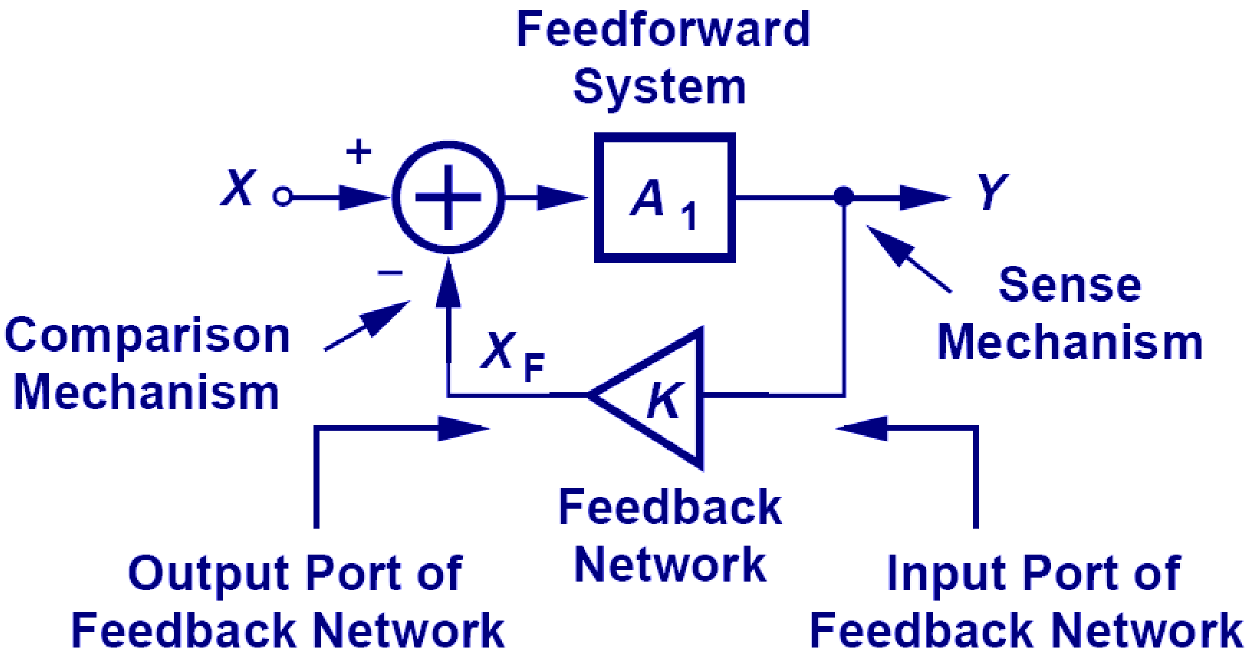
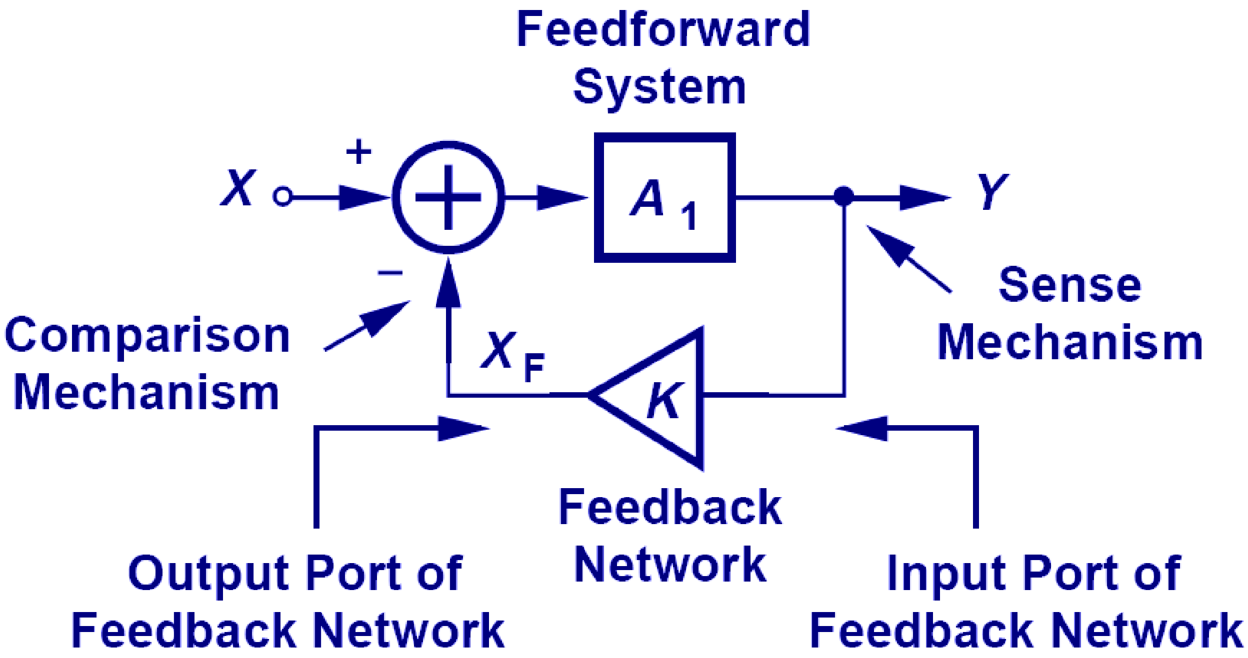
Chapter 12 Feedback

Negative Feedback System



- A negative feedback system consists of four components:
- 1) feedforward system
- 2) sense mechanism
- 3) feedback network
- 4) comparison mechanism

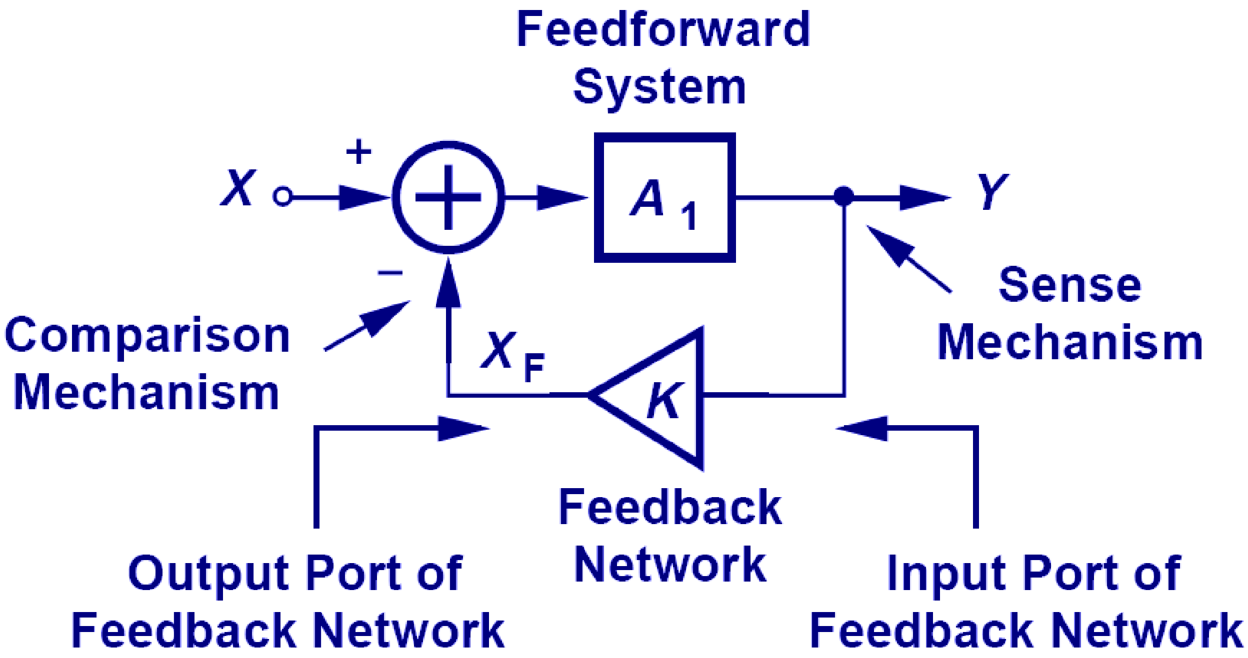
Close-loop Transfer Function



$$X_F = KY$$

$$\begin{aligned} Y &= A_1(X - X_F) \\ &= A_1(X - KY) \end{aligned}$$

Close-loop Transfer Function

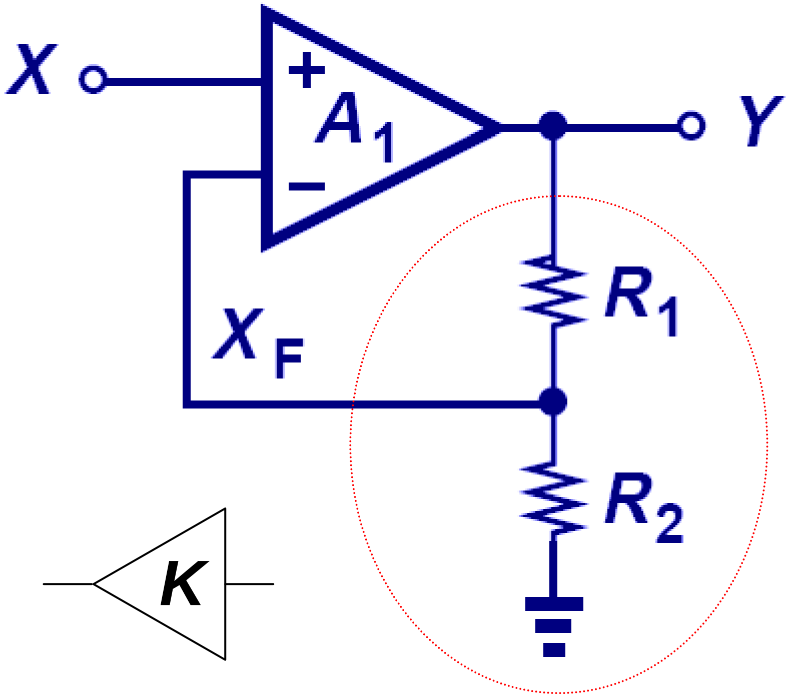


$$Y = A_1(X - KY)$$

$$Y + A_1 KY = A_1 X$$

$$\boxed{\frac{Y}{X} = \frac{A_1}{1 + KA_1}}$$

Example 12.1: Feedback

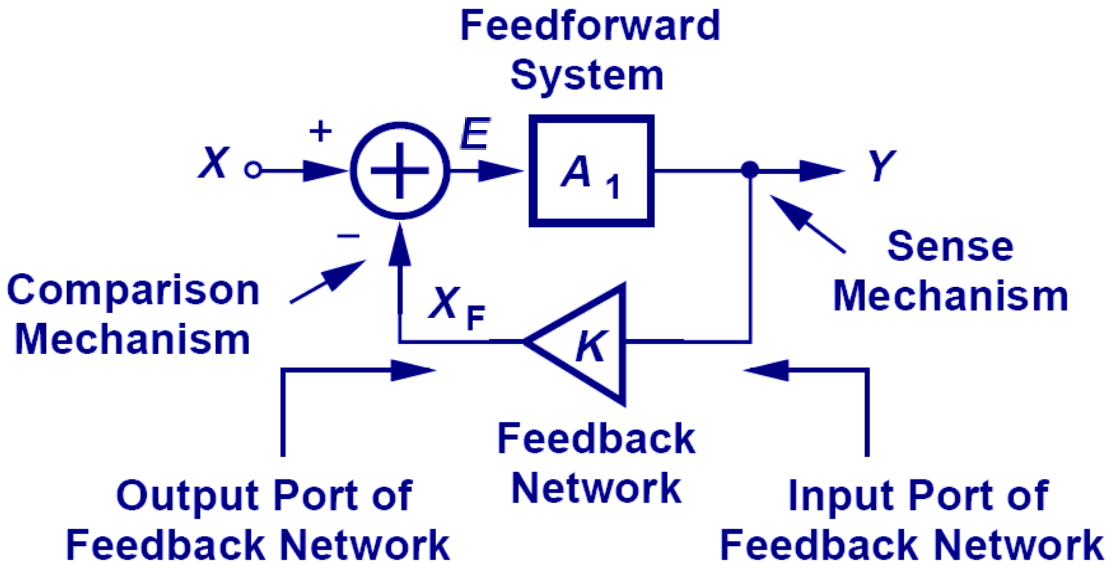


$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{Y}{X} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

- **A_1 is the feedforward network,**
  R_1 and R_2 provide the sensing and feedback capabilities, and comparison is provided by differential input of A_1.

Comparison Error



$$X_F = KY$$

$$\because Y = \frac{A_1}{1+KA_1}X$$

$$X_F = \frac{KA_1}{1+KA_1}X$$

$$E = X - X_F$$

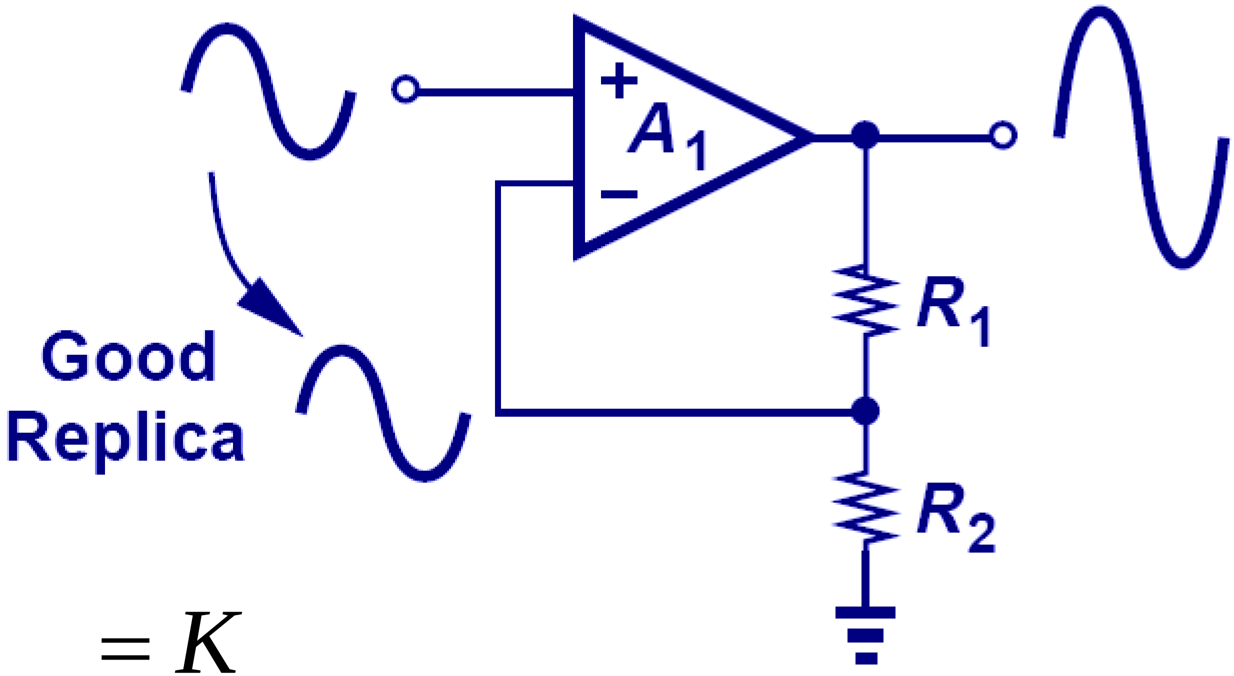
$$= \frac{X}{1+A_1K}$$

$$E \approx 0 \quad (\because A_1K \gg 1)$$

$$X \approx X_F \quad (\because E = X - X_F)$$

- As A_1K increases, the error between the input and feedback signal decreases. Or the feedback signal approaches a good replica of the input.

