Chapter 11 Frequency Response

Chapter Outline

Fundamental Concepts

- Bode's Rules
- Association of Poles with Nodes
- Miller's Theorem

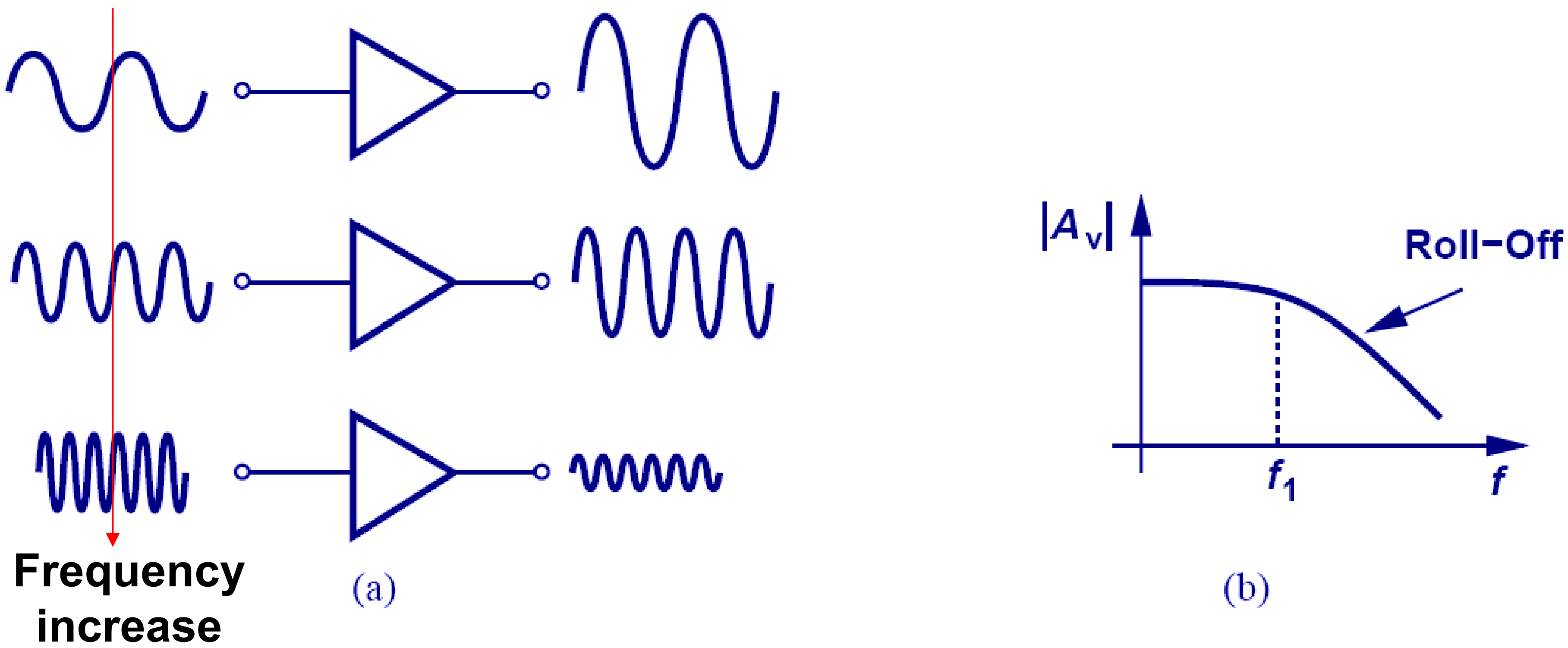
High-Frequency Models of Transistors

- Bipolar Model
- MOS Model
- Transit Frequency

Frequency Response of Circuits

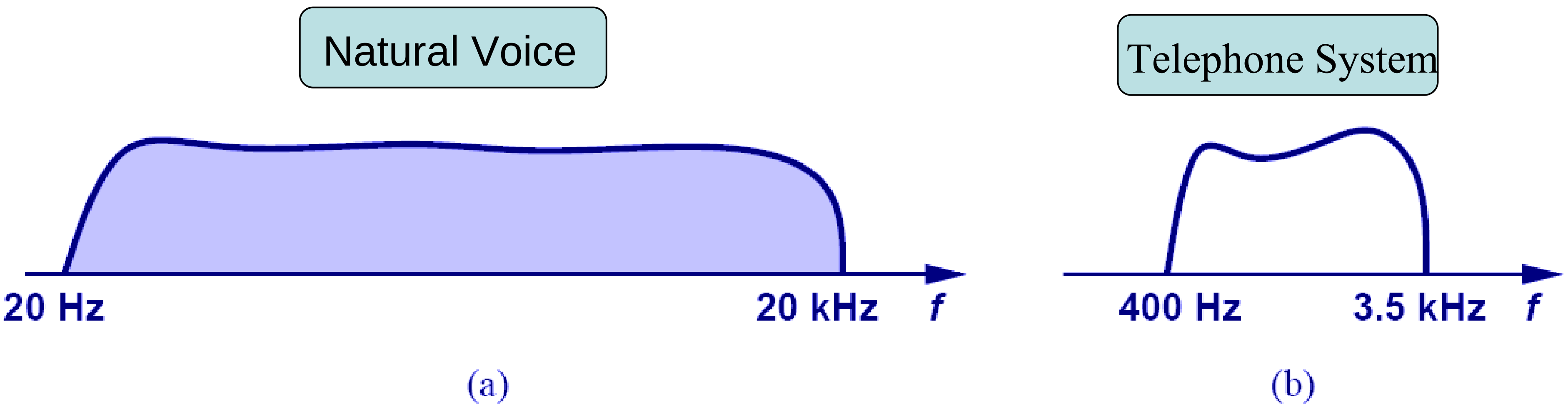
- CE/CS Stages
- CB/CG Stages
- Followers
- Cascode Stage
- Differential Pair

High Frequency Roll-off of Amplifier



- As frequency of operation increases, the gain of amplifier decreases. This chapter analyzes this problem.

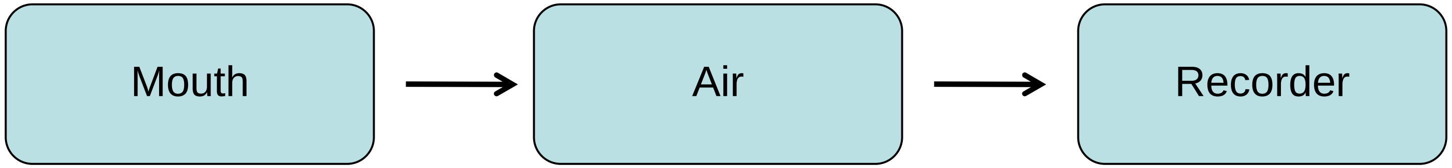
Example: Human Voice I



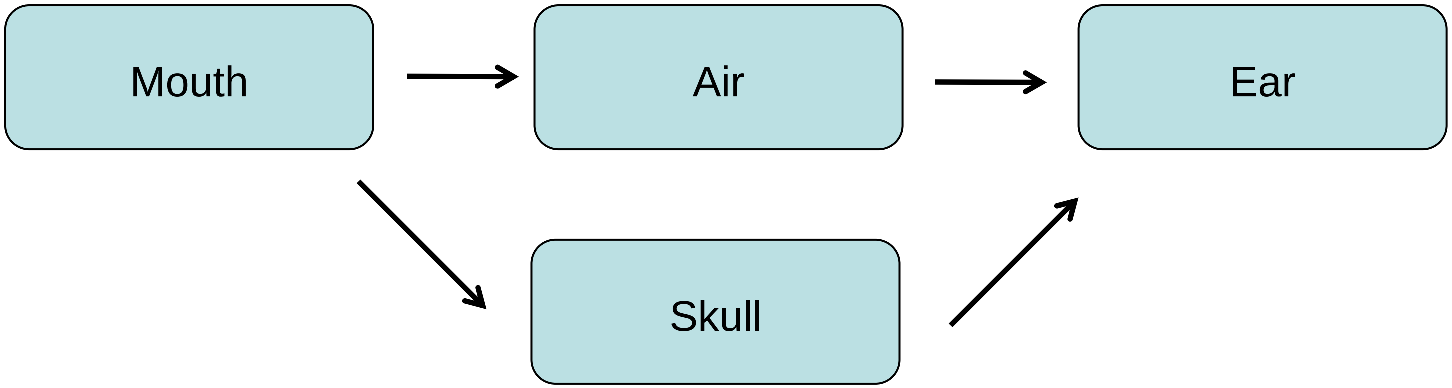
- Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.

Example: Human Voice II

Path traveled by the human voice to the voice recorder

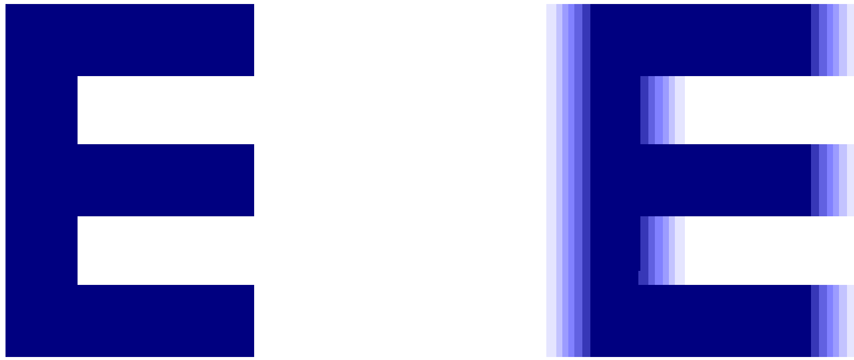


Path traveled by the human voice to the human ear



- **Since the paths are different, the results will also be different.**

Example: Video Signal



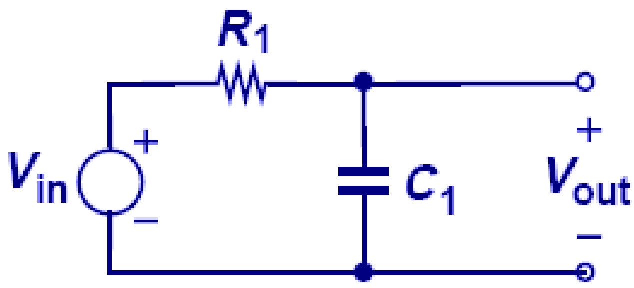
(a) (b)

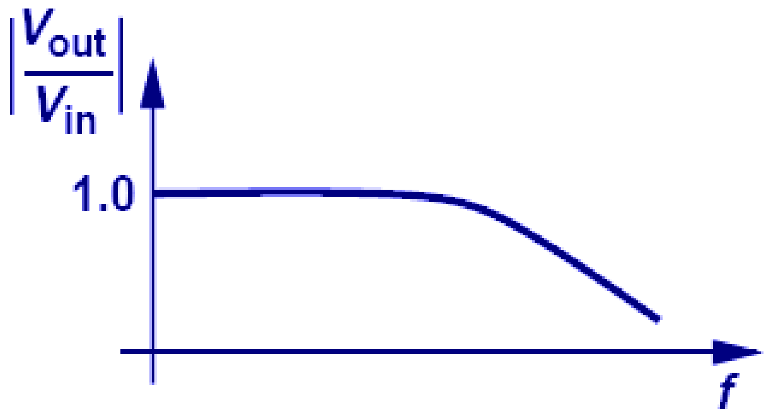
High Bandwidth

Low Bandwidth

- **Video signals without sufficient bandwidth become fuzzy as they fail to abruptly change the contrast of pictures from complete white into complete black.**

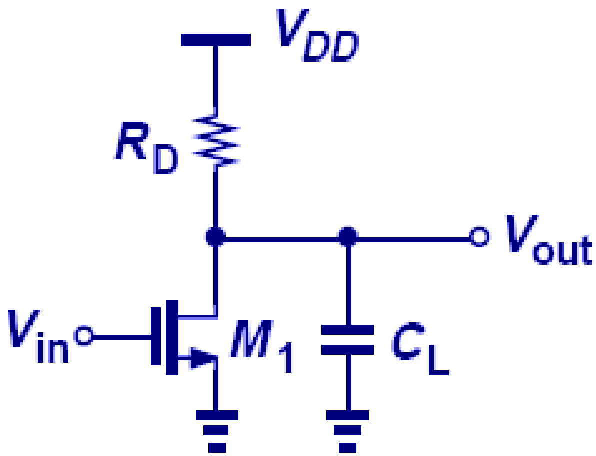
Gain Roll-off: Simple Low-pass Filter





- In this simple example, as frequency increases the impedance of C_1 decreases and the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.

Gain Roll-off: Common Source



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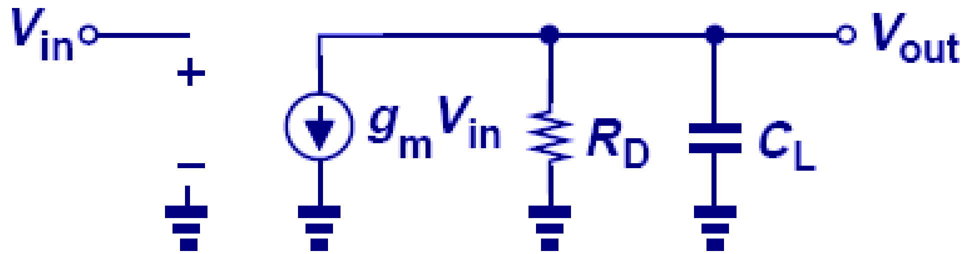
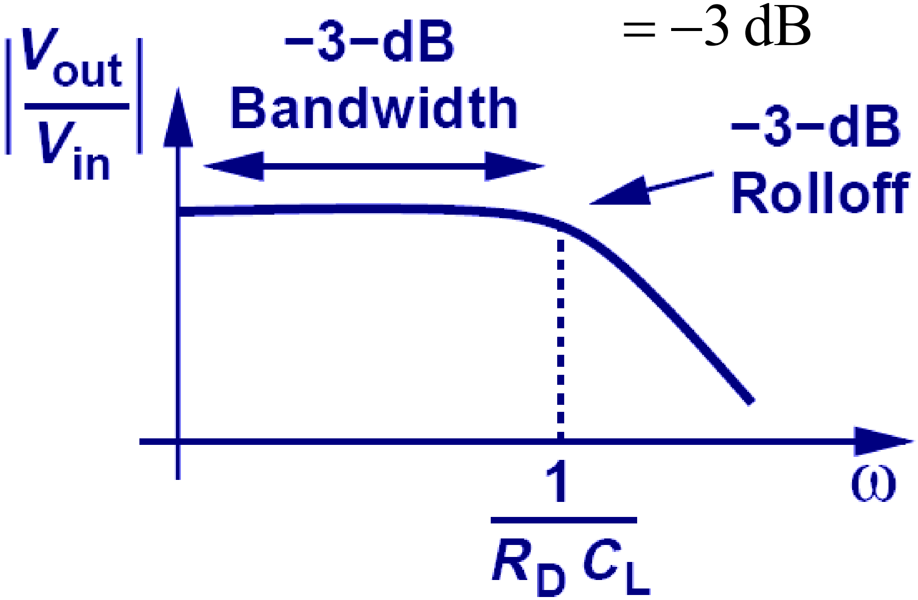


(b)

$$V_{out} = -g_m V_{in}\left(R_D \,||\, \frac{1}{C_L s}\right)$$

- **The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will "steal" away some signal current and shunt it to ground.**

Frequency Response of the CS Stage

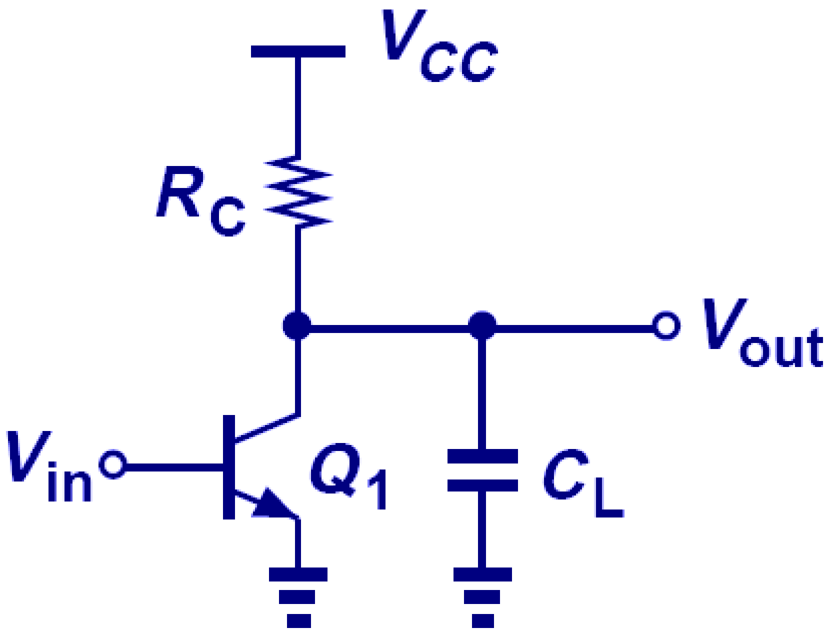


$$A = 20\log(1/\sqrt{2})$$
$$= -3 \text{ dB}$$

$$H(s) = \frac{V_{out}}{V_{in}}(s)$$
$$= -g_m\left(R_D \left\| \frac{1}{C_L s}\right.\right)$$
$$= \frac{-g_m R_D}{R_D C_L s + 1}$$
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$
$$\Rightarrow g_m R_D \quad @\, \omega = 0$$

- **At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease. A special frequency is $\omega = 1/(R_D C_L)$, where the gain drops by 3dB.**

Example: Figure of Merit



$$F.O.M. = \frac{Gain \times Bandwidth}{Power\ Consumption}$$

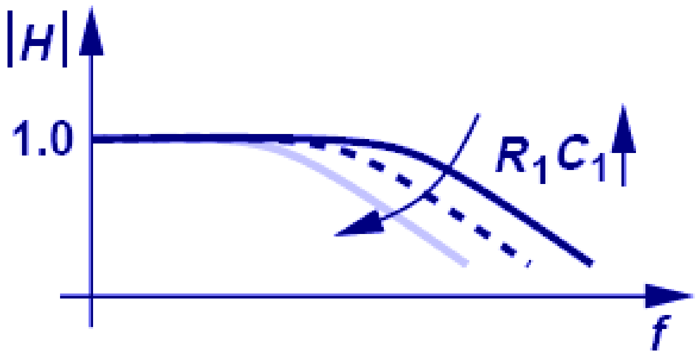
$$= \frac{g_m R_C \times \dfrac{1}{R_C C_L}}{I_C V_{CC}}$$

$$= \frac{\dfrac{I_C}{V_T} R_C \times \dfrac{1}{R_C C_L}}{I_C V_{CC}}$$

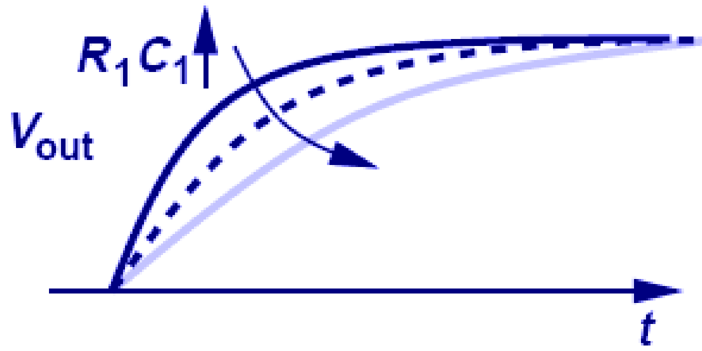
$$\boxed{= \frac{1}{V_T V_{CC} C_L}}$$

- This metric quantifies a circuit's gain, bandwidth, and power dissipation. In the bipolar case, low temperature, supply, and load capacitance mark a superior figure of merit.

Example: Relationship between Frequency Response and Step Response



(a)



(b)

$$\left|H(s = j\omega)\right| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0 \left(1 - \exp\frac{-t}{R_1 C_1}\right) u(t)$$

- **The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower.***

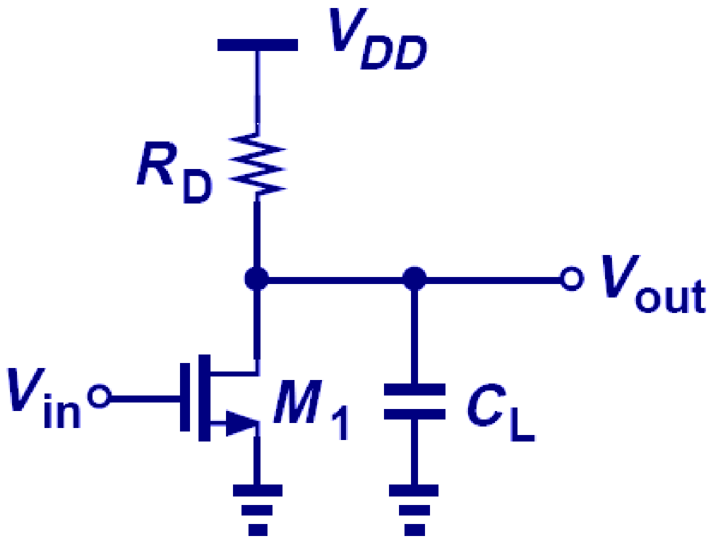
Bode Plot

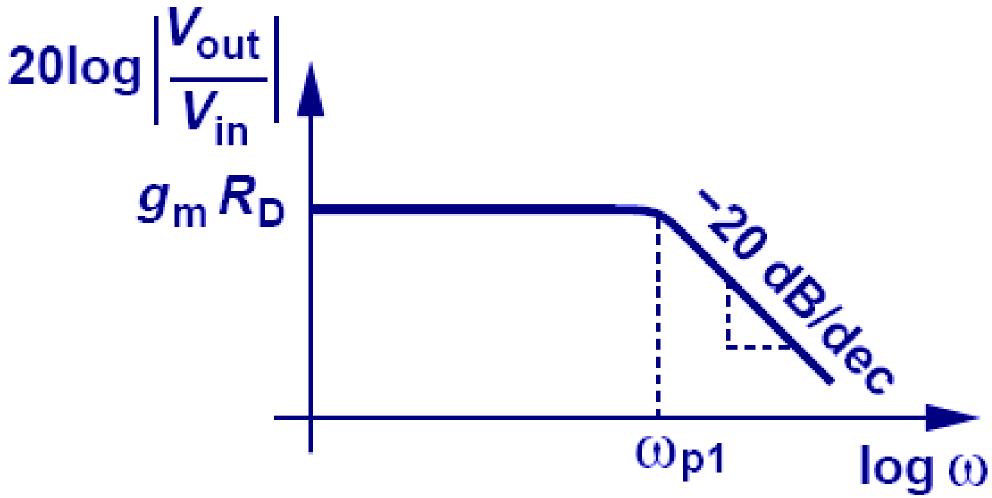
Transfer function

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

- **When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.**
- **When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec**

Example: Bode Plot





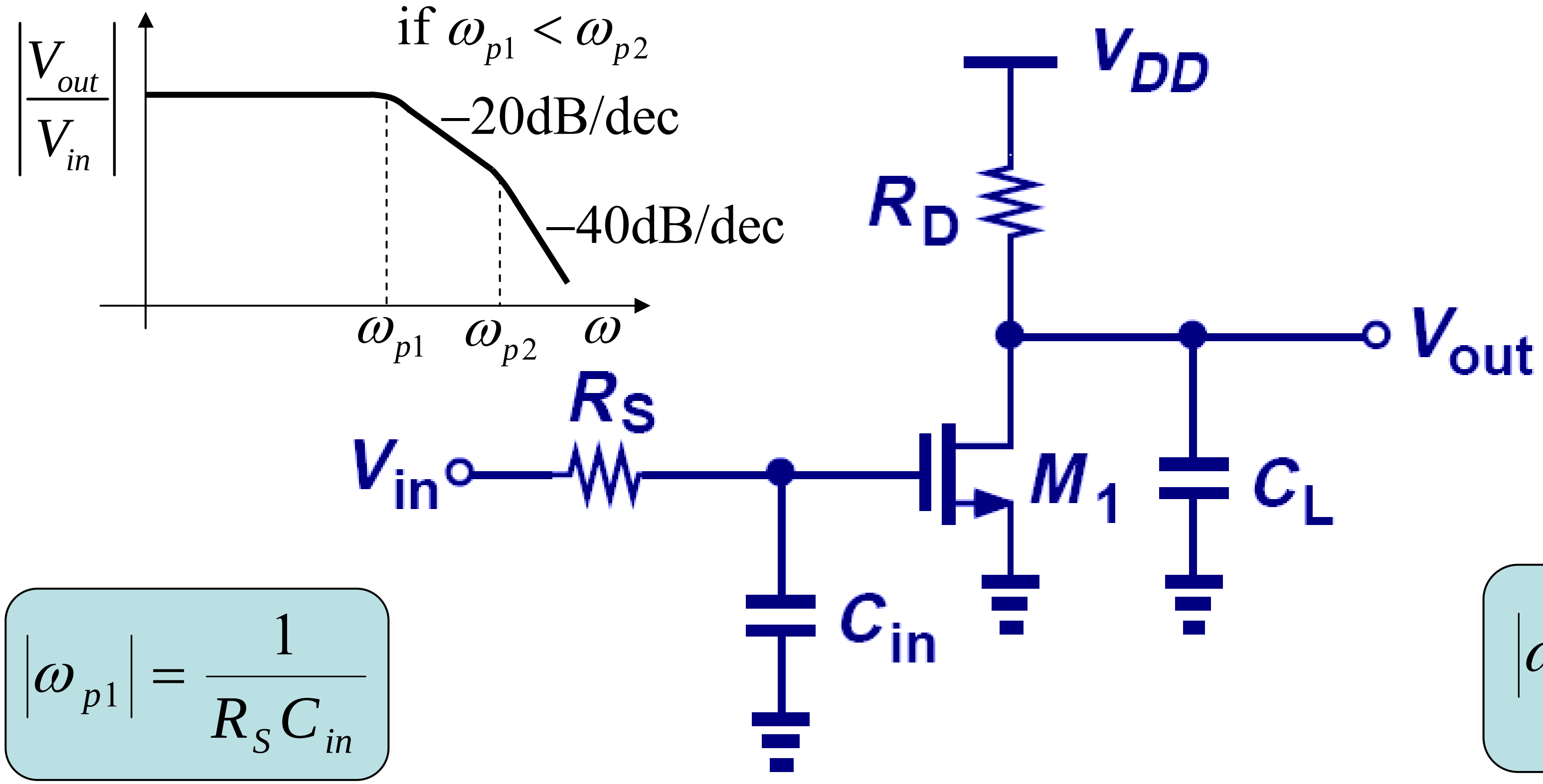
$$\because \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

$$\left|\frac{V_{out}}{V_{in}}\right|_{\omega=10\omega_{p1}} = \frac{g_m R_D}{\sqrt{100+1}} \approx \frac{g_m R_D}{10}$$

$$\left|\omega_{p1}\right| = \frac{1}{R_D C_L}$$

- The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}.

