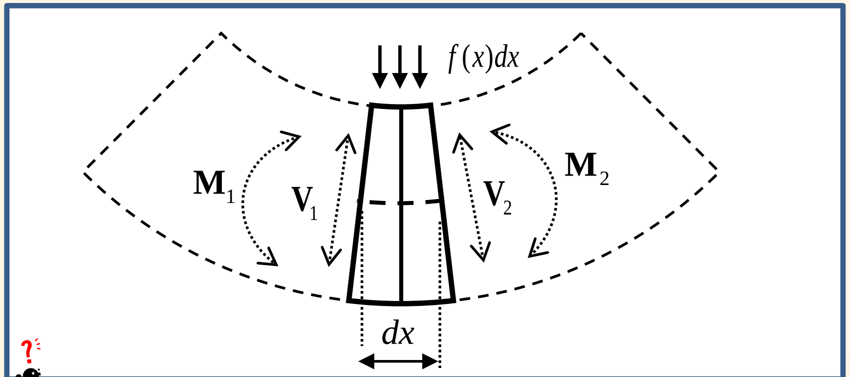
Summary : Sign Convention & Equations





is it ok to assume arbitrary direction for V, M?

the direction of two unknowns should be assumed → 'a criteria' is required

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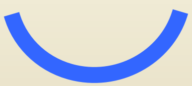
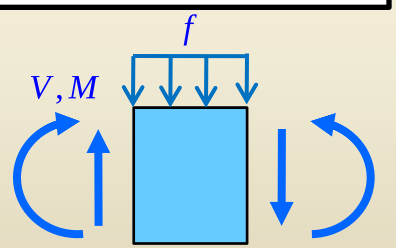
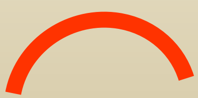
'a criteria ' for this bending



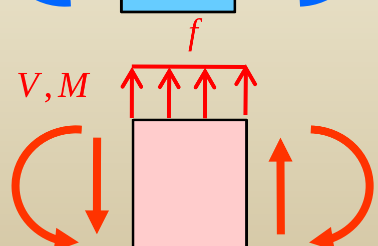
We will call this 'positive'



'another criteria' for this bending



We will call this 'negative'



recall and if we use vector

$\sigma \mathbf{i}_1 = -\sigma_{xx}$

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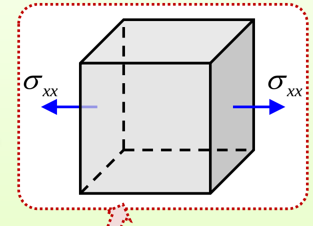

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$\left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right) dydz$

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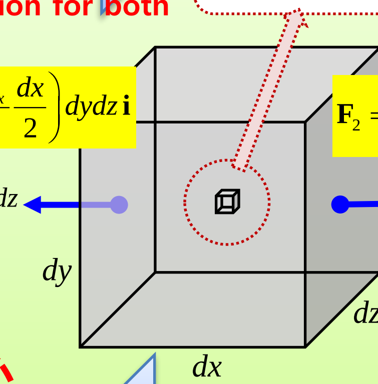

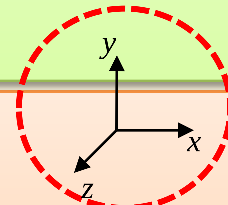

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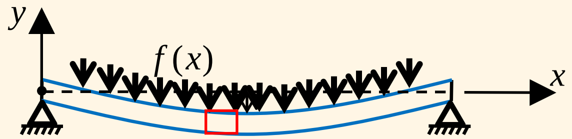
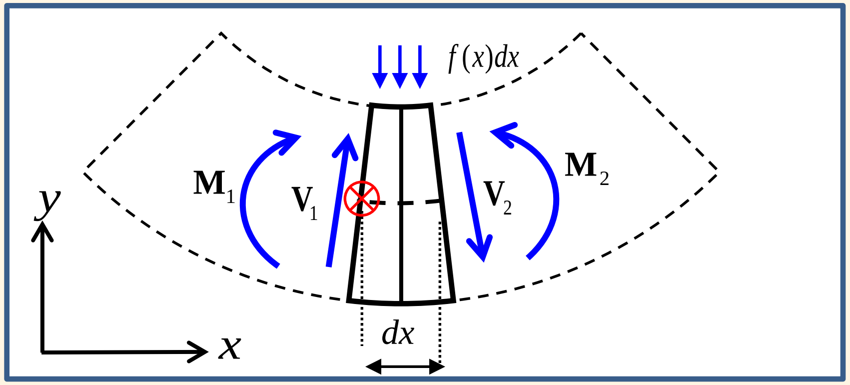
Summary : Sign Convention & Equations





