Chapter 6.
Single-parameter Disaggregated Models

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Contents

6.1. A model with discrete types

- Gross benefit: $U_i(q)$
- Net benefit: $U_i(q) - P(q)$
- Firm's profit contribution

$$\text{Profit Contribution} = \sum_{i=1}^{m} f_i \cdot [P(q_i) - C(q_i)]$$

- Constraints
  - Participation constraint $U_i(q_i) - P(q_i) \geq U_i(0) - P(0) \equiv 0\,.$
  - Compatibility constraint

$$U_i(q_i) - P(q_i) \geq U_i(q_j) - P(q_j), \qquad \text{for each } j \neq i\,.$$

6.1. A model with discrete types

- Characterization of an optimal tariff
  1. If P_i ; $q_i \geq 0$, type i is an active customer. Then:

$$v_i(q_i) - c(q_i) = \sum_{j \neq i} \lambda_{ji} \frac{f_j}{f_i} [v_j(q_i) - v_i(q_i)] ,$$

  2. If type i's net benefit is positive, participation constraint is not binding. Then:

$$f_i = \sum_{j \neq i} [\lambda_{ij} f_i - \lambda_{ji} f_j] .$$

- Assumption: an active customer type i only binding incentive-compatibility constraint is the one for type j=i -1
- OR: λ_{ij} is positive only for j=i -1

6.1. A model with discrete types

- Def.

$$\hat{\lambda}_i = f_i \lambda_{i,i-1}$$

$$\hat{\lambda}_i - \hat{\lambda}_{i+1} = f_i \,, \quad \text{and} \quad \hat{\lambda}_{m+1} = 0 \,,$$

$$\hat{\lambda}_i = \bar{F}_i \,, \quad \text{where} \quad \bar{F}_i \equiv \sum_{j \geq i} f_j \,.$$

- Then:

$$v_i(q_i) = c(q_i) + \frac{\bar{F}_{i+1}}{f_i}[v_{i+1}(q_i) - v_i(q_i)] \,.$$

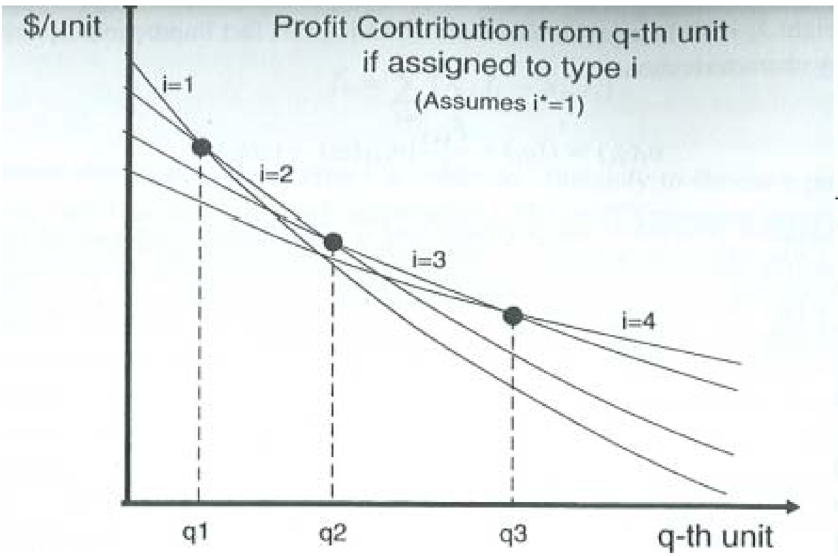
- Other form:

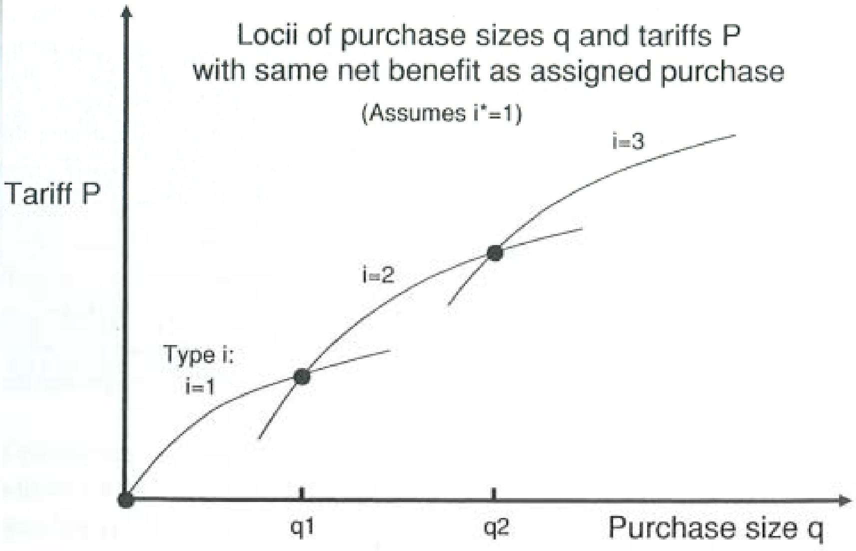
$$P_i = P_{i-1} + U_i(q_i) - U_i(q_{i-1}) \,,$$

$$[v_i(q_i) - c(q_i)]\bar{F}_i = [v_{i+1}(q_i) - c(q_i)]\bar{F}_{i+1} \,.$$

$p_i = v_i(q_i)$ is optimal marginal price

6.1. A model with discrete types



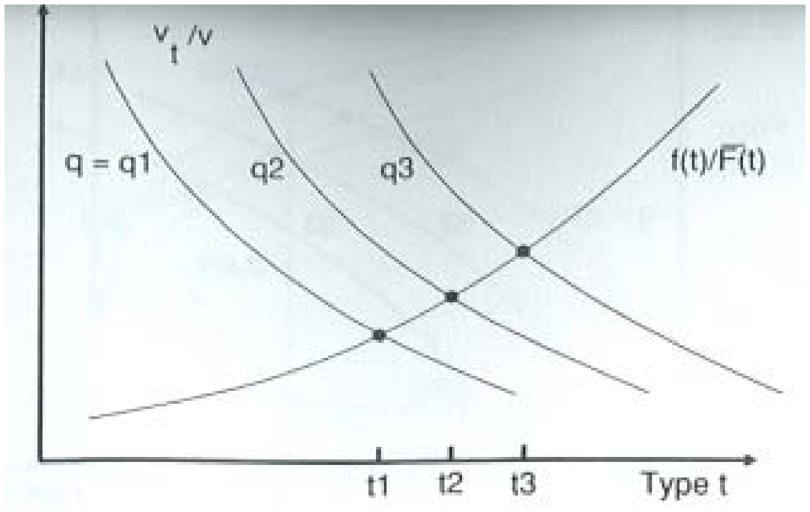


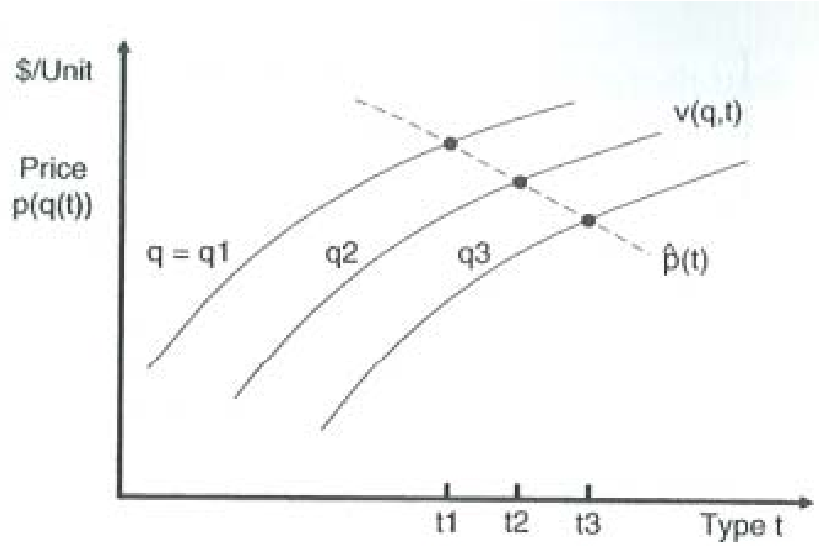
- Extension to a continuum of types

$$v(q(t),t) = c(q(t)) + \frac{\bar{F}(t)}{f(t)} \cdot \frac{\partial v}{\partial t}(q(t),t)\,.$$

6.1. A model with discrete types

- Marginal cost : c = 0
- Hazard rate: $f(t)/\bar{F}(t)$ is increasing
- $v_t(q,t)/v(q,t)$ is decreasing in t but increasing in q.