Pole Identification Example I

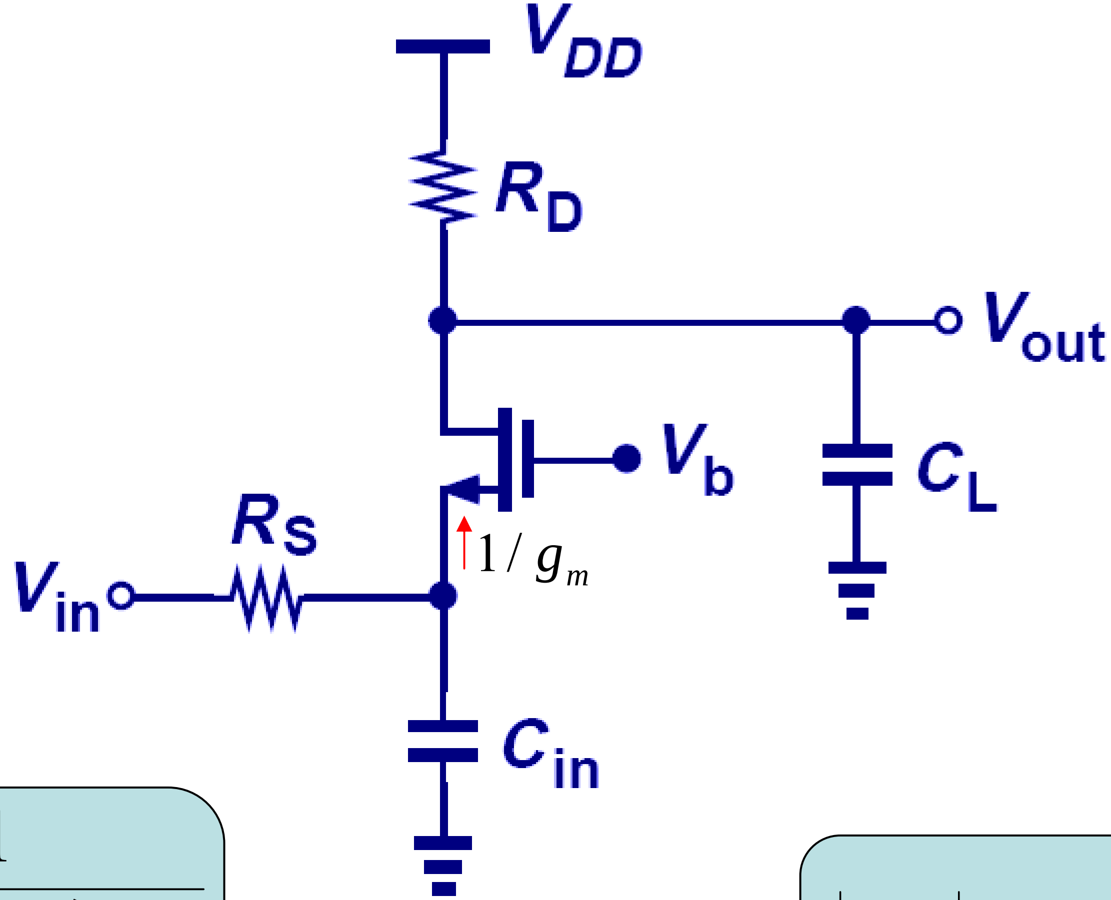


$$\left|\omega_{p1}\right| = \frac{1}{R_S C_{in}}$$

$$\left|\omega_{p2}\right| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

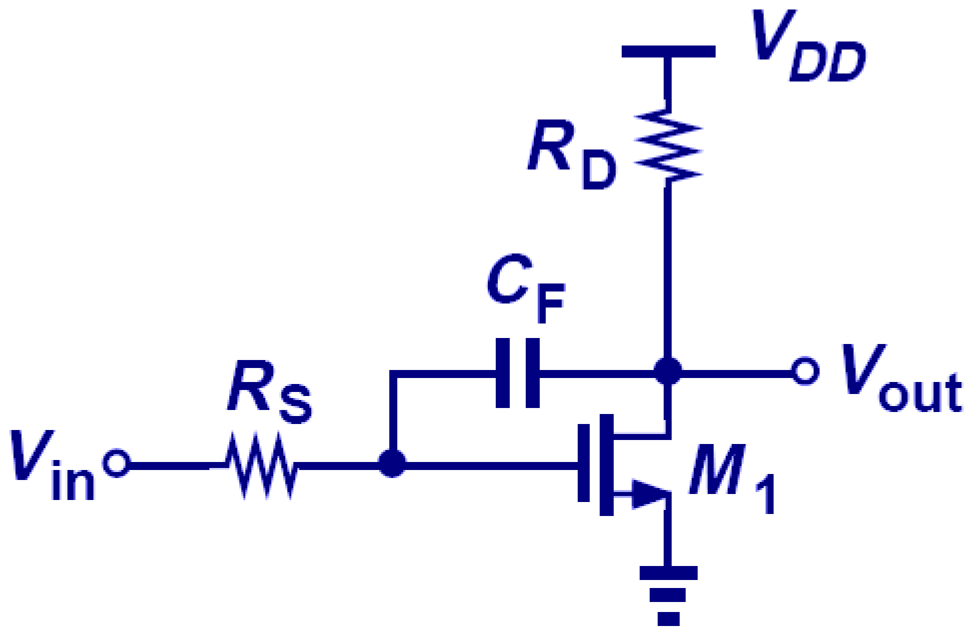
Pole Identification Example II



$$|\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_{in}}$$

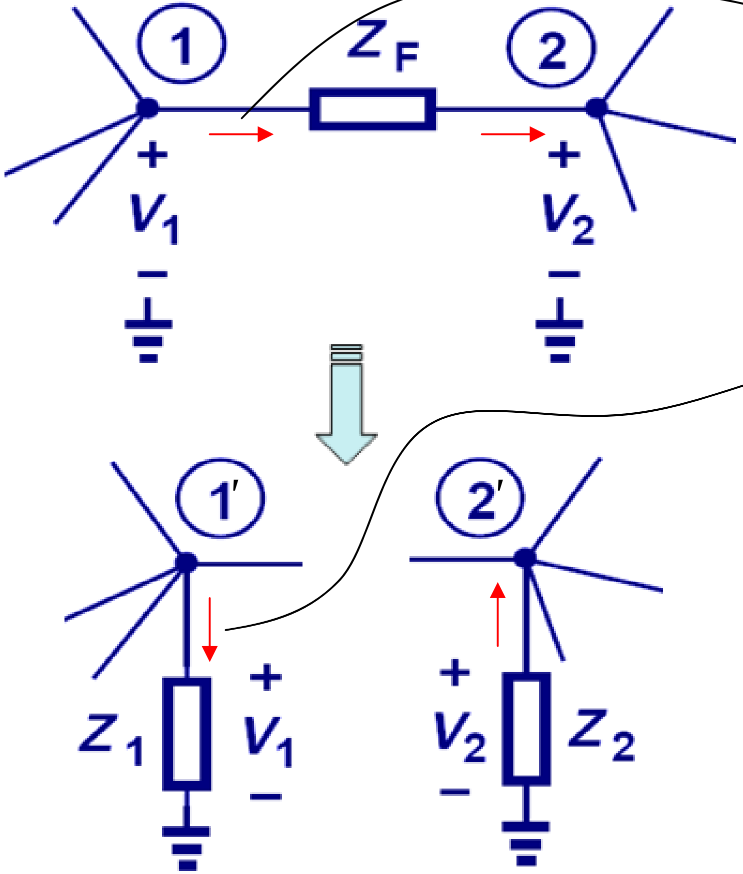
$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



- The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.

Miller's Theorem



$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}, \qquad \frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}$$

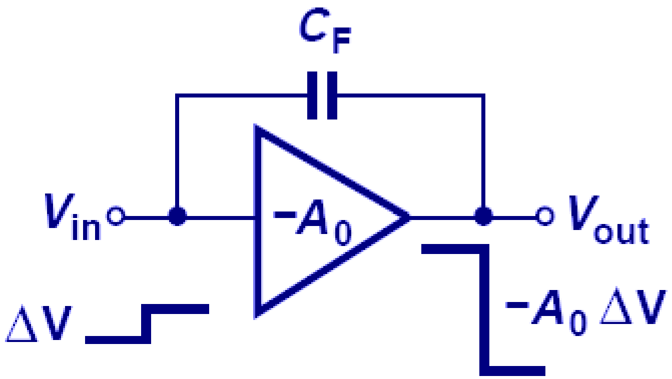
$$\Rightarrow Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v}$$

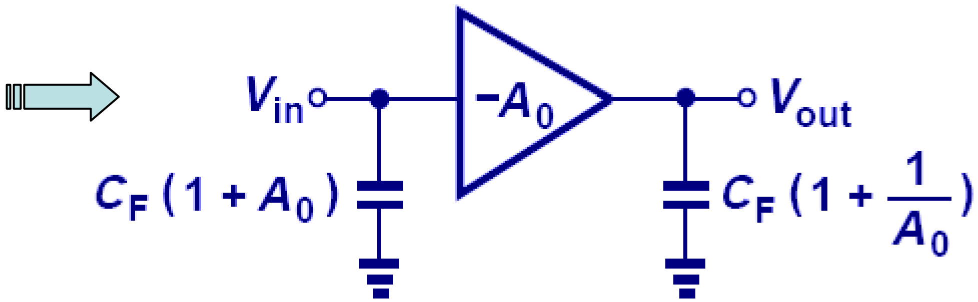
$$Z_2 = Z_F \frac{-V_2}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}$$

- **If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.**

Miller Multiplication

$$Z_1 = \frac{Z_F}{1-A_v} = \frac{1}{(1+A_0)C_F s}$$

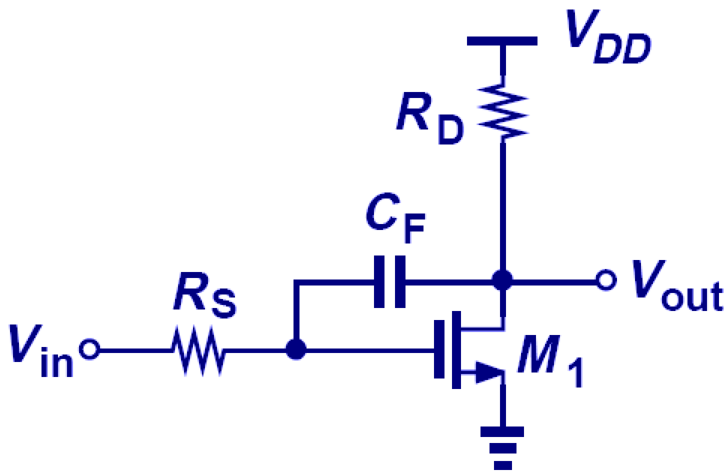


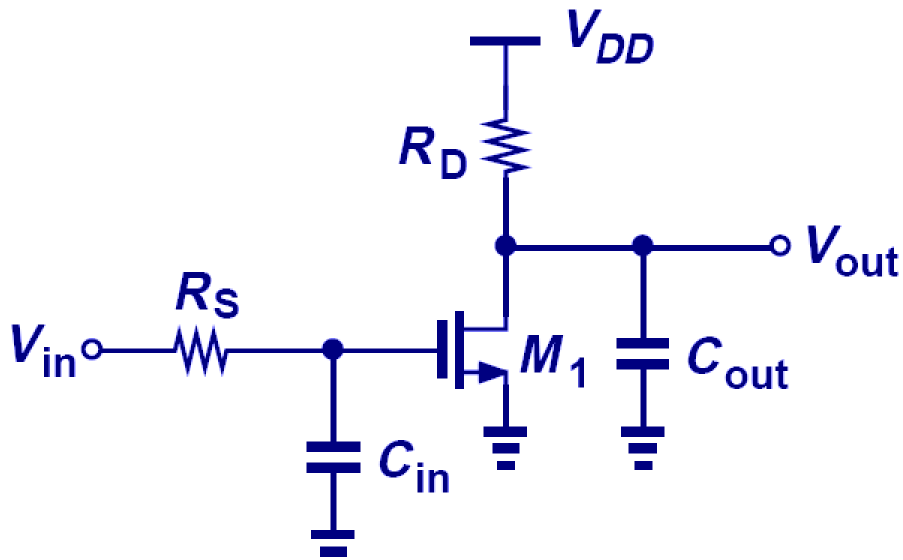


$$Z_2 = \frac{Z_F}{1-\frac{1}{A_v}} = \frac{1}{\left(1+\frac{1}{A_0}\right)C_F s}$$

- With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Example: Miller Theorem

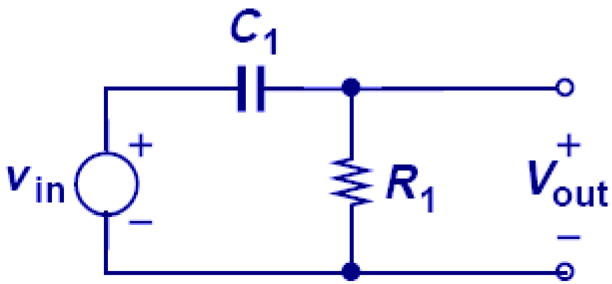




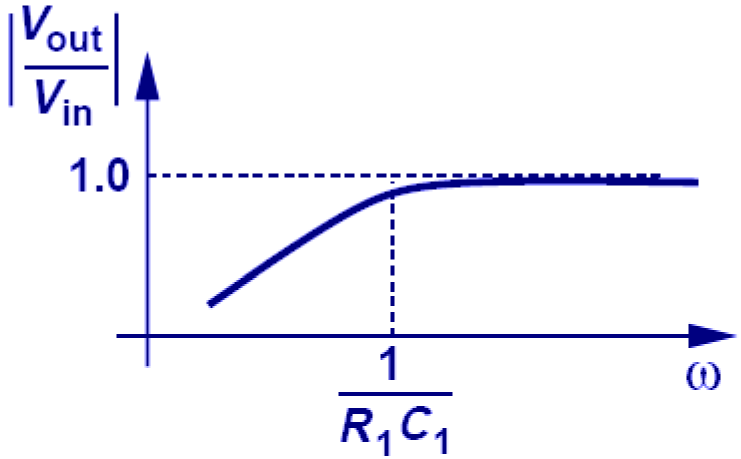
$$\omega_{in} = \frac{1}{R_S(1+g_m R_D)C_F}$$

$$\omega_{out} = \frac{1}{R_D\left(1+\frac{1}{g_m R_D}\right)C_F}$$

High-Pass Filter Response



(a)



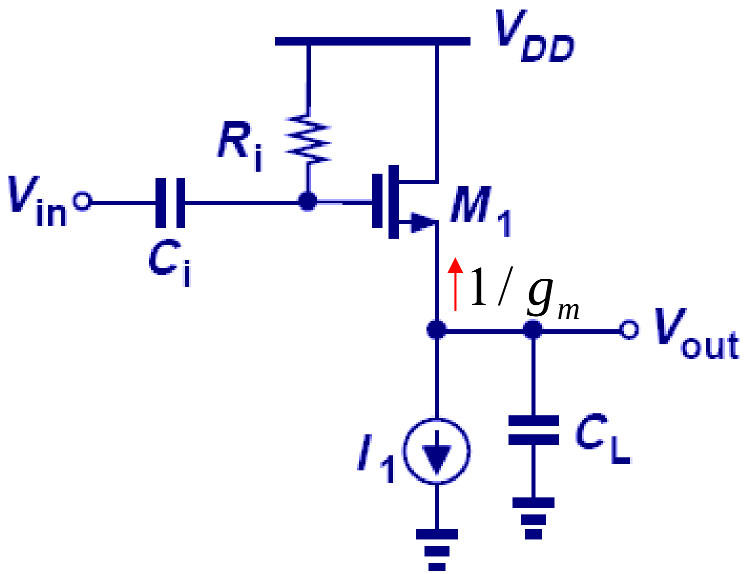
(b)

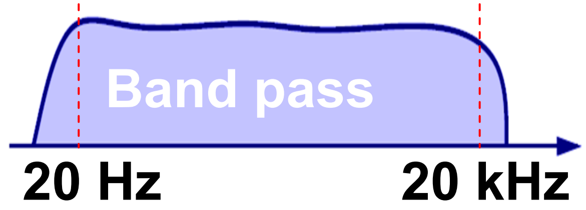
$$\frac{V_{out}}{V_{in}} = \frac{R_1C_1s}{R_1C_1s+1}$$

$$\boxed{\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1C_1\omega}{\sqrt{R_1^2C_1^2\omega_1^2+1}}}$$

- The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.

Example: Audio Amplifier





$$R_i = 100\ \text{k}\Omega$$
$$g_m = 1/200\ \Omega$$

$$\frac{1}{R_i C_i} \leq 2\pi \times (20\text{Hz})$$

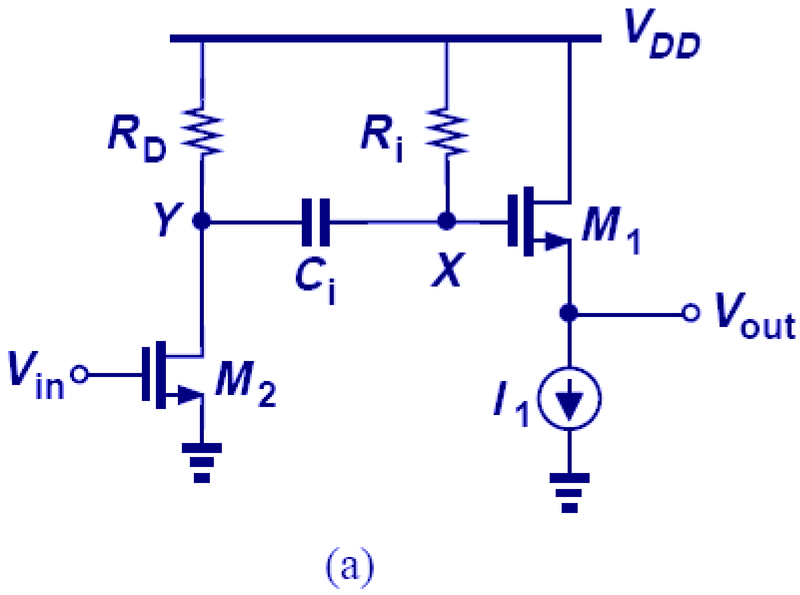
$$\Rightarrow C_i \geq \frac{1}{100k \times 2\pi \times 20} = 79.6\text{nF}$$

$$\omega_{p,out} = \frac{g_m}{C_L} \geq 2\pi \times (20\text{kHz})$$

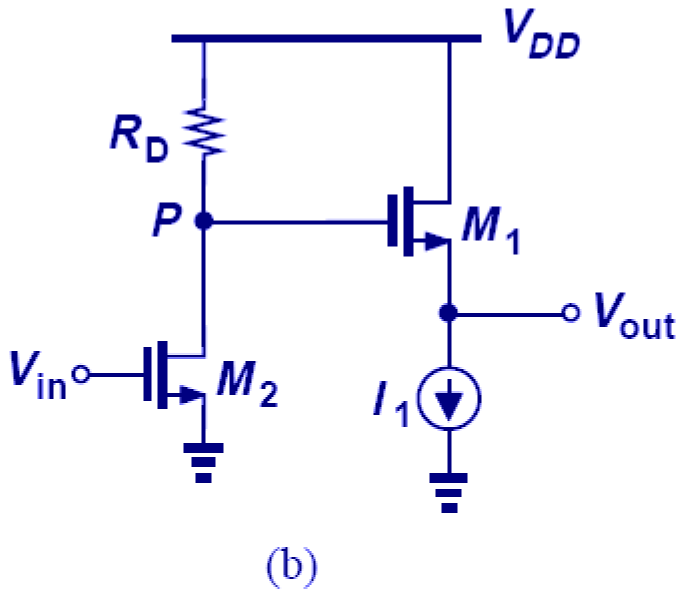
$$\Rightarrow C_L \leq \frac{1}{200 \times 2\pi \times 20\text{k}} = 39.8\text{nF}$$

- In order to successfully pass audio band frequencies (20 Hz-20 kHz), large input and small output capacitances are needed.

Capacitive Coupling vs. Direct Coupling



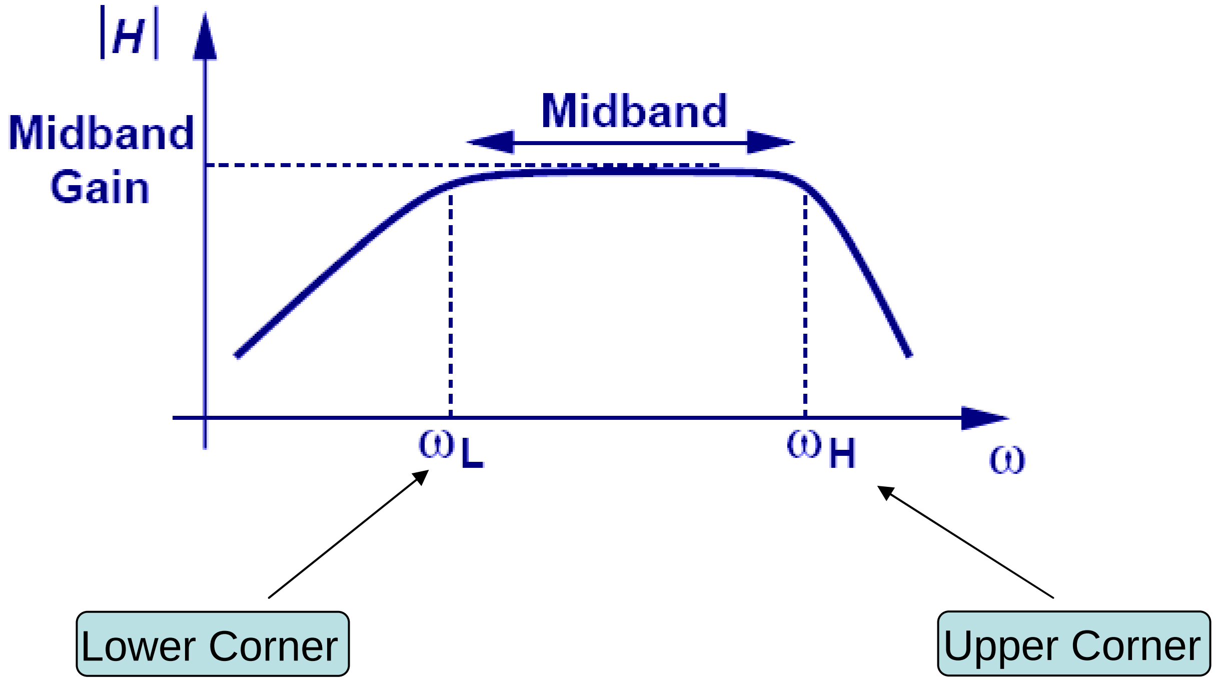
Capacitive Coupling



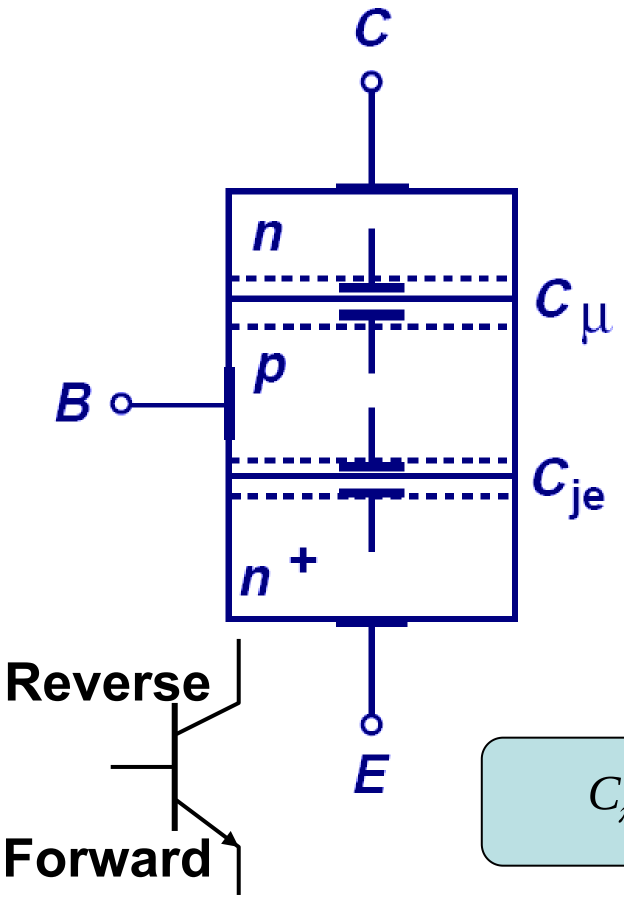
Direct Coupling

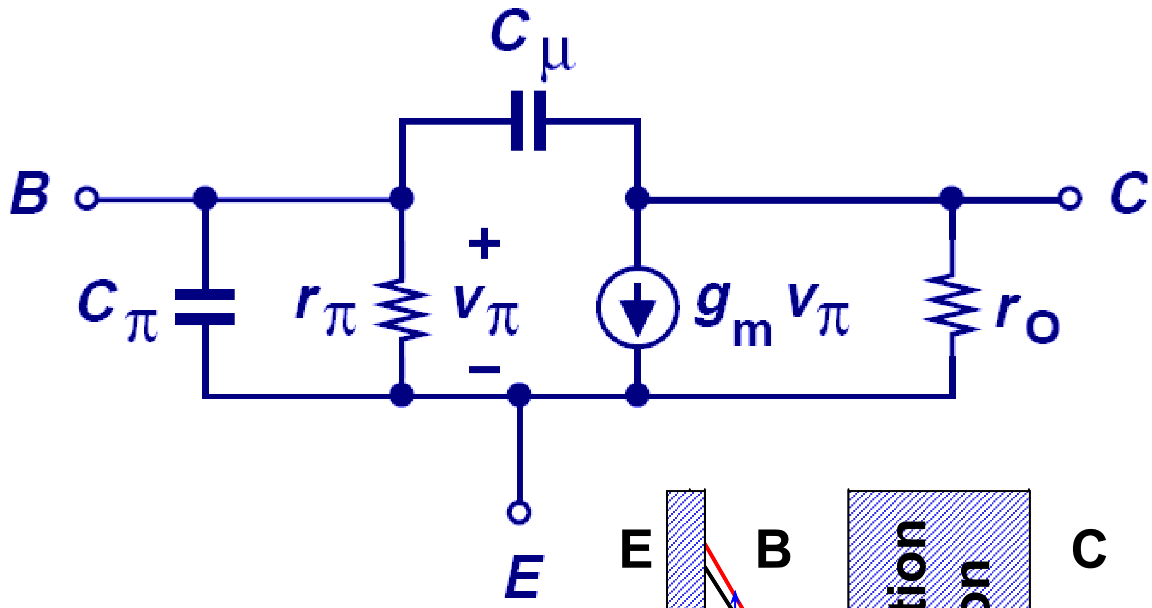
- **Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.**
- **This technique allows independent bias conditions between stages. Direct coupling does not.**