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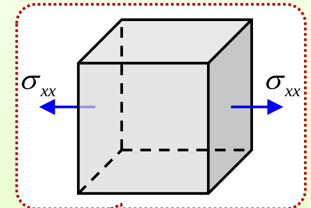

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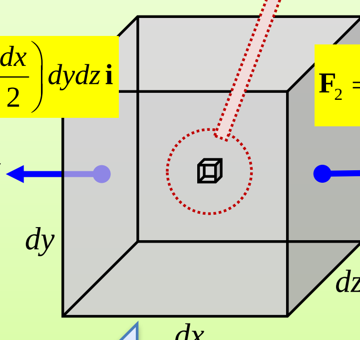

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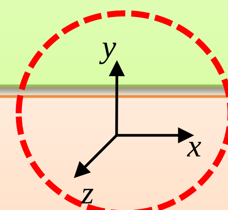

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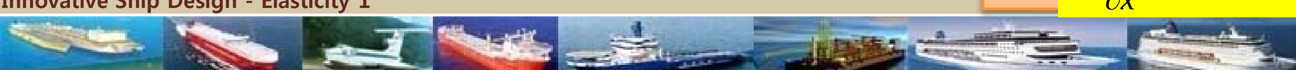
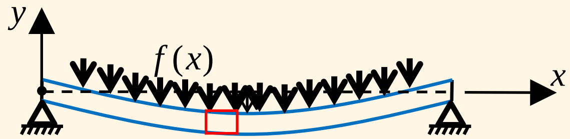
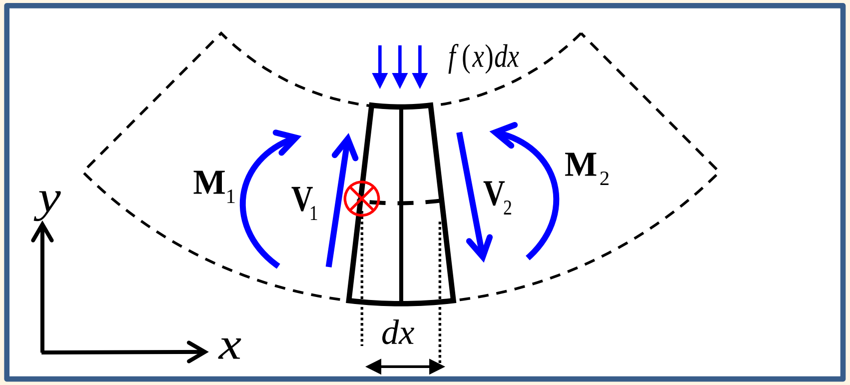


Summary : Sign Convention & Equations





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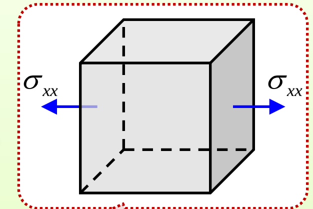

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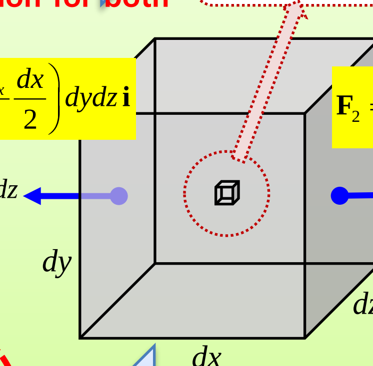

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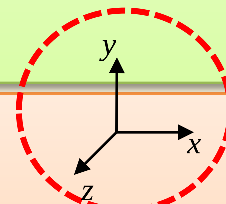