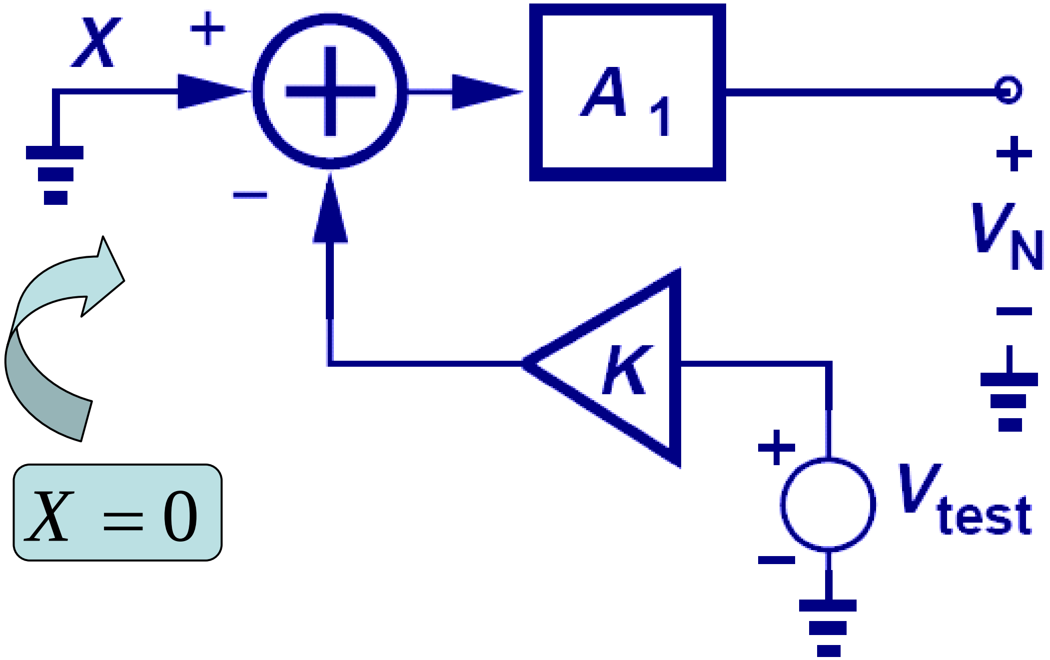
Comparison Error



$$= K$$

$$Y\frac{R_2}{R_1+R_2} = X_F \quad \because X \approx X_F \quad \frac{Y}{X} \approx 1 + \frac{R_1}{R_2} = \frac{1}{K}$$

Loop Gain

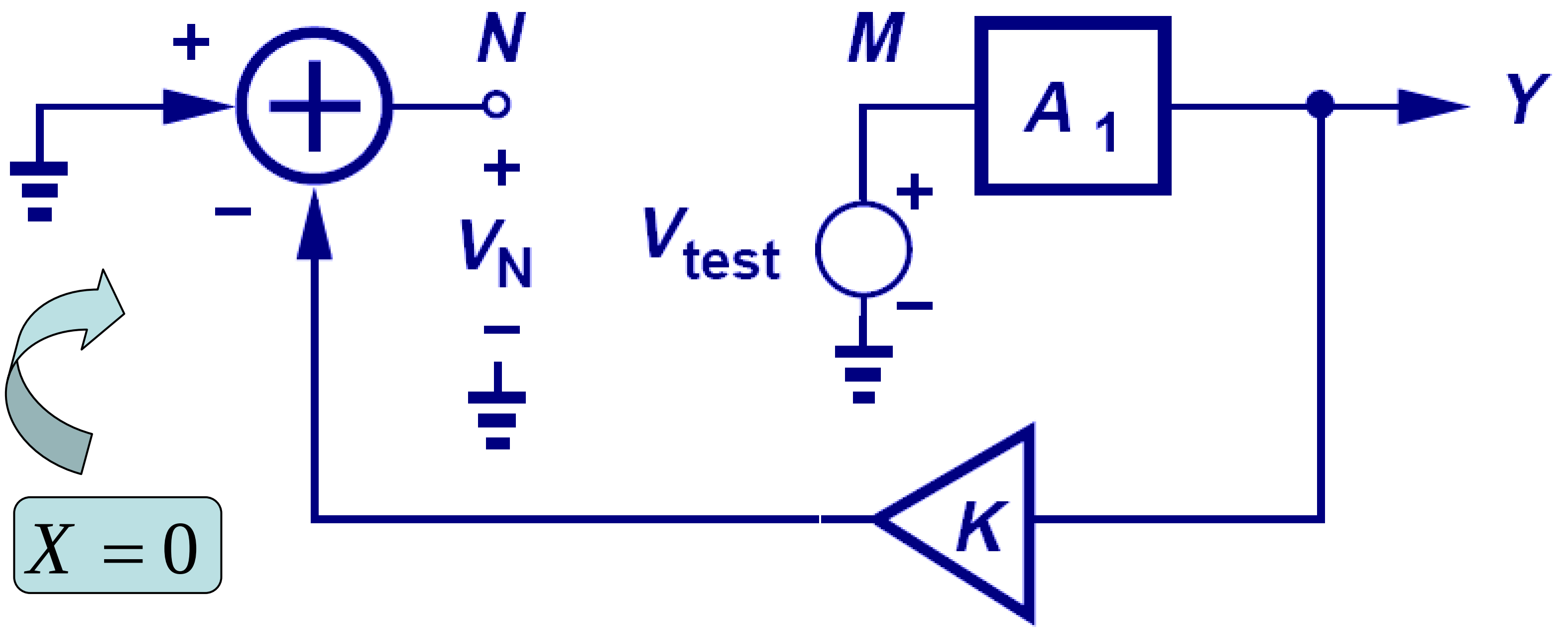


$$X = 0$$

$$KA_1 = -\frac{V_N}{V_{test}}$$

- **When the input is grounded, and the loop is broken at an arbitrary location, the loop gain is measured to be KA$_1$.**

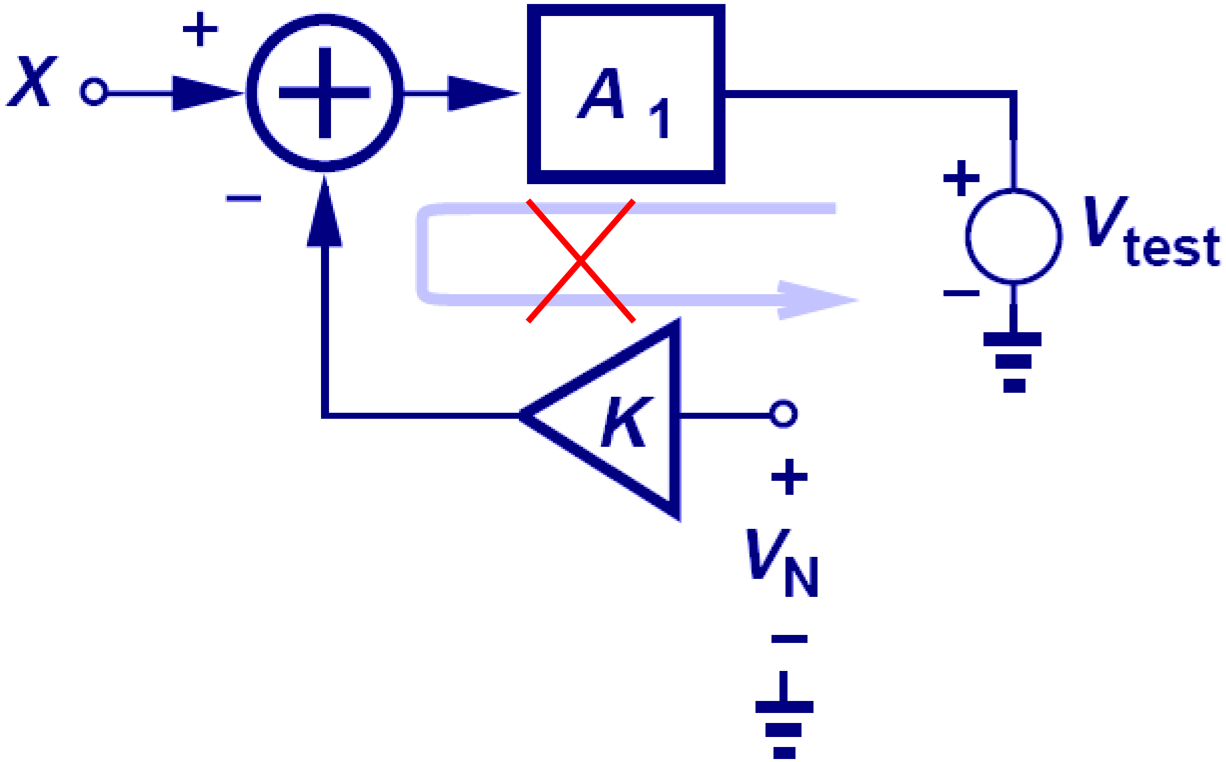
Example 12.3: Alternative Loop Gain Measurement



$X = 0$

$$V_N = -KA_1V_{test}$$

Incorrect Calculation of Loop Gain



- Signal naturally flows from the input to the output of a feedforward/feedback system. If we apply the input the other way around, the "output" signal we get is not a result of the loop gain, but due to poor isolation.

Gain Desensitization

$$E = X - X_F$$

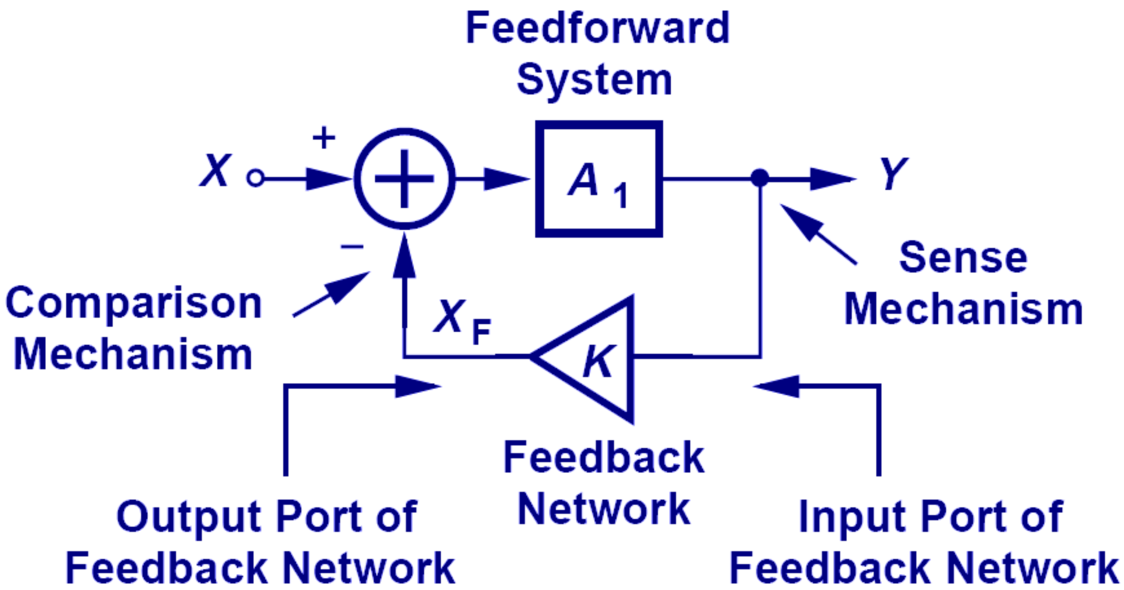
$$= \frac{X}{1 + A_1 K}$$

$$\text{if } A_1 K \gg 1$$

$$E \approx 0$$

$$\therefore X \approx X_F$$

$$X_F = KY \qquad \therefore \frac{Y}{X_F} \approx \frac{Y}{X}$$

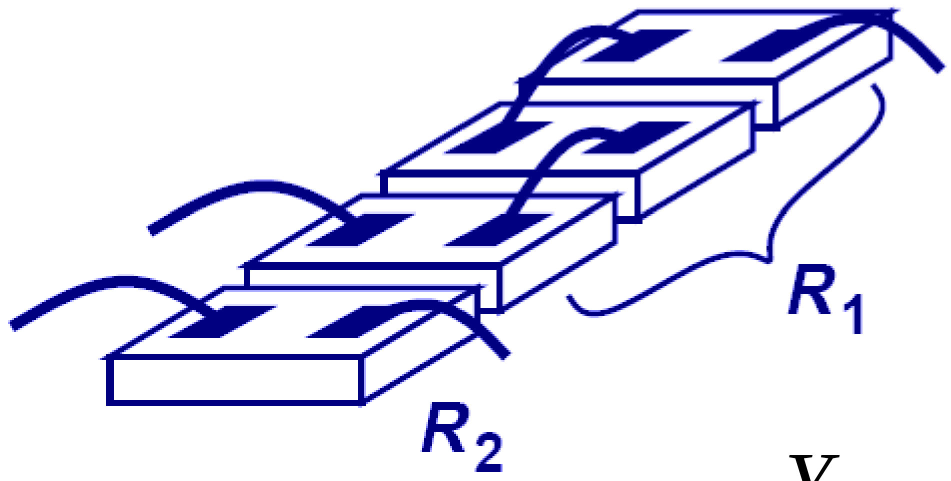


$$\boxed{A_1 K >> 1} \Rightarrow \boxed{\frac{Y}{X} \approx \frac{1}{K}}$$

ex: $g_m R_D$ $\quad g_m = \underline{\mu C_{ox}} \frac{W}{L} (V_{GS} - \underline{V_{TH}})$

- A large loop gain is needed to create a precise gain, one that does not depend on A_1, which can vary by ±20%.
- Fabrication process and temperature give some effect on the gain.

Ratio of Resistors

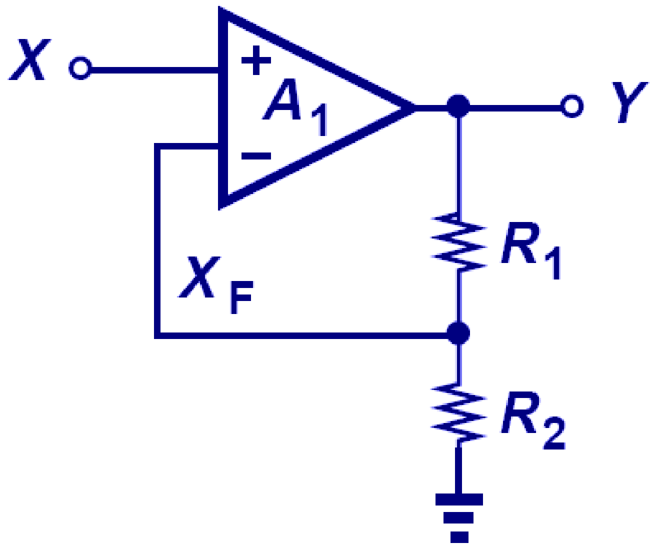


$$\frac{Y}{X} \approx 1 + \frac{R_1 \nearrow}{R_2 \nearrow}$$

- When two resistors are composed of the same unit resistor, their ratio is very accurate. Since when they vary, they will vary together and maintain a constant ratio.

Example 12.4: Gain Desensitization

- Determine the actual gain if A_1=1000. Determine the percentage change in the gain if A_1 drops to 500.