Typical Frequency Response

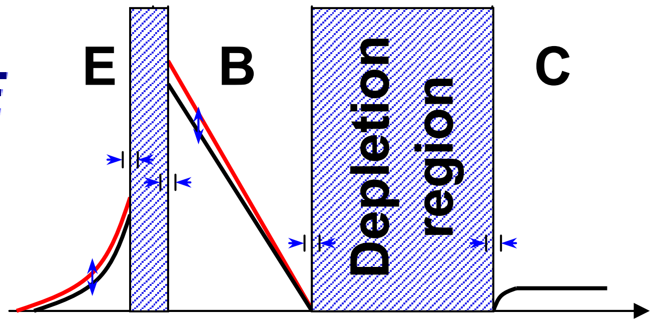


High-Frequency Bipolar Model



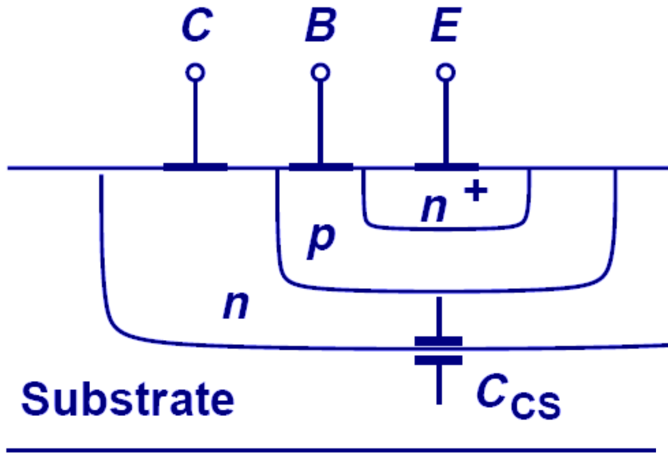


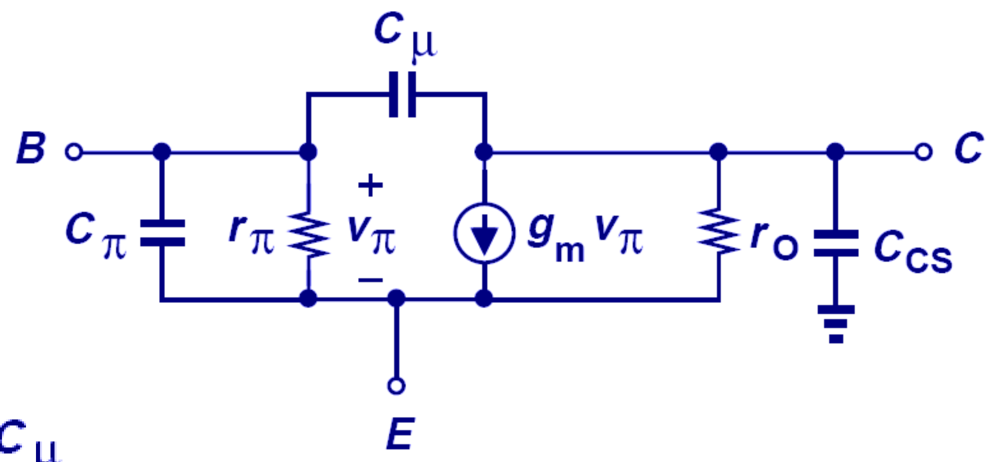
$$C_\pi = C_b + C_{je}$$

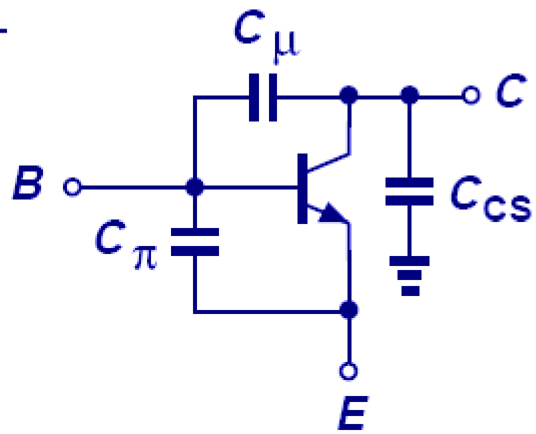


- At high frequency, capacitive effects come into play. C_b represents diffusion capacitance at the forward biased BE junction, whereas C_μ and C_{je} are the junction capacitances.

High-Frequency Model of Integrated Bipolar Transistor

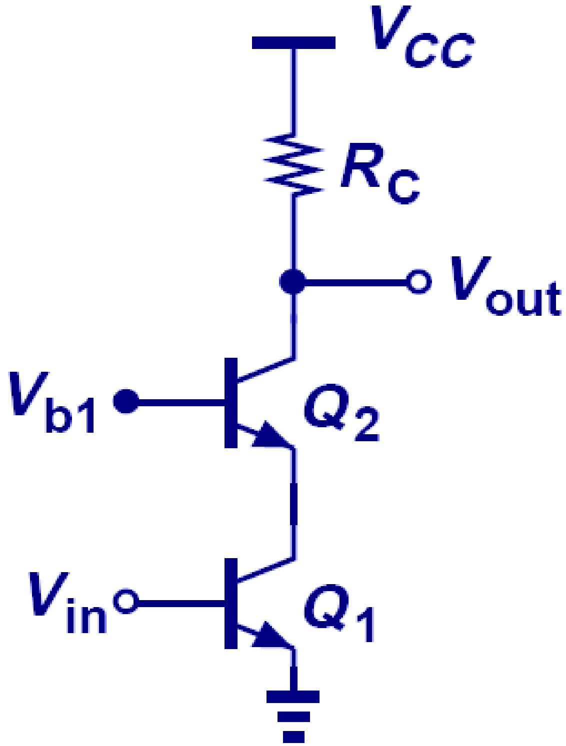


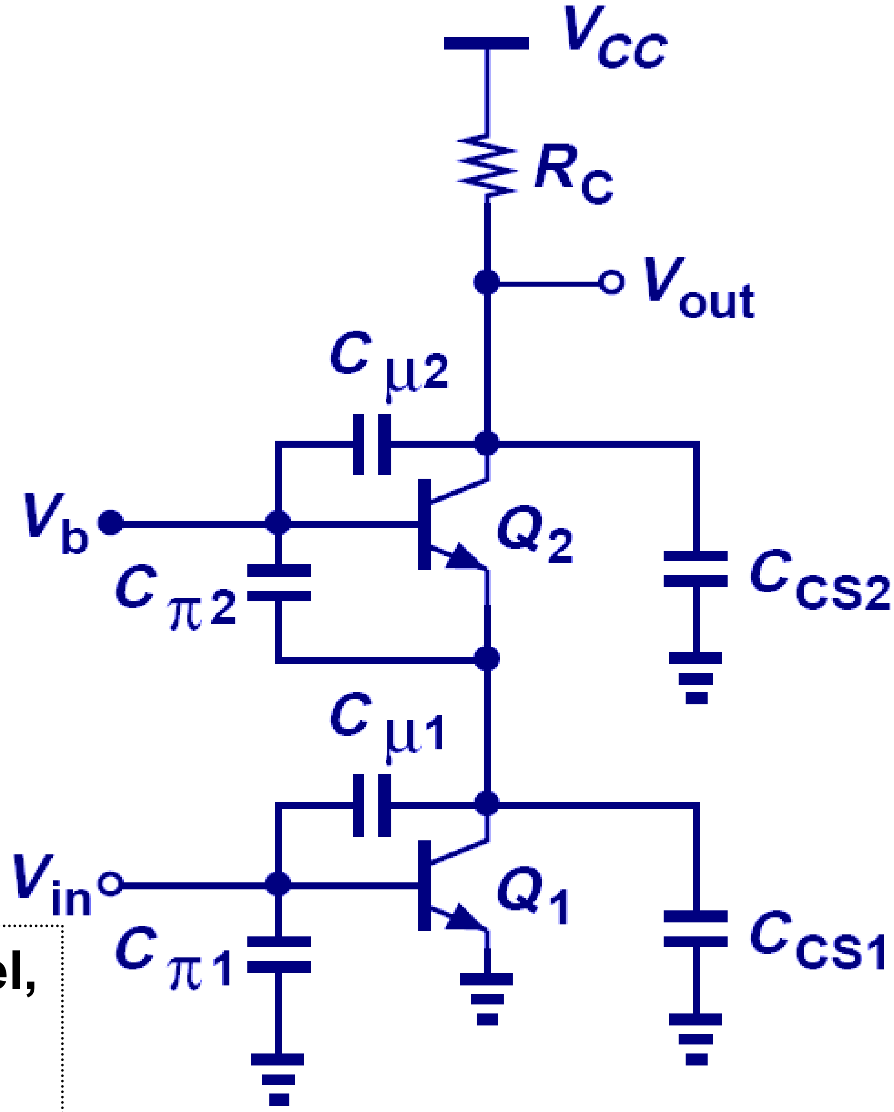




- **Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely C_{CS}.**

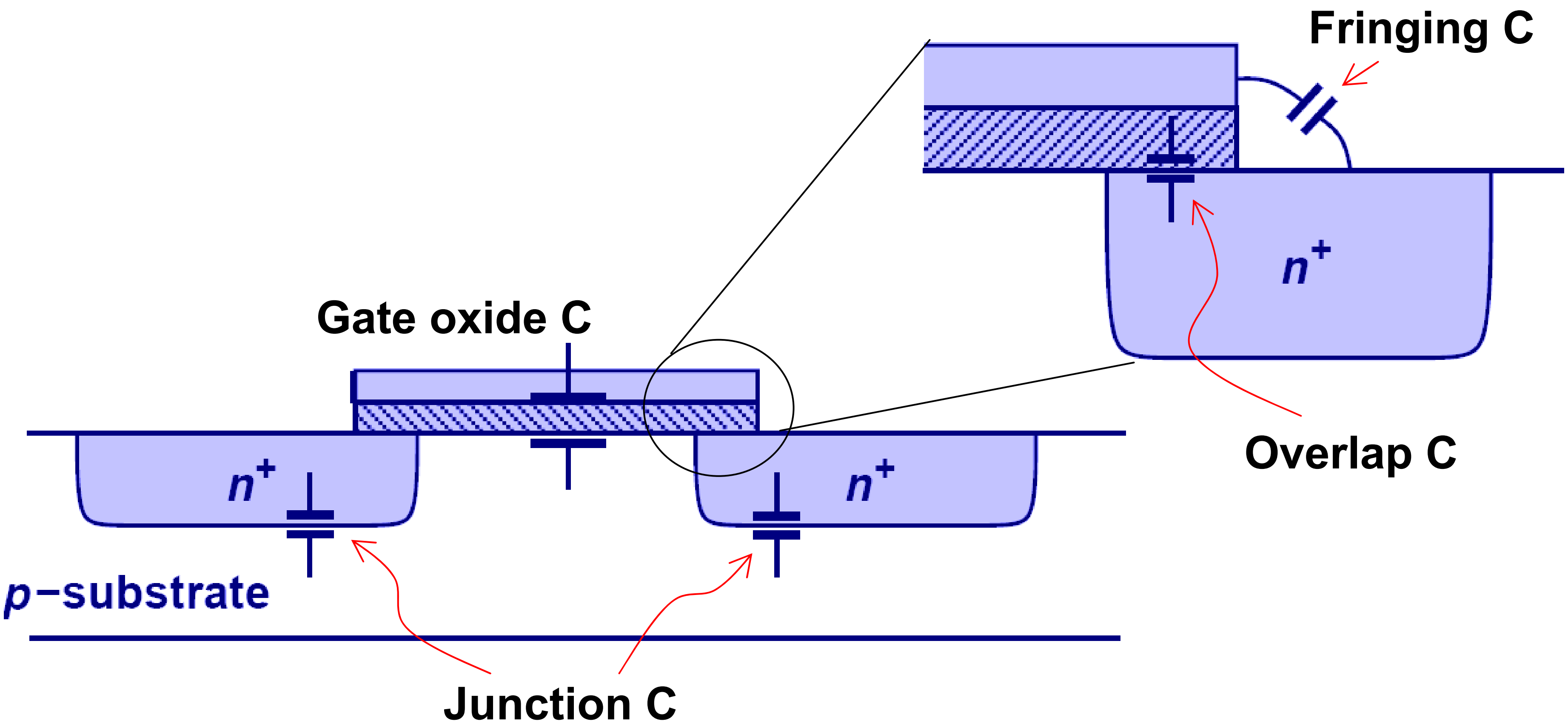
Example: Capacitance Identification





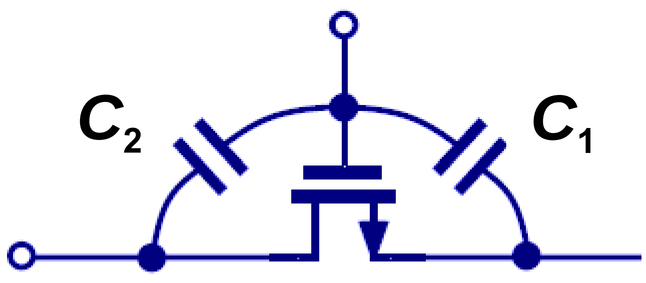
- C_{CS1} and $C_{\pi 2}$ appear in parallel, and so do $C_{\mu 2}$ and C_{CS2} .

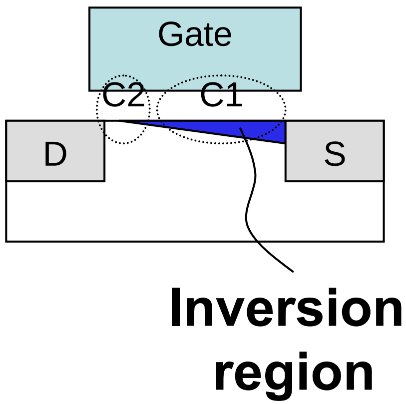
MOS Intrinsic Capacitances

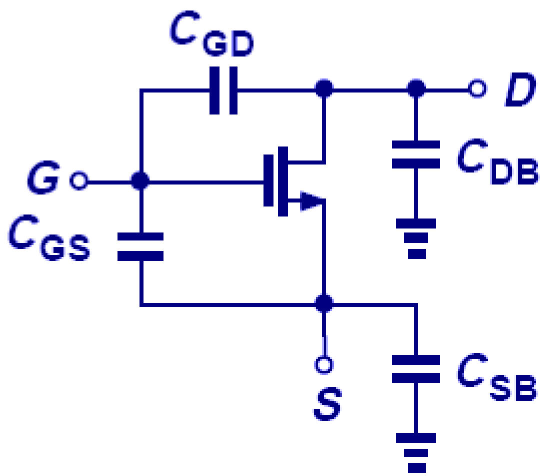


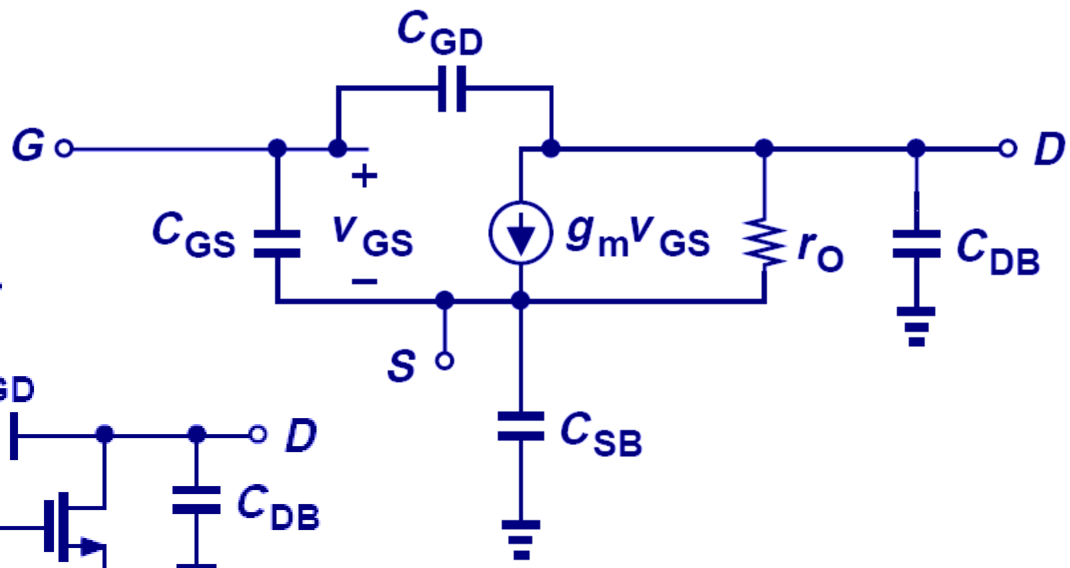
- For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model







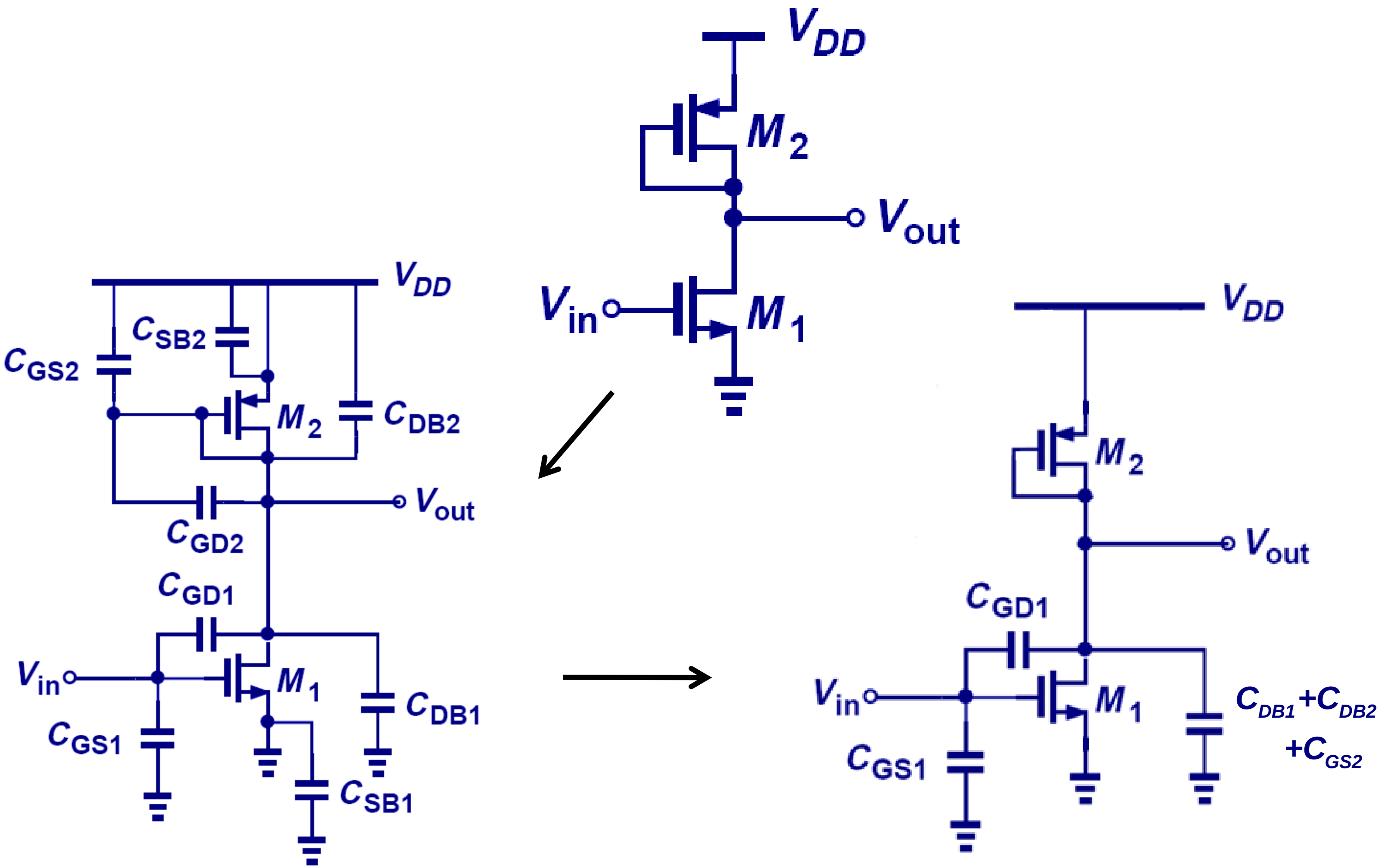


$$C_{GS} = C_1 + C_{overlap,GS}$$

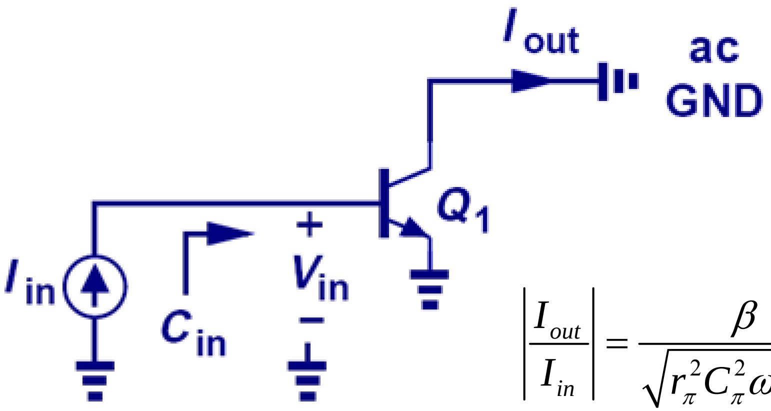
$$C_{GD} = C_2 + C_{overlap,GD}$$

- **The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_1 \sim 2/3C_{gate}$, and $C_2 \sim 0$. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD}.**

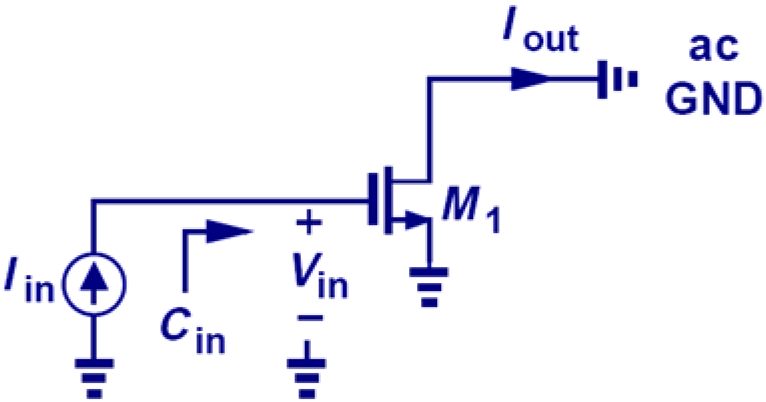
Example: Capacitance Identification



Transit Frequency (or Cut-off frequency)



$$\left|\frac{I_{out}}{I_{in}}\right| = \frac{\beta}{\sqrt{r_\pi^2 C_\pi^2 \omega_T^2 + 1}}$$



$$Z_{in} = \frac{1}{C_\pi s} \| r_\pi, \quad I_{out} = g_m I_{in} Z_{in}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = \frac{g_m r_\pi}{r_\pi C_\pi s + 1} = \frac{\beta}{r_\pi C_\pi s + 1}$$

$$\left|\frac{I_{out}}{I_{in}}\right| = 1 \quad \Rightarrow \quad r_\pi^2 C_\pi^2 \omega_T^2 = \beta^2 - 1 \approx \beta^2$$

$$\Rightarrow \omega_T = 2\pi f_T \approx \frac{g_m}{C_\pi}$$

The transit frequency of MOSFETs is obtained in a similar fashion.

$$\omega_T = 2\pi f_T \approx \frac{g_m}{C_{GS}}$$

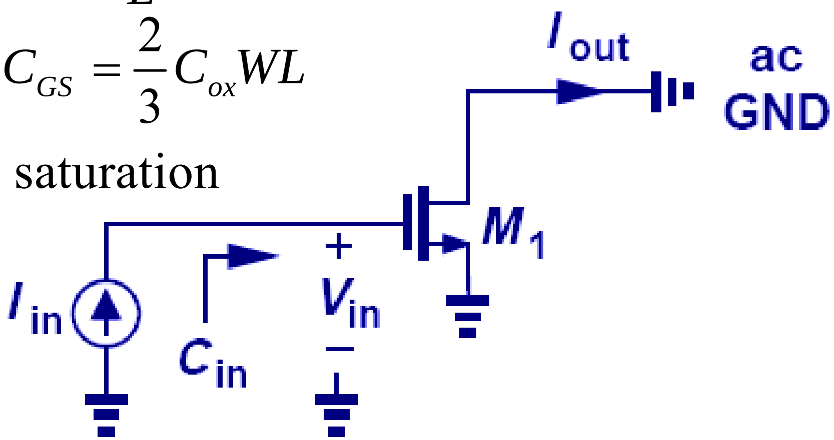
- **Transit frequency, f_T, is defined as the frequency where the current gain from input to output drops to 1.**

Example: Transit Frequency Calculation

$$\because g_m = \frac{W}{L}\mu C_{ox}(V_{GS} - V_{TH})$$

$$\because C_{GS} = \frac{2}{3}C_{ox}WL$$

in saturation



$$L: 1\mu m \to 65nm$$

$$(V_{GS} - V_{TH}): 400mV \to 100mV$$

From Problem 11.28,

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$\Rightarrow \frac{f_{T,today}}{f_{T,1980s}} = \frac{100}{\left(65\times10^{-9}\right)^2} \Big/ \frac{400}{\left(1\times10^{-6}\right)^2} \approx 59$$

If $\mu_n = 400\ cm^2/(V\cdot s)$,

$$f_{T,today} \approx 226\ GHz \quad @\ 65nm$$

- **The minimum channel length of MOSFETs has been scaled from 1μm in the late 1980s to 65nm today. Also, the inevitable reduction of the supply voltage has reduced the gate-source overdrive voltage from about 400mV to 100mV. By what factor has the f_T of MOSFETs increased?**

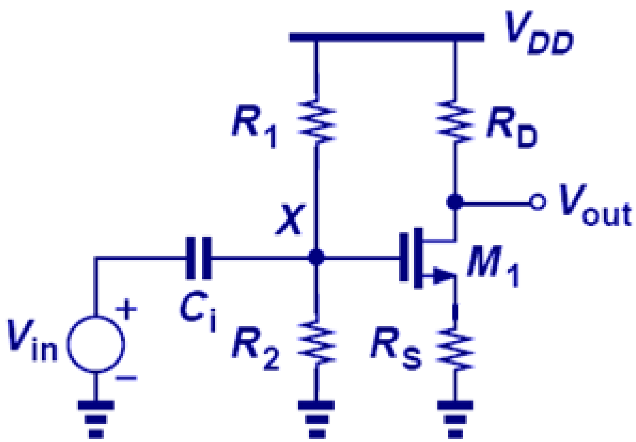
Analysis Summary

- The frequency response refers to the magnitude of the transfer function.
- Bode’s approximation simplifies the plotting of the frequency response if poles and zeros are known.
- In general, it is possible to associate a pole with each node in the signal path.
- Miller’s theorem helps to decompose floating capacitors into grounded elements.
- Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.