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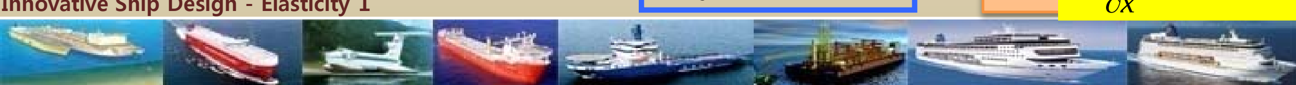
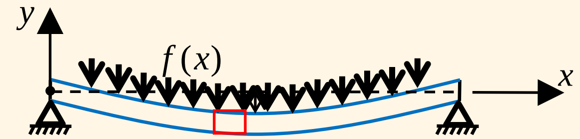
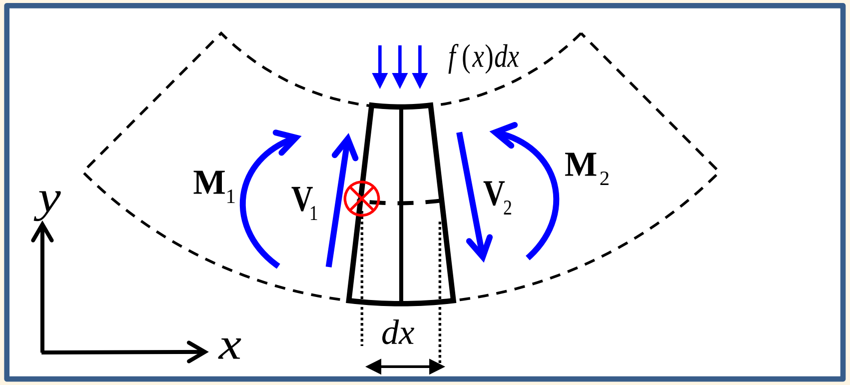


Summary : Sign Convention & Equations





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•moment equilibrium

$$\sum \mathbf{M}_z = \mathbf{M}_1 + \mathbf{M}_2 + d\mathbf{x} \times \mathbf{V}_2 + \frac{1}{2}d\mathbf{x} \times \left(\mathbf{f}(x) \cdot dx\right) = 0$$

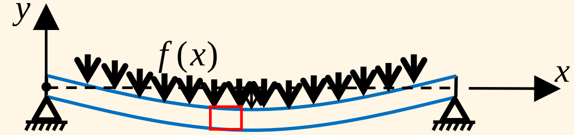
$$-M\mathbf{k} + \left(M + \frac{\partial M}{\partial x}dx\right)\mathbf{k} + (dx\,\mathbf{i}) \times \left(-\left(V + \frac{\partial V}{\partial x}dx\right)\mathbf{j}\right) + \left(\frac{1}{2}dx\,\mathbf{i}\right) \times \left(-f(x)\mathbf{j}\right) = 0$$

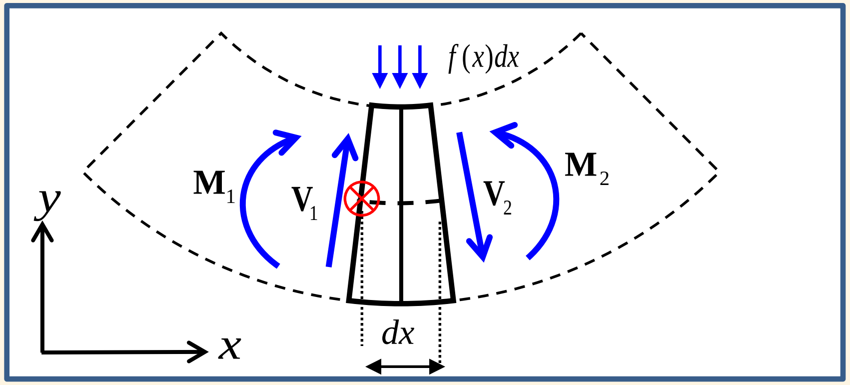
$$\left(-M + M + \frac{\partial M}{\partial x}dx - Vdx - \frac{\partial V}{\partial x}(dx)^2 - \frac{1}{2}f(x)(dx)^2\right)\mathbf{k} = 0$$

$$\left(\frac{\partial M}{\partial x}dx - Vdx\right)\mathbf{k} = 0 \qquad ,\left(\because (dx)^2 \approx 0\right)$$

$$\therefore \frac{dM}{dx} = V(x)$$

Summary : Sign Convention & Equations





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•**recall,**

$$\mathbf{M} = EI\frac{d^2y}{dx^2}\mathbf{k} \qquad , M = EI\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