Nominal gain $\frac{1}{K} = 4$

$$\frac{Y}{X} = \frac{A_1}{1 + A_1 K}$$

$$\frac{Y}{X} = 3.984 \ (A_1 = 1000)$$

$$\frac{Y}{X} = 3.968 \ (A_1 = 500)$$

-0.4% drop

Merits of Negative Feedback

- 1) **Bandwidth enhancement**
- 2) **Modification of I/O Impedances**
- 3) **Linearization**

$$\frac{Y}{X} = \frac{A_1 \searrow}{1 + KA_1 \searrow}$$

Bandwidth Enhancement

Open Loop

$$A_1(s) = \frac{A_0}{1+\frac{s}{\omega_0}}$$

ω_0 : -3 dB BW

Negative Feedback

$$\because \frac{Y}{X} = \frac{A_1}{1+KA_1}$$

Closed Loop

gain

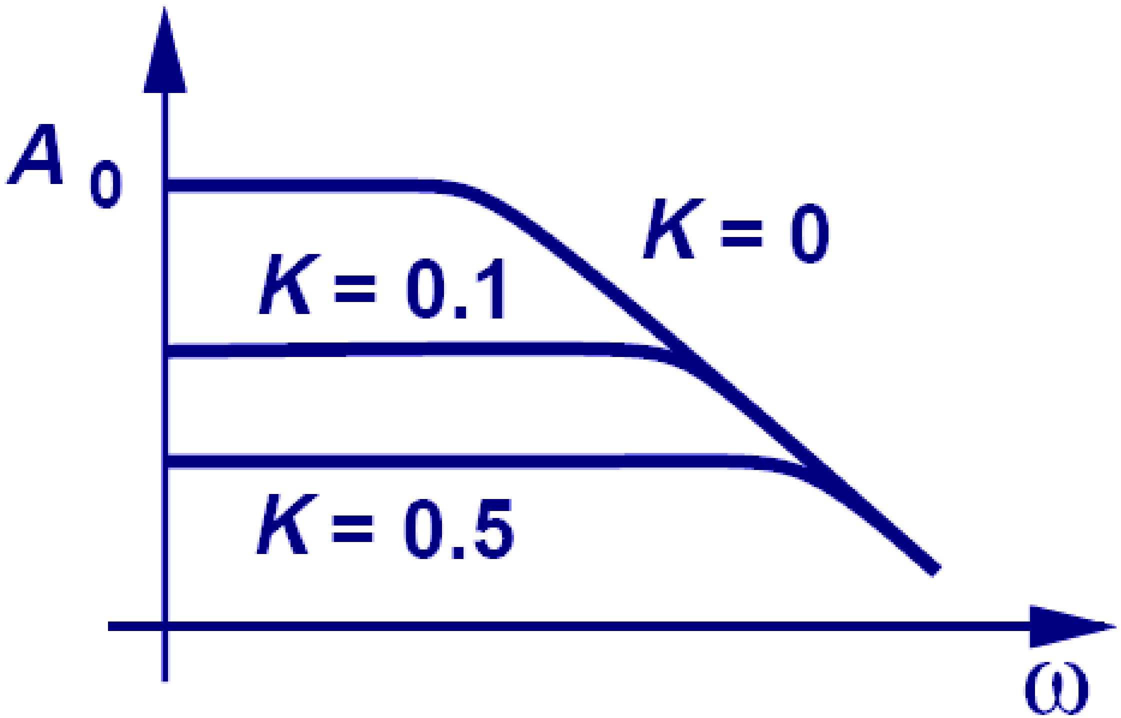
$$\frac{Y}{X}(s) = \frac{\frac{A_0}{1+KA_0}}{1+\frac{s}{(1+KA_0)\omega_0}}$$

bandwidth

$$\because \frac{Y}{X} = \frac{\frac{A_0}{1+s/\omega_0}}{1+K\frac{A_0}{1+s/\omega_0}} = \frac{A_0}{1+KA_0+s/\omega_0} = \frac{A_0/1+KA_0}{1+s/\omega_0(1+KA_0)}$$

- **Although negative feedback lowers the gain by $(1+KA_0)$, it also extends the bandwidth by the same amount.**

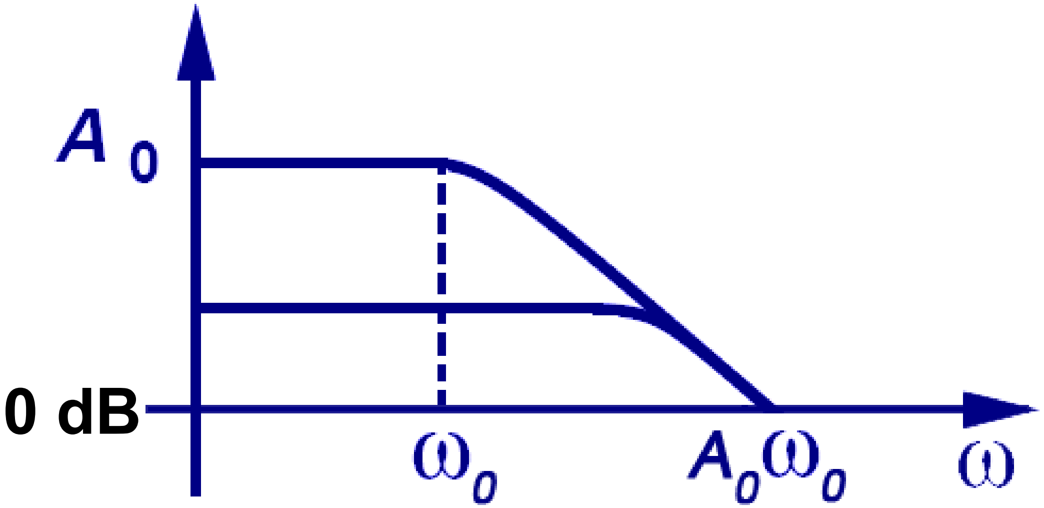
Bandwidth Extension Example



- **As the loop gain increases, we can see the decrease of the overall gain and the extension of the bandwidth.**

Example 12.6: Unity-gain bandwidth

$$\left|\frac{Y}{X}(j\omega)\right| = \frac{A_0 / (1+KA_0)}{\sqrt{1+\omega^2 / (1+KA_0)^2\omega_0^2}} \qquad 1 = \frac{A_0 / (1+KA_0)}{\sqrt{1+\omega_u^2 / (1+KA_0)^2\omega_0^2}}$$



$$\omega_u = \omega_0\sqrt{A_0^2-(1+KA_0)^2}$$

$$\longleftarrow 1 \ll KA_0$$

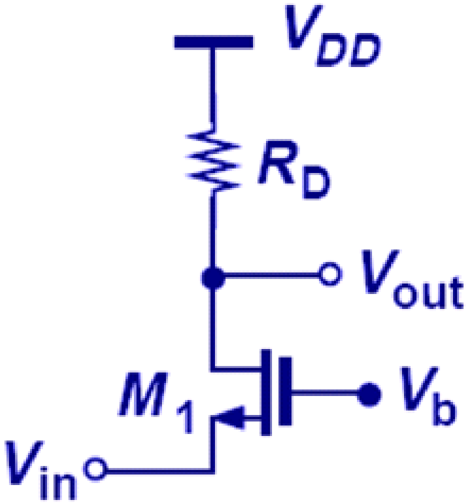
$$\approx \omega_0\sqrt{A_0^2-K^2A_0^2}$$

$$\longleftarrow K^2 \ll 1$$

$$\approx \omega_0 A_0$$

- **We can see the unity-gain bandwidth remains independent of K, if $KA_0 \gg 1$ and $K^2 \ll 1$**

