Nonlinear pricing

Chapter 4. Tariff Design

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Contents

The Price Schedule of a Monopolist

- For a single product, a monopolist would ordinarily choose the price to maximize the profit contribution, which is the profit margin times the demand at the chosen price.
- Demand Profile: $$N(p,q) = \sum_{x \geq q} n(p,x)\,.$$
- Aggregate Demand with uniform price: $$\bar{D}(p) = \sum_{q} n(p,q)q = \sum_{q=\delta,2\delta,\ldots} N(p,q)\delta\,,$$
- Aggregate Demand with price schedule p(q): $$Q = \sum_{q} N(p(q),q)\delta\,,$$
- Shorthand notation Demand Profile: $$N(p,q) = \#\,\{i \mid D_i(p) \geq q\}\,.$$

The Price Schedule of a Monopolist

- To ensure that N(p(q),q) will in fact be the realized demand for the q –th increment requires that a customer who would purchase at least q units at the uniform price p = p(q) will also purchase at least q units when offered the entire schedule of prices for increments.
- total variable cost: $C(Q) = cQ$
- total profit contribution expected from price schedule p(q):

$$\text{Pft} \equiv \sum_q N(p(q), q) \cdot [p(q) - c]\delta \, .$$

- increment's profit contribution:

$$R(p(q), q) \equiv N(p(q), q) \cdot [p(q) - c] \, .$$

Example 4.1

Table 4.1

Demand Profile for Example 4.1

Optimal Tariff for Marginal Cost $c = \$1$

		$N(p,q)$					
p q :	1	2	3	4	5 units		$\bar{D}(p)$
\$2/unit	90	75	55	30	5		255
\$3	80	65	45	20	0		210
\$4	65	50	30	5	0		150
\$5	45	30	10	0	0		85
$p(q)$:	\$4	\$4	\$3	\$3	\$2/unit		\$4
$P(q)$:	\$4	\$8	\$11	\$14	\$16		
$R(p(q), q)$:	\$195	\$150	\$90	\$40	\$5		
Total Profit :						\$480	\$450
'CS'(q) :	\$45	\$30	\$40	\$5	\$0	\$120	\$85
'TS'(q) :						\$600	\$535

"inverse elasticity rule"

- The demand profile summarizes the heterogeneity among customers at the coarsest level of aggregation that still allows analysis of nonlinear tariffs.
- In applications it is often useful to represent this information in terms of the price elasticities of demands for different units.

$$N(p^\circ, q)[p^\circ - c] > N(p, q)[p - c]\,.$$

$$dN \;\equiv\; N(p^\circ, q) - N(p, q) \;>\; 0$$

$$dp \equiv p^\circ - p < 0$$

$$\frac{p^\circ - c}{p^\circ} > \left[\frac{dN/N}{-dp/p}\right]^{-1}$$

$$\frac{p(q) - c}{p(q)} \approx \frac{1}{\eta(p(q), q)}\,.$$

price that maximizes the profit contribution's necessary condition

$$N(p(q), q) + \frac{\partial N}{\partial p}(p(q), q) \cdot [p(q) - c] = 0\,.$$

Maximization of the profit contribution from the q -th unit via optimal selection of the marginal price