High Frequency Circuit Analysis Procedure

- Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).
- Calculate the midband gain by replacing the capacitors with short circuits while still neglecting transistor capacitances
- Include transistor capacitances.
- Merge capacitors connected to AC grounds and omit those that play no role in the circuit.
- Determine the high-frequency poles and zeros.
- Plot the frequency response using Bode's rules or exact analysis.

Frequency Response of CS Stage

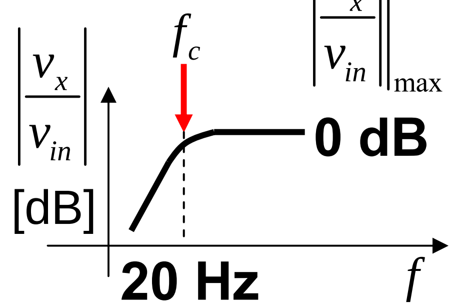


$$\frac{V_X}{V_{in}}(s) = \frac{R_1 \| R_2}{R_1 \| R_2 + \frac{1}{C_i s}} = \frac{(R_1 \| R_2) C_i s}{(R_1 \| R_2) C_i s + 1}$$

$$\left| \frac{V_X}{V_{in}} \right|_{f=f_c} = \frac{(R_1 \| R_2) C_i \omega_c}{\sqrt{(R_1 \| R_2)^2 C_i^2 \omega_c^2 + 1}} = \frac{1}{\sqrt{2}}$$

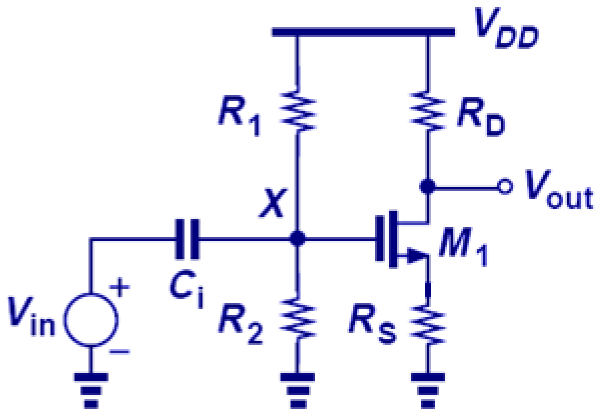
$$\text{Thus, } \underbrace{\frac{1}{2\pi\left[(R_1 \| R_2) C_i\right]}}_{f_c} < f_{sig,\min}$$

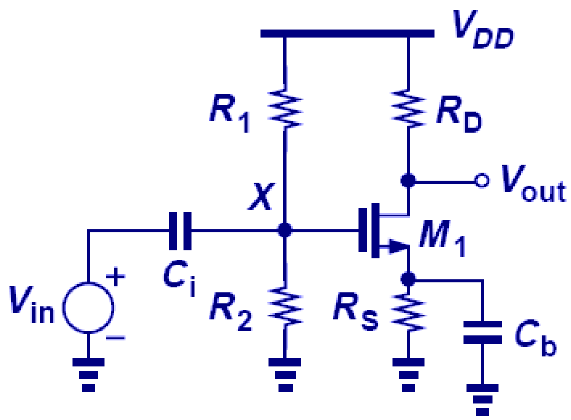
$$\left| \frac{v_x}{v_{in}} \right|_{\max} = 1$$

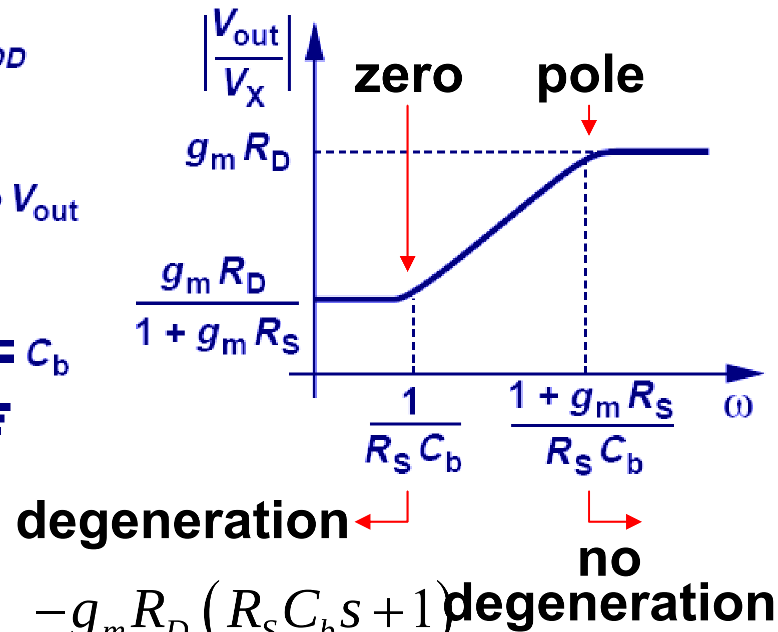


- C_i **acts as a high pass filter.**
- **Lower corner frequency must be lower than the lowest signal frequency $f_{sig,min}$ (20 Hz in audio applications).**

Frequency Response of CS Stage with Bypassed Degeneration





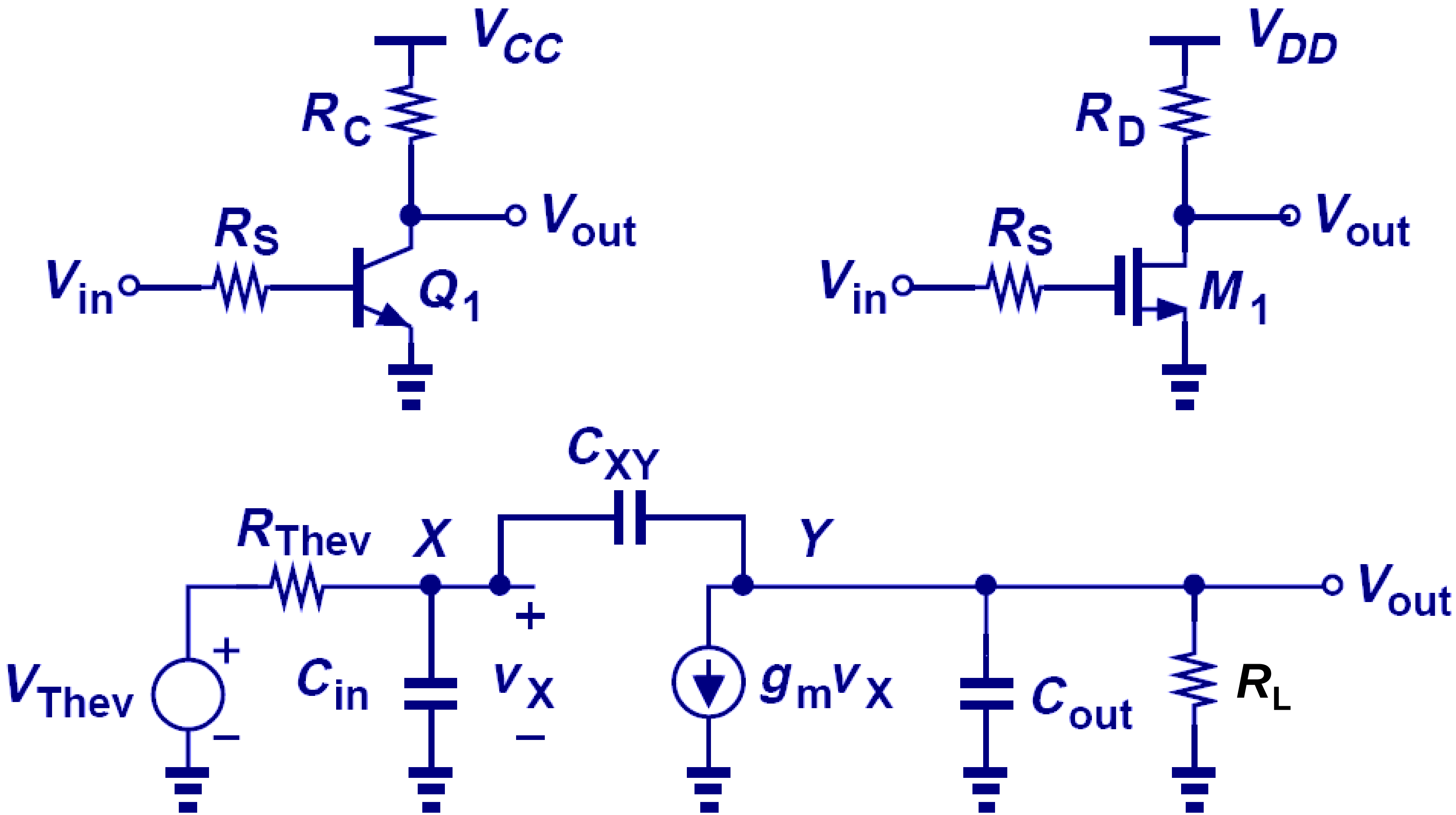


$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S + \dfrac{1}{g_m}}$$

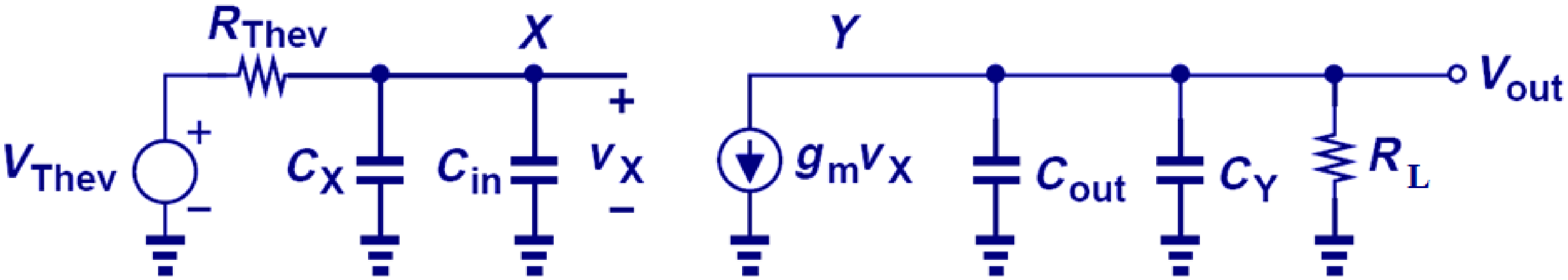
$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S \parallel \dfrac{1}{C_b s} + \dfrac{1}{g_m}} = \frac{-g_m R_D \left(R_S C_b s + 1\right)}{R_S C_b s + g_m R_S + 1}$$

- In order to increase the midband gain, a capacitor C_b is placed in parallel with R_s.
- The pole frequency must be well below the lowest signal frequency to avoid the effect of degeneration.

Unified Model for CE and CS Stages



Unified Model Using Miller's Theorem



$$\left|\omega_{p,in}\right| = \frac{1}{R_{Thev}\left[C_{in} + \left(1 + g_m R_L\right) C_{XY}\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[C_{out} + \left(1 + \frac{1}{g_m R_L}\right) C_{XY}\right]}$$

CE Stage

$$V_{Thev} = V_{in}\frac{r_\pi}{r_\pi + R_S}$$

$$R_{Thev} = R_S \,||\, r_\pi$$

$$C_X = C_\mu\,(1 + g_m R_L)$$

$$C_Y = C_\mu\,(1 + \frac{1}{g_m R_L})$$

CS Stage

$$V_{Thev} = V_{in}$$

$$R_{Thev} = R_S$$

$$C_X = C_{GD}\,(1 + g_m R_L)$$

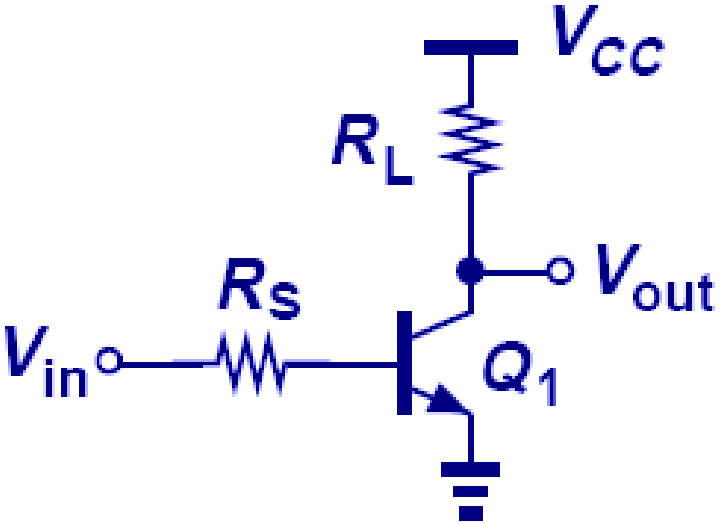
$$C_Y = C_{GD}\,(1 + \frac{1}{g_m R_L})$$

$r_\pi \rightarrow \infty$

in MOSFETs

Example: CE Stage

- **(a) Calculate the input and output poles if R_L=2 kΩ. Which node appears as the speed bottleneck?**



$$\left|\omega_{p,in}\right| = \frac{1}{(R_S \| r_\pi)\left[C_\pi + (1+g_m R_L)C_\mu\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[C_{CS} + \left(1+\frac{1}{g_m R_L}\right)C_\mu\right]}$$

$$R_S = 200\ \Omega,\quad I_C = 1\ \text{mA}$$
$$\beta = 100,\quad C_\pi = 100\ \text{fF}$$
$$C_\mu = 20\ \text{fF},\quad C_{CS} = 30\ \text{fF}$$

$$\left|\omega_{\text{p,in}}\right| = 2\pi \times (516\ \text{MHz})$$
$$\left|\omega_{\text{p,out}}\right| = 2\pi \times (1.59\ \text{GHz})$$

Example: CE Stage – cont'd

- **(b) Is it possible to choose R_L such that the output pole limits the bandwidth?**

$$|\omega_{p,in}| > |\omega_{p,out}|$$

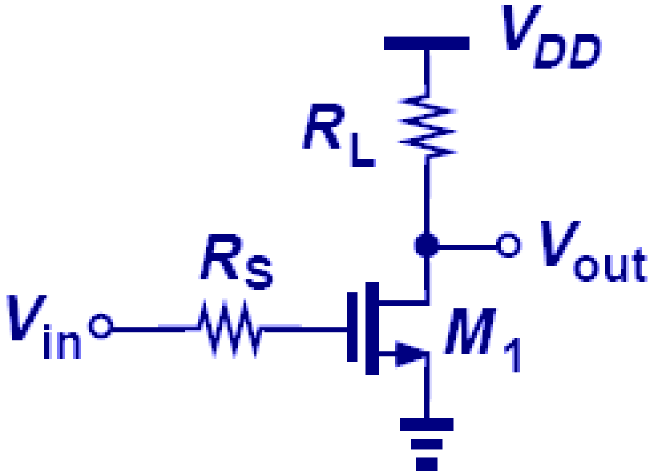
$$\Rightarrow \frac{1}{(R_S \| r_\pi)\left[C_\pi + (1 + g_m R_L) C_\mu\right]} > \frac{1}{R_L\left[C_{CS} + \left(1 + \frac{1}{g_m R_L}\right) C_\mu\right]}$$

$$\text{If } \mathrm{g_m R_L} \gg 1,$$

$$\Rightarrow \left[C_{CS} + C_\mu - g_m (R_S \| r_\pi) C_\mu\right] R_L > (R_S \| r_\pi) C_\pi$$

With the values assumed in this example, the left-hand side is negative, implying that no solution exists. Thus, the input pole remains the speed bottleneck.

Example: Half Width CS Stage



$W \downarrow 2X$

bias current $\downarrow 2X$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \downarrow 2X$$

capacitances $\downarrow 2X$

$$\left|\omega_{p,in}\right| = \frac{1}{R_S\left[\frac{C_{in}}{2} + \left(1 + \frac{g_m R_L}{2}\right)\frac{C_{XY}}{2}\right]}$$

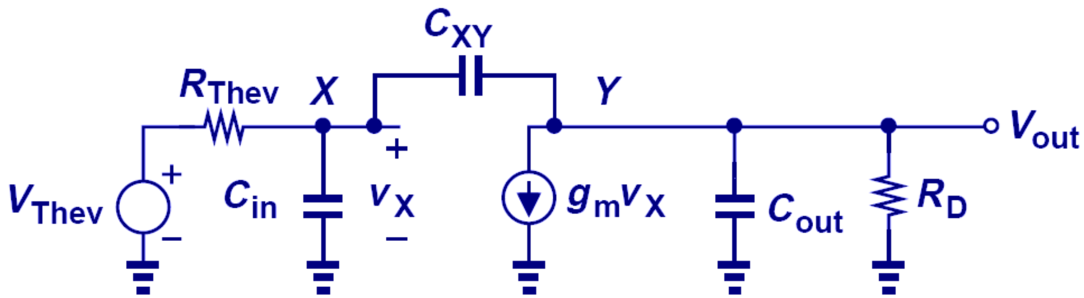
$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[\frac{C_{out}}{2} + \left(1 + \frac{2}{g_m R_L}\right)\frac{C_{XY}}{2}\right]}$$

bandwidth $\uparrow 2X$

gain $\downarrow 2X$

gain•bandwidth $\rightarrow$ constant

Direct Analysis of CE and CS Stages



$$\text{At node Y: } (V_X - V_{out})C_{XY}s = g_m V_X + V_{out}\left(\frac{1}{R_L} + C_{out}s\right) \Rightarrow V_X = V_{out}\frac{C_{XY}s + \frac{1}{R_L} + C_{out}s}{C_{XY}s - g_m}$$

$$\text{At node X: } (V_{out} - V_X)C_{XY}s = V_X C_{in}s + \frac{V_X - V_{Thev}}{R_{Thev}}$$

$$\Rightarrow V_{out}C_{XY}s - \left(C_{XY}s + C_{in}s + \frac{1}{R_{Thev}}\right)\frac{C_{XY}s + \frac{1}{R_L} + C_{out}s}{C_{XY}s - g_m}V_{out} = \frac{-V_{Thev}}{R_{Thev}}$$

$$\Rightarrow \boxed{\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1} \quad \text{where } \begin{aligned} a &= R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out}), \\ b &= (1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out}) \end{aligned}}$$

Direct Analysis of CE and CS Stages – cont'd

$$|\omega_z| = \frac{g_m}{C_{XY}}$$

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

$$if \;\; \omega_{p2} \gg \omega_{p1} \quad \Rightarrow \quad \omega_{p1}^{-1} + \omega_{p2}^{-1} \approx \omega_{p1}^{-1}$$

Dominant-pole approximation

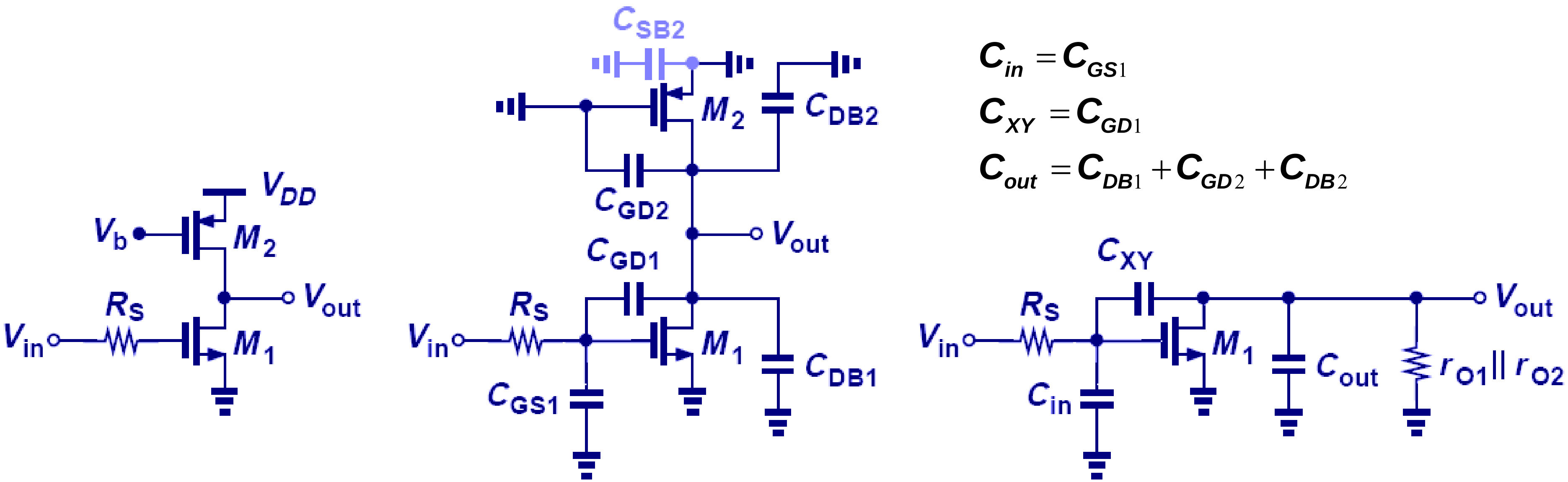
$$\Rightarrow b = \frac{1}{\omega_{p1}}$$

$$\left|\omega_{p1}\right| = \frac{1}{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}$$

$$|\omega_{p2}| = \frac{b}{a} = \frac{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

- **Direct analysis yields different pole locations and an extra zero.**

Example: Dominant-pole approximation



$$C_{in} = C_{GS1}$$

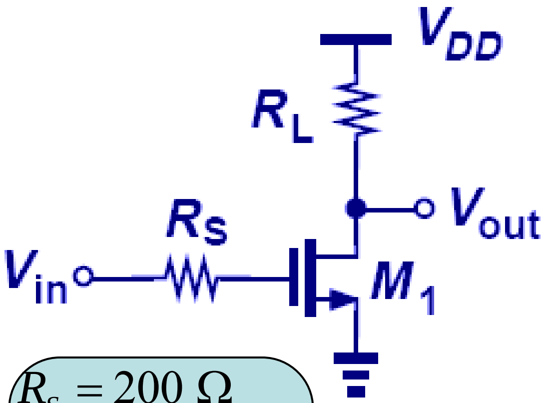
$$C_{XY} = C_{GD1}$$

$$C_{out} = C_{DB1} + C_{GD2} + C_{DB2}$$

$$\omega_{p1} \approx \frac{1}{\left[1+g_{m1}\left(r_{O1}\,\|\,r_{O2}\right)\right]C_{XY}R_S + R_S C_{in} + \left(r_{O1}\,\|\,r_{O2}\right)\left(C_{XY}+C_{out}\right)}$$

$$\omega_{p2} \approx \frac{\left[1+g_{m1}\left(r_{O1}\,\|\,r_{O2}\right)\right]C_{XY}R_S + R_S C_{in} + \left(r_{O1}\,\|\,r_{O2}\right)\left(C_{XY}+C_{out}\right)}{R_S\left(r_{O1}\,\|\,r_{O2}\right)\left(C_{in}C_{XY}+C_{out}C_{XY}+C_{in}C_{out}\right)}$$

Example: Comparison Between Different Methods



$R_S = 200\ \Omega$

$C_{GS} = 250\text{ fF}$

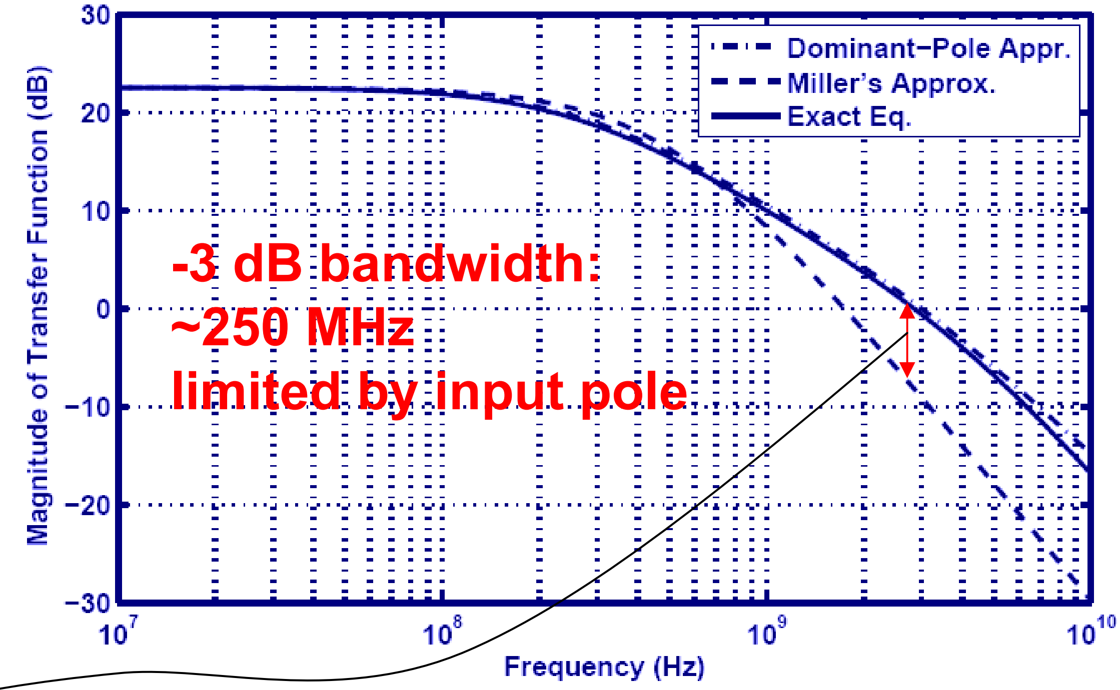
$C_{GD} = 80\text{ fF}$

$C_{DB} = 100\text{fF}$

$g_m = (150\ \Omega)^{-1}$

$\lambda = 0$

$R_L = 2\text{ k}\Omega$



This error arises because we have multiplied C_{GD} by the midband gain $(1+g_m R_L)$ rather than the gain at high frequencies.