↗

$$\frac{d^3y}{dx^3} = \frac{1}{EI} \cdot \frac{dM}{dx} = \frac{1}{EI} \cdot V(x)$$

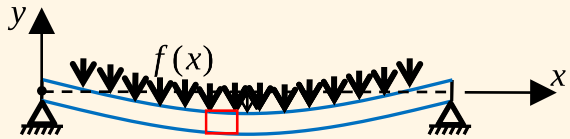
↓

$$\frac{d^4y}{dx^4} = \frac{1}{EI} \cdot \frac{dV}{dx} = -\frac{1}{EI} \cdot f(x)$$

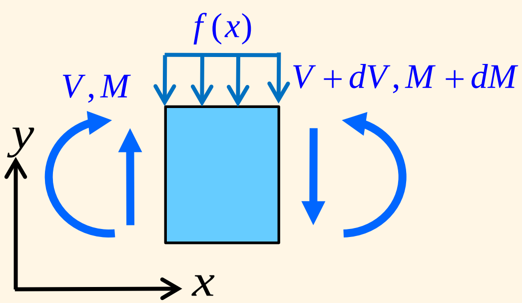
↗

$$\therefore EI\frac{d^4y}{dx^4} = -f(x)$$

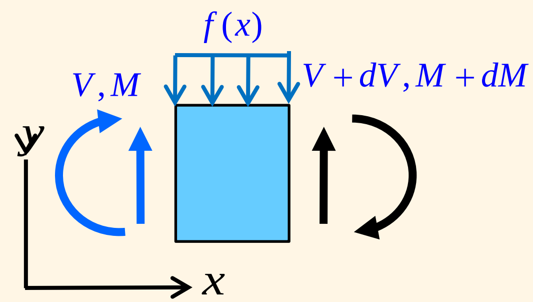
the equation is derived with a 'positive' direction convention



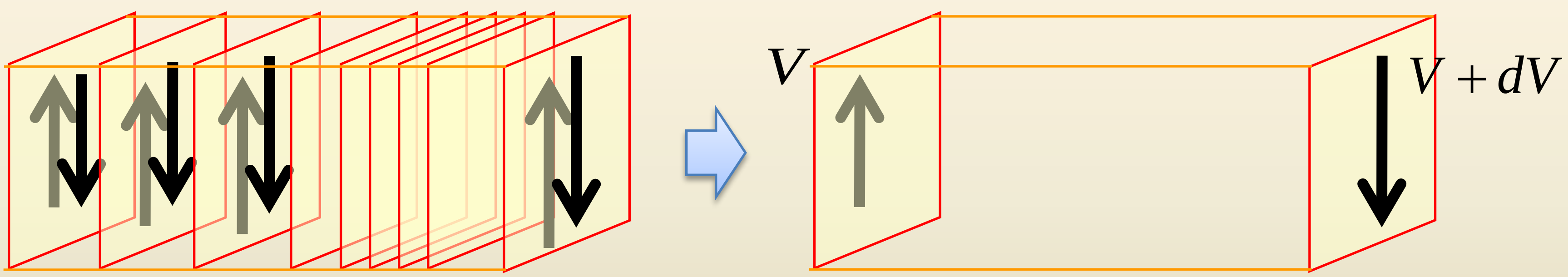
relationships between loads, shear forces, and bending moments



the direction is reasonable?