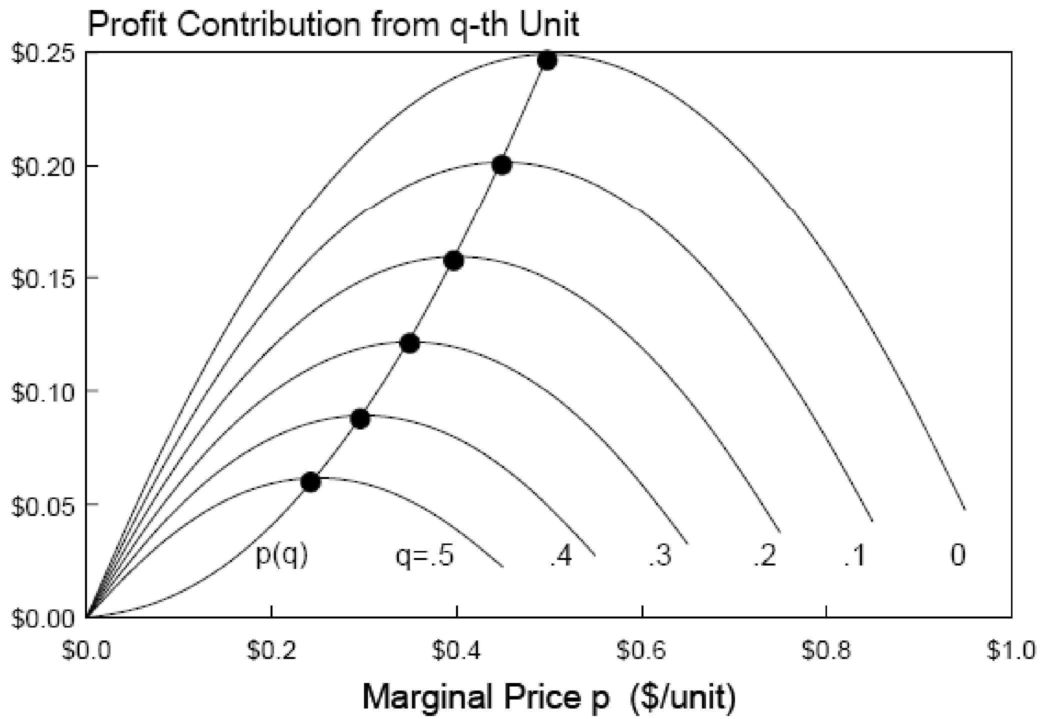

Figure 1: Maximization of the profit contribution from the q -th unit via optimal selection of the marginal price $p(q)$. Assumes $N(p, q) = 1 - p - q$ and $c = 0$.

the marginal price schedule for several values of the parameters

$$U(q,x;t,s) = qt + xs - \frac{1}{2}[q^2 + 2aqx + x^2]$$

$$D(p,p^*;t,s) = ([t-p] - a[s-p^*])/[1-a^2]$$

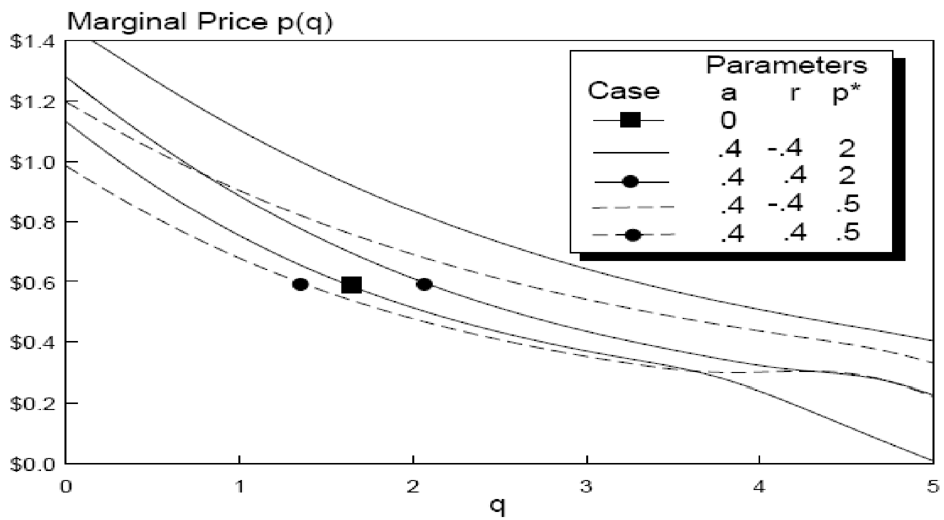

Figure 2: The marginal price schedule for several values of the parameters in Example 4.3.

4.2. Extensions and Qualifications -Variable marginal cost

- total cost

$$\mathcal{C}(\mathcal{Q}) = C_1(Q) + \sum_i C_2(q_i)\,.$$

$$Q = \sum_i q_i$$

- relevant marginal cost for an individual purchase q

$$c(q) = C_1'(Q) + C_2'(q)\,,$$

- feasible condition

$$\int_0^\infty N(p(q), q)\, dq \le Q\,.$$

Decreasing Price Schedules and the Ironing Procedure

necessary condition for optimality

$$N(p(q),q) + \frac{\partial N}{\partial p}(p(q),q)\cdot[p(q)-c(q)] = 0,$$

$$\int_A^B \left\{ N(p,q) + \frac{\partial N}{\partial p}(p,q)\cdot[p-c(q)] \right\} dq = 0$$