Miller's

$$|\omega_{p,in}| = 2\pi \times (571\text{ MHz})$$
$$|\omega_{p,out}| = 2\pi \times (428\text{ MHz})$$

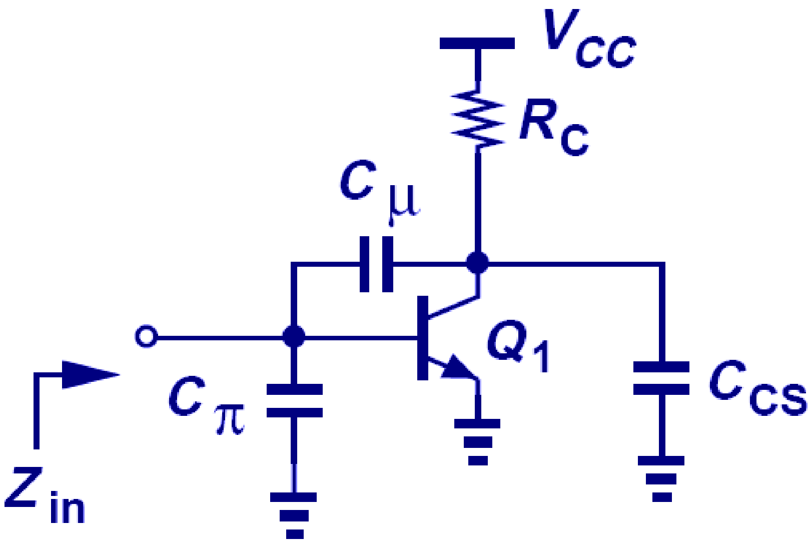
Exact

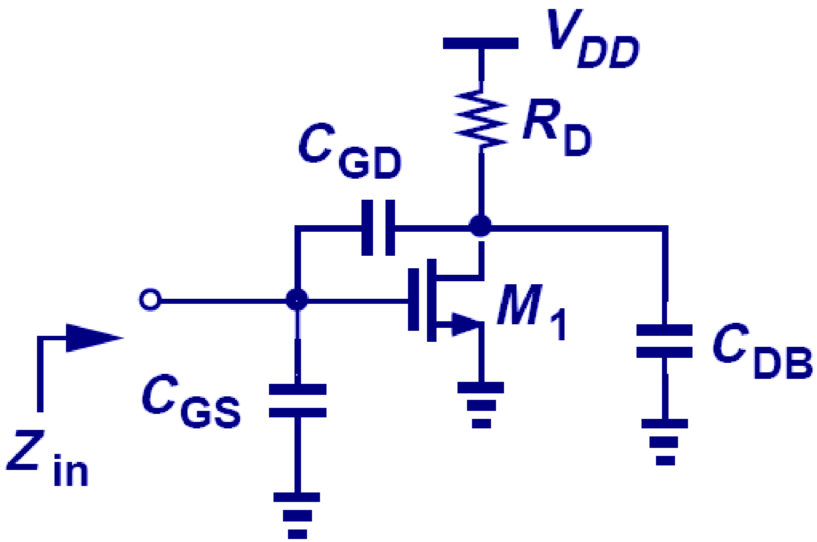
$$|\omega_{p,in}| = 2\pi \times (264\text{ MHz})$$
$$|\omega_{p,out}| = 2\pi \times (4.53\text{ GHz})$$

Dominant Pole

$$|\omega_{p,in}| = 2\pi \times (249\text{ MHz})$$
$$|\omega_{p,out}| = 2\pi \times (4.79\text{ GHz})$$

Input Impedance of CE and CS Stages

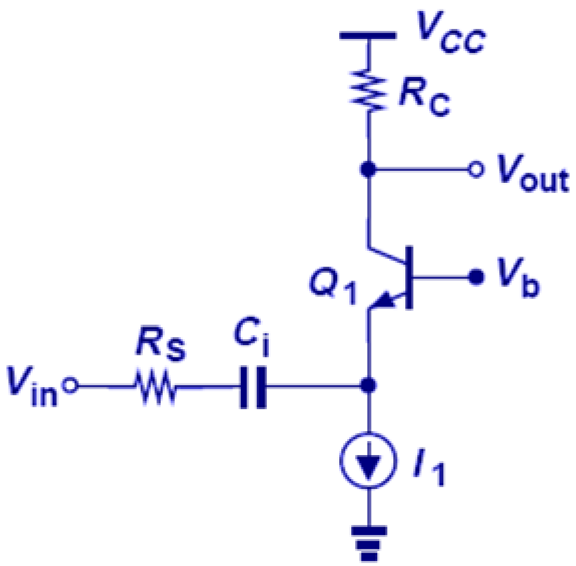


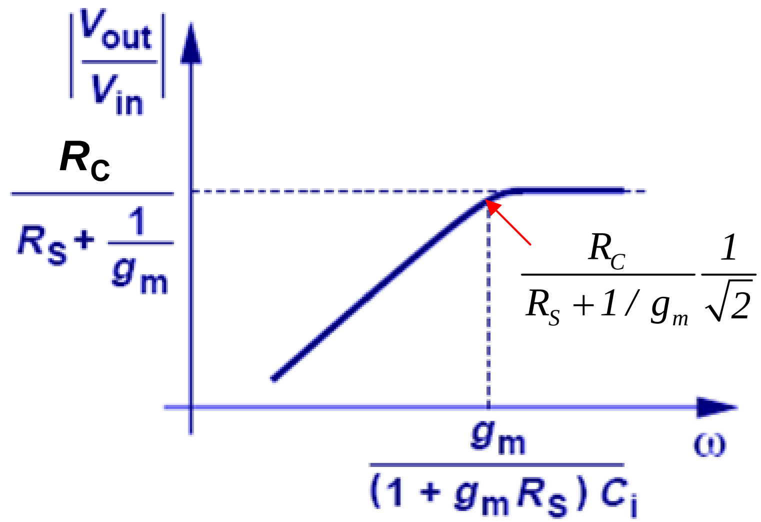


$$Z_{in} \approx \frac{1}{\left[C_\pi + (1 + g_m R_C) C_\mu\right] s} \parallel r_\pi$$

$$Z_{in} \approx \frac{1}{\left[C_{GS} + (1 + g_m R_D) C_{GD}\right] s}$$

Low Frequency Response of CB and CG Stages

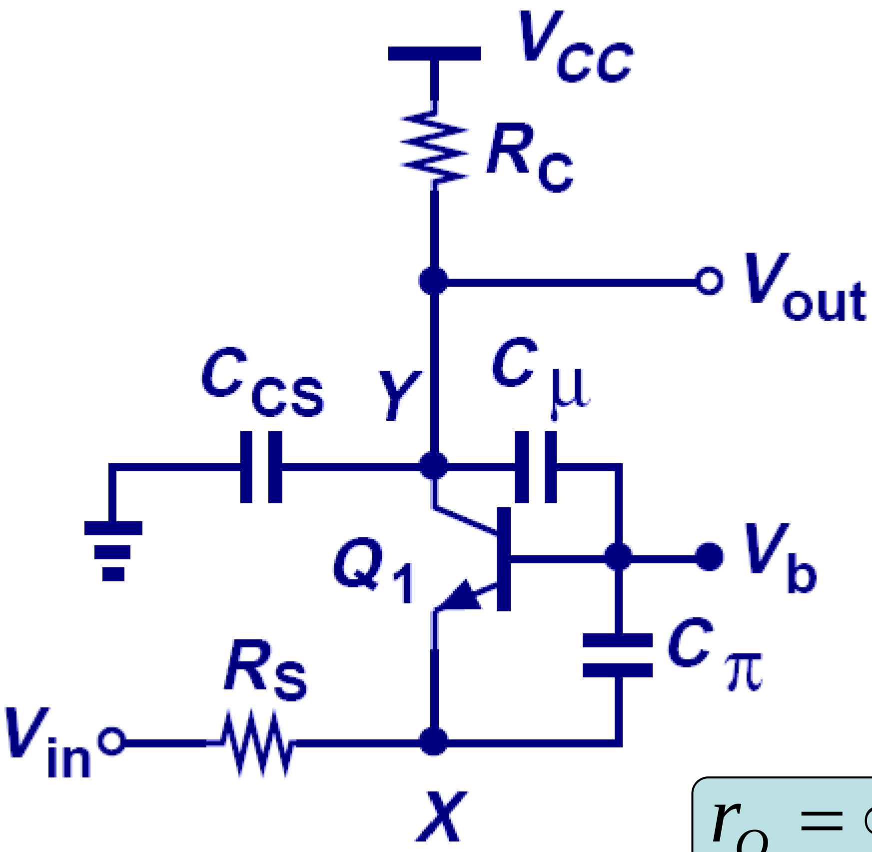




$$\frac{V_{out}}{V_{in}}(s) = \frac{R_C}{R_S + (C_i s)^{-1} + 1/g_m} = \frac{g_m R_C C_i s}{(1 + g_m R_S) C_i s + g_m}$$

- **As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).**

Frequency Response of CB Stage



$r_O = \infty$

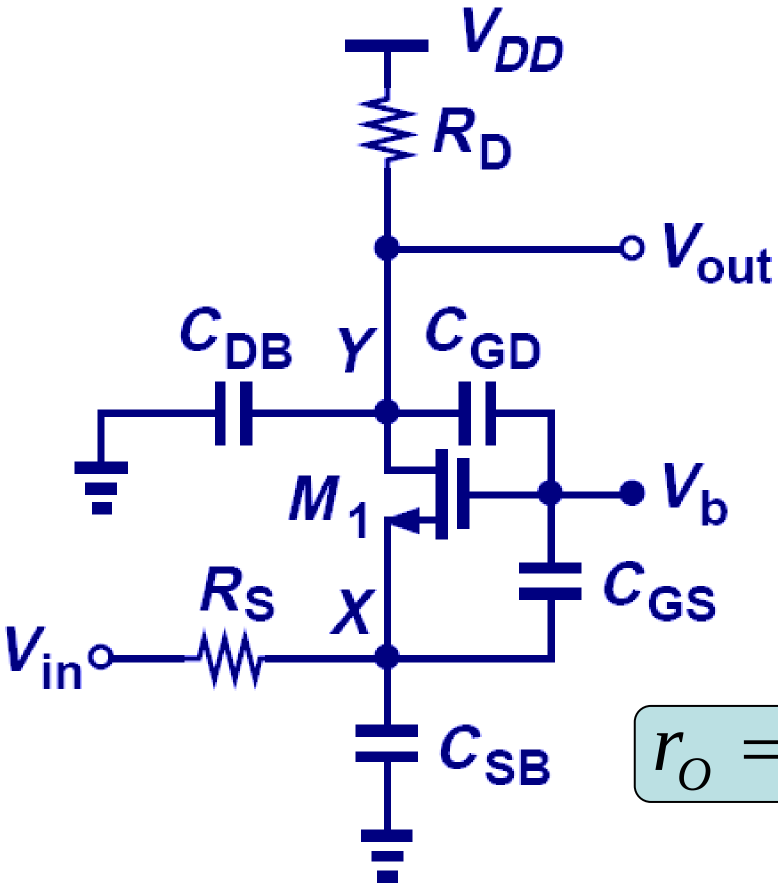
$$\omega_{p,X} = \frac{1}{\left(R_S \,\|\, \dfrac{1}{g_m}\right) C_X}$$

$$C_X = C_\pi$$

$$\omega_{p,Y} = \frac{1}{R_L C_Y}$$

$$C_Y = C_\mu + C_{CS}$$

Frequency Response of CG Stage



$$r_O = \infty$$

$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

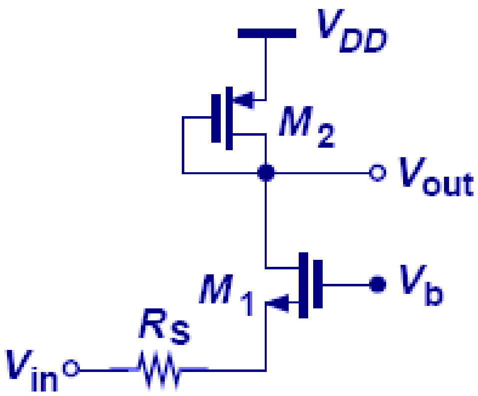
$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_L C_Y}$$

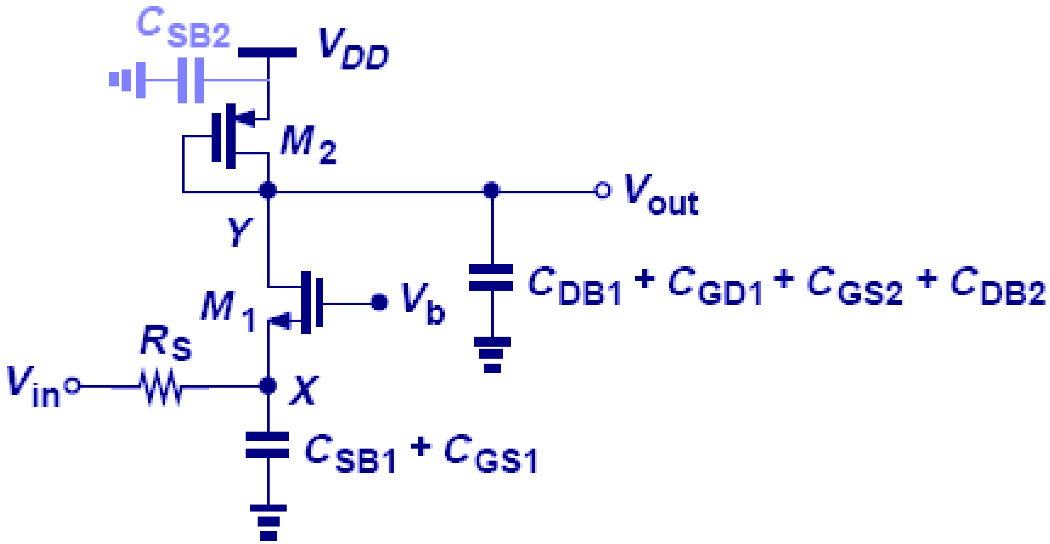
$$C_Y = C_{GD} + C_{DB}$$

- **Similar to a CB stage, the input pole is on the order of f_T, so rarely a speed bottleneck.**

Example: CG Stage Pole Identification



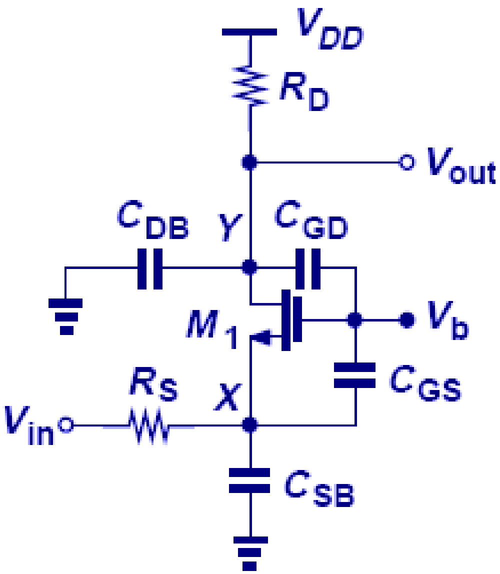
(a)

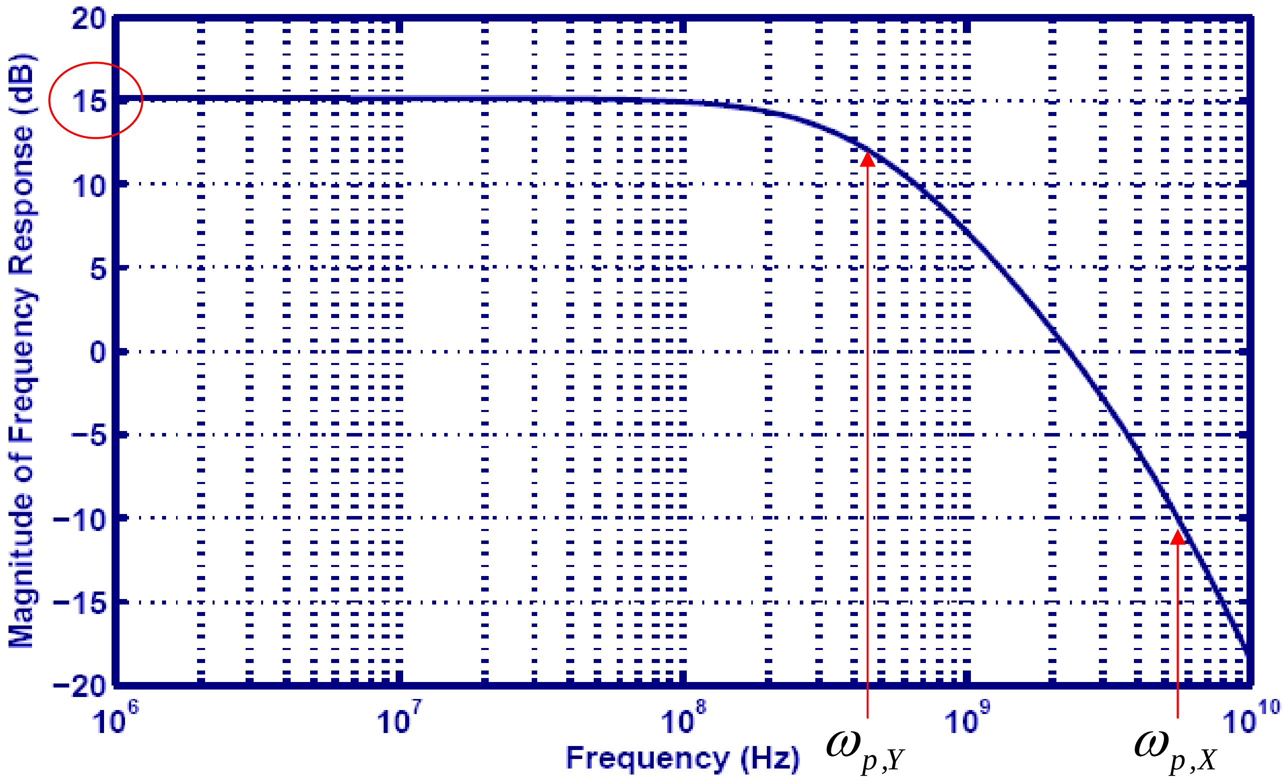


(b)

$$\omega_{p,X} = \frac{1}{\left(R_S \,\|\, \frac{1}{g_{m1}}\right)\left(C_{SB1} + C_{GS1}\right)} \qquad \omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left(C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2}\right)}$$

Example: Frequency Response of CG Stage



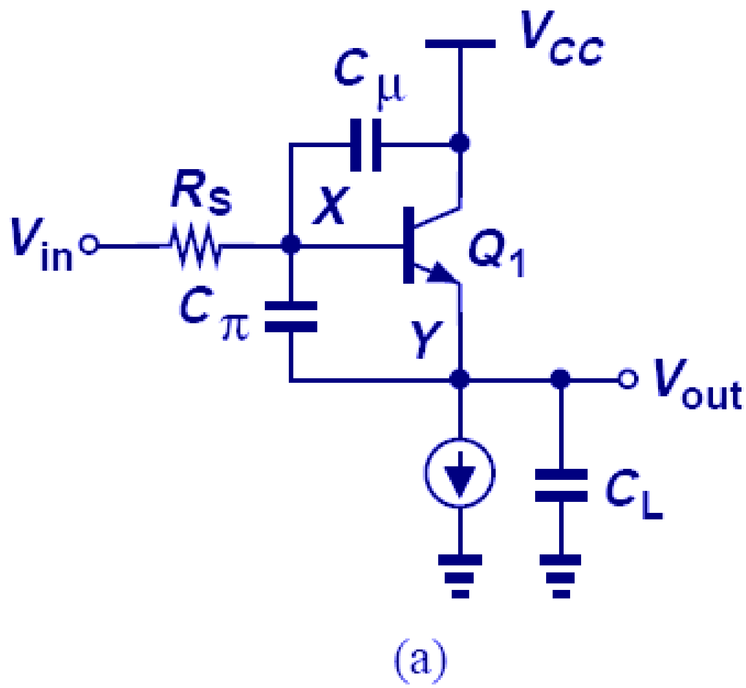


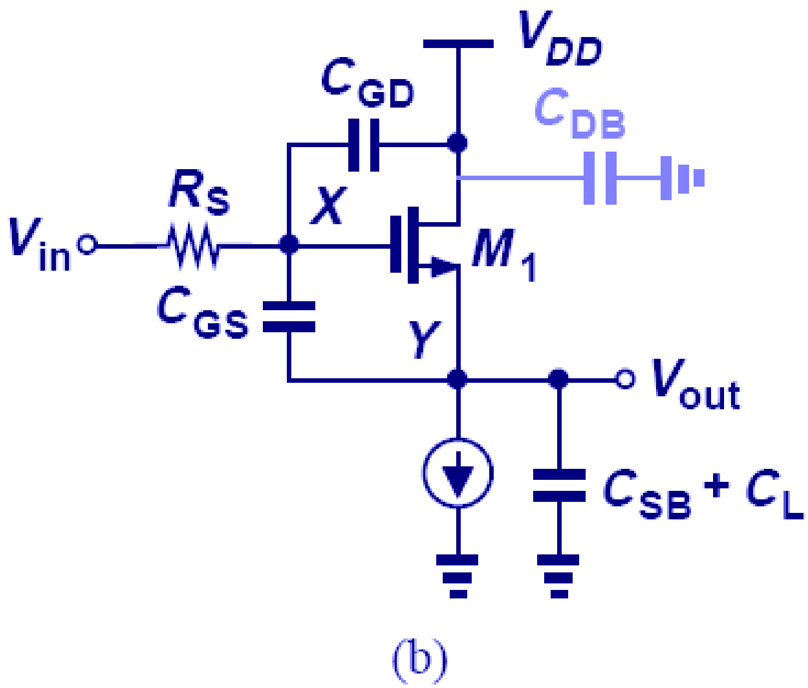
$$R_S = 200\ \Omega$$
$$C_{GS} = 250\ \text{fF}$$
$$C_{GD} = 80\ \text{fF}$$
$$C_{DB} = 100\ \text{fF}$$
$$g_m = (150\ \Omega)^{-1}$$
$$\lambda = 0$$
$$R_D = 2\ \text{k}\Omega$$

$$|\omega_{p,X}| = 1 / \left(R_S \,\|\, \frac{1}{g_m} \right) C_X = 2\pi \times (5.31\ \text{GHz})$$

$$|\omega_{p,Y}| = R_L / C_Y = 2\pi \times (442\ \text{MHz})$$

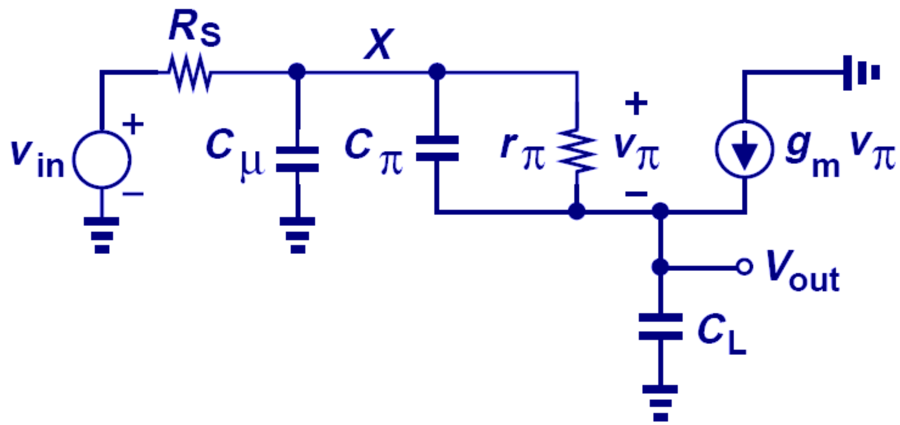
Emitter and Source Followers





- **The following will discuss the frequency response of emitter and source followers using direct analysis.**
- **Emitter follower is treated first and source follower is derived easily by allowing r_π to go to infinity.**

Direct Analysis of Emitter Follower



At node X: $$\frac{V_{out}+V_\pi-V_{in}}{R_S}+\left(V_{out}+V_\pi\right)C_\mu s+\frac{V_\pi}{r_\pi}+V_\pi C_\pi s=0$$

At output node: $$\frac{V_\pi}{r_\pi}+V_\pi C_\pi s+g_m V_\pi=V_{out}C_L s \quad\Rightarrow\quad V_\pi=\frac{V_{out}C_L s}{\frac{1}{r_\pi}+C_\pi s+g_m}$$

$$\frac{V_{out}}{V_{in}}=\frac{1+\frac{C_\pi}{g_m}s}{as^2+bs+1}$$

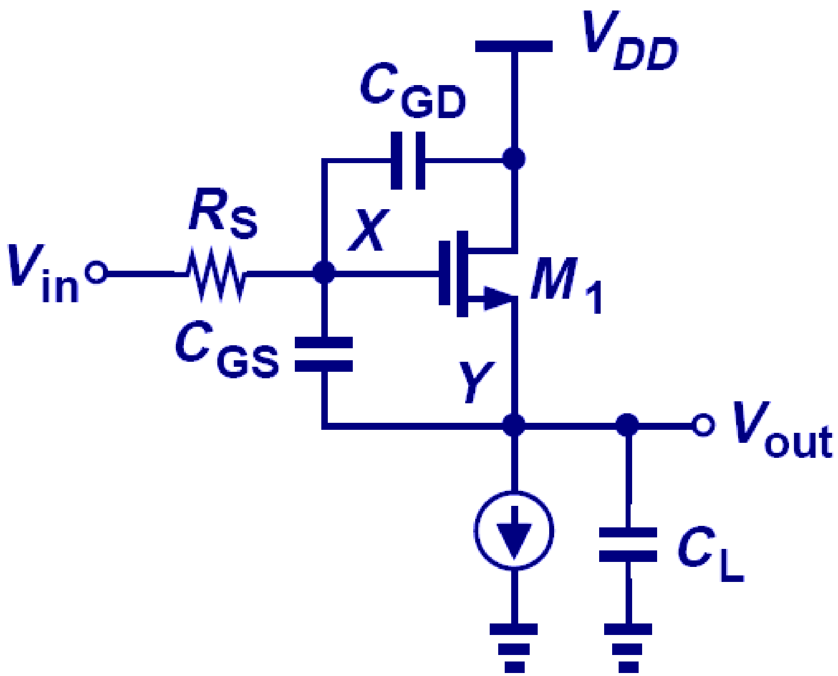
with $r_\pi \gg g_m^{-1}$

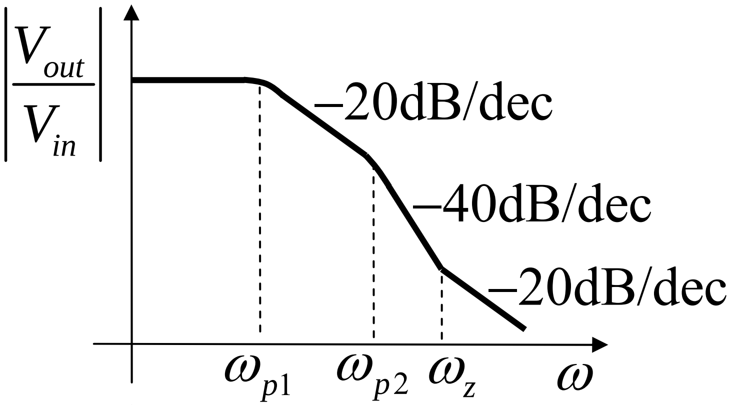
where $$a=\frac{R_S}{g_m}\left(C_\mu C_\pi+C_\mu C_L+C_\pi C_L\right)$$