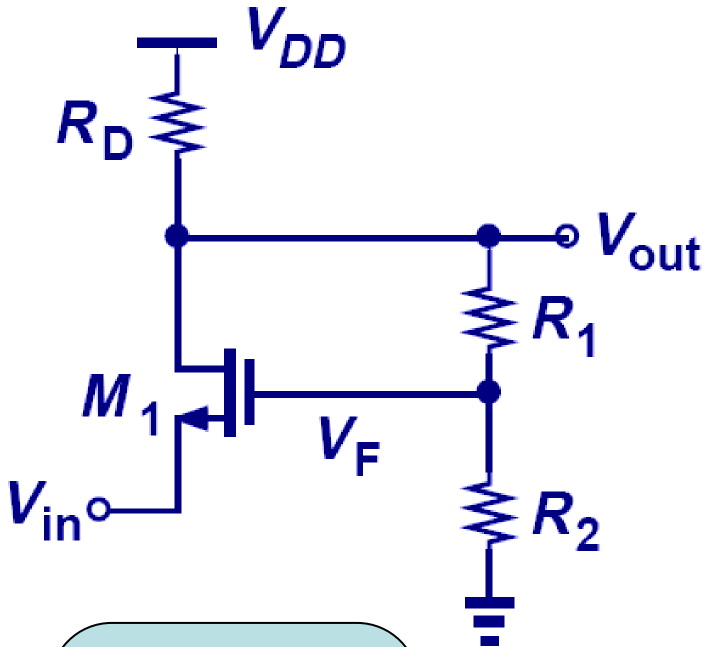
Example12.7: Open Loop Parameters



$$A_0 \approx g_m R_D$$

$$R_{in} = \frac{1}{g_m}$$

$$R_{out} = R_D$$

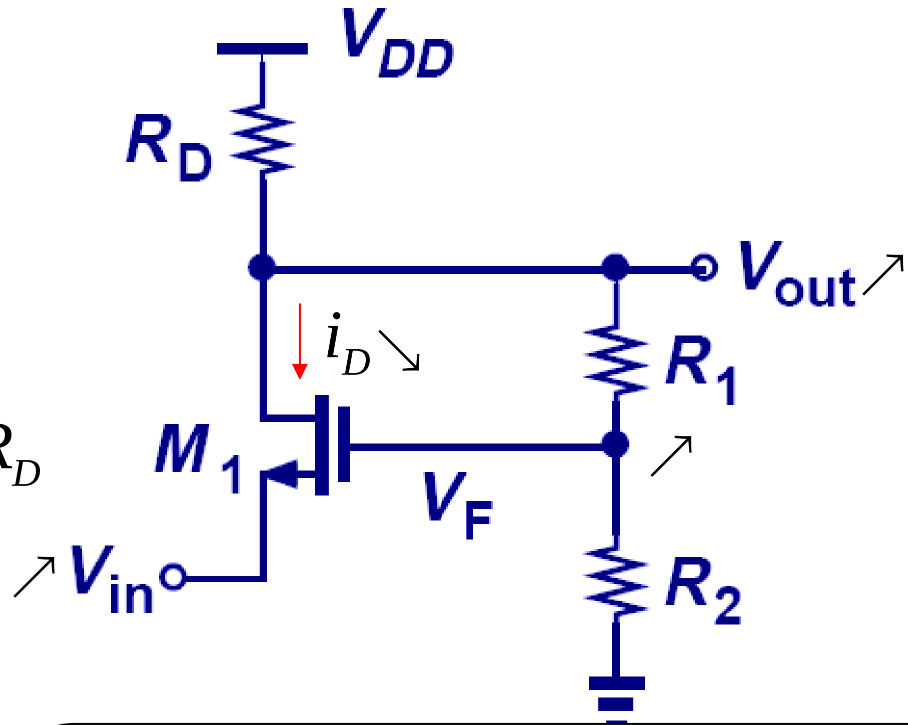


$$A_0 = ?$$

$$R_{in} = ?$$

$$R_{out} = ?$$

Example12.7: Closed Loop Voltage Gain



$$i_D = g_m(v_G - v_S)$$

$$A_0 \approx g_m R_D$$

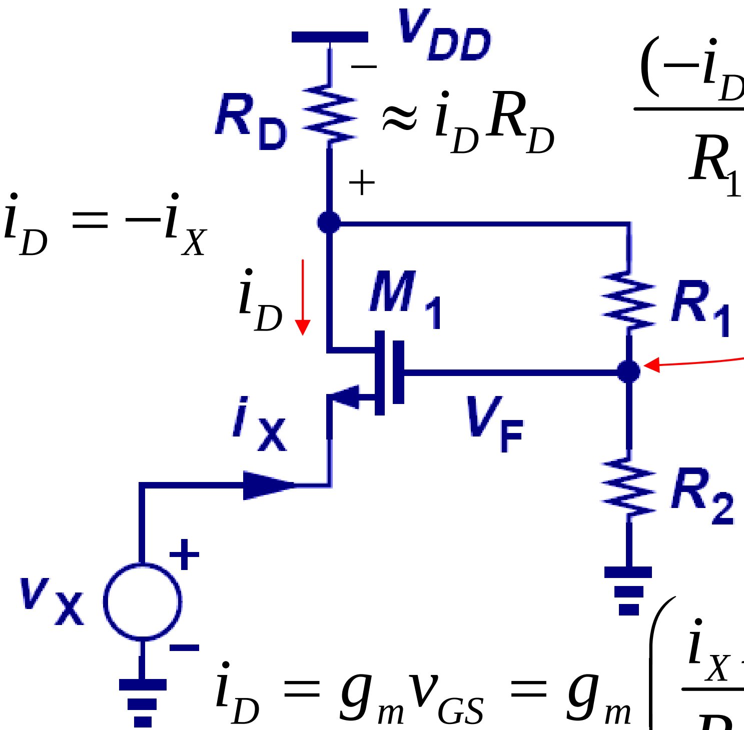
because of $R_1 + R_2 >> R_D$

$$\frac{v_{out}}{v_{in}} = \frac{A_0}{1 + KA_0}$$

Assuming $R_1 + R_2 >> R_D$,

$$\frac{v_{out}}{v_{in}} = \frac{g_m R_D}{1 + \frac{R_2}{R_1 + R_2} g_m R_D}$$

Example12.7: Closed Loop I/O Impedance

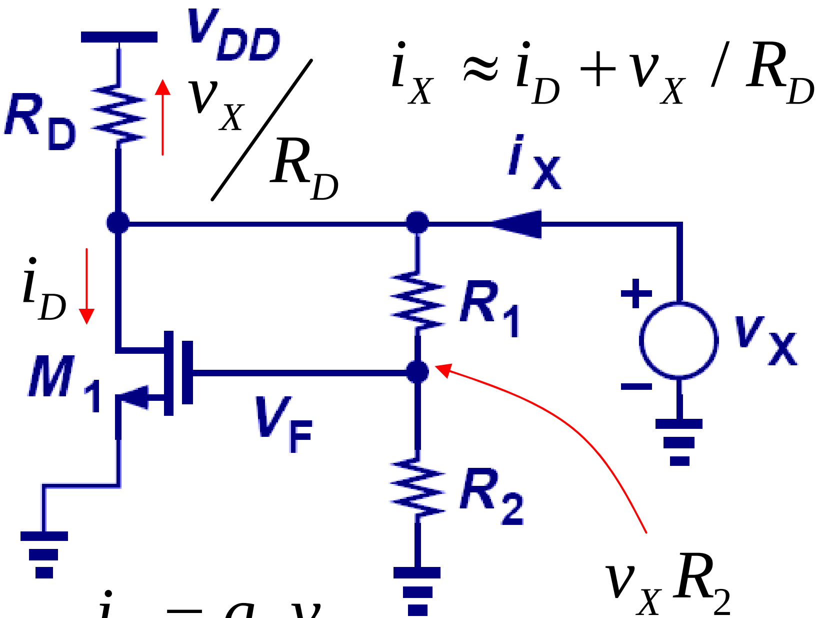


$$\approx i_D R_D$$

$$\frac{(-i_D R_D)R_2}{R_1+R_2}$$

$$i_D = -i_X$$

$$i_D = g_m v_{GS} = g_m\left(\frac{i_X R_D R_2}{R_1+R_2} - v_X\right)$$



$$i_X \approx i_D + v_X / R_D$$

$$v_X / R_D$$

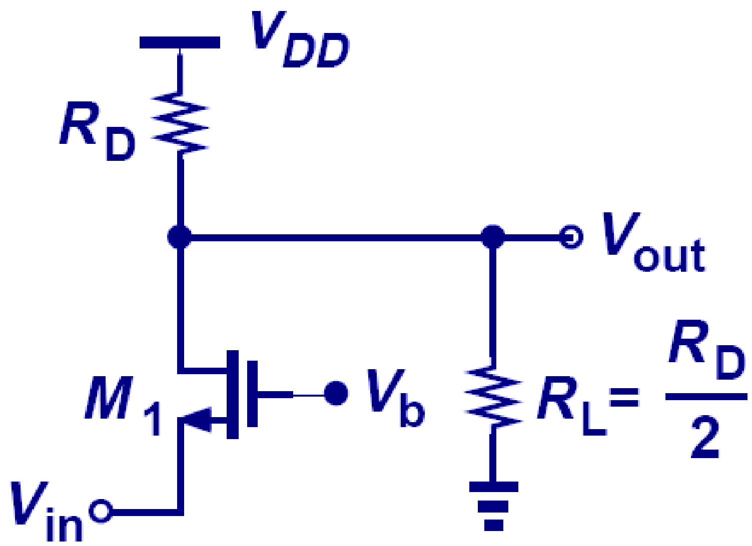
$$i_D = g_m v_{GS}$$

$$\frac{v_X R_2}{R_1+R_2}$$

$$R_{in} = \frac{1}{g_m}\left(1+\frac{R_2}{R_1+R_2} g_m R_D\right)$$

$$R_{out} = \frac{R_D}{1+\frac{R_2}{R_1+R_2} g_m R_D}$$

Example: Load Desensitization



$$\frac{g_m(R_D/3)}{1+\frac{R_2}{R_1+R_2}g_m(R_D/3)}$$

W/O Feedback
Large Difference

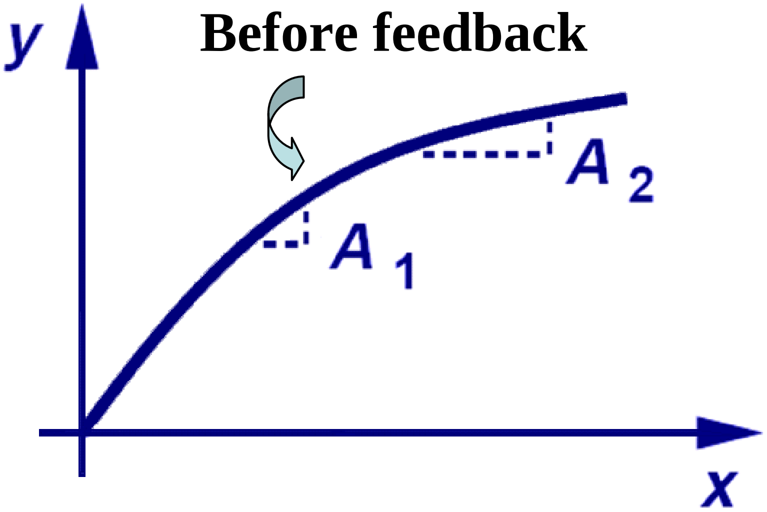
$$g_m R_D \rightarrow g_m R_D/3$$

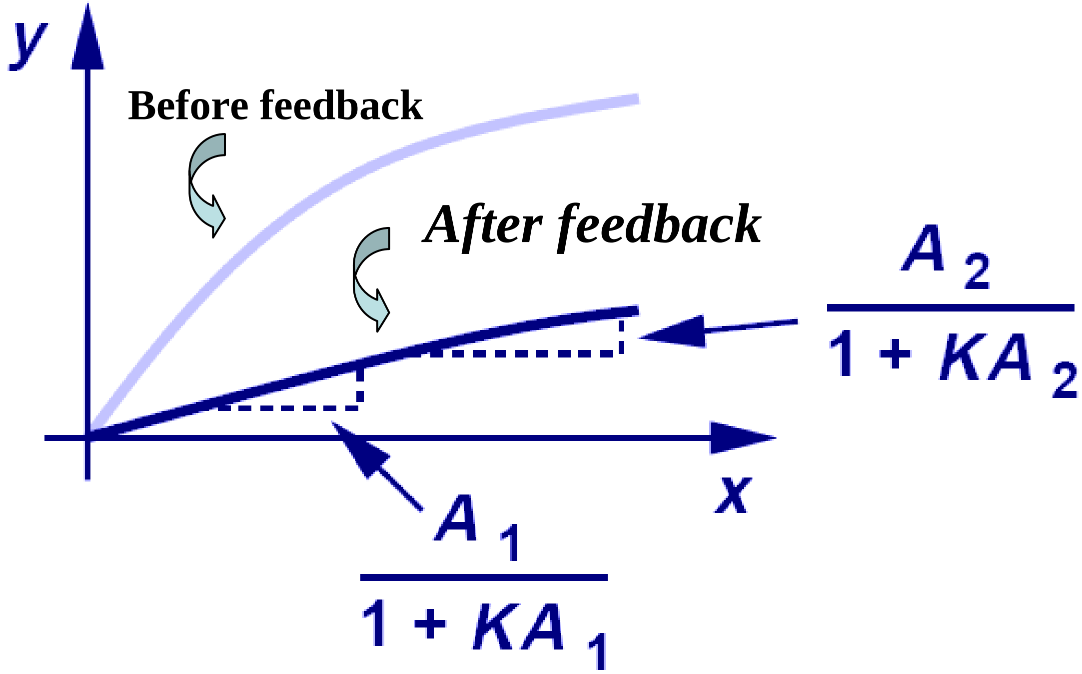
drops by factor of three

With Feedback
Small Difference

$$\frac{g_m R_D}{1+\frac{R_2}{R_1+R_2}g_m R_D} \rightarrow \frac{g_m R_D}{3+\frac{R_2}{R_1+R_2}g_m R_D}$$

Linearization



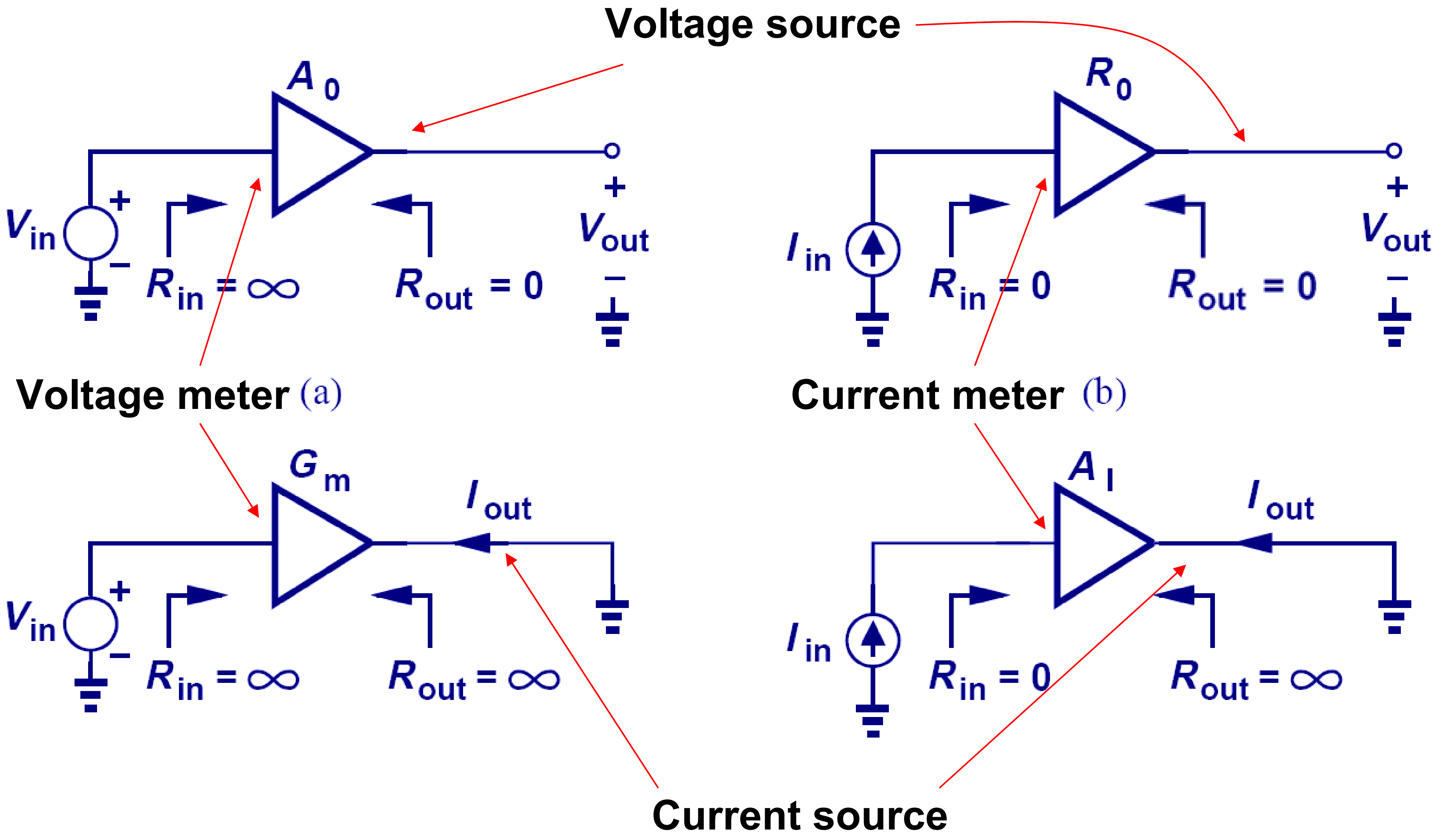


$$\text{Gain at } x_1 = \frac{A_1}{1+KA_1}$$

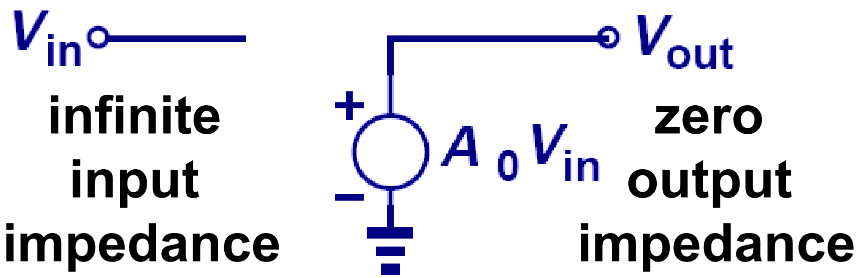
$$\frac{1/K}{1/KA_1+1} = \frac{1}{K}\frac{1}{(1+1/KA_1)} = \frac{1}{K}\left(1+\frac{1}{KA_1}\right)^{-1} \approx \frac{1}{K}\left(1-\frac{1}{KA_1}\right)$$

$$\text{Gain at } x_2 = \frac{A_2}{1+KA_2} \approx \frac{1}{K}\left(1-\frac{1}{KA_2}\right)$$

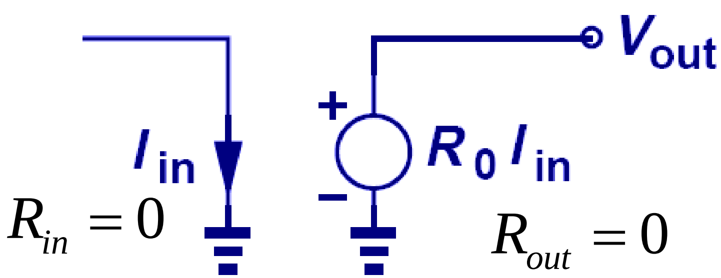
Four Types of Amplifiers



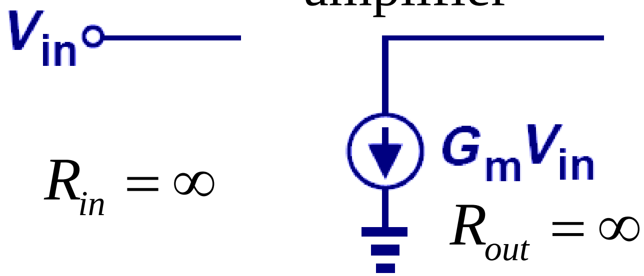
Ideal Models of the Four Amplifier Types



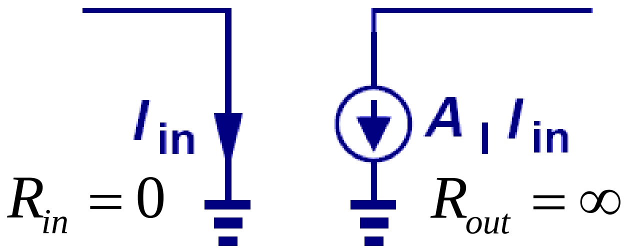
(a) Voltage amplifier



(b) Transresistance amplifier

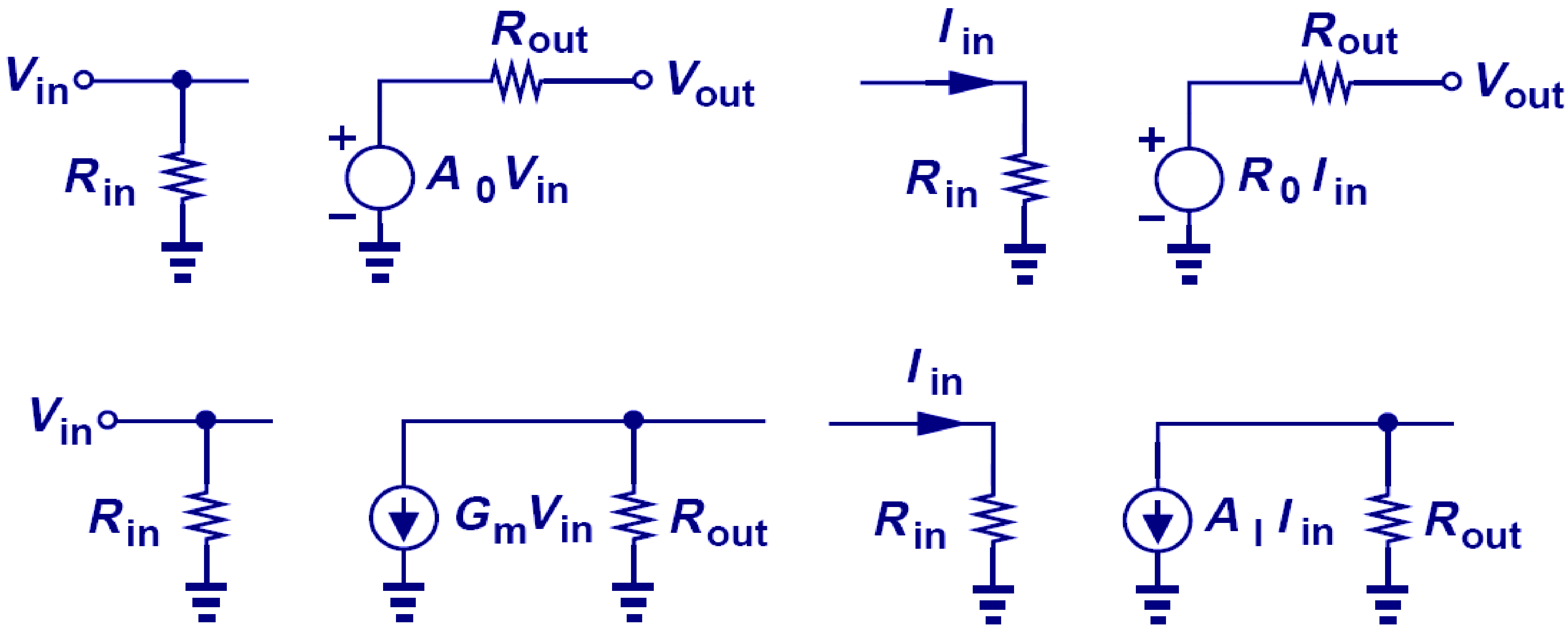


(c) Transconductance amplifier

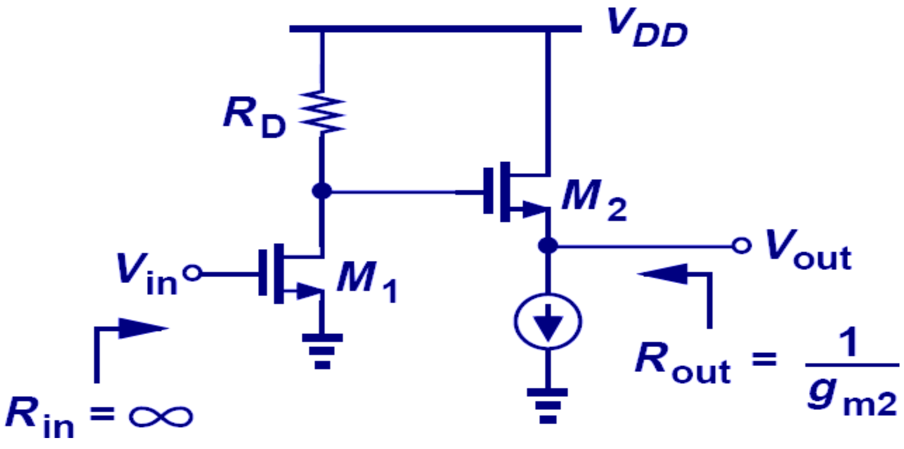


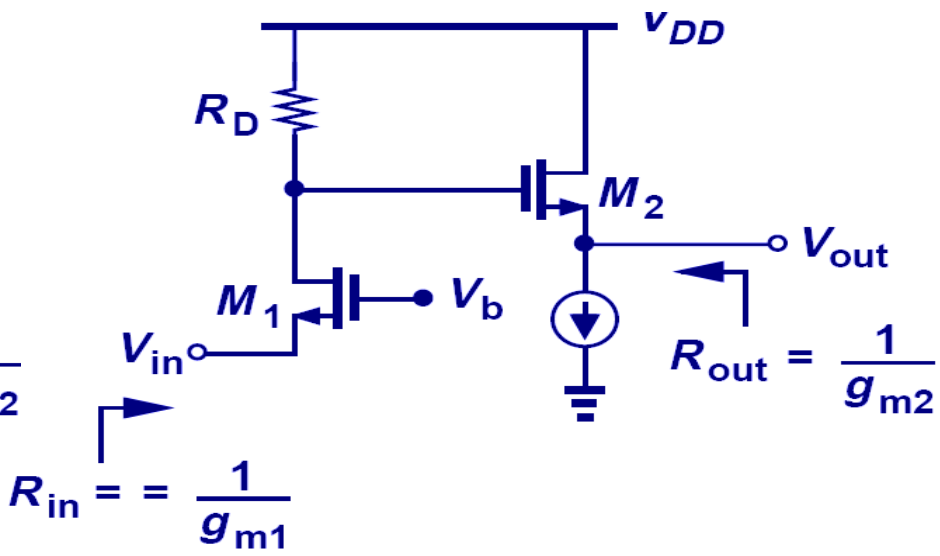
(d) Current amplifier

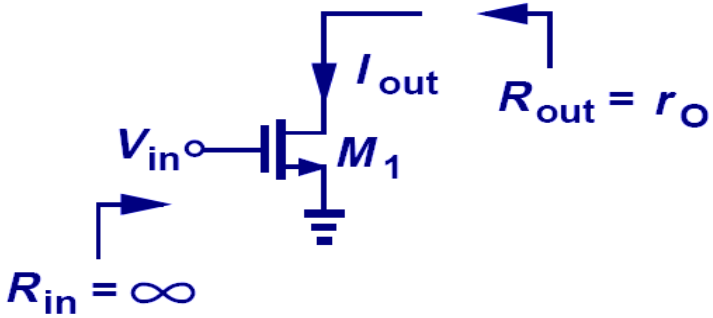
<u>Realistic</u> Models of the Four Amplifier Types