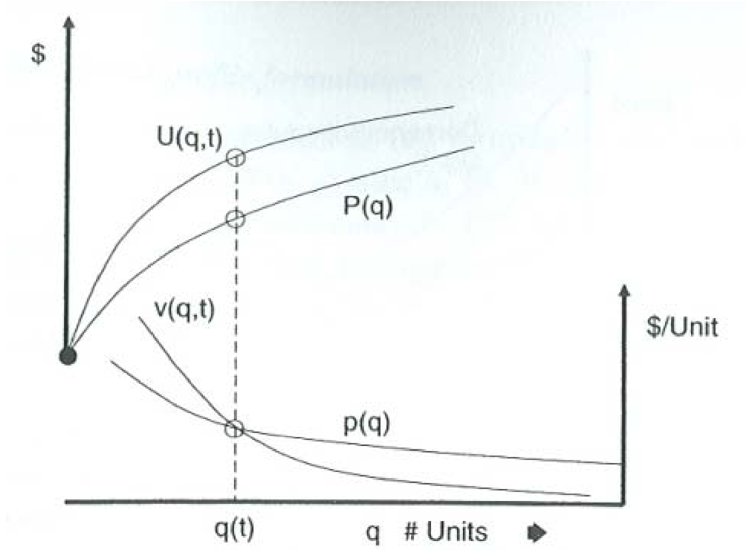


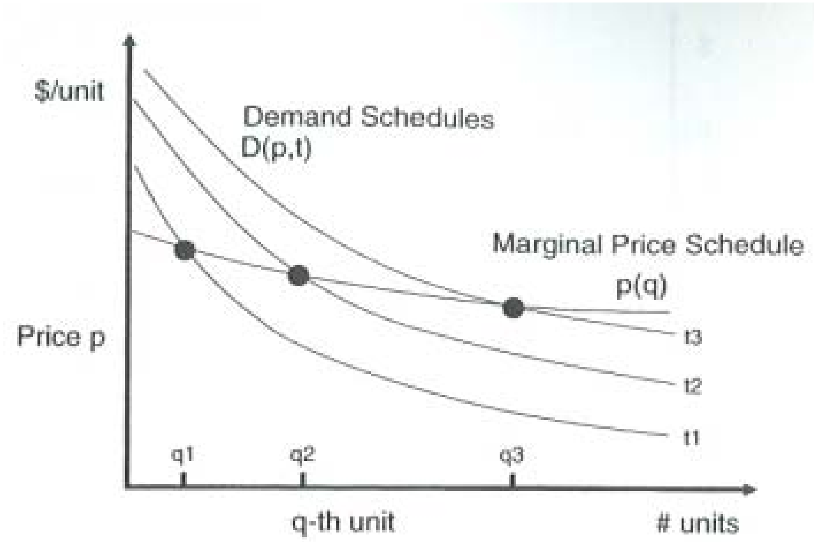


6.2. Model with one-dimensional types

- Net benefit:

$$U_i(q) - P(q)$$





6.2. Model with one-dimensional types

- Connection to the demand-profile formulation

$$N(p,q) = \# \{ t \mid v(q,t) \geq p \} = \# \{ t \mid D(p,t) \geq q \} \ ,$$

- Customer's rank: r = F(t)
- Refer to section 4. Condition for profit-maximizing

$$N(p,q) + N_p(p,q) \cdot [p - c] = 0$$

- Equivalent condition is expressed:

$$\bar{F}(t) - [f(t)/v_t(q,t)][v(q,t) - c] = 0 \, ,$$

6.3. Two-part tariffs

- Uniform price + fixed fee
- Def. t* is lowest type of customer among subscriber. (market penetration)
- Profit contribution

$$PS = P \cdot \bar{F}(t_*) + [p - c] \cdot \int_{t_*}^{\infty} D(p, t)\, dF(t)\,.$$