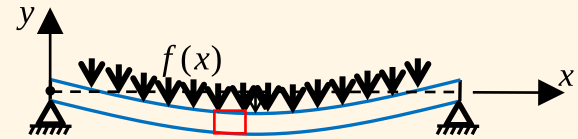


Shear force



ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006, p300~306

Summary : Sign Convention & Equations



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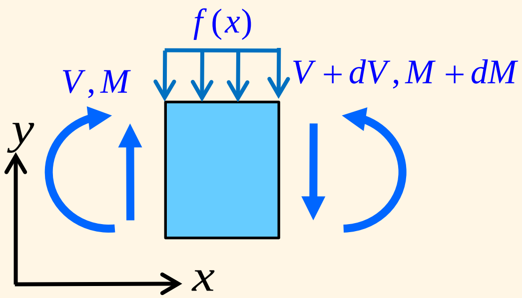

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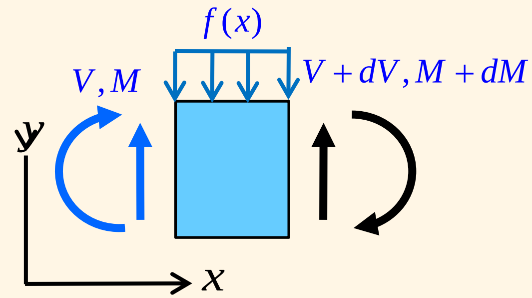
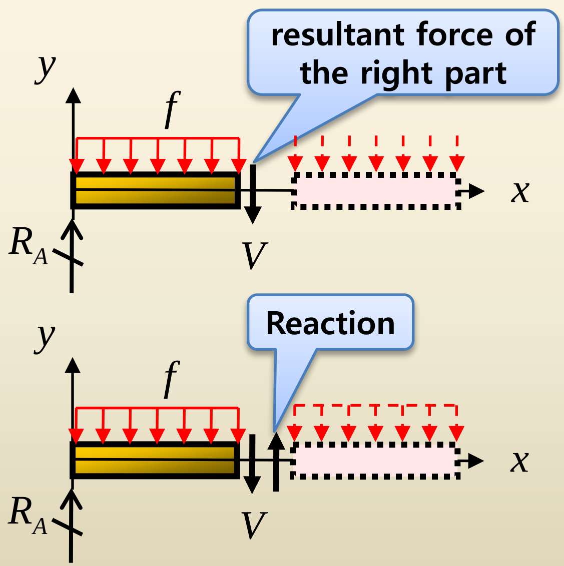
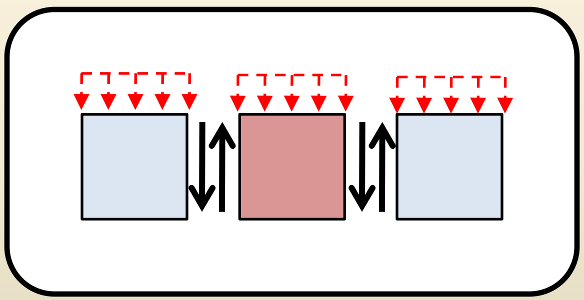






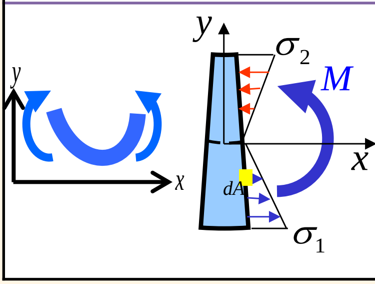
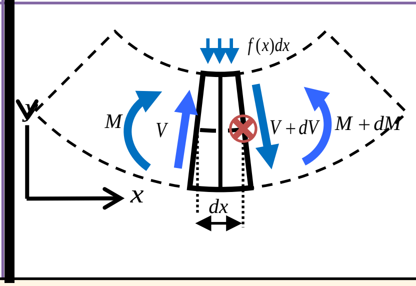
$\varepsilon \Rightarrow \sigma = E \cdot \varepsilon \Rightarrow y\sigma dA \to dM$