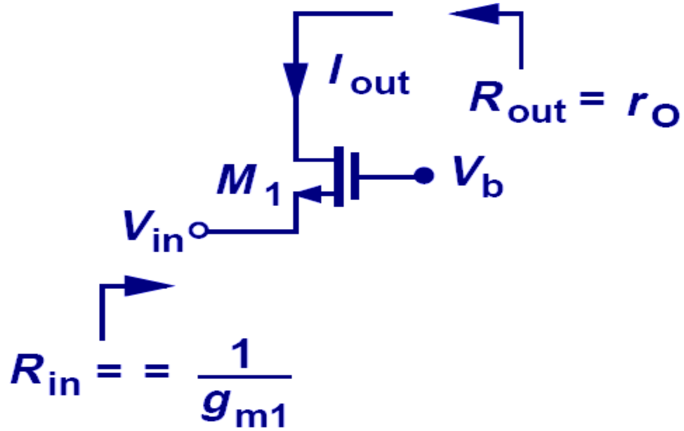


Examples of the Four Amplifier Types

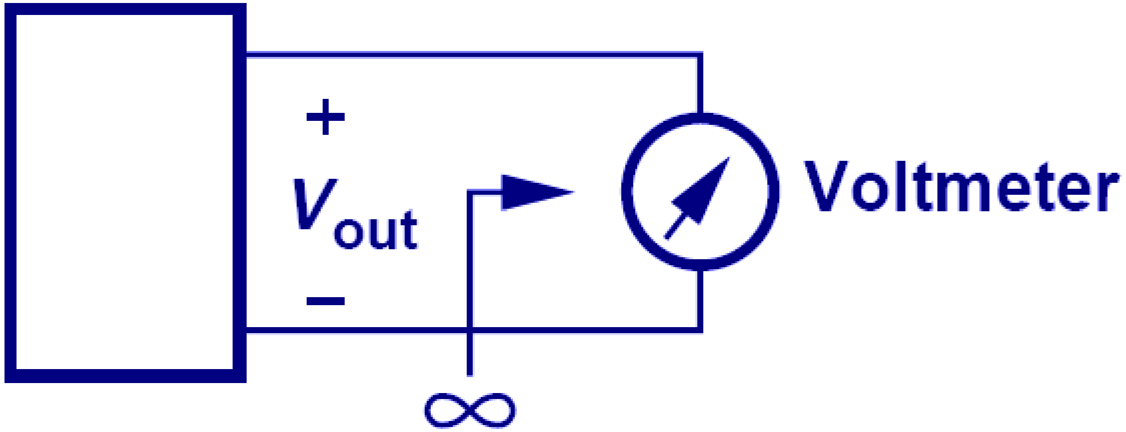






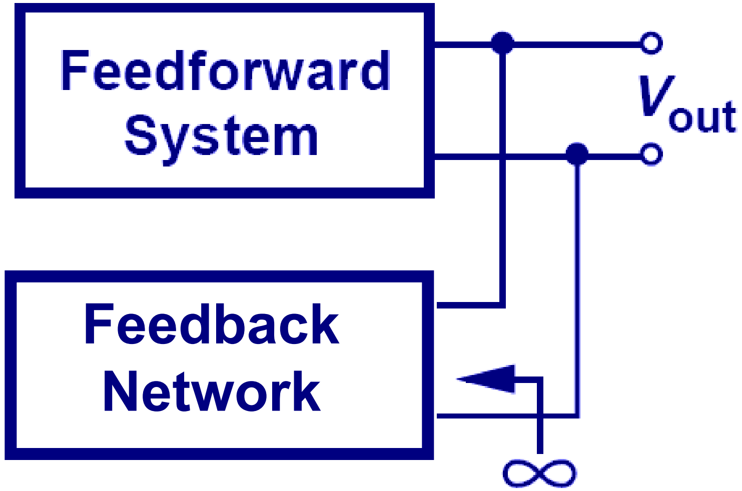


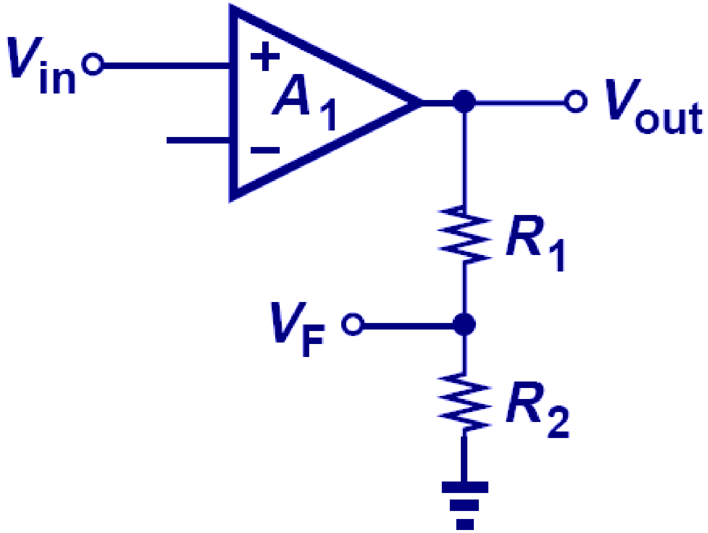
Sensing a Voltage



➢ **In order to sense a voltage across two terminals, a voltmeter with ideally infinite impedance is used.**

Sensing and Returning a Voltage

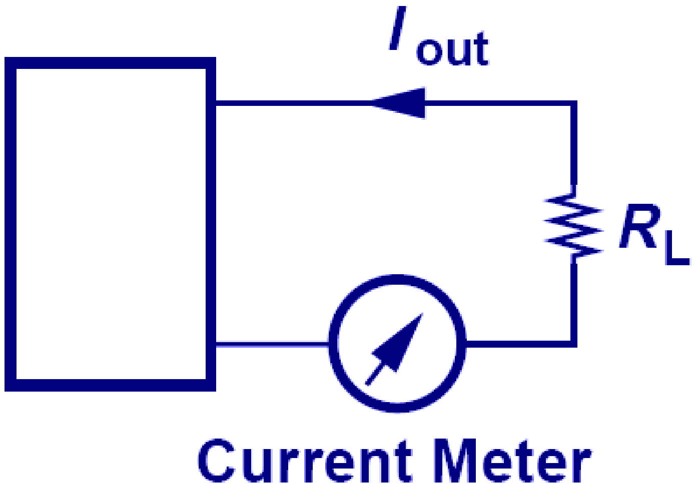


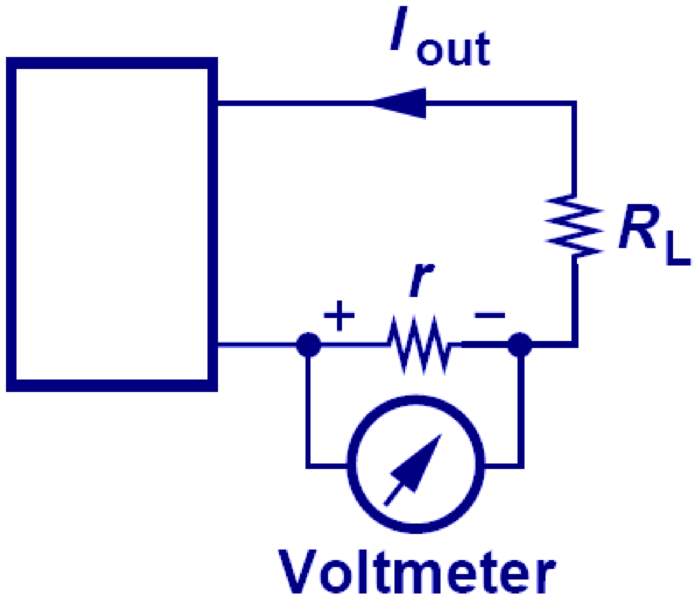


$$R_1 + R_2 \approx \infty$$

- **Similarly, for a feedback network to correctly sense the output voltage, its input impedance needs to be large.**
- **R_1 and R_2 also provide a means to return the voltage.**

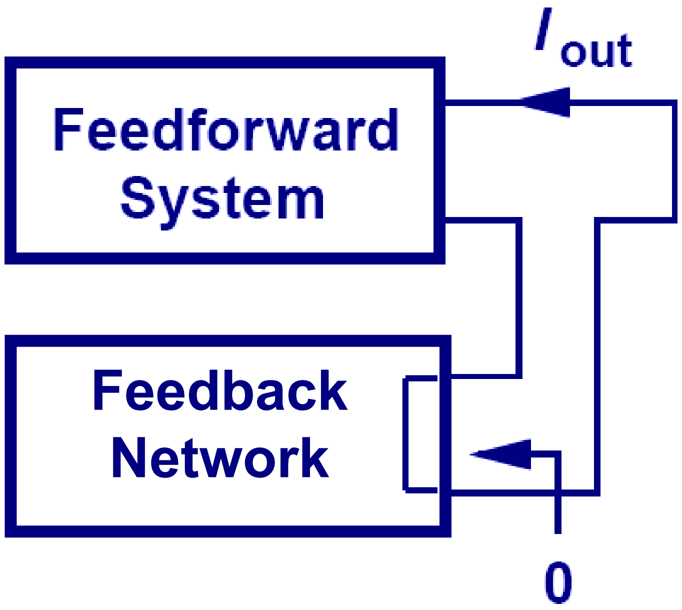
Sensing a Current

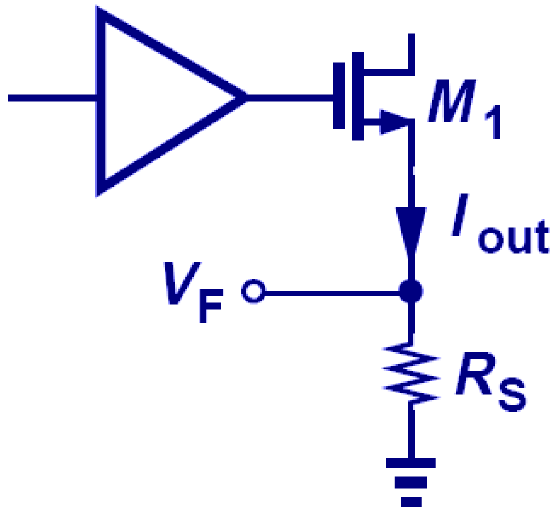




- A current is measured by inserting a current meter with ideally zero impedance in series with the conduction path.
- The current meter is composed of a small resistance r in parallel with a voltmeter.

Sensing and Returning a Current

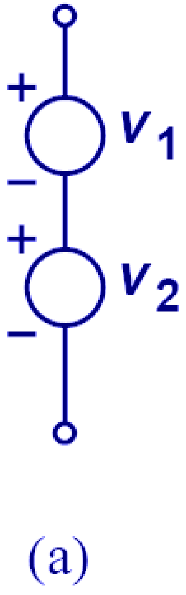




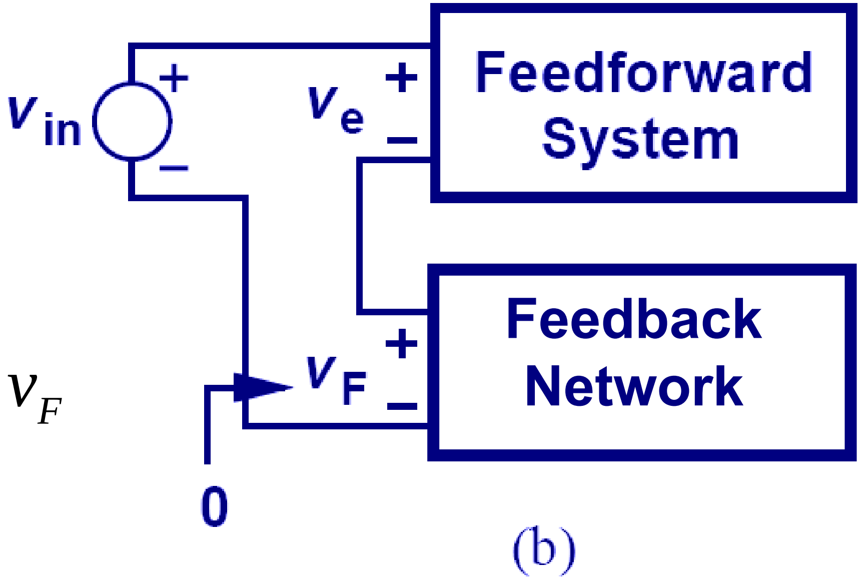
$$R_S \approx 0$$

- Similarly for a feedback network to correctly sense the current, its input impedance has to be small.
- R_S has to be small so that its voltage drop will not change I_{out}.

Addition of Two Voltage Sources

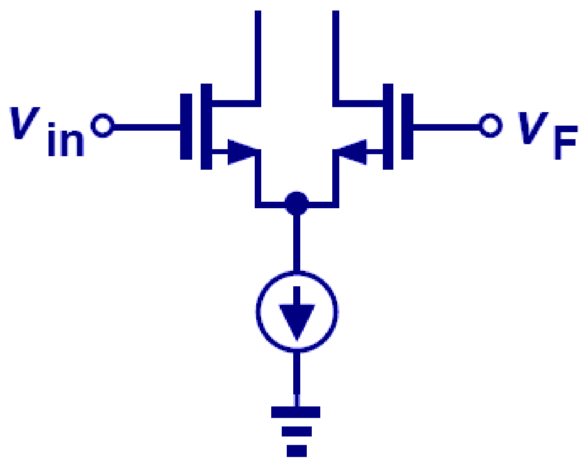


$$v_e = v_{in} - v_F$$

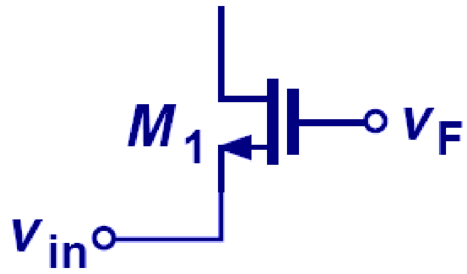


- **In order to add or substrate two voltage sources, we place them in series. So the feedback network is placed in series with the input source.**

Practical Circuits to Subtract Two Voltage Sources



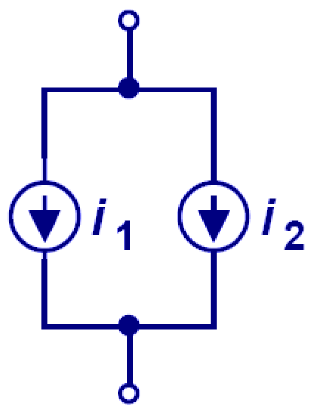
(c)



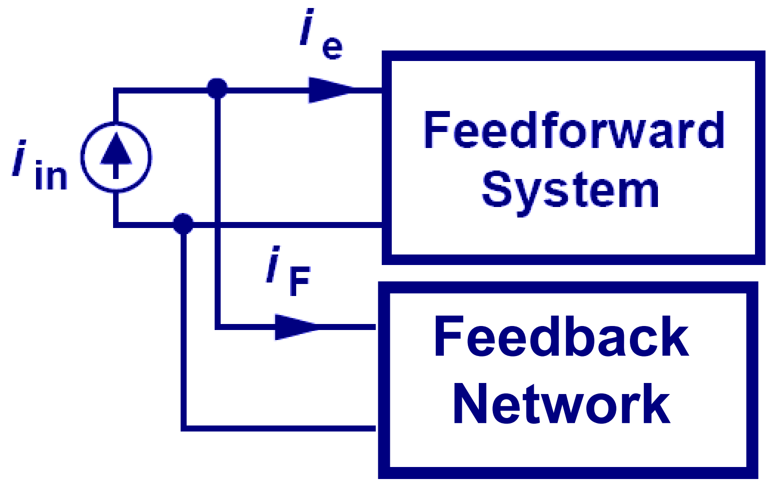
(d)

- **Although not directly in series, V_{in} and V_F are being subtracted since the resultant currents, differential and single-ended, are proportional to the difference of V_{in} and V_F.**

Addition of Two Current Sources



(a)

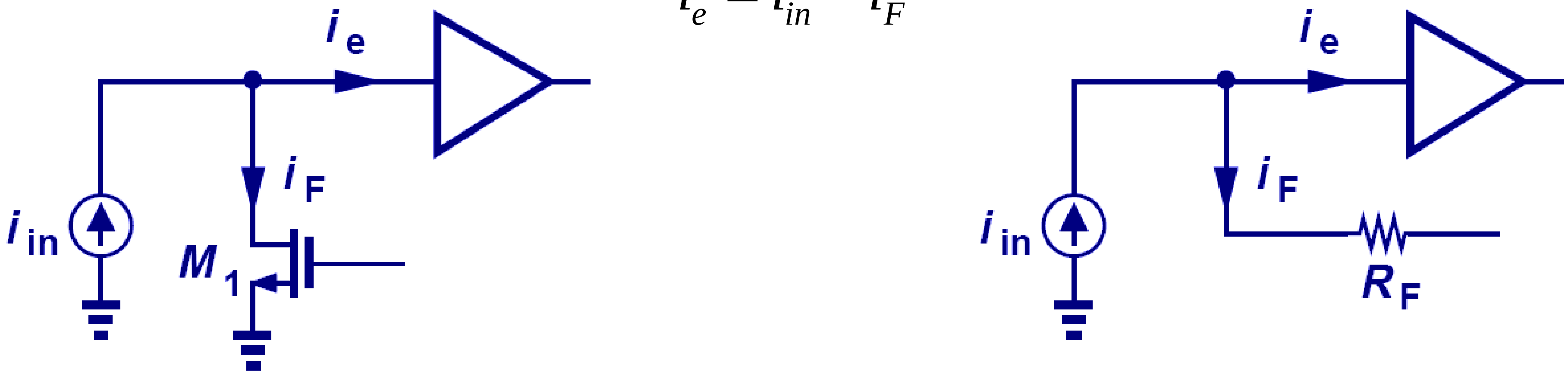


(b)

- In order to add two current sources, we place them in parallel. So the feedback network is placed in parallel with the input signal.