- Customer's surplus

$$\int_p^{\infty} D(\pi, t)\, d\pi - P\,.$$

In type t*, CS = 0 then:

$$P = \int_p^{\infty} D(\pi, t_*)\, d\pi\,.$$

6.3. Two-part tariffs

- Profit maximizing monopoly
  - Condition for optimal choice of price

$$\frac{p-c}{p} = \frac{1}{\bar{\eta}(p,t_*)}\left[1 - \bar{F}(t_*)\frac{D(p,t_*)}{\bar{D}(p,t_*)}\right].$$

  - Where

$$\bar{D}(p,t_*) = \int_{t_*}^{\infty} D(p,t)\,dF(t)$$

and $\bar{\eta}(p,t_*) = -p\bar{D}_p(p,t_*)/\bar{D}(p,t_*)$

  - Ordinary uniform pricing $\frac{p-c}{p} = \frac{1}{\bar{\eta}(p,t_*)}$

6.3. Two-part tariffs

- Regulated monopoly

  Objective fun = $\text{CS} + [1 + \lambda]\text{PS},$

  - Condition for optimal choice of price

$$\frac{p-c}{p} = \frac{\alpha}{\bar{\eta}(p, t_*)}\left[1 - \bar{F}(t_*)\frac{D(p, t_*)}{\bar{D}(p, t_*)}\right].$$

  - Ordinary uniform pricing

$$\frac{p-c}{p} = \frac{\alpha}{\bar{\eta}(p, t_*)},$$

6.3. Two-part tariffs

- Exp. 6-1

Demand fun. D(p,r) = r.[A-p]/B. then

$$P = \frac{r_*}{2B}[A-p]^2 \qquad \text{so} \qquad r_*(P) = 2BP/[A-p]^2$$

$$\text{Profit} = \frac{1}{2}[p-c][A-p][1-r_*(P)^2] + P[1-r_*(P)] .$$

The two-part tariff that maximizes this profit is:

$$p = c + \frac{1}{2}[1-r_*][A-c] , \qquad \text{and} \qquad P = \frac{r_*}{2B}[A-p]^2 ,$$

where $r_* = \frac{5}{4} - \sqrt{17/16} \approx 1 - 0.78078$.

Table 6.1 Example 6.1: Optimal Two-Part Tariffs

c/A	p/A	$10 \times PB/A^2$	Pft/A^2
.00	.39	.407	.15
.10	.45	.330	.12
.20	.51	.261	.09
.30	.57	.200	.07
.40	.63	.147	.05
.50	.70	.102	.04
.60	.76	.065	.02
.70	.82	.037	.01
.80	.88	.016	.01
.90	.94	.004	.00
1	1	0	0

6.3. Two-part tariffs

- The optimality condition as an average
  - The optimality condition for the marginal price

$$\int_{t_*}^{\infty} \left[[p-c] \cdot D_p(p,t) + \alpha \frac{\bar{F}(t)}{f(t)} \cdot D_t(p,t) \right] dF(t) = 0 .$$

  - The optimality condition for the marginal types

$$\int_{p}^{\infty} \left[[\pi-c] \cdot D_p(\pi,t_*) + \alpha \frac{\bar{F}(t_*)}{f(t_*)} \cdot D_t(\pi,t_*) \right] d\pi = 0 ,$$

6.4. Multipart tariffs

- Equivalent to the menu of optional two-part tariffs
- n-1 two-part tariffs, in order $P_i < P_{i+1}$ and $p_i > p_{i+1}$

  ($P_0 = 0$ and $p_0 = \text{inf.}$)

$$PS_i(t) = [p_i - c] \cdot D(p_i, t) + P_i \,, \qquad CS_i(t) = \int_{p_i}^{\infty} D(p,t)\,dp - P_i \,,$$

- Aggregates:

$$PS = \sum_{i=1}^{n-1} \int_{t_i}^{t_{i+1}} PS_i(t)\,dF(t) \,, \qquad CS = \sum_{i=1}^{n-1} \int_{t_i}^{t_{i+1}} CS_i(t)\,dF(t) \,.$$