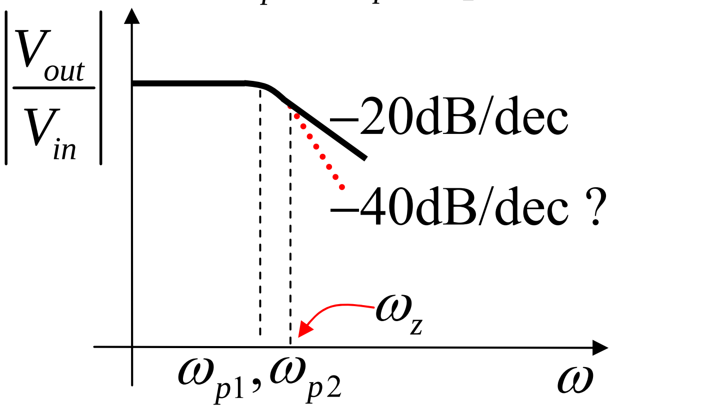
$$b=R_S C_\mu+\frac{C_\pi}{g_m}+\left(1+\frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}$$

$$\left|\omega_z\right|=\frac{g_m}{C_\pi}\approx f_T$$

Direct Analysis of Source Follower Stage





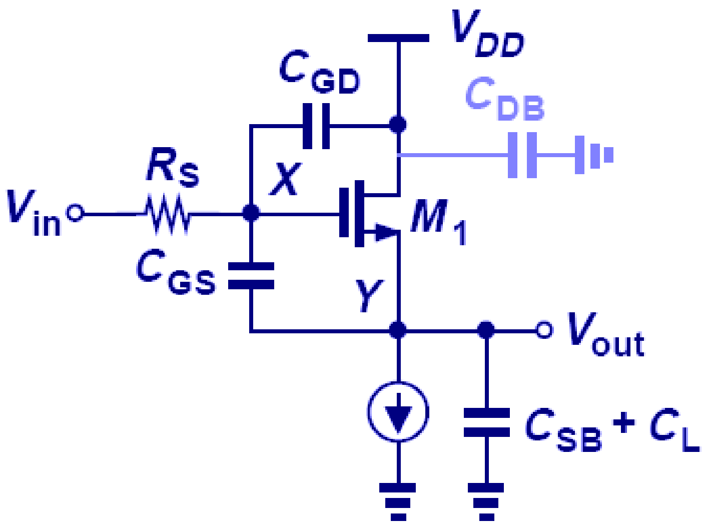


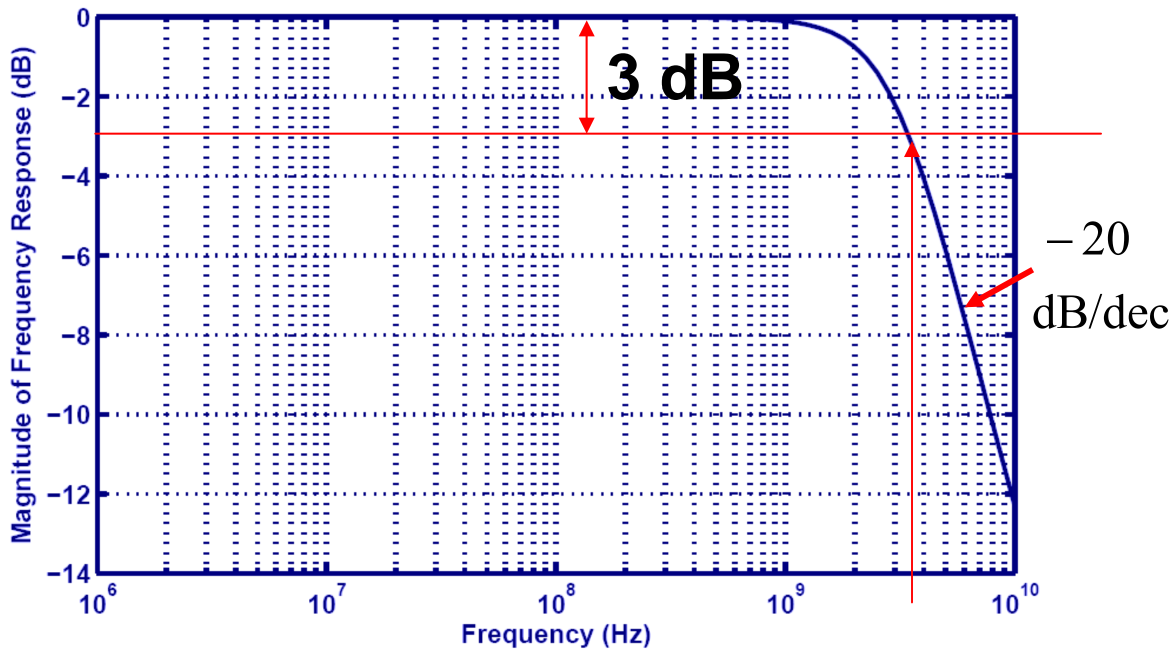
$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m} s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_m}\left(C_{GD}C_{GS} + C_{GD}\left(C_{SB} + C_L\right) + C_{GS}\left(C_{SB} + C_L\right)\right)$$

$$b = R_S C_{GD} + \frac{C_{GD} + C_{SB} + C_L}{g_m}$$

Example: Frequency Response of Source Follower





$R_S = 200\ \Omega$

$C_L = 100\ \text{fF}$

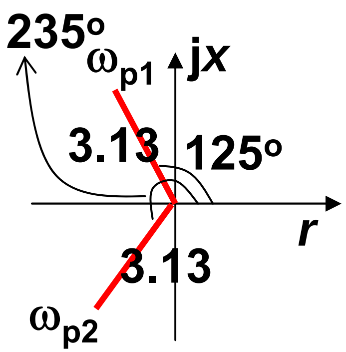
$C_{GS} = 250\ \text{fF}$

$C_{GD} = 80\ \text{fF}$

$C_{DB} = 100\ \text{fF}$

$g_m = (150\ \Omega)^{-1}$

$\lambda = 0$



$$\omega = r + jx = |\omega| e^{j\theta}$$

$$\theta = \tan^{-1}\left(\frac{x}{r}\right)$$

$$a = 2.58\times10^{-21}\, s^{-2}$$

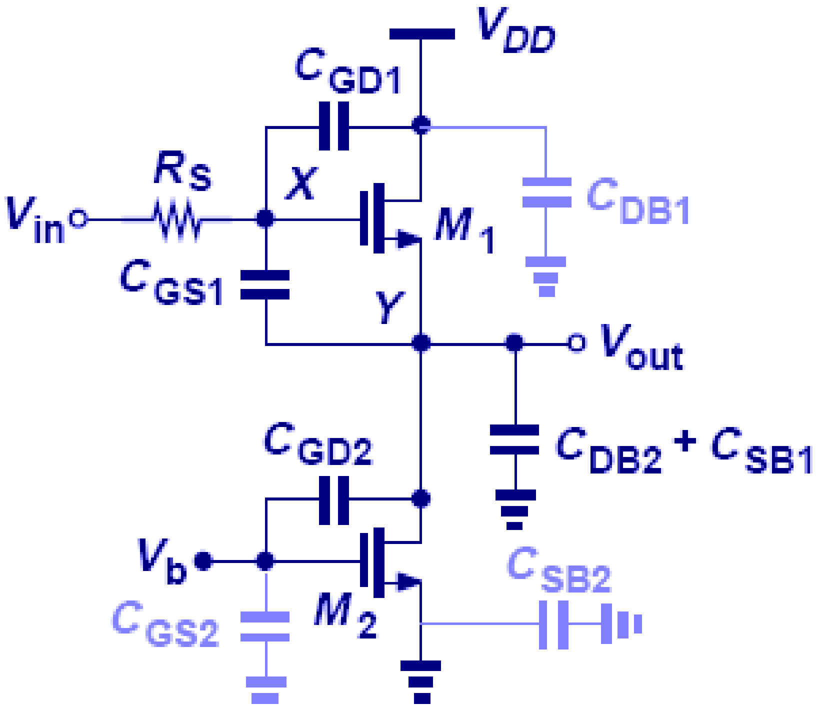
$$b = 5.8\times10^{-11}\, s$$

$$\omega_z = g_m / C_{GS} = 2\pi\times(4.24\ \text{GHz})$$

$$\omega_{p1} = 2\pi\left[-1.79\ \text{GHz} + j(2.57\ \text{GHz})\right]$$

$$\omega_{p2} = 2\pi\left[-1.79\ \text{GHz} - j(2.57\ \text{GHz})\right]$$

Example: Source Follower

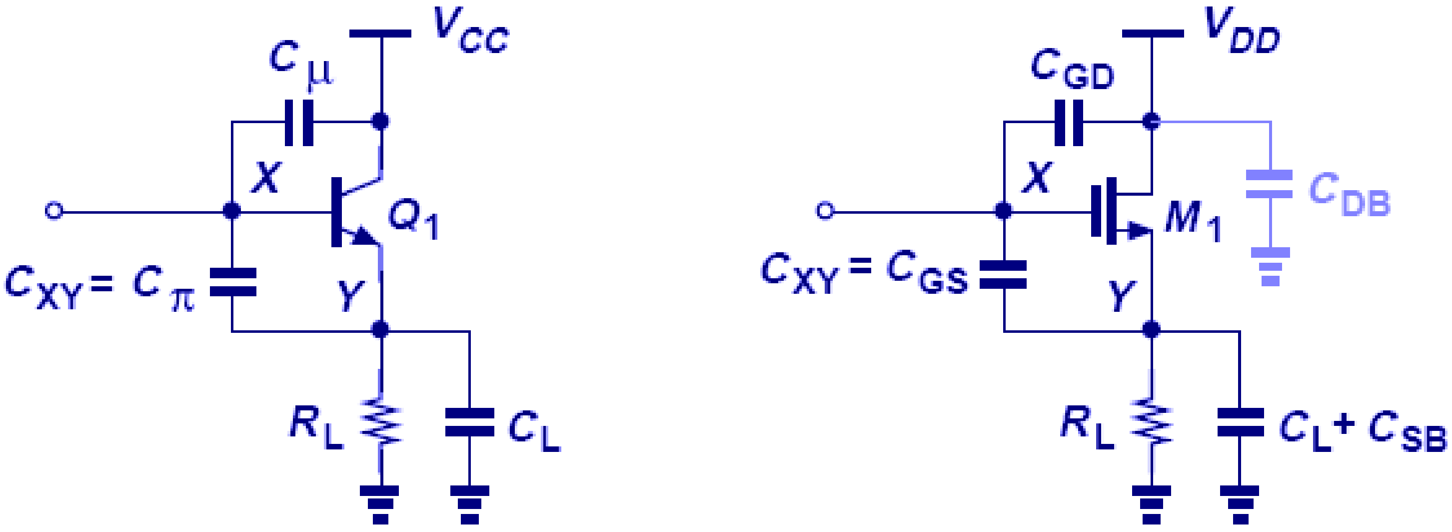


$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m} s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_{m1}}\left[C_{GD1}C_{GS1} + (C_{GD1} + C_{GS1})(C_{SB1} + C_{GD2} + C_{DB2})\right]$$

$$b = R_S C_{GD1} + \frac{C_{GD1} + C_{SB1} + C_{GD2} + C_{DB2}}{g_{m1}}$$

Input Capacitance of Emitter or Source Follower

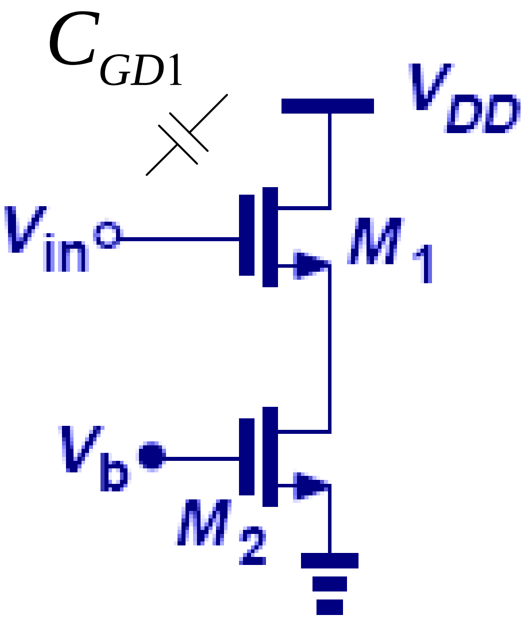


(a) (b)

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} \quad \Rightarrow \quad C_X = (1 - A_v) C_{XY} = \frac{1}{1 + g_m R_L} C_{XY}$$

$$\therefore \boxed{C_{in} = \left(C_\mu \ \ or \ \ C_{GD}\right) + \frac{C_{XY}}{1 + g_m R_L}}$$

Example: Source Follower Input Capacitance

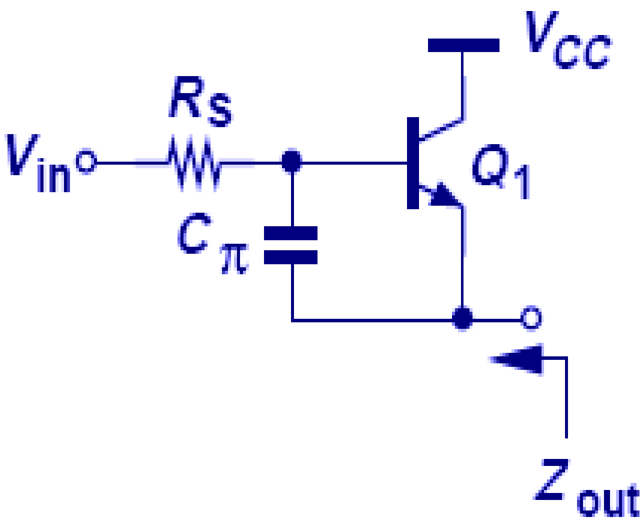


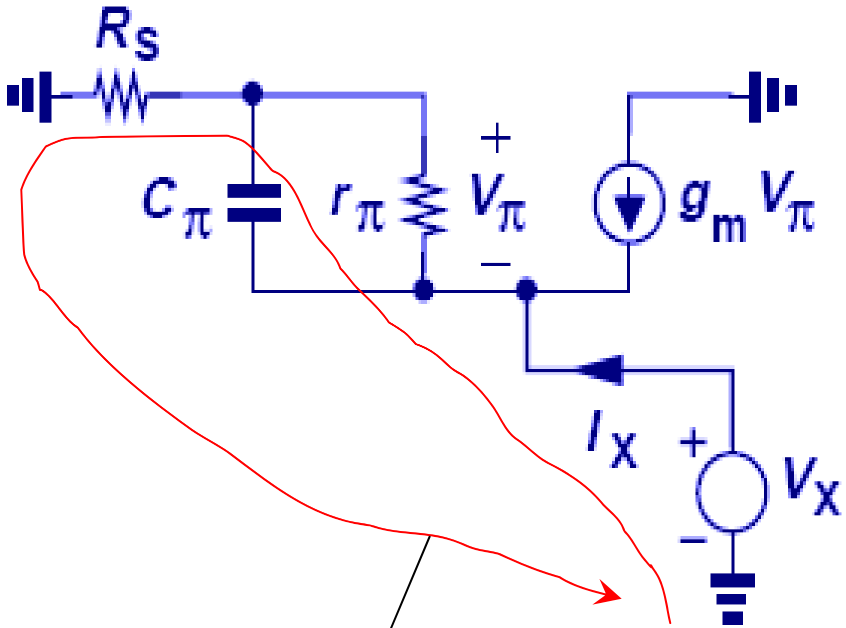
$$A_v = \frac{r_{O1} \| r_{O2}}{r_{O1} \| r_{O2} + \dfrac{1}{g_{m1}}}$$

$$\Rightarrow C_{in} = C_{GD1} + (1 - A_v) C_{GS1}$$

$$= \boxed{C_{GD1} + \frac{1}{1 + g_{m1}(r_{O1} \| r_{O2})} C_{GS1}}$$

Output Impedance of Emitter Follower





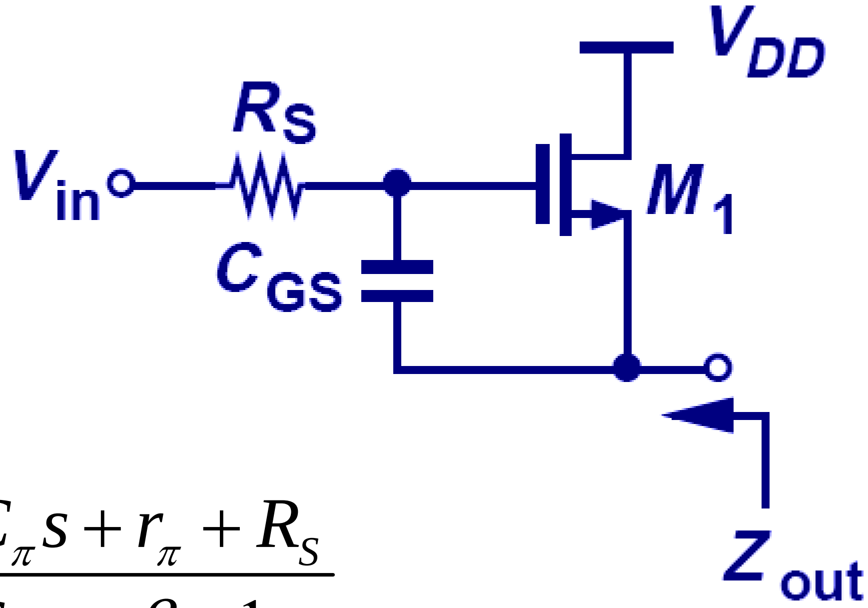
$$\left(I_X + g_m V_\pi\right)\left(r_\pi \,\Big\|\, \frac{1}{C_\pi s}\right) = -V_\pi$$

$$\Rightarrow V_\pi = -I_X \frac{r_\pi}{r_\pi C_\pi s + \beta + 1}$$

$$\left(I_X + g_m V_\pi\right) R_S - V_\pi = V_X$$

$$\Rightarrow \boxed{\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}}$$

Output Impedance of Source Follower

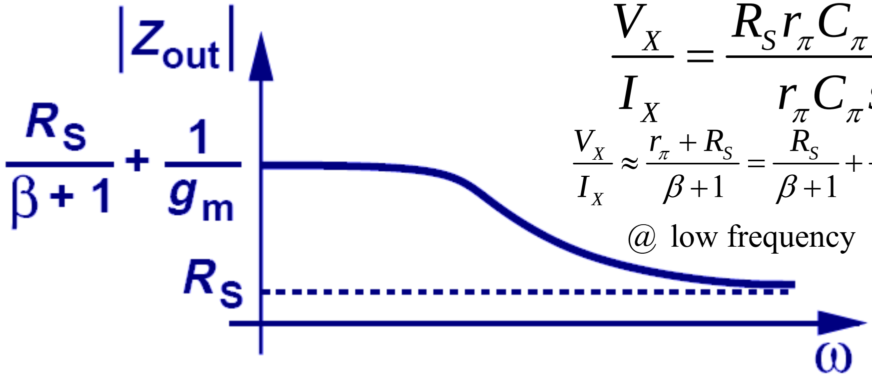


$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

with $r_\pi \to \infty$ in MOSFETs and $g_m \cdot r_\pi = \beta$

$$\boxed{\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}} \Leftarrow \frac{V_X}{I_X} = \frac{r_\pi\left(R_S C_\pi s + 1\right) + R_S}{r_\pi\left(C_\pi s + g_m\right) + 1}$$

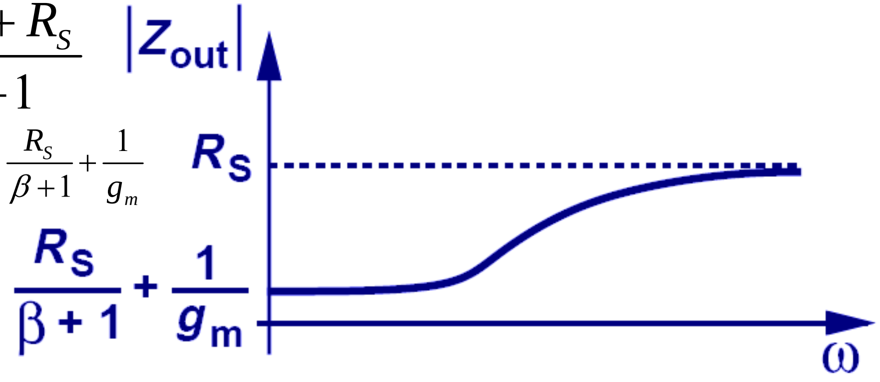
Active Inductor

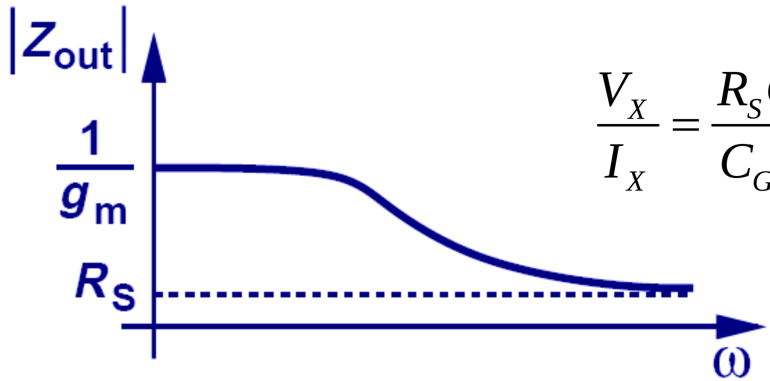


$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

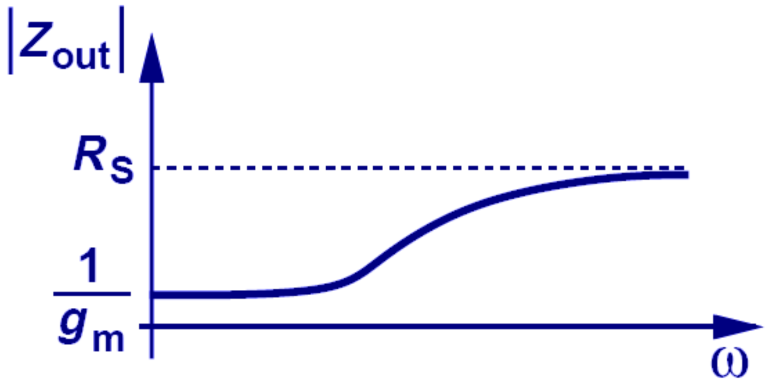
$$\frac{V_X}{I_X} \approx \frac{r_\pi + R_S}{\beta + 1} = \frac{R_S}{\beta + 1} + \frac{r_\pi}{g_m r_\pi + 1} \approx \frac{R_S}{\beta + 1} + \frac{1}{g_m}$$

@ low frequency



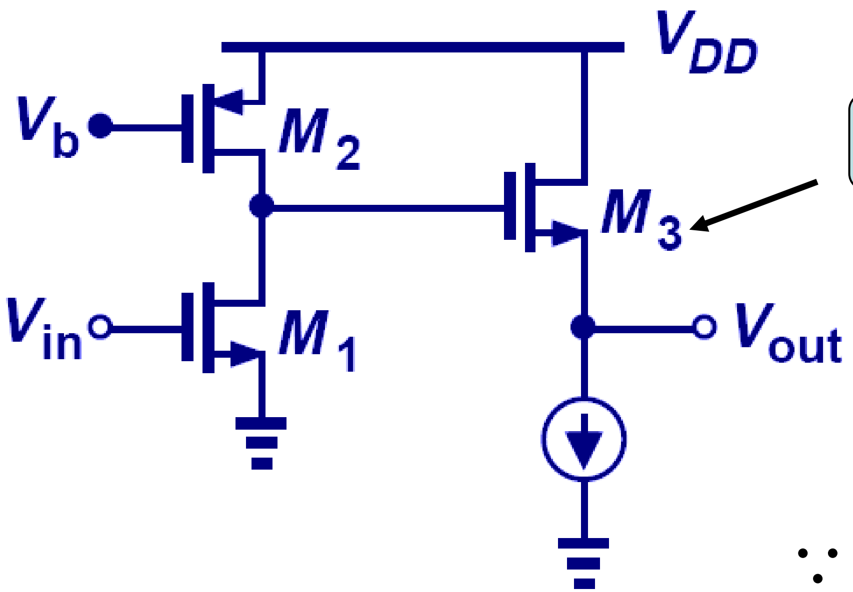


$$\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

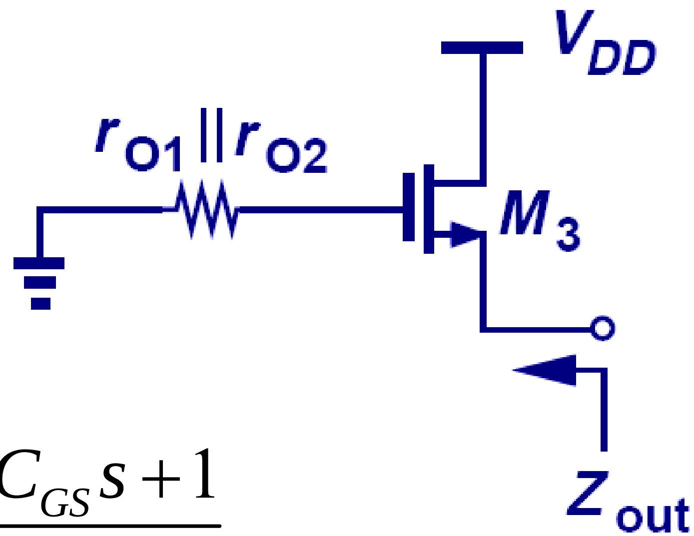


- **The plot above shows the output impedance of emitter and source followers. Since a follower's primary duty is to lower the driving impedance ($R_S > 1/g_m$), the "active inductor" characteristic on the right is usually observed.**