Practical Circuits to Subtract Two Current Sources

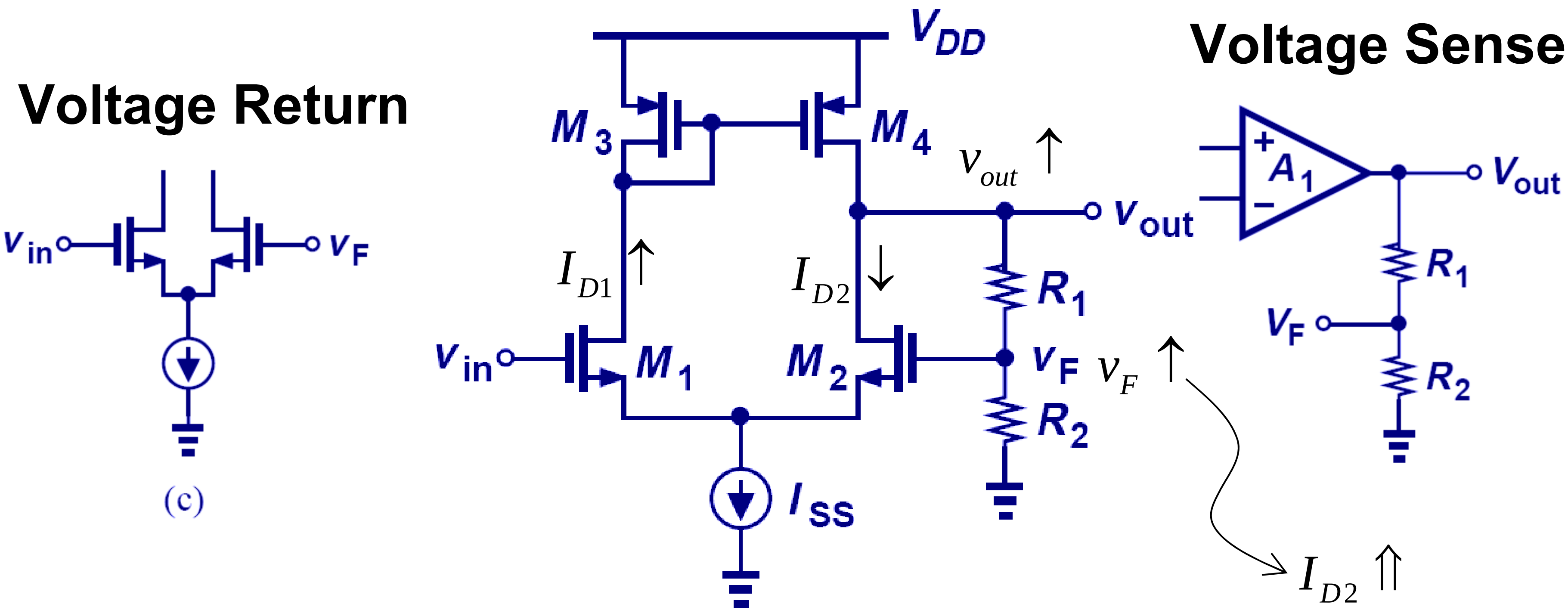
$$i_e = i_{in} - i_F$$



(c) (d)

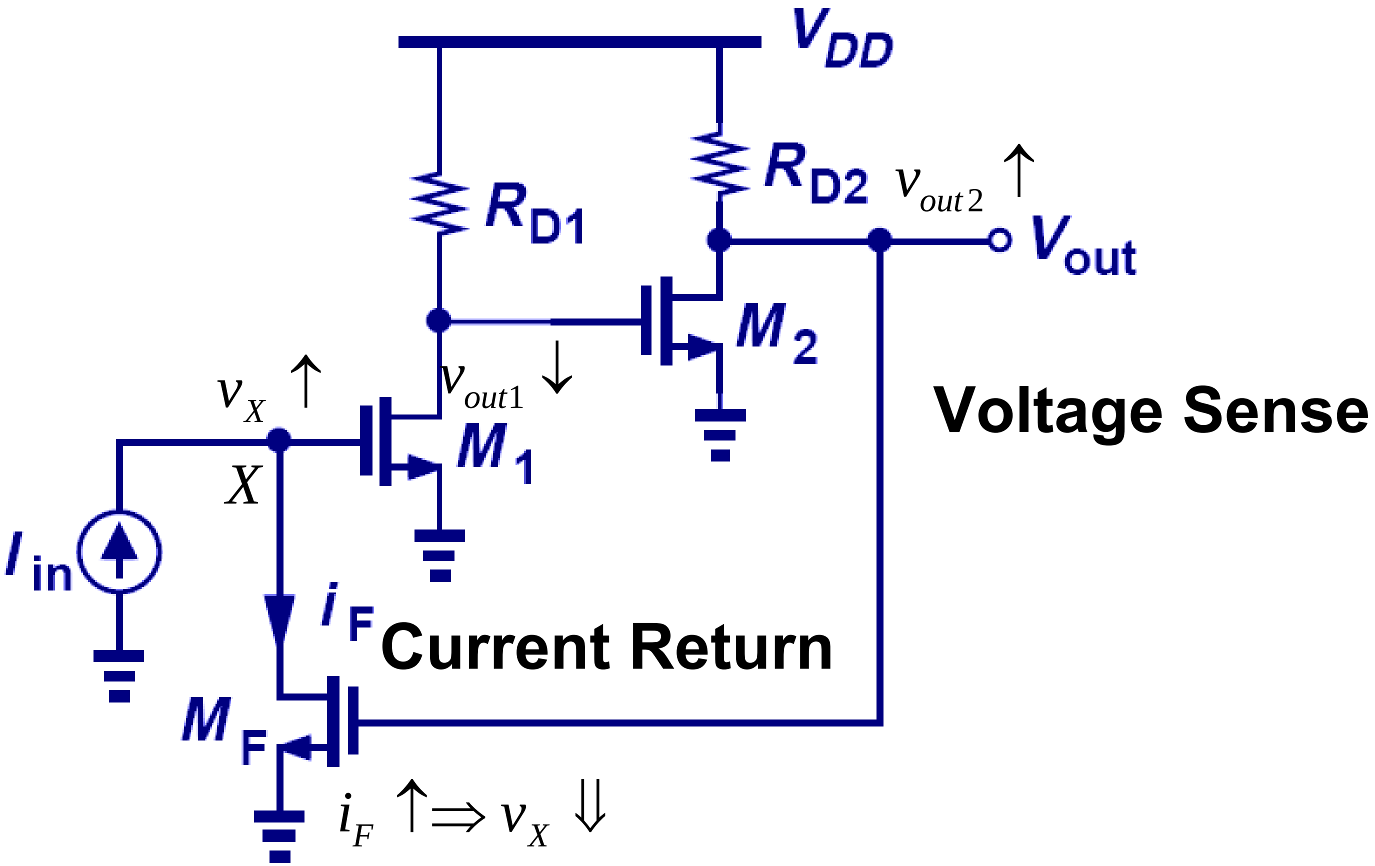
- Since M_1 and R_F are in parallel with the input current source, their respective currents are being subtracted. Note, R_F has to be large enough to approximate a current source.

Example 12.10: Sense and Return



- R_1 and R_2 sense and serve as the feedback network.
- M_1 and M_2 are part of the op-amp and also act as a voltage subtractor.

Example 12.11: Feedback Factor



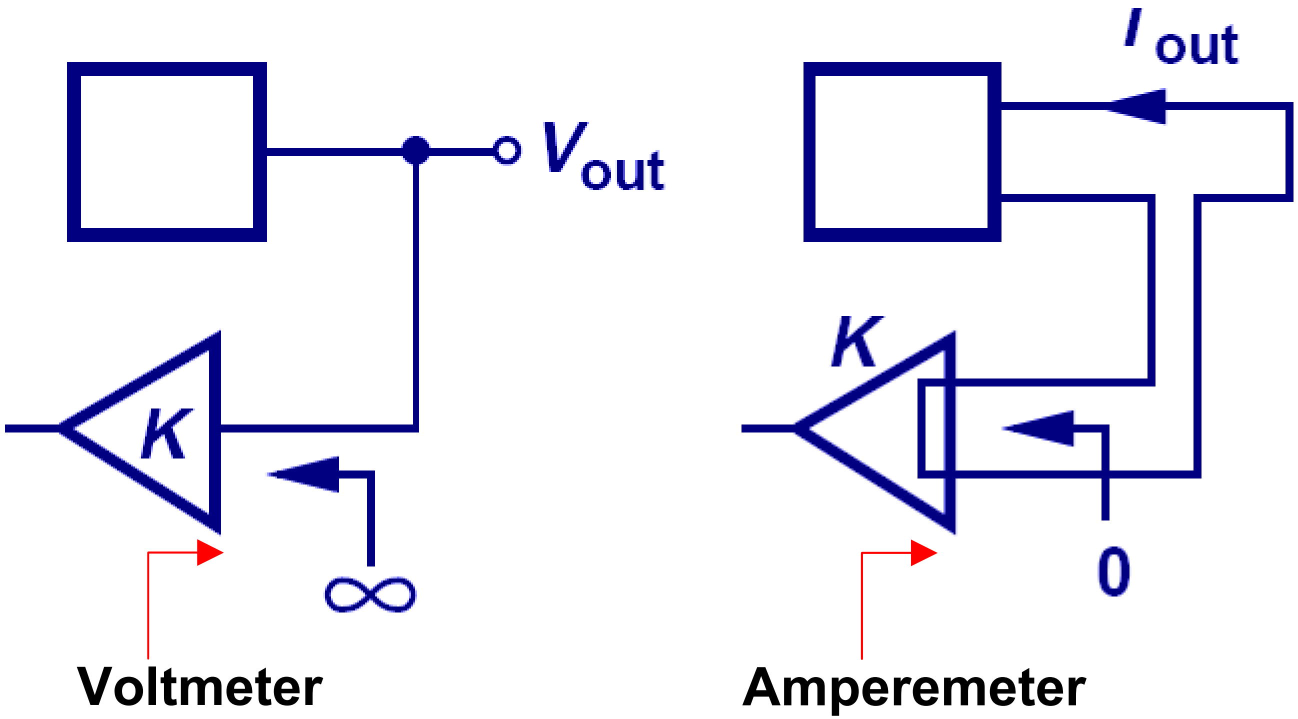
$$i_F \uparrow \Rightarrow v_X \Downarrow$$

$$K = \frac{i_F}{v_{out}} = g_{mF}$$

Topics in Last and Today's Lectures

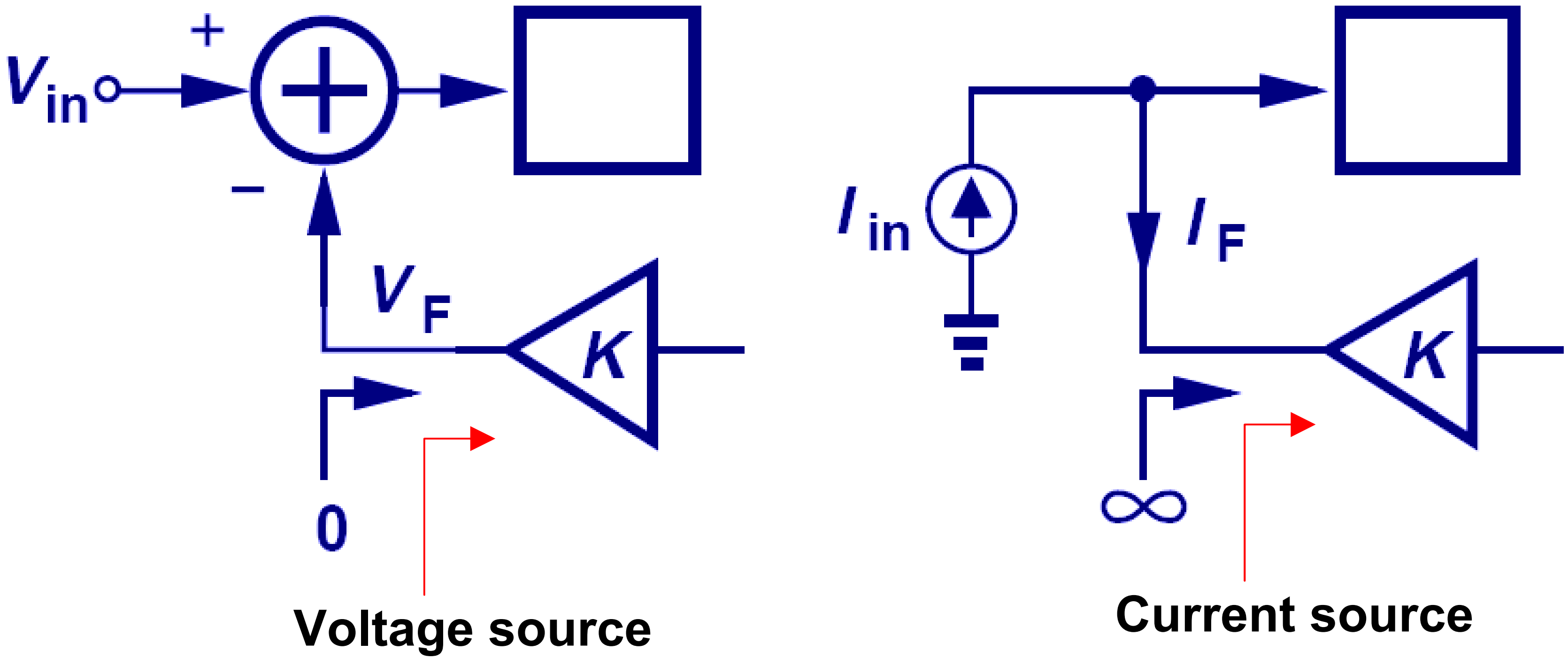
- **12.3 Types of Amplifiers**
  - Simple Amplifier Models
  - Examples of Amplifier Types
- **12.4 Sense and Return Techniques**
- **12.5 Polarity of Feedback**
- **12.5 Feedback Topologies**
  - Voltage-Voltage Feedback
  - Voltage-Current Feedback
  - Current-Voltage Feedback
  - Current-Current Feedback

Input Impedance of an Ideal Feedback Network



- To sense a voltage, the input impedance of an ideal feedback network must be infinite.
- To sense a current, the input impedance of an ideal feedback network must be zero.