6.4. Multipart tariffs

- Condition that type t_i is indifferent between the tariff i or i+1, is $CS_i(t_i) = CS_i(t_{i+1})$. Then

$$P_i - P_{i-1} = \int_{p_i}^{p_{i-1}} D(p, t_i)\, dp\,.$$

$$P_i = P_o + \sum_{j \le i} \int_{p_j}^{p_{j-1}} D(p, t_j)\, dp\,,$$

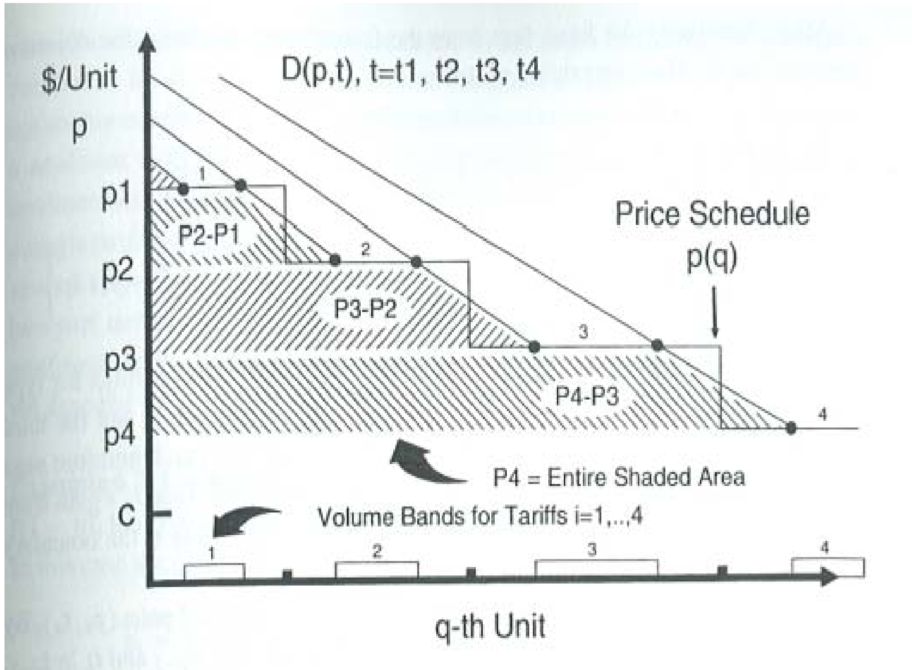


FIG. 6.7 A block-declining price schedule. Due to the increment $P_i - P_{i-1}$ in the fixed fee, type t_i is indifferent between the two-part tariffs $\langle P_{i-1}, p_{i-1} \rangle$ and $\langle P_i, p_i \rangle$.

6.4. Multipart tariffs

- Ramsey pricing: Objective fun.

$$\sum_{i=1}^{n-1} \left\{ \int_{t_i}^{t_{i+1}} \left(\int_{p_i}^{\infty} D(p,t)\,dp + [1+\lambda][p_i - c] \cdot D(p_i,t) \right) dF(t) + \lambda \bar{F}(t_i) \int_{p_i}^{p_{i-1}} D(p,t_i)\,dp \right\}.$$

  - Condition for the optimal marginal price p_i

$$\int_{t_i}^{t_{i+1}} \left\{ [p_i - c] \cdot D_p(p_i,t) + \alpha \frac{\bar{F}(t)}{f(t)} \cdot D_t(p_i,t) \right\} dF(t) = 0.$$

  - Condition for the optimal boundary type t_i

$$\int_{p_i}^{p_{i-1}} \left\{ [p - c] \cdot D_p(p,t_i) + \alpha \frac{\bar{F}(t_i)}{f(t_i)} \cdot D_t(p,t_i) \right\} dp = 0.$$

6.4. Multipart tariffs

- Exp.6-2
  - D(p,t) = t[1-p], type uniformly distributed
  - α= 1; c = 0. Then

$$p_i = 1 - [t_i + t_{i+1}]/2 \qquad \text{and} \qquad t_i = 1 - [p_i + p_{i-1}]/2\,.$$