Example: Output Impedance



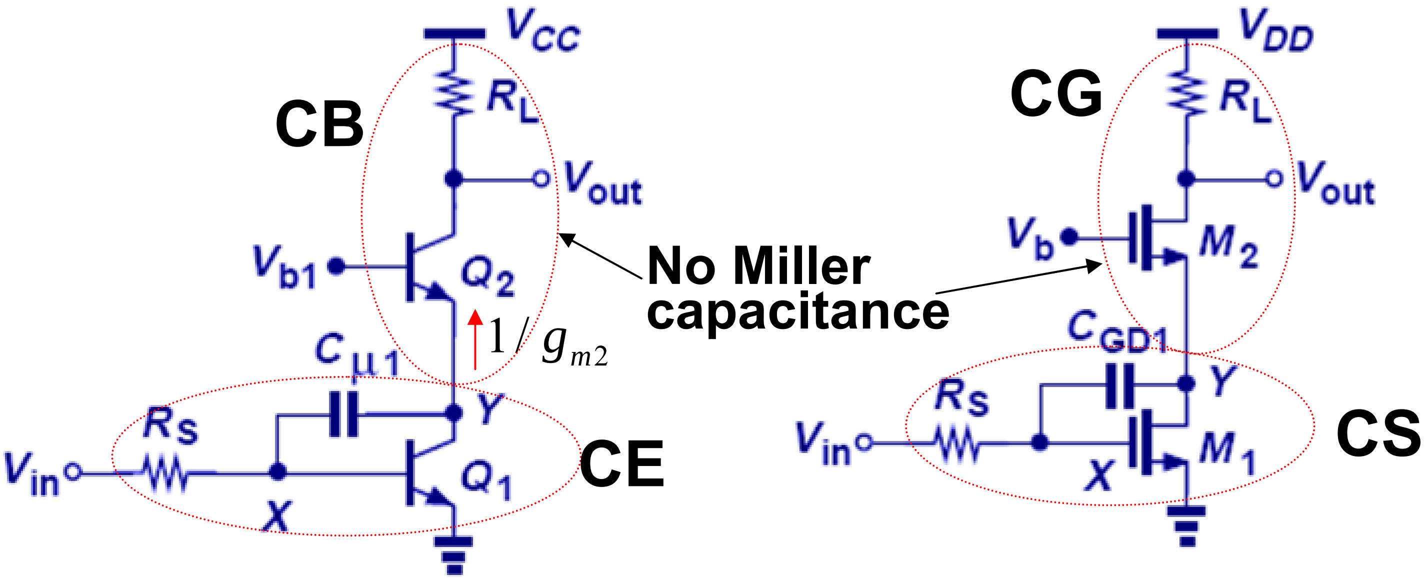
$r_{O3} = \infty$



$$\because \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

$$\frac{V_X}{I_X} = \frac{(r_{O1} \| r_{O2}) C_{GS3} s + 1}{C_{GS3} s + g_{m3}}$$

Frequency Response of Cascode Stage



Assuming $r_o = \infty$ for all transistors,

$$A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1$$

$$C_x = \left(1 - A_{v,XY}\right) C_{XY} \approx 2 \cdot C_{XY}$$

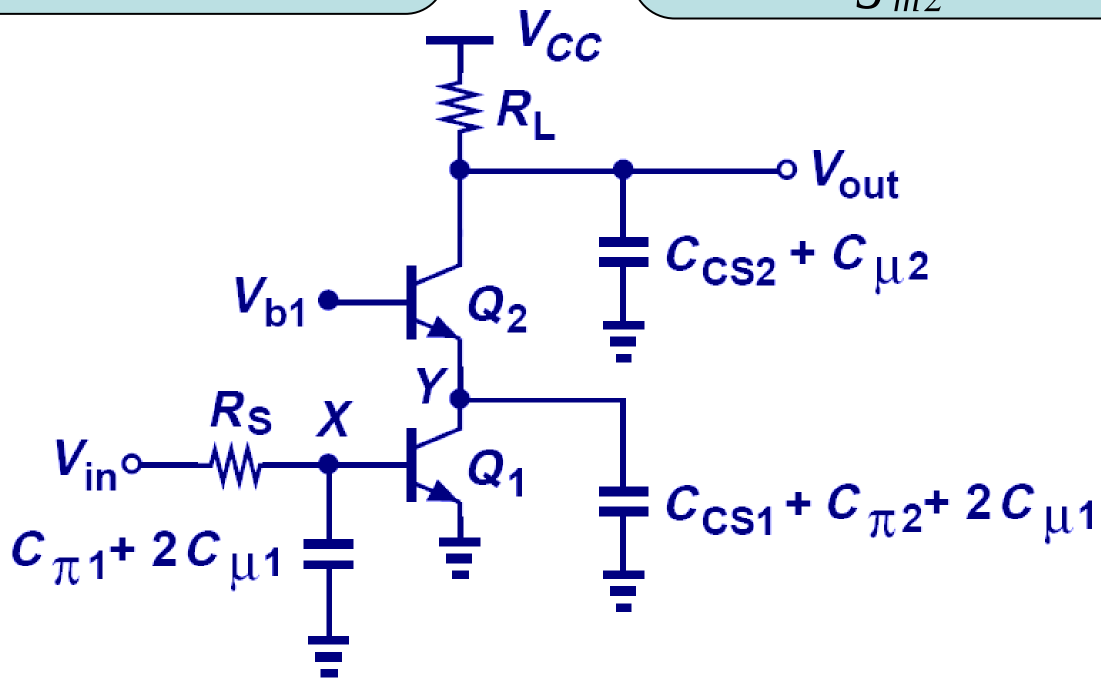
Smaller input capacitance than in CE or CS.

- **For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.**

Poles of Bipolar Cascode

$$\omega_{p,X} = \frac{1}{(R_S \parallel r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}(C_{CS1} + C_{\pi 2} + 2C_{\mu 1})}$$

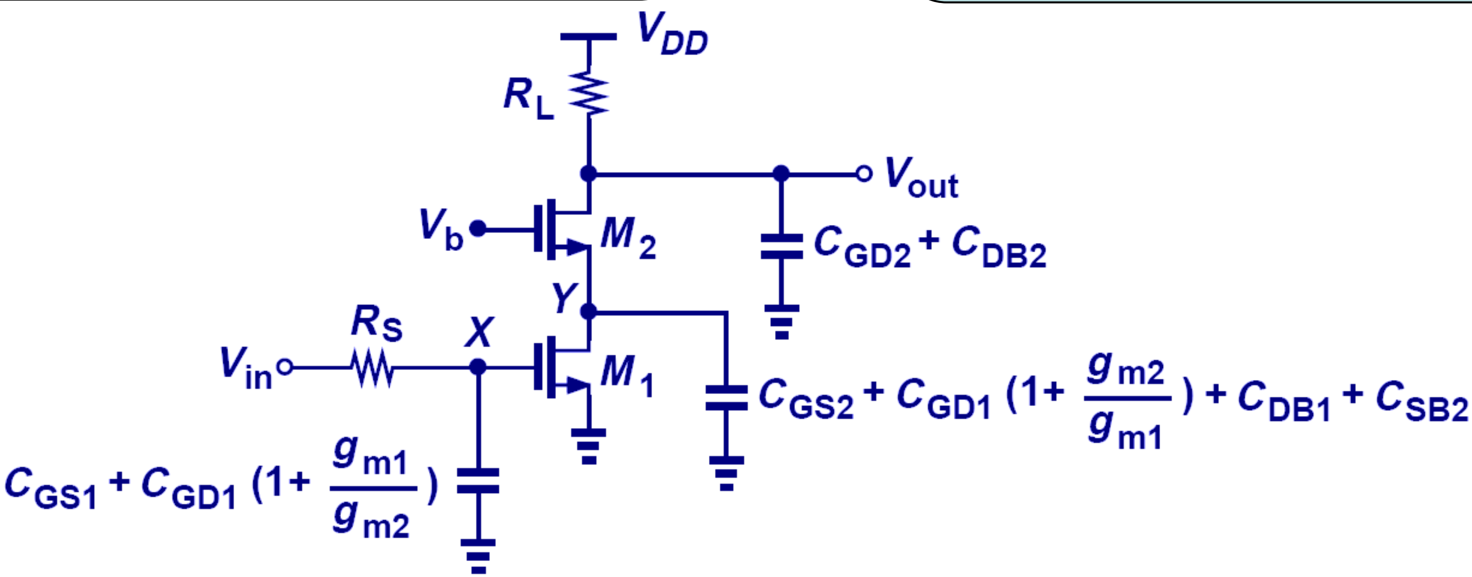


$$\omega_{p,out} = \frac{1}{R_L(C_{CS2} + C_{\mu 2})}$$

Poles of MOS Cascode

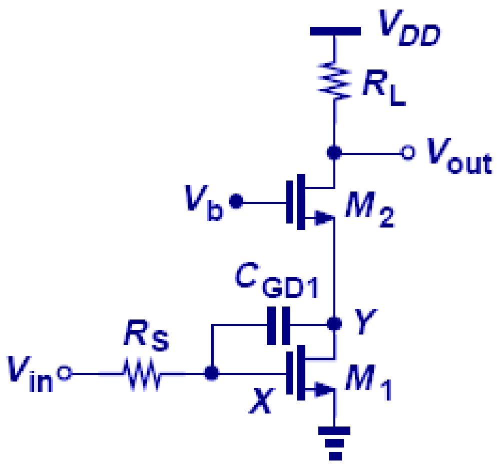
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

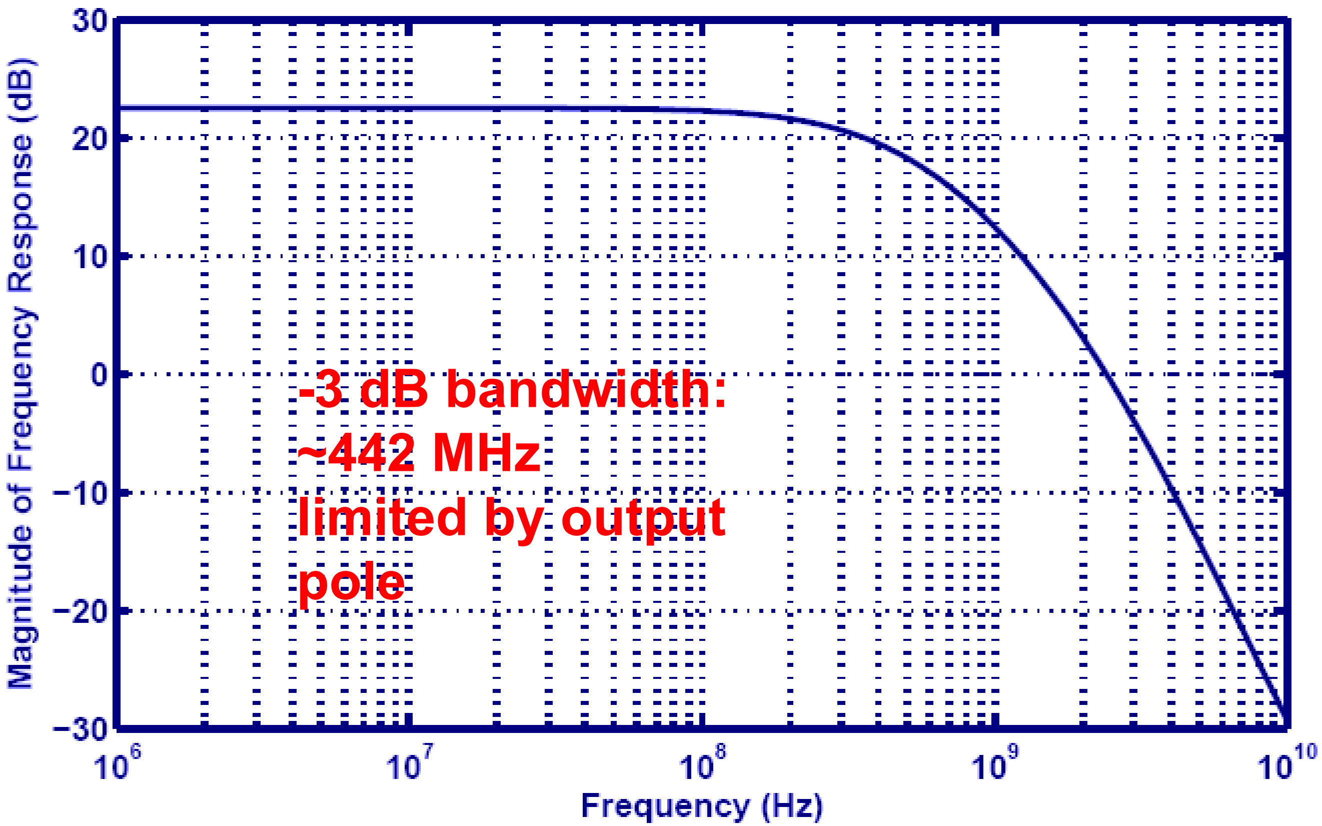
$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1}\right]}$$

Example: Frequency Response of Cascode





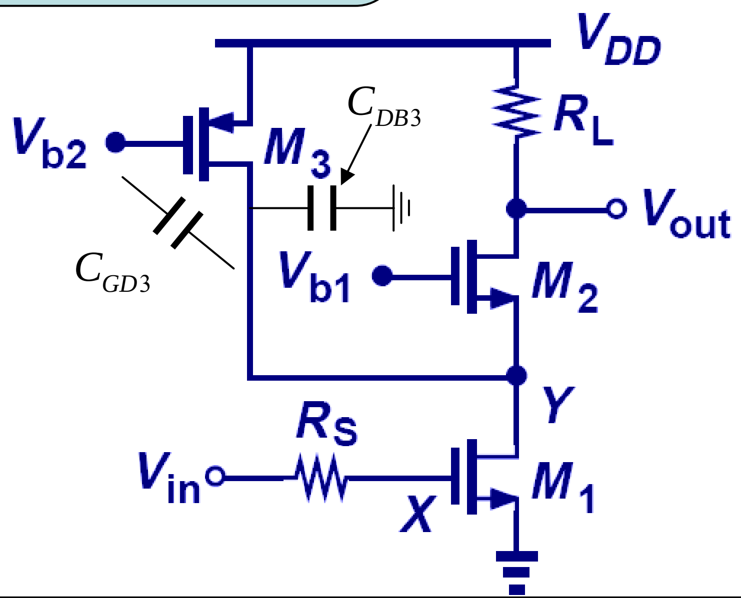
$$R_S = 200\ \Omega$$
$$C_{GS} = 250\ \text{fF}$$
$$C_{GD} = 80\ \text{fF}$$
$$C_{DB} = 100\ \text{fF}$$
$$g_m = (150\ \Omega)^{-1}$$
$$\lambda = 0$$
$$R_L = 2\ \text{k}\Omega$$

$$|\omega_{p,X}| = 2\pi \times (1.95\ \text{GHz})$$
$$|\omega_{p,Y}| = 2\pi \times (1.73\ \text{GHz})$$
$$|\omega_{p,out}| = 2\pi \times (442\ \text{MHz})$$

MOS Cascode Example

$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$

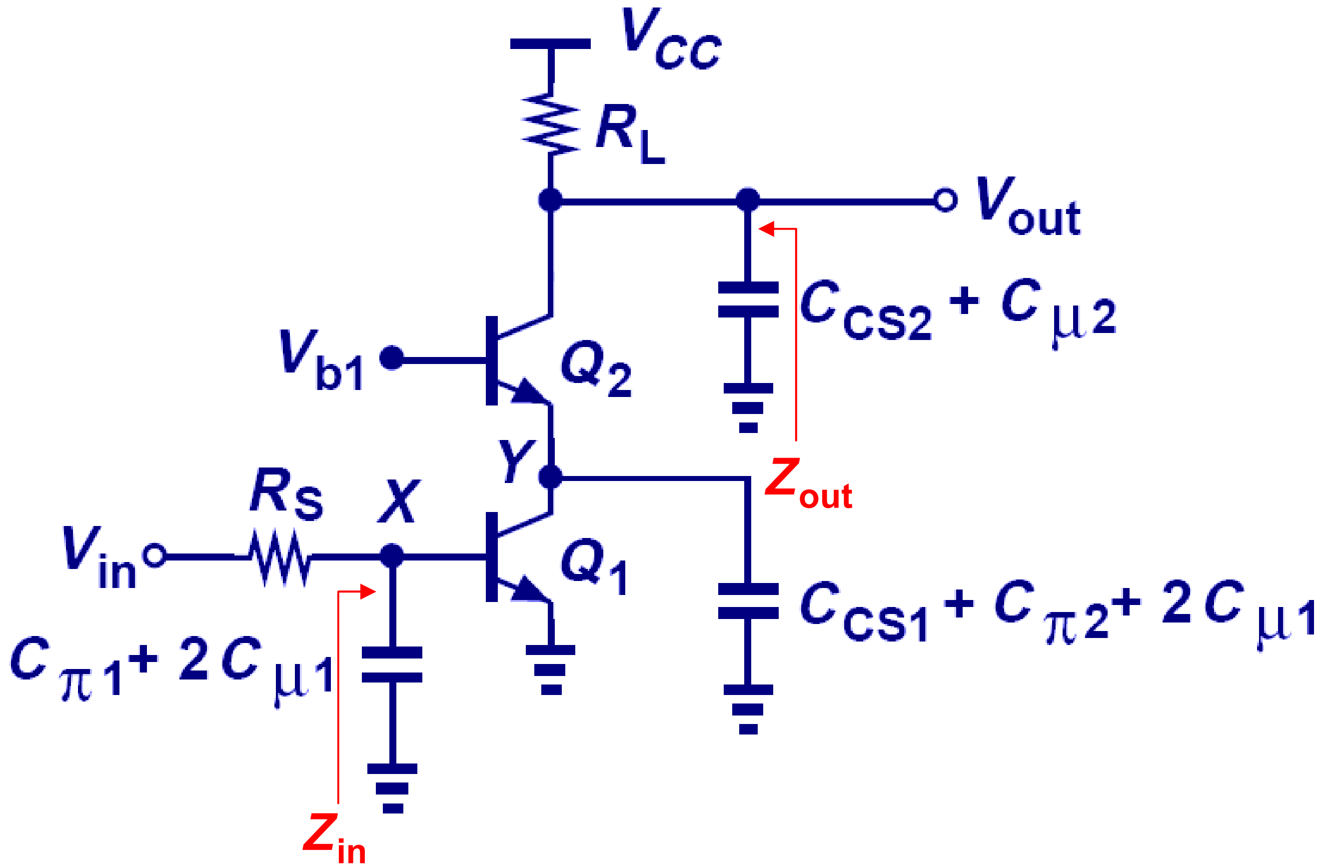


The addition of M_3:
$\rightarrow I_{D2}\downarrow \rightarrow g_{m2}\downarrow \rightarrow \omega_{p,X}\downarrow$

M_3: Constant current source

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{SB2} + C_{GD3} + C_{DB3}\right]}$$

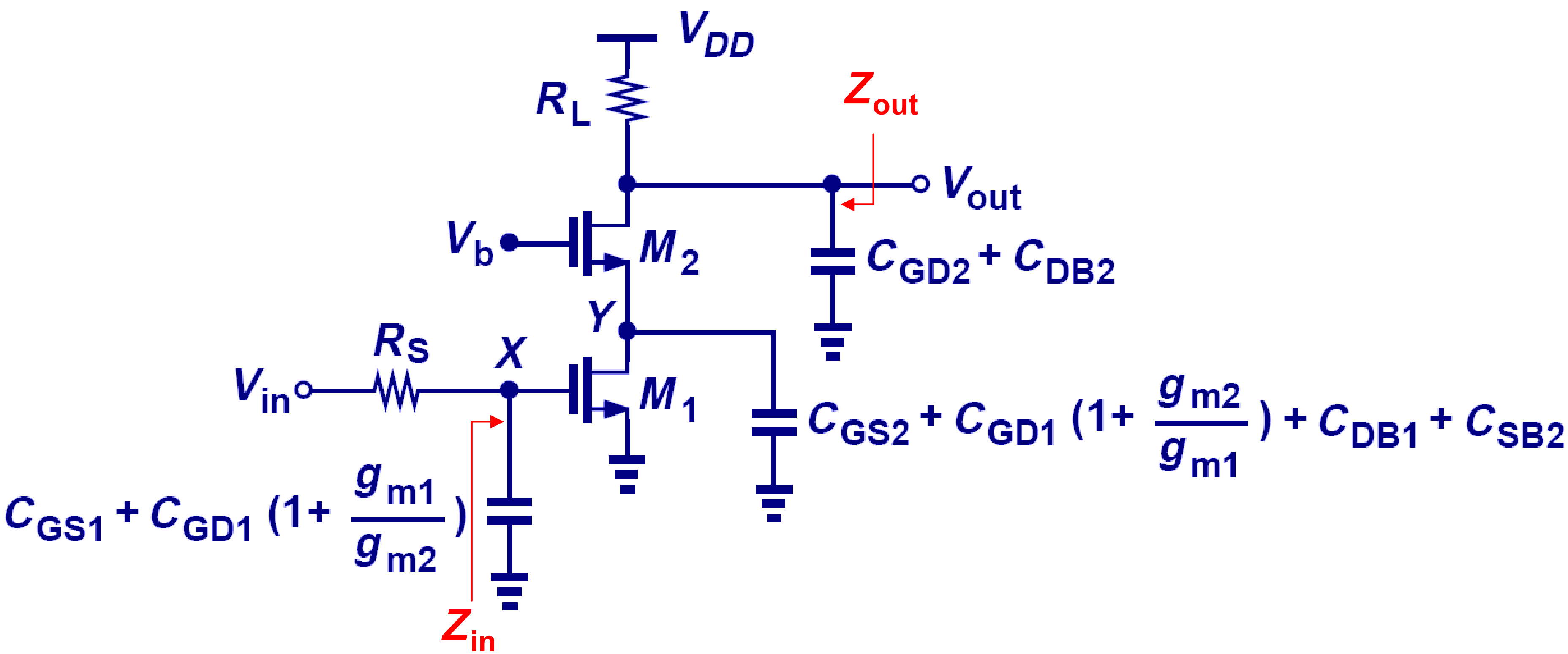
I/O Impedance of Bipolar Cascode



$$Z_{in} = r_{\pi 1} \parallel \frac{1}{\left(C_{\pi 1} + 2C_{\mu 1}\right)s}$$

$$Z_{out} = R_L \parallel \frac{1}{\left(C_{\mu 2} + C_{CS2}\right)s}$$

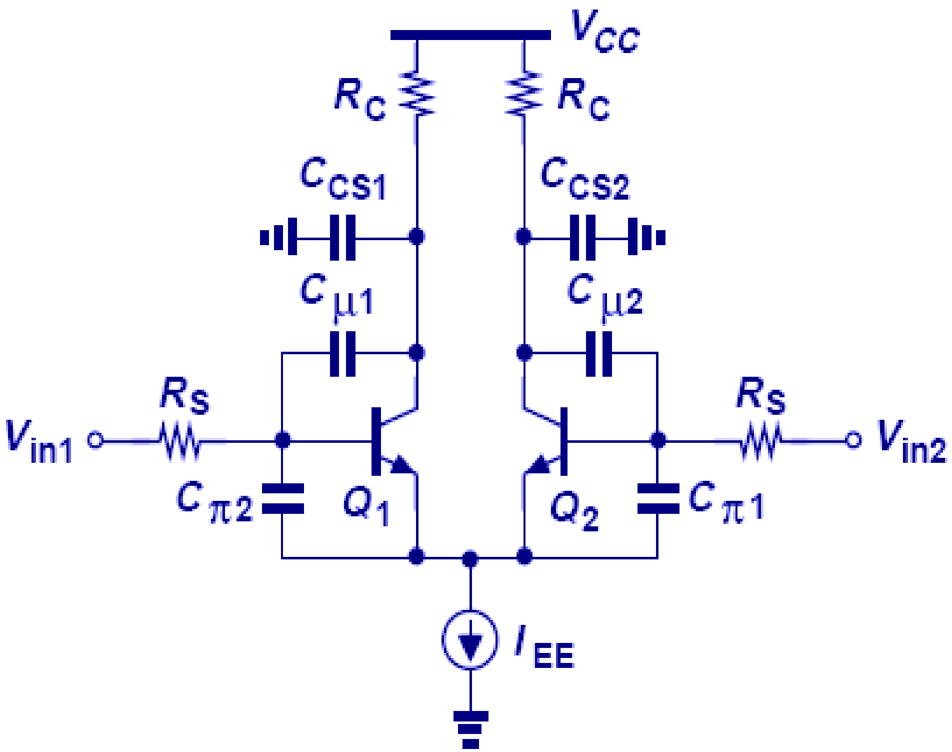
I/O Impedance of MOS Cascode

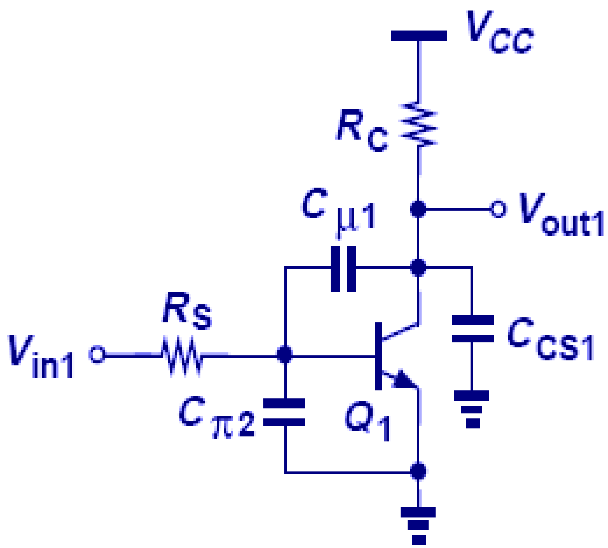


$$Z_{in} = \frac{1}{\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]s}$$

$$Z_{out} = R_L \,\|\, \frac{1}{\left(C_{GD2} + C_{DB2}\right)s}$$

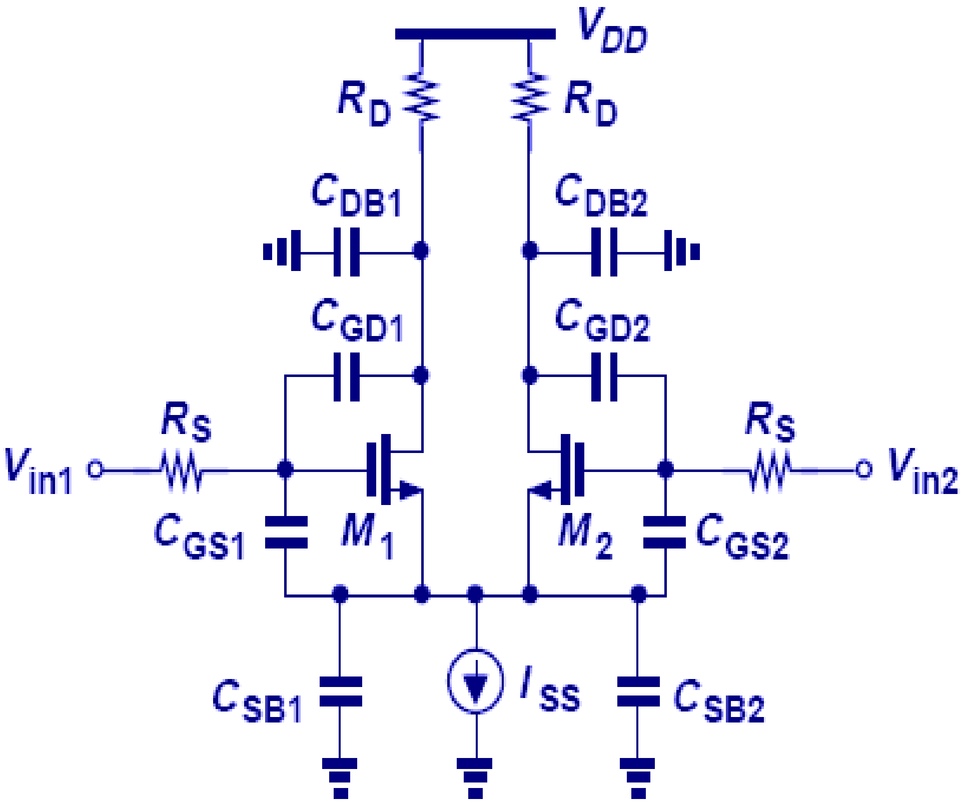
Bipolar Differential Pair Frequency Response