Output Impedance of an Ideal Feedback Network



- To return a voltage, the output impedance of an ideal feedback network must be zero.
- To return a current, the output impedance of an ideal feedback network must be infinite.

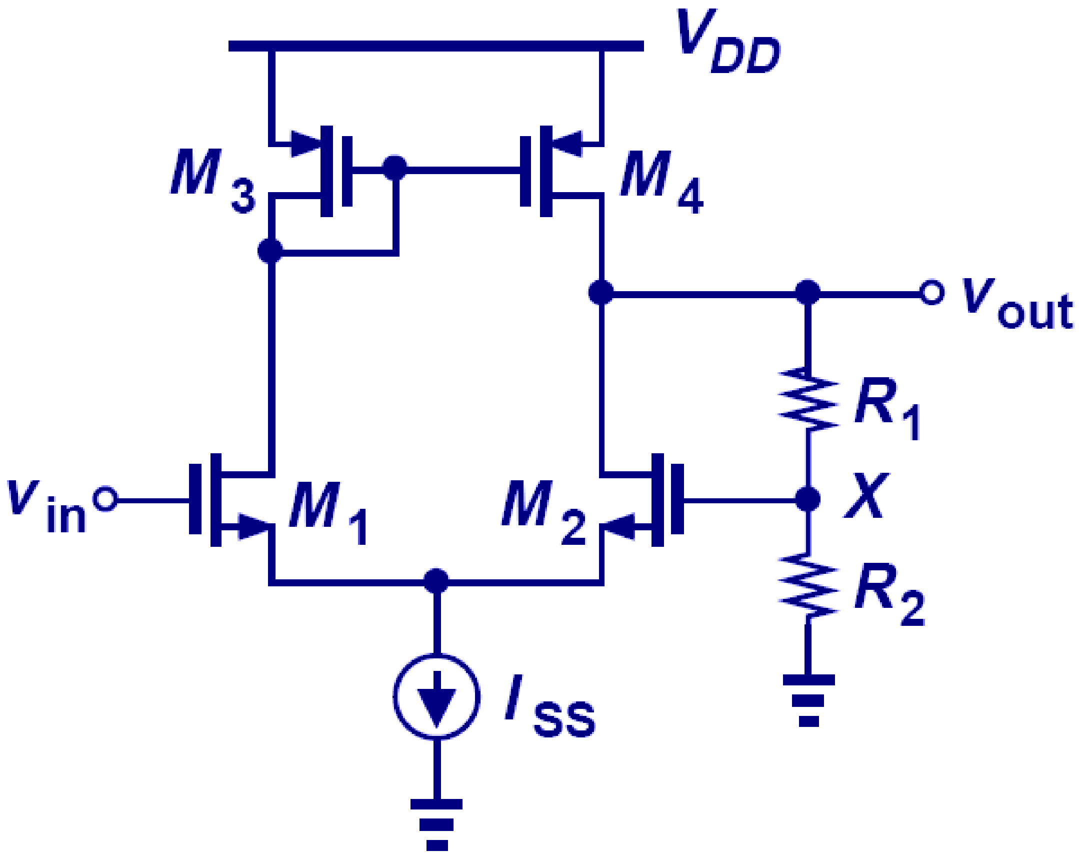
Determining the Polarity of Feedback

- 1) Assume the input goes either up or down.
- 2) Follow the signal through the loop.
- 3) Determine whether the returned quantity enhances or opposes the original change.

Negative feedback

Positive feedback

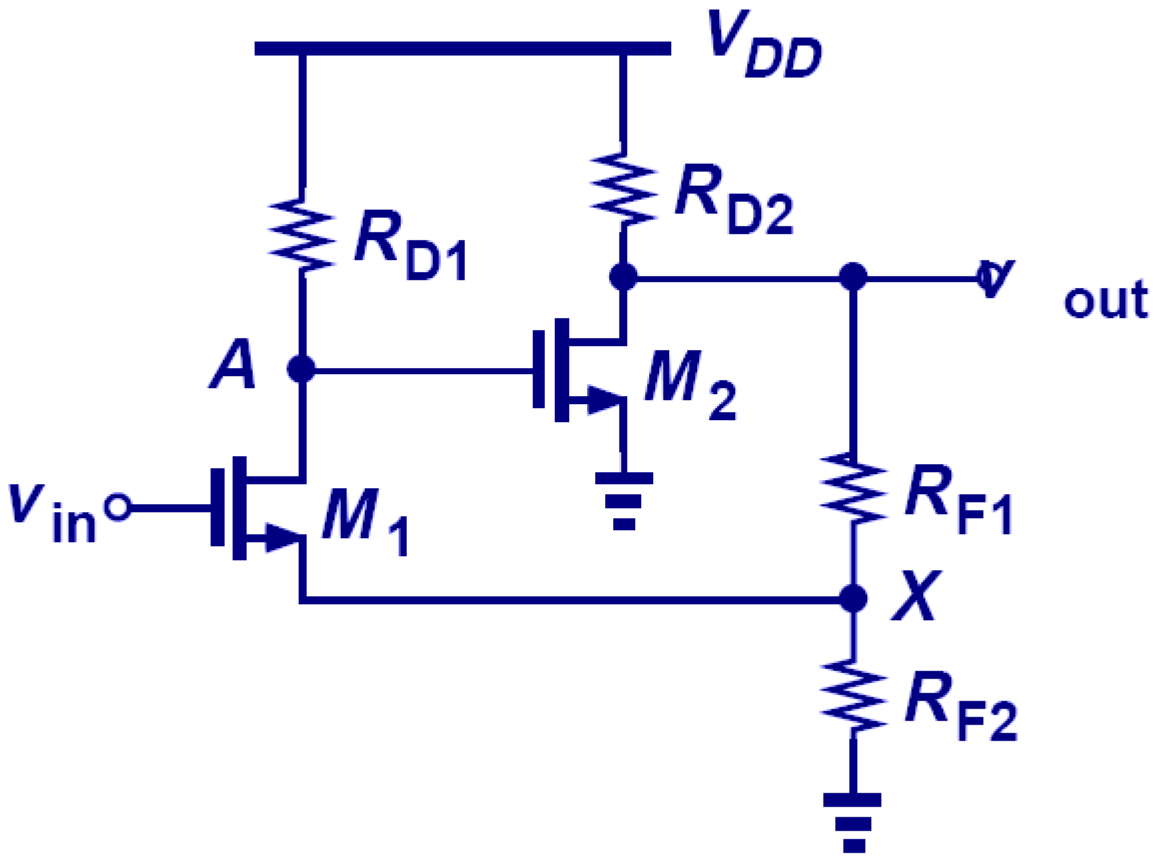
Example 12.12: Polarity of Feedback



$$V_{in} \uparrow \Rightarrow I_{D1} \uparrow, I_{D2} \downarrow \Rightarrow V_{out} \uparrow, V_x \uparrow \Rightarrow I_{D2} \uparrow, I_{D1} \downarrow$$

Negative Feedback

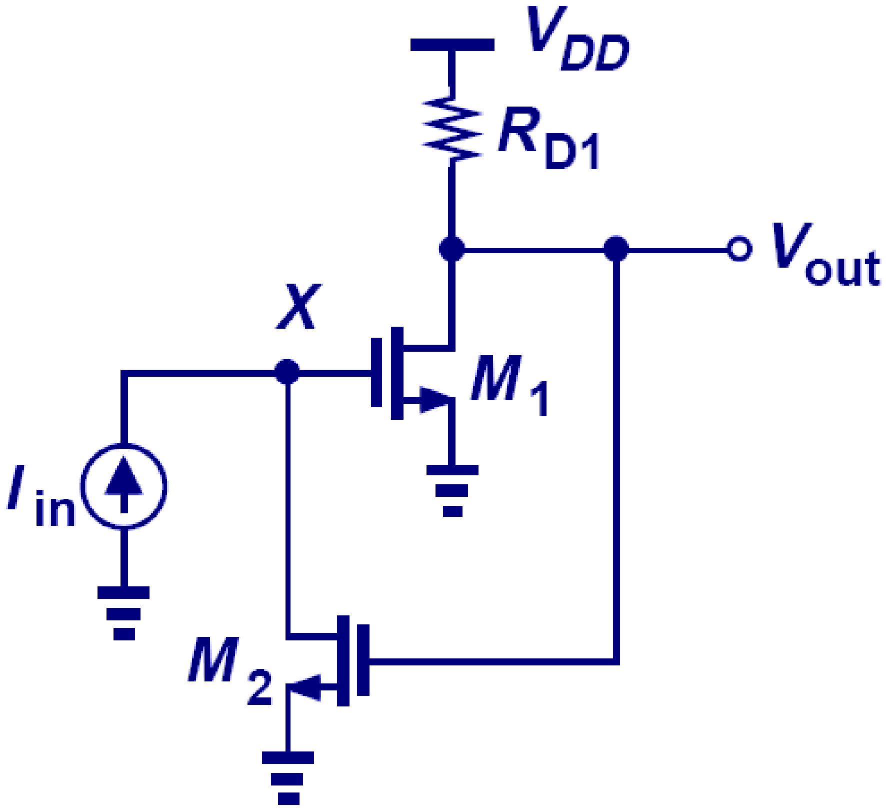
Example 12.13: Polarity of Feedback



$$V_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_A\downarrow \Rightarrow V_{out}\uparrow, V_x\uparrow \Rightarrow I_{D1}\downarrow, V_A\uparrow$$

Negative Feedback

Example 12.14: Polarity of Feedback

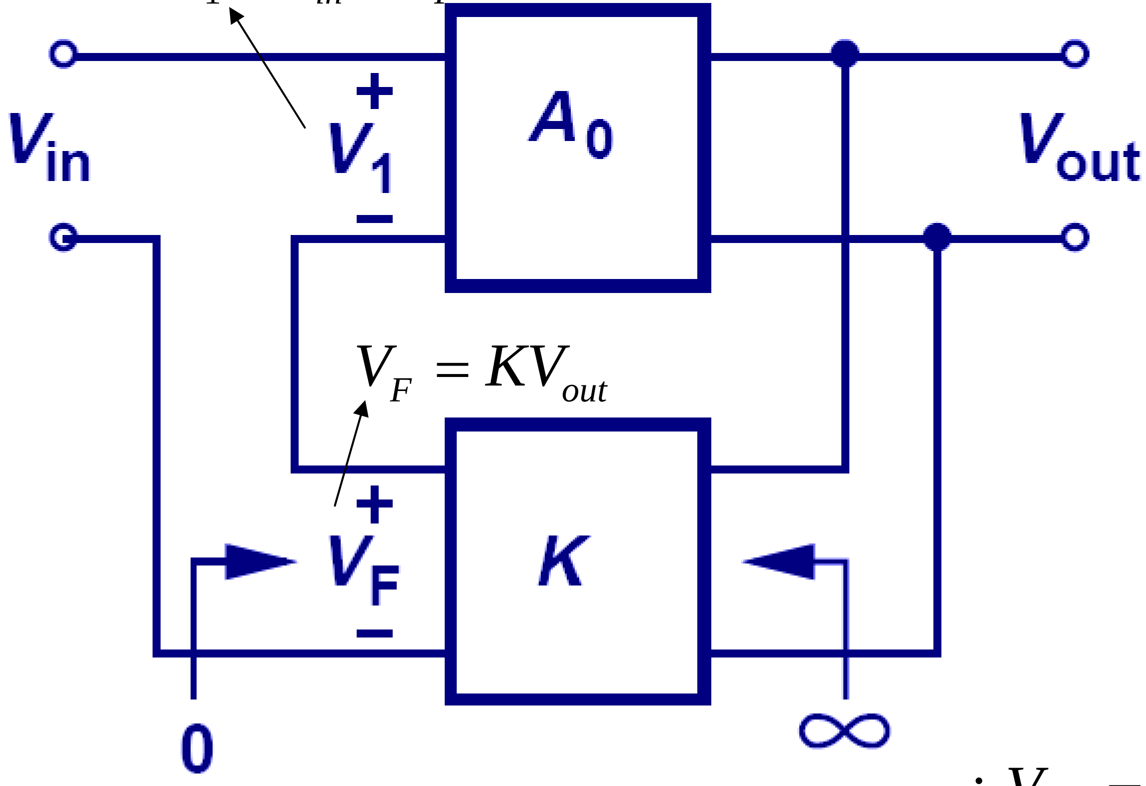


$$I_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_X\uparrow \Rightarrow V_{out}\downarrow, I_{D2}\downarrow \Rightarrow I_{D1}\uparrow, V_X\uparrow$$

Positive Feedback

Voltage-Voltage Feedback

$$V_1 = V_{in} - V_F$$



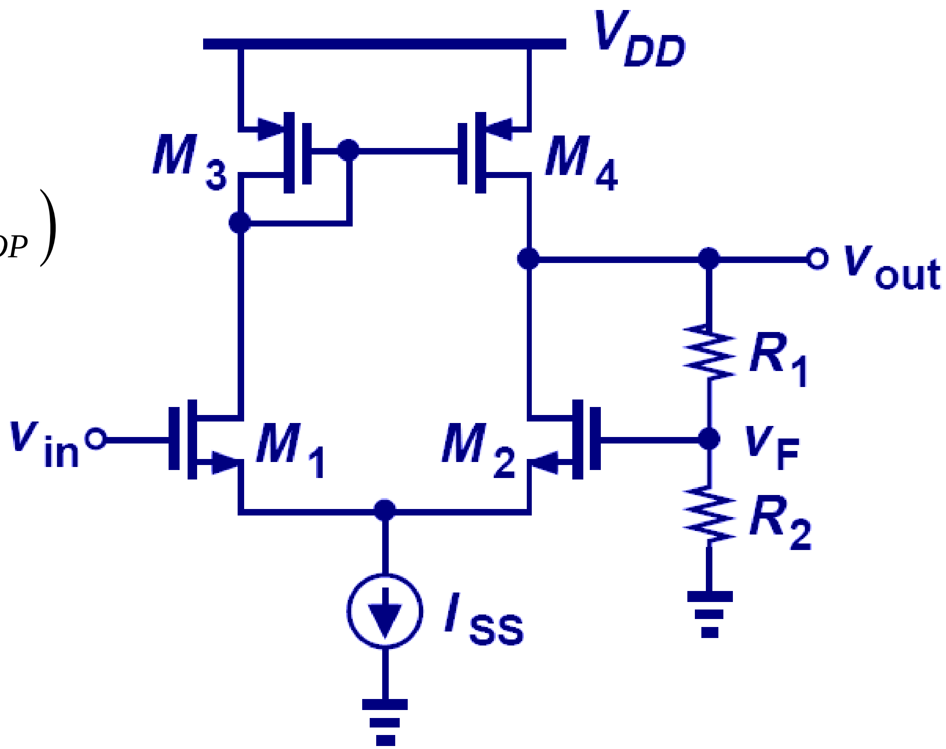
$$V_{out} = A_0 V_1$$

$$\therefore V_{out} = A_0 (V_{in} - K V_{out})$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + K A_0}$$

Example 12.15: Voltage-Voltage Feedback

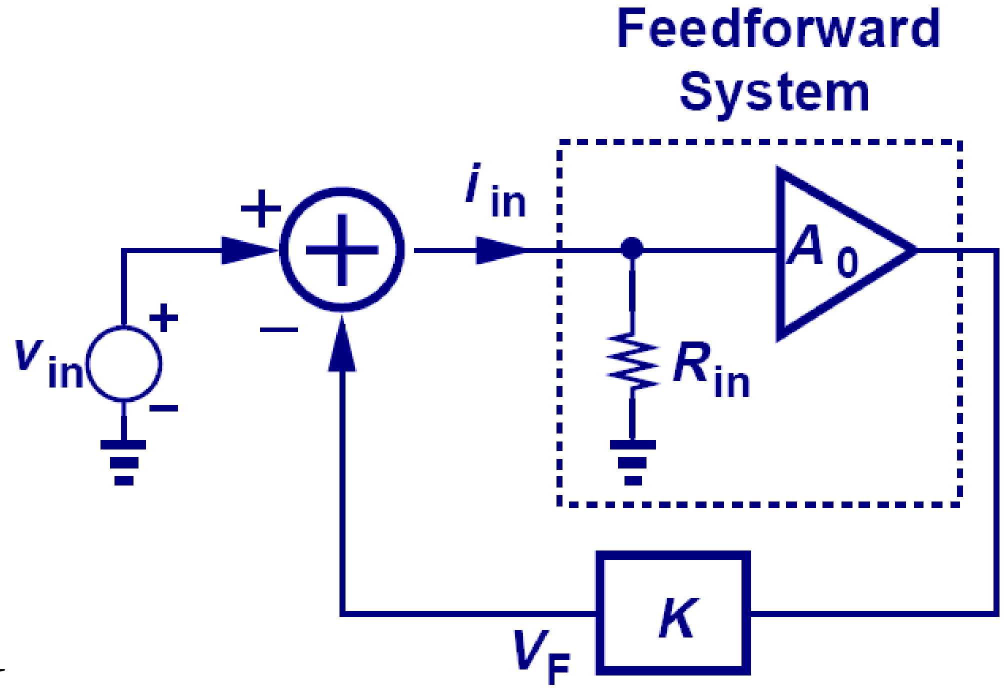
$$\because \frac{v_{out}}{v_{in1} - v_{in2}} \approx g_{mN}\left(r_{ON} \| r_{OP}\right)$$