  - Boundary condition: $t_n = 1$; $p_0 = 1$. Then

$$t_i = \frac{i - .5}{n - .5} \qquad \text{and} \qquad p_i = 1 - \frac{i}{n - .5}\,.$$

$$P_i = \frac{1}{3} \cdot \frac{[i - .5]i[i + .5]}{[n - .5]^3}\,.$$

$$\mathrm{PS}(n) = \frac{1}{6} \cdot \left[1 - \frac{1}{4(n - .5)^2}\right]\,.$$

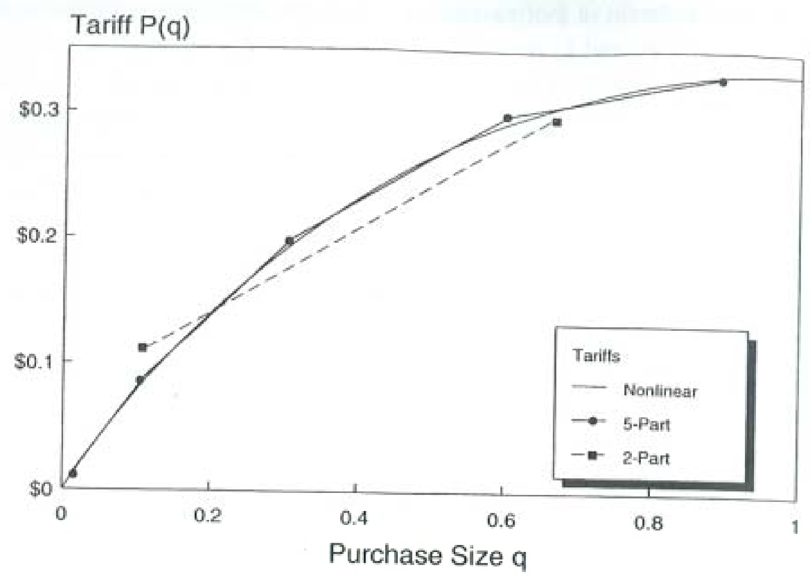


FIG. 6.8 The optimal nonlinear, 5-part, and two-part tariffs for Example 6.2.

6.4. Multipart tariffs

- A demand-profile formulation
  - Consumer's surplus is in section 5. Producer's surplus:

$$PS = \sum_{i=1}^{n-1} \left\{ N(p_i, q_i) \cdot \int_{p_i}^{p_{i-1}} q(p; p_i, q_i)\, dp + [p_i - c] \cdot \int_{q_i}^{r_i} q\, dN(p_i, q) \right\},$$

  - Condition for the optimal marginal price p_i

$$\int_{q_i}^{r_i} \{ \alpha N(p_i, q) + N_p(p_i, q) \cdot [p_i - c] \}\, dq = 0,$$

  - Condition for the optimal boundary type t_i

$$\int_{p_i}^{p_{i-1}} \{ \alpha N(p, q_i) + N_p(p, q(p; p_i, q_i)) \cdot [p - c] \}\, dp = 0,$$

6.5. Nonlinear tariff p(t)

- Producer's surplus

$$\mathrm{PS} = \int_0^\infty [p(t) - c] \cdot D(p(t), t)\, dF(t) - \int_0^\infty \bar{F}(t) \cdot D(p(t), t)\, dp(t)\,,$$

  - p(t) is the limit of the price p_i
  - t_i is the limit of market segment boundary t_i.

- Customer's surplus

$$W(p,t) = \int_p^\infty D(p,t)\, dp$$

6.5. Nonlinear tariff p(t)

- Ramsey pricing problem:
  - Objective fun.

$$\int_0^\infty \{[W(p(t),t) + [1+\lambda][p(t)-c]\cdot D(p(t),t)]\, f(t) \\ -\lambda \bar{F}(t)\cdot D(p(t),t)p'(t)\}\, dt\,,$$

  - Euler necessary condition for the optimal *p(t)*

$$[p(t)-c]\cdot D_p(p(t),t)f(t) + \alpha\bar{F}(t)\cdot D_t(p(t),t) = 0\,,$$

6.6. Some example