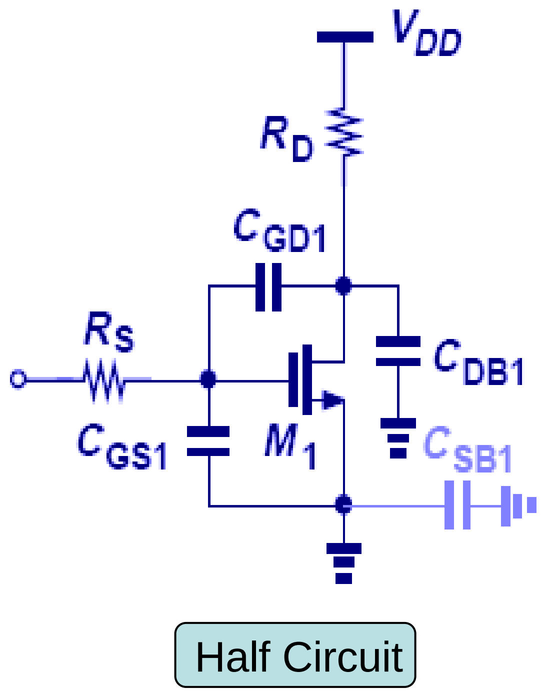




- Since bipolar differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

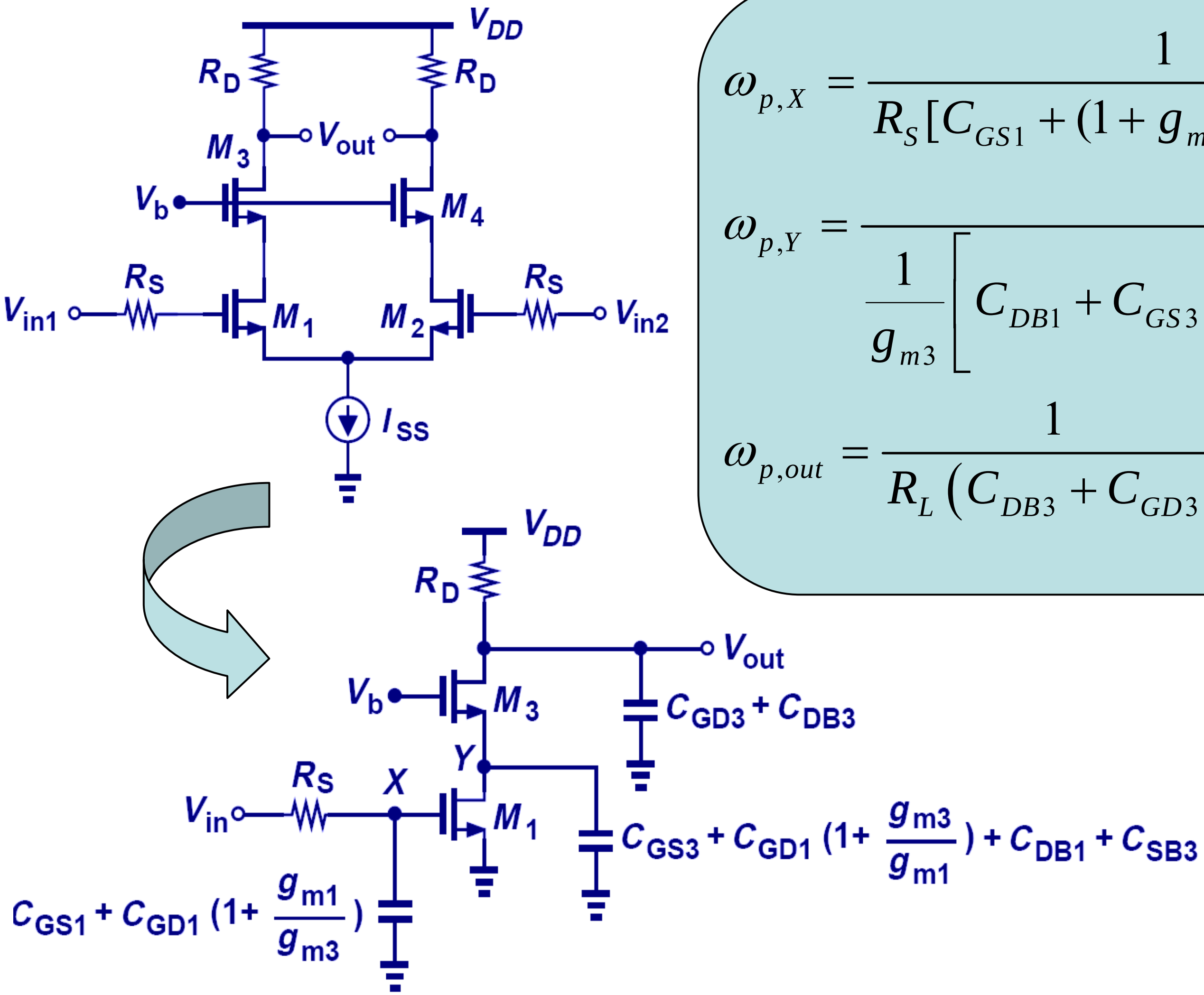
MOS Differential Pair Frequency Response





- Since MOS differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

Example: MOS Differential Pair

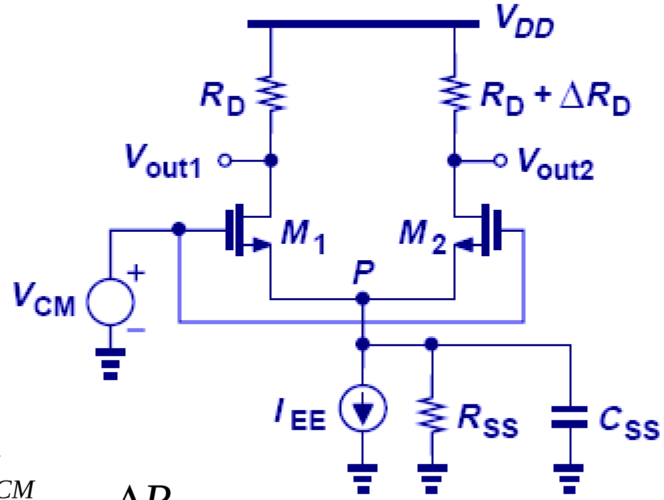


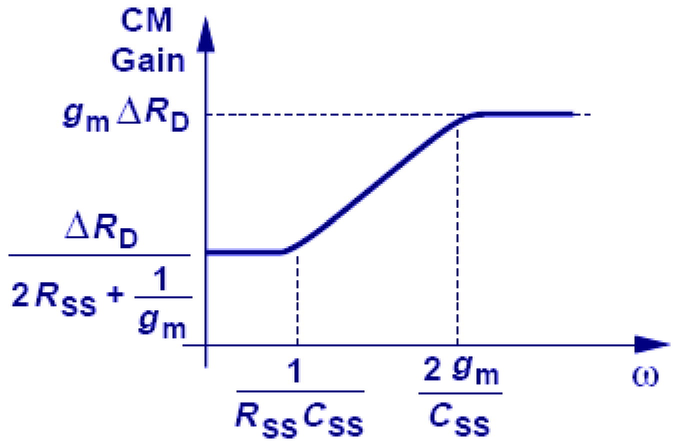
$$\omega_{p,X} = \frac{1}{R_S [C_{GS1} + (1 + g_{m1} / g_{m3}) C_{GD1}]}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m3}} \left[C_{DB1} + C_{GS3} + C_{SB3} + \left(1 + \frac{g_{m3}}{g_{m1}} \right) C_{GD1} \right]}$$

$$\omega_{p,out} = \frac{1}{R_L \left(C_{DB3} + C_{GD3} \right)}$$

Common Mode Frequency Response





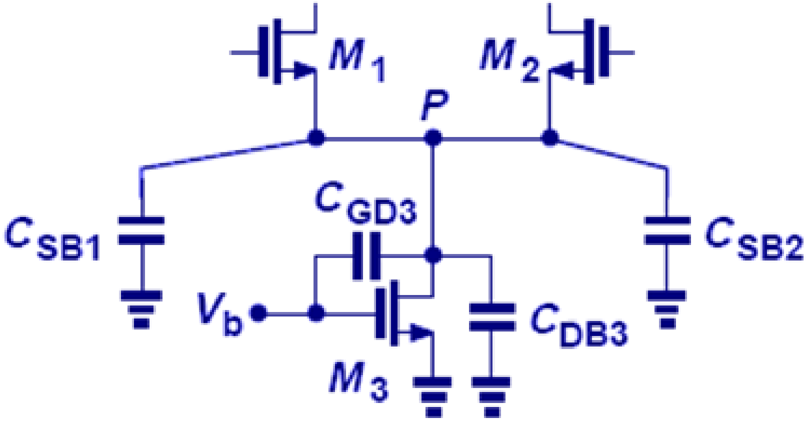
$$\because \Delta V_{out} = \frac{\Delta V_{CM}}{1/g_m + 2R_{SS}} \Delta R_D$$

@ low frequency

$$\left|\frac{\Delta V_{out}}{\Delta V_{CM}}\right| = \frac{\Delta R_D}{\dfrac{1}{g_m} + 2\left(R_{SS} \,\Big\|\, \dfrac{1}{C_{SS}s}\right)} = \frac{g_m \Delta R_D \left(R_{SS}C_{SS}s + 1\right)}{R_{SS}C_{SS}s + 2g_m R_{SS} + 1}$$

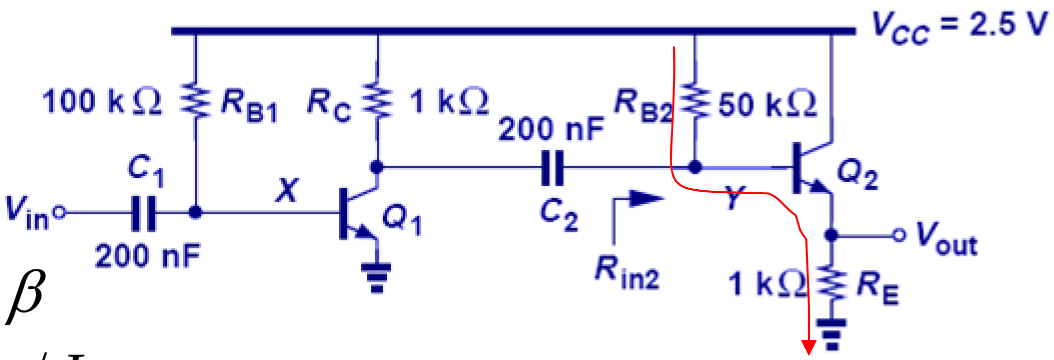
- **C_{ss} will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.**

Tail Node Capacitance Contribution



- **Source-Body Capacitance of M_1, M_2**
- **Drain-Body Capacitance of M_3**
- **Gate-Drain Capacitance of M_3**

Example: Capacitive Coupling



$$I_S = 5\times 10^{-16}\text{ A}$$
$$\beta = 100$$
$$V_A = \infty$$

$$\because g_m r_\pi = \beta$$
$$\because g_m = V_T / I_C$$

$$\because I_{C2} = \beta I_{B2}$$

For Q_1, assuming $V_{BE1} = 800$ mV,

$$I_{C1} = \beta \frac{V_{CC} - V_{BE1}}{R_{B1}} = 1.7\text{ mA}$$

$$\Rightarrow V_{BE1} = V_T \ln\left(I_{C1}/I_{S1}\right) = 748\text{ mV}$$

$$\Rightarrow I_{C1} = 1.75\text{ mA} \Rightarrow g_{m1} = \left(14.9\ \Omega\right)^{-1}$$

$$\Rightarrow r_{\pi 1} = 14.9\text{ k}\Omega$$

For Q_2, assuming $V_{BE2} = 800$mV,

$$V_{CC} = I_{B2}R_{B2} + V_{BE2} + R_E I_{C2}$$

$$\Rightarrow I_{C2} = \frac{V_{CC} - V_{BE2}}{R_{B2}/\beta + R_E} = 1.13\text{ mA}$$

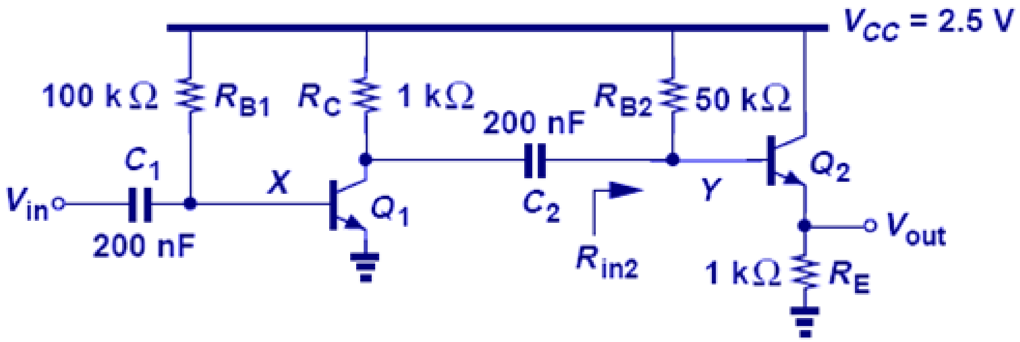
$$V_{BE2} = V_T \ln(I_{C2}/I_{S2}) = 0.88\text{ V}$$

Iteration yields

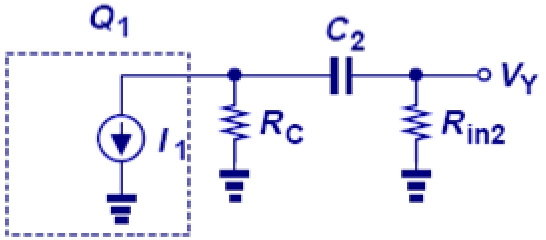
$$I_{C2} = 1.17\text{ mA},\ g_{m2} = \left(22.2\ \Omega\right)^{-1}$$

$$\Rightarrow r_{\pi 2} = 2.22\text{ k}\Omega$$

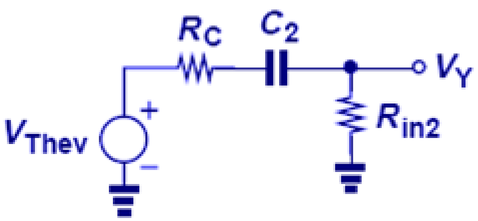
Example: Capacitive Coupling – cont'd

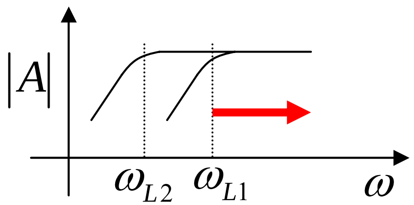


$$\omega_{L1} = \frac{1}{(r_{\pi 1} \| R_{B1}) C_1} = 2\pi \times (542\ \text{Hz})$$



$$R_{in2} = R_{B2} \| [r_{\pi 2} + (\beta + 1) R_E]$$

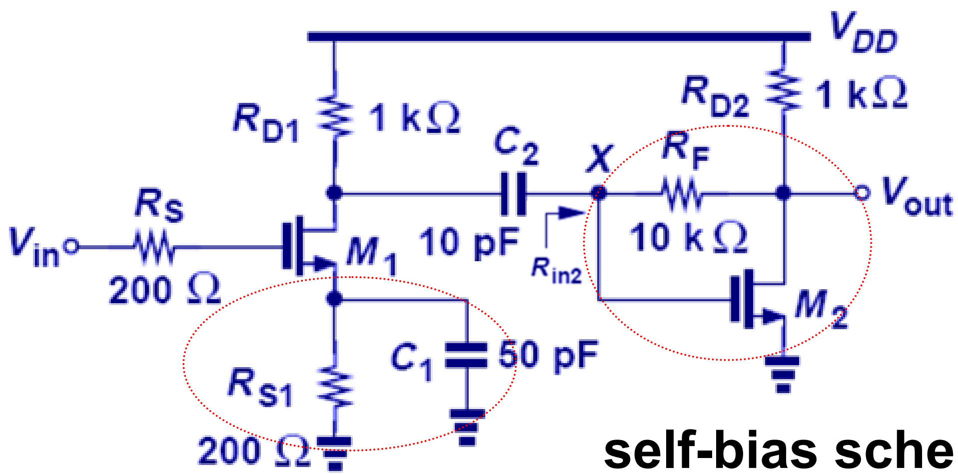




ω_{L1} dominates the low-frequency response

$$\omega_{L2} = \frac{1}{(R_C + R_{in2}) C_2} = \pi \times (22.9\ \text{Hz})$$

Example: IC Amplifier – Low Frequency Behavior



self-bias scheme

degenerated stage

$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$A_{v2} \approx -g_{m2} R_{D2} = -6.67$$

$$\Rightarrow R_{in2} = 1.30\ \text{k}\Omega$$

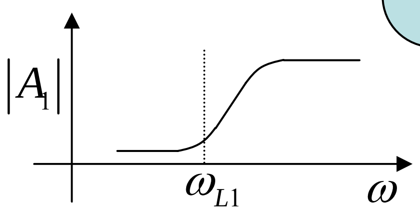
$$\omega_{L1} = \frac{1}{\left(R_{S1} \,\|\, \dfrac{1}{g_{m1}}\right) C_1}$$

$$= \frac{g_{m1} R_{S1} + 1}{R_{S1} C_1}$$

$$= 2\pi \times (42.4\ \text{MHz})$$

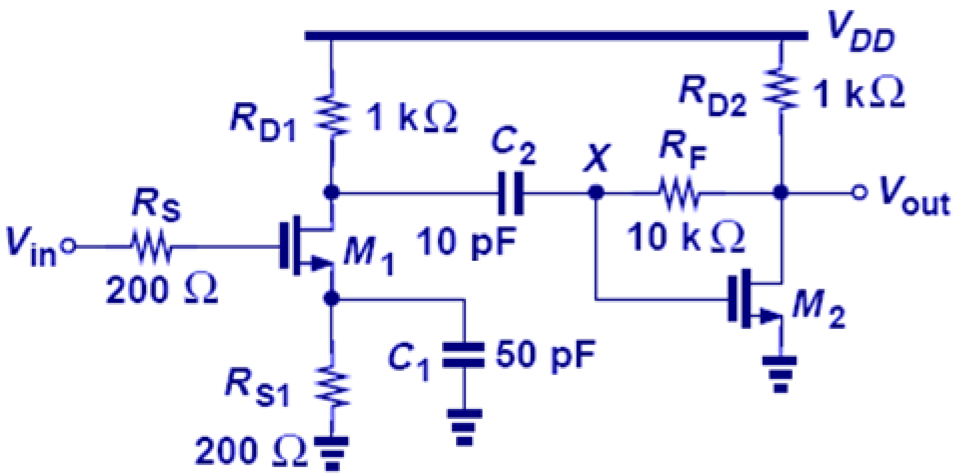
$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2}) C_2}$$

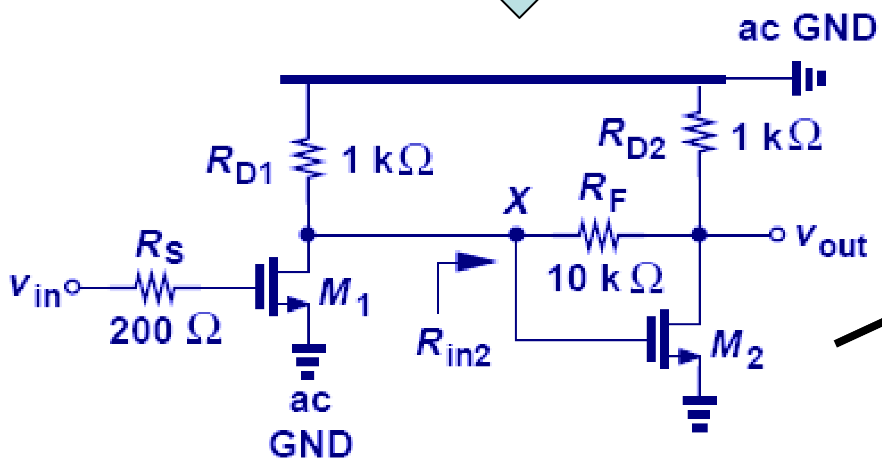
$$= 2\pi \times (6.92\ \text{MHz})$$



low-frequency cut-off at ω_{L1}

Example: IC Amplifier – Midband Behavior





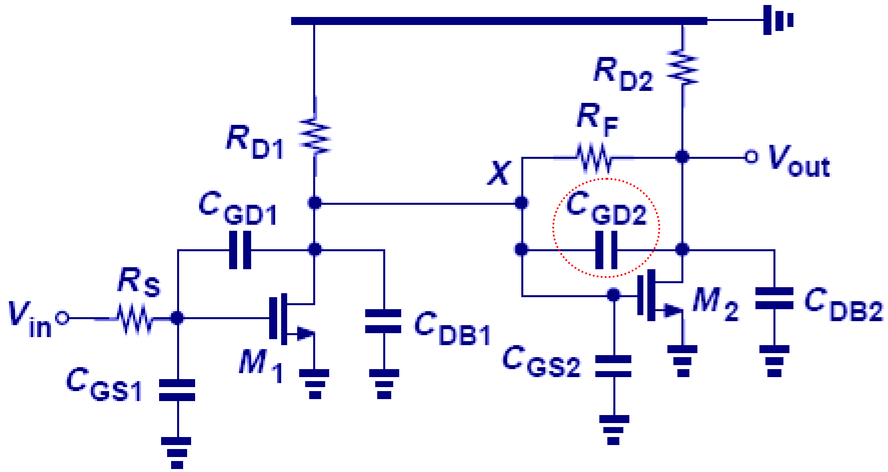
$$\frac{v_{out}}{v_{in}} = \left(\frac{v_X}{v_{in}}\right) \times \left(\frac{v_{out}}{v_X}\right)$$

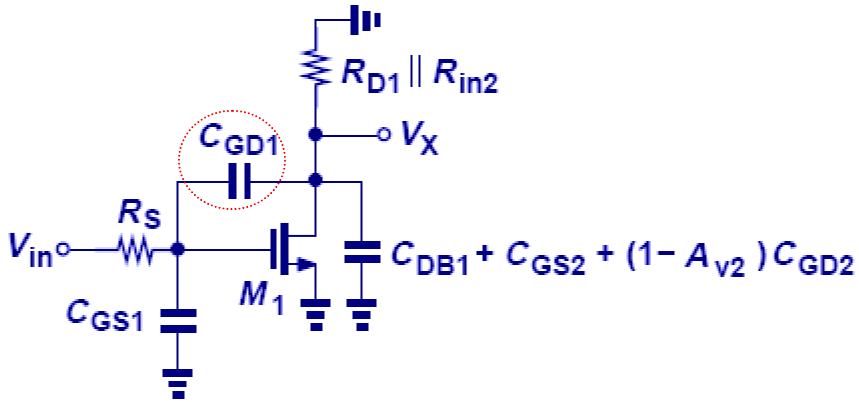
$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel R_{in2}\right) = -3.77$$

$$\frac{v_{out}}{v_X} \approx -g_{m2}R_{D2} = -6.67$$

$$\Rightarrow \frac{v_{out}}{v_{in}} \approx 25.1$$

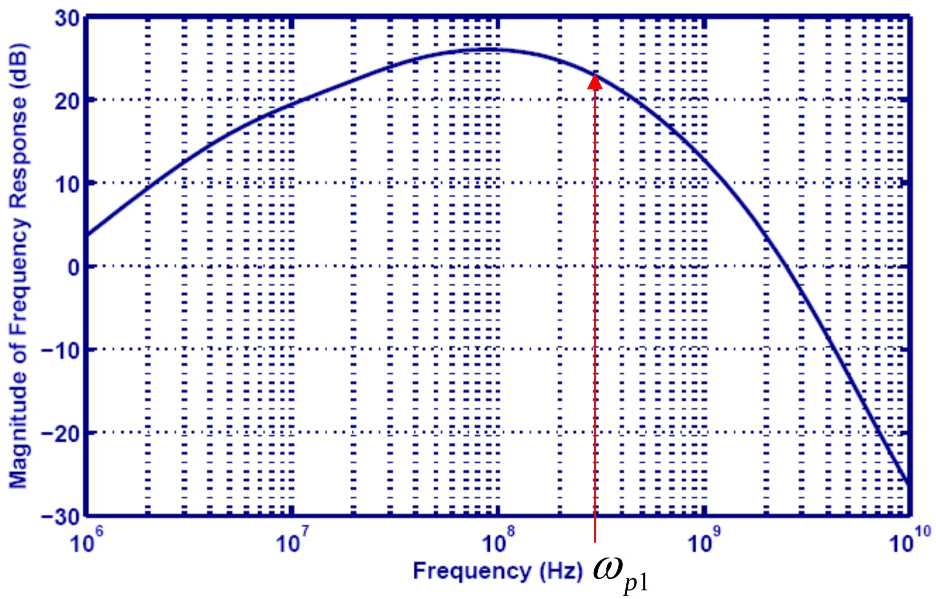
Example: IC Amplifier – High Frequency Behavior





$$\left|\omega_{p1}\right| = 2\pi \times (308\ \text{MHz})$$

$$\left|\omega_{p2}\right| = 2\pi \times (2.15\ \text{GHz})$$



With Miller effect,

$$\left(1 - A_{v2}^{-1}\right) C_{GD2} \approx 1.15 \cdot C_{GD2}$$

$$\left|\omega_{p3}\right| = \frac{1}{R_{L2}(1.15 \cdot C_{GD2} + C_{DB2})} = 2\pi \times (1.21\ \text{GHz})$$