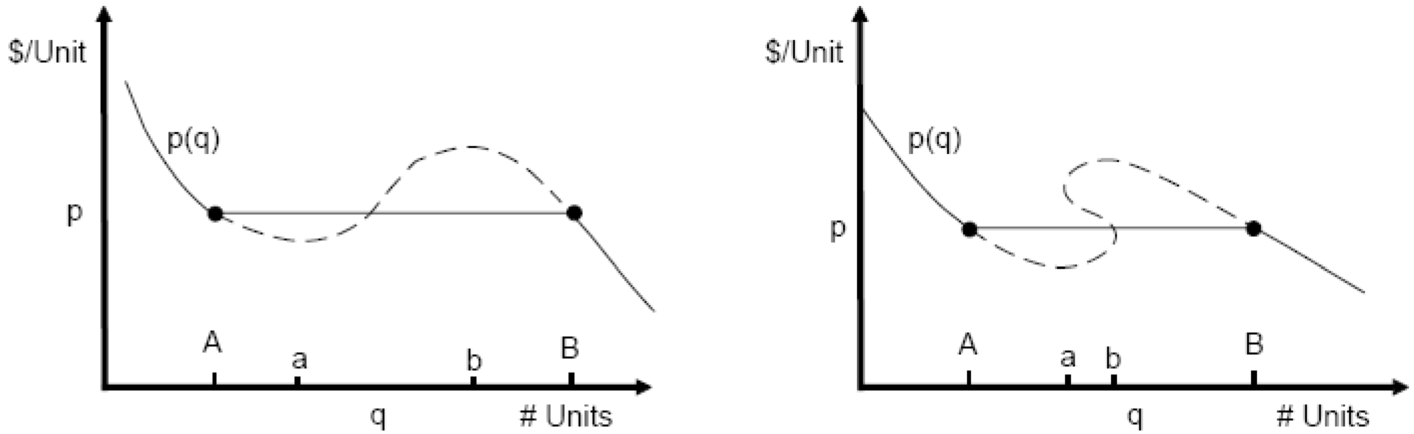

Figure 3: Construction of a nonincreasing price schedule via the ironing procedure. The horizontal segment is selected so that the optimality condition is satisfied on average.

Predictive Power of Demand Profile

- The exposition above assumes that the demand profile is an adequate predictor of customers'purchase behavior in response to a nonlinear price schedule.

- To exclude these problematic cases, theoretical analyses of nonlinear pricing use assumptions sufficient to assure that the optimal price schedule intersects each customer's demand function once, from below.

4.3. The Bundling Interpretation

•An alternative view construes nonlinear pricing as an instance of bundling.

•Products are said to be bundled if the charge for a purchase of several products in combination is less than the sum of the charges for the components. Bundling applies to products that are diverse (such as the options on a new automobile),

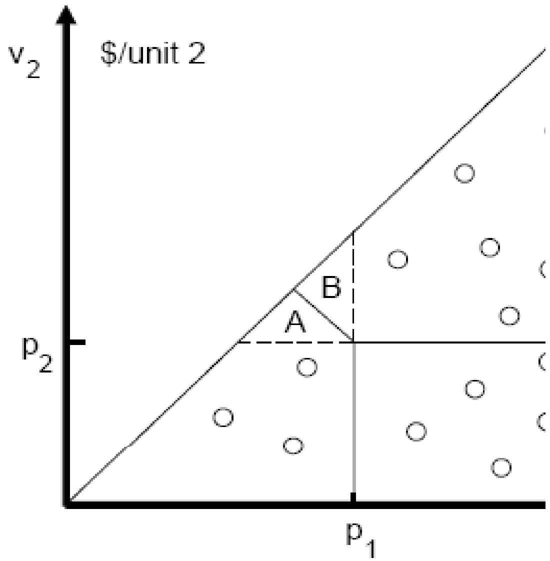

Figure 6: Bundling analysis of nonlinear pricing.
Note: ∘ indicates valuations of one type of customer.

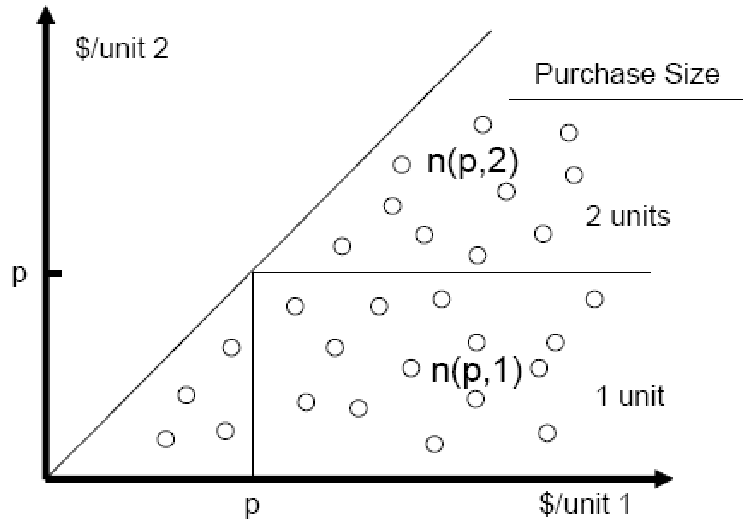

Figure 7: Construction of the demand profile from the distribution of customers' valuations. Note: $n(p,1) = 14$ and $n(p,2) = 10$ customers.