Summary : Sign Convention & Equations

convention

Derived

ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006

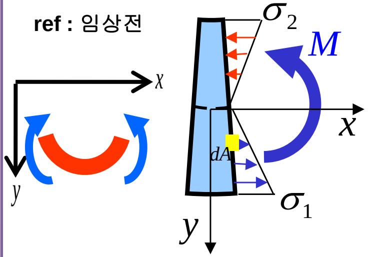
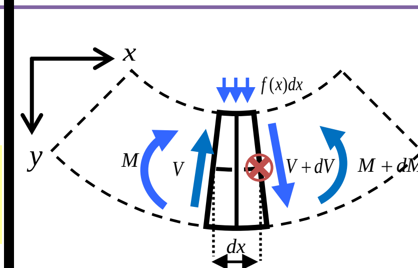
deformation (ref : Gere)	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)
	= ⊕	+	+	$-\kappa y$ or $-\frac{y}{\rho}$	-	comp.	⊖	$-\sigma y dA$	$M=-\int_A y\sigma dA$ $=-\int_A y(-\frac{E}{\rho}y)dA$ $M=\frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI}=\frac{d^2y}{dx^2}$	 $V-f(x)dx-(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $-M+f(x)dx\cdot\frac{1}{2}dx+(M+dM)-Vdx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=-f(x)$
			-		+	tension				

not match!

modified

What is the difference comparing with other books?

임상전 편저, 재료역학, 2002년 ,문운당 (Timoshenko S., Young D.H., Elements of strength of materials, 5th edition, Van Nostrand, 1968

	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)
ref : 임상전 	≠ ⊕	-	-	κy or $\frac{y}{\rho}$	-	comp.	⊕	$\sigma y dA$	$\frac{d^2y}{dx^2}=\pm\frac{M}{EI}$ ⇩ $\frac{d^2y}{dx^2}=-\frac{M}{EI}$	 $-V+f(x)dx+(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $M-(M+dM)+Vdx-f(x)dx\cdot\frac{1}{2}dx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=f(x)$
			+		+	tension				

match

modify considering the curvature

all sign convention is same
except y-axis in opposite direction

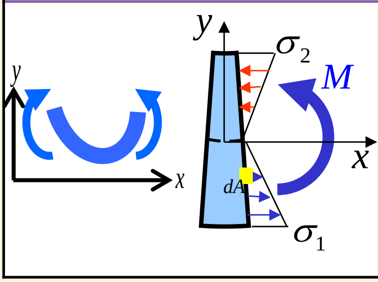
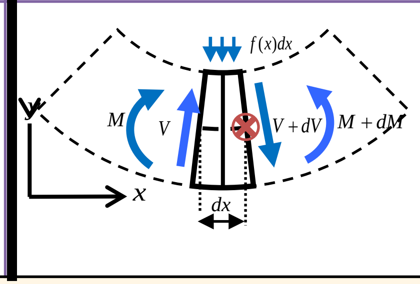
ε ⇨ $\sigma = E \cdot \varepsilon$ ⇨ $y\sigma dA \rightarrow dM$

Summary : Sign Convention & Equations

convention

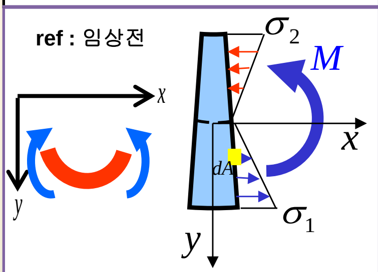
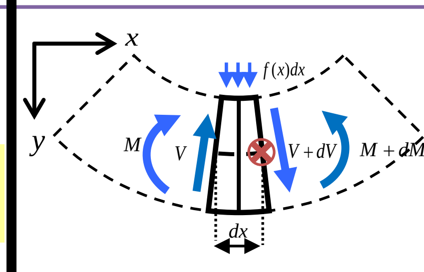
Derived

ref : Gere J.M., Mechanics of Materials, 6th edition, Thomson, 2006

deformation (ref : Gere)	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)	
	⇔ = ⊕	+	+ -	$-\kappa y$ or $-\dfrac{y}{\rho}$	- +	comp. tension	⊖	$-\sigma y dA$ modified	$M=-\int_A y\sigma dA$ $=-\int_A y(-\frac{E}{\rho}y)dA$ $M=\frac{E}{\rho}\int_A y^2 dA$ $\frac{M}{EI}=\frac{d^2y}{dx^2}$		$V-f(x)dx-(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $-M+f(x)dx\cdot\frac{1}{2}dx+(M+dM)-Vdx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=-f(x)$

not match!