Assuming $R_1 + R_2 >> (r_{ON} \| r_{OP})$,

$$\frac{V_{out}}{V_{in}} = \frac{\overbrace{g_{mN}(r_{ON} \| r_{OP})}^{A_0}}{1 + \underbrace{\frac{R_2}{R_1 + R_2}}_{K} \overbrace{g_{mN}(r_{ON} \| r_{OP})}^{A_0}}$$

Input Impedance of a V-V Feedback

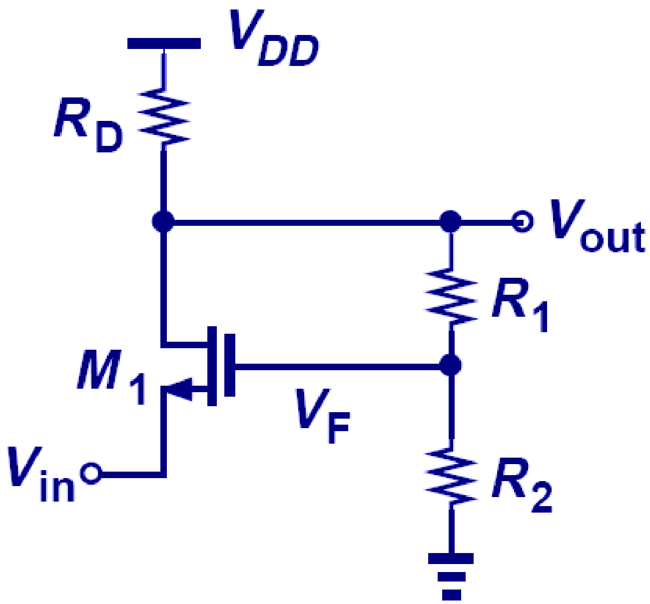


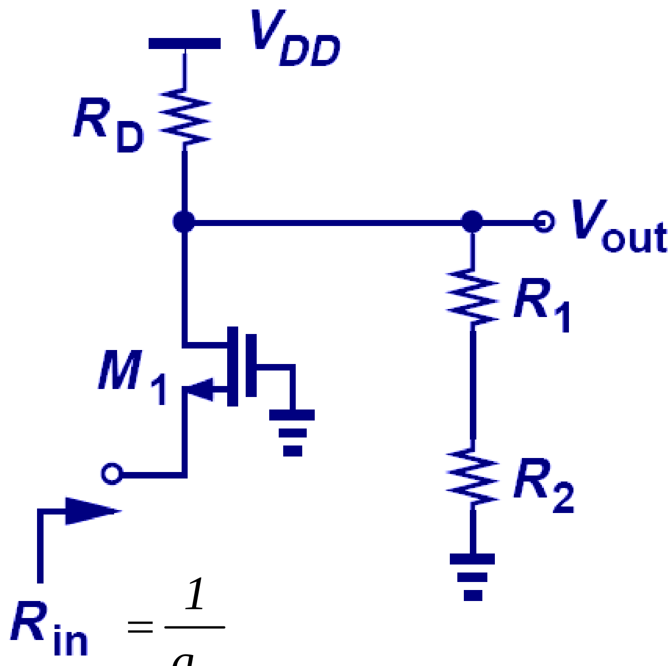
$$I_{in}R_{in} = V_{in} - V_F$$
$$= V_{in} - (I_{in}R_{in})A_0K$$

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + A_0K)$$

- The impedance modification brings the circuit closer to an ideal voltage amplifier

Example12.16: V-V Feedback Input Impedance





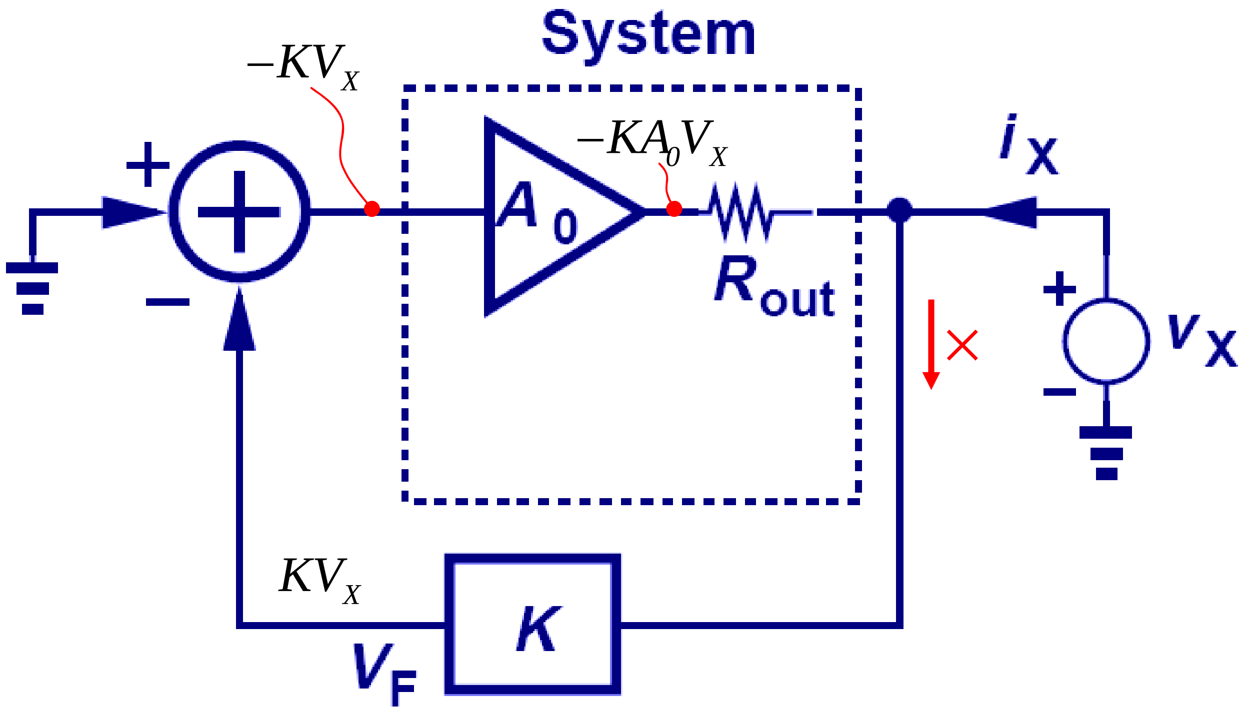
$$R_{in} = \frac{1}{g_m}$$

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + A_0 K)$$

Assuming $R_1 + R_2 >> R_D$,

$$\frac{V_{in}}{I_{in}} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1 + R_2} g_m R_D\right)$$

Output Impedance of a V-V Feedback

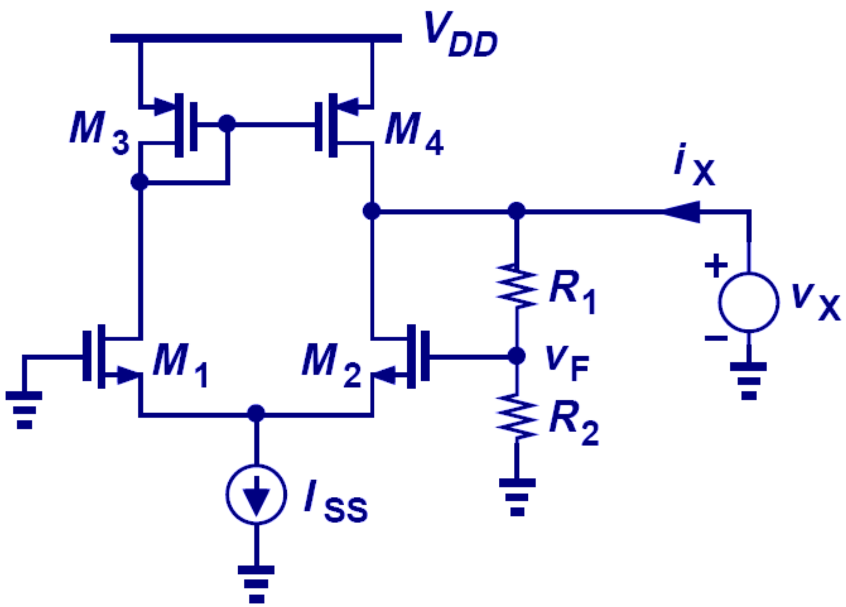


$$I_X = \frac{V_X - (-KA_0V_X)}{R_{out}}$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{(1 + KA_0)}$$

- A better voltage source

Example 12.17: V-V Feedback Output Impedance



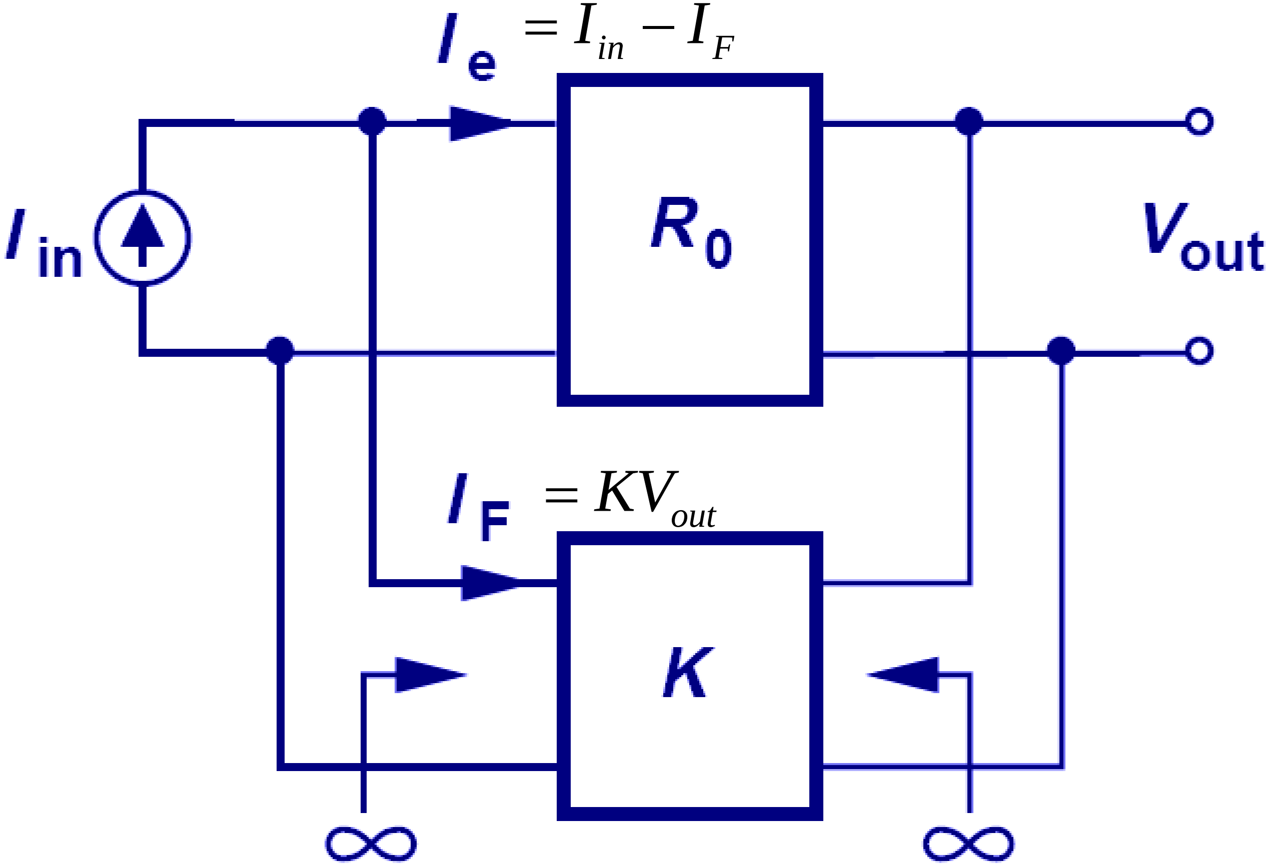
$$\because \frac{V_X}{I_X} = \frac{R_{out}}{(1 + KA_0)}$$

Assuming $R_1 + R_2 >> (r_{ON} \parallel r_{OP})$,

$$R_{out,closed} = \frac{r_{ON} \parallel r_{OP}}{1 + R_2 / (R_1 + R_2) \cdot g_{mN} \cdot (r_{ON} \parallel r_{OP})}$$

$$\approx \left(1 + \frac{R_1}{R_2}\right) \frac{1}{g_{mN}}$$

Voltage-Current Feedback



$$I_e = I_{in} - I_F$$

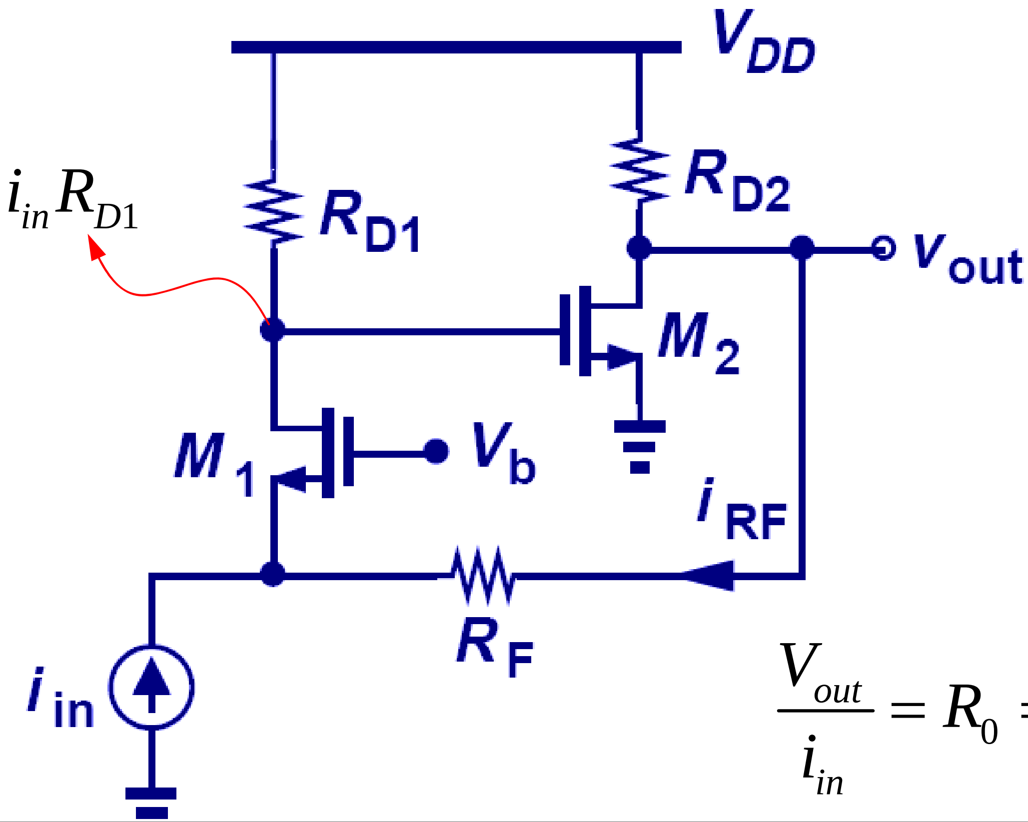
$$V_{out} = I_e R_0$$

$$I_F = KV_{out}$$

$$\therefore V_{out} = (I_{in} - KV_{out})R_0$$

$$\frac{V_{out}}{I_{in}} = \frac{R_O}{1 + KR_O}$$

Example 12.18: Voltage-Current Feedback