4.4. Fixed Costs and Fixed Fees

- In important cases the firm incurs a fixed cost for each customer served.
- fixed fee

$$P_\circ = \int_0^{q_*} [\hat{p}(q, q_*) - p_*] \, dq \, .$$

$$P(q_*) = P_\circ + p_* q_*$$

- Total profit contribution

$$N(p_*, q_*) \cdot [P(q_*) - C(q_*)] + \int_{q_*}^{\infty} N(p(q), q) \cdot [p(q) - c(q)] \, dq$$

- The necessary condition for an optimal choice

$$P(q_*) = C(q_*) + [q_* / K(q_*)] \cdot \int_0^{q_*} \frac{\partial \hat{p}}{\partial q_*}(q, q_*) \, dq$$

4.5. Multipart Tariffs

- Multipart tariffs take many forms
  - The simplest is a two-part tariff comprising a fixed fee plus a uniform price for every unit purchased.
  - An n-part tariff is usually presented as a fixed fee plus n-1 different "block declining" marginal prices that apply in different intervals or volume bands.
- An appropriate specification of the demand profile in terms of volume bands:

$$\bar{N}(p,[q_i,r_i]) \equiv \bar{N}(p,i) \equiv \sum_{q_i \le q \le r_i} N(p,q)\delta$$

$$\bar{N}(p,i) = \int_{q_i}^{r_i} N(p,q)\,dq$$

- profit contribution:

$$\bar{N}(p_i,i) \cdot [p_i - c]\,.$$

Example 4.5 :

Table 4.2

Demand Profile for Example 4.5

$c = 1$		$\bar{N}(p, i)$		
p	q :	[0, 2]	[3, 5]	Total
$2/unit		165	90	
$3		145	65	
$4		115	35	
$5		75	10	
$p(q)$:		$4	$3/unit	
Profit:		$345	$130	$475

The Approximate Formulation

- The profit contribution

$$N_*(P(q_1), q_1) \cdot [P(q_1) - C(q_1)] + \sum_{i=1}^{n-1} \bar{N}(p_i, [q_i, q_{i+1}]) \cdot [p_i - c] \, .$$

- Fixed fee Pi

$$P_{i-1} + p_{i-1} q_i = P(q_i) = P_i + p_i q_i$$

$$P_i = P_{i-1} + [p_{i-1} - p_i] \cdot q_i \, .$$

Example 4.6

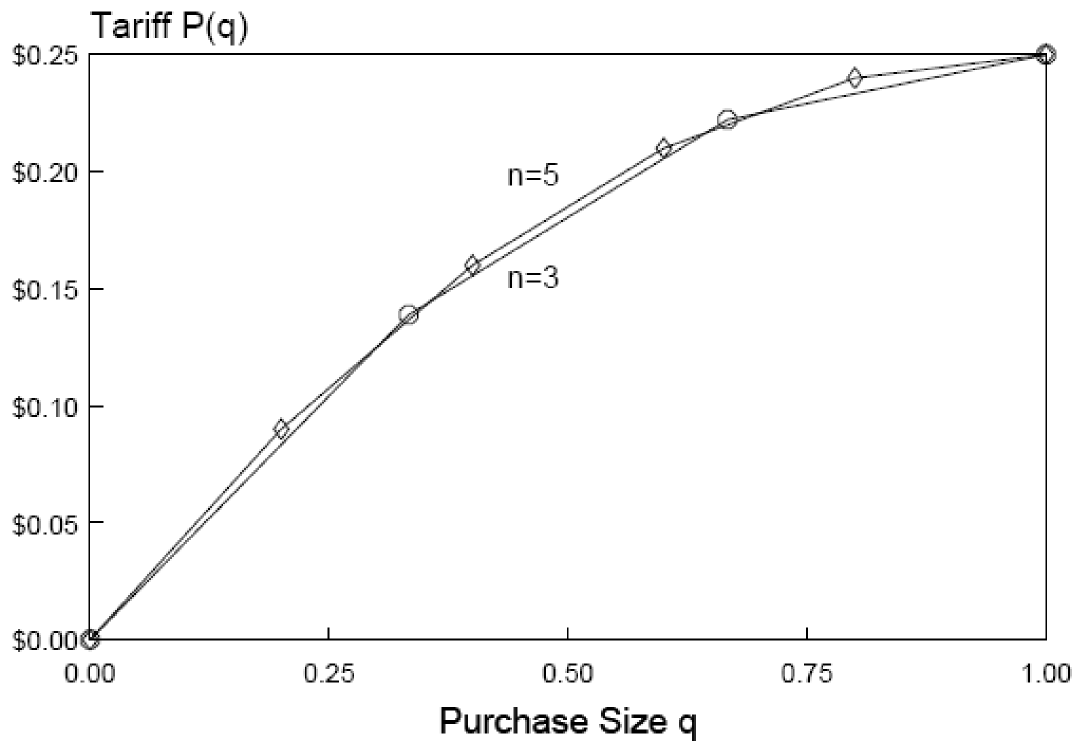

Figure 8: Example 4.6: Approximate multipart tariffs for $n = 3$ and $n = 5$.

4.6. Summary