임상전 편저, 재료역학, 2002년 ,문운당 (Timoshenko S., Young D.H., Elements of strength of materials, 5th edition, Van Nostrand, 1968

ref : 임상전	B.M.	K =1/ρ	y	ε	σ	check	dM= yσdA	dM	$M=\int_A dM$	relation btw V, M, f(x)	
	⇔ ≠ ⊕	-	- +	κy or $\dfrac{y}{\rho}$	- +	comp. tension	⊕	$\sigma y dA$	$\frac{d^2y}{dx^2}=\oplus\frac{M}{EI}$ ⇩ $\frac{d^2y}{dx^2}=\ominus\frac{M}{EI}$		$-V+f(x)dx+(V+dV)=0$ $\Rightarrow \frac{dV}{dx}=-f(x)$ $M-(M+dM)+Vdx-f(x)dx\cdot\frac{1}{2}dx=0$ $\Rightarrow \frac{dM}{dx}=V(x)$ $EI\frac{d^4y}{dx^4}=f(x)$

match

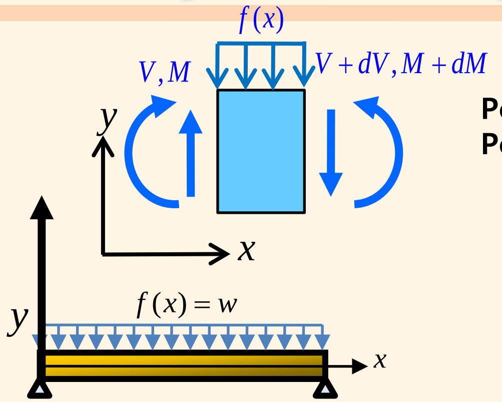
modify considering the curvature

all sign convention is same
except y-axis in opposite direction

What is different with the solution when the direction of y axis is reversed?

Simple Integration

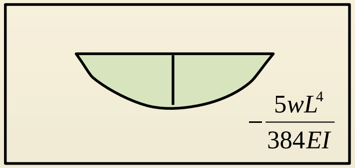
$$y(0)=0 \qquad y(L)=0$$
$$y''(0)=0 \qquad y''(L)=0$$



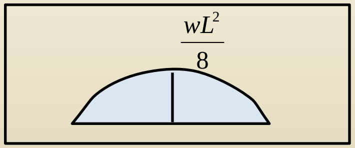
Positive Bending Moment
Positive Curvature

$$EI\frac{d^4y(x)}{dx^4}=-w$$

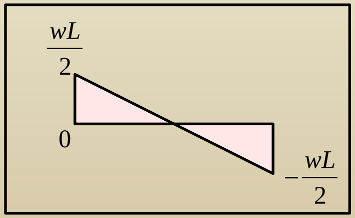
$$y(x)=-\frac{wx}{24EI}(L^3-2Lx^2+x^3)$$



$$M(x)=\frac{wLx}{2}-\frac{wx^2}{2}$$

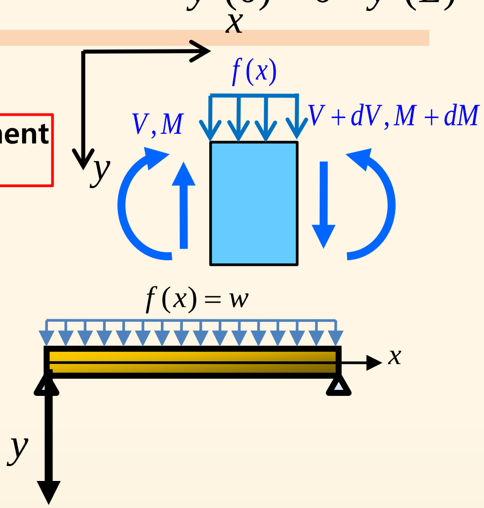


$$V(x)=\frac{wL}{2}-wx$$

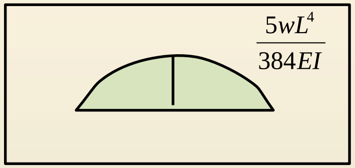


Positive Bending Moment
Negative Curvature

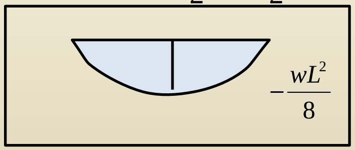
$$EI\frac{d^4y(x)}{dx^4}=w$$



$$y(x)=\frac{wx}{24EI}(L^3-2Lx^2+x^3)$$

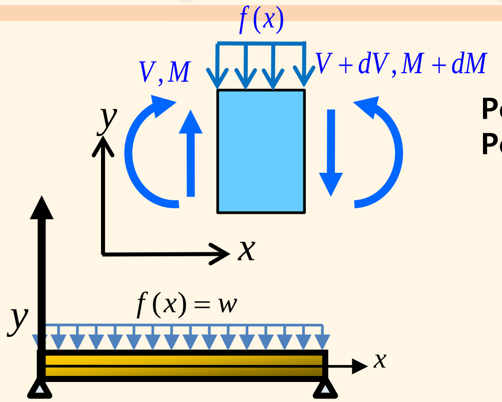


$$M(x)=-\frac{wLx}{2}+\frac{wx^2}{2}$$



Simple Integration

$$y(0)=0 \quad y(L)=0$$
$$y''(0)=0 \quad y''(L)=0$$

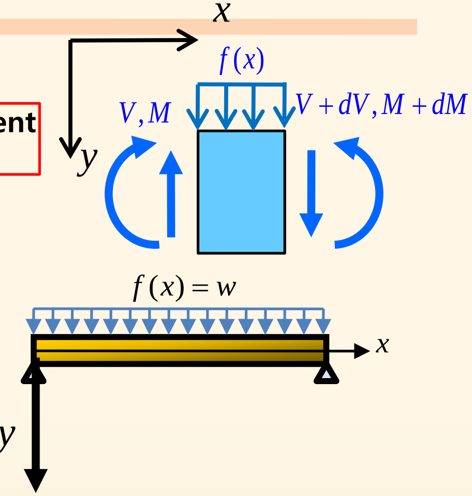


Positive Bending Moment
Positive Curvature

$$EI\frac{d^4y(x)}{dx^4}=-w$$

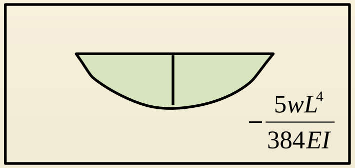
Positive Bending Moment
Negative Curvature

$$EI\frac{d^4y(x)}{dx^4}=w$$

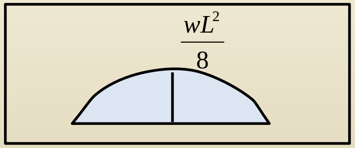


The solution is correct to the physical phenomenon as long as it is interpreted by the sign convention used.

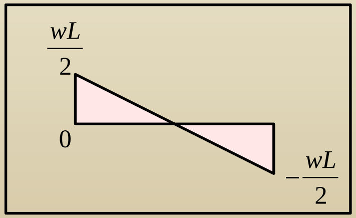
$$y(x)=-\frac{wx}{24EI}(L^3-2Lx^2+x^3)$$



$$M(x)=\frac{wLx}{2}-\frac{wx^2}{2}$$



$$V(x)=\frac{wL}{2}-wx$$



match

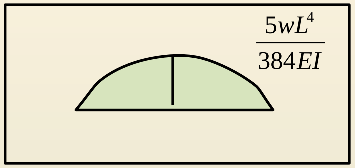
match

$$y'(x)=\frac{w}{24EI}(L^3-6Lx^2+4x^3)$$

$$y''(x)=\frac{w}{24EI}(-12Lx+12x^2)$$

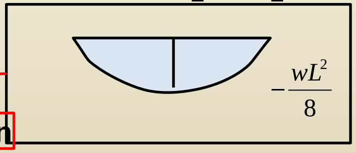
$$=\frac{w}{2EI}(-Lx+x^2)$$

$$y(x)=\frac{wx}{24EI}(L^3-2Lx^2+x^3)$$



Since 'Negative Curvature' should be with 'Positive Bending Moment'

$$M(x)=-\frac{wLx}{2}+\frac{wx^2}{2}$$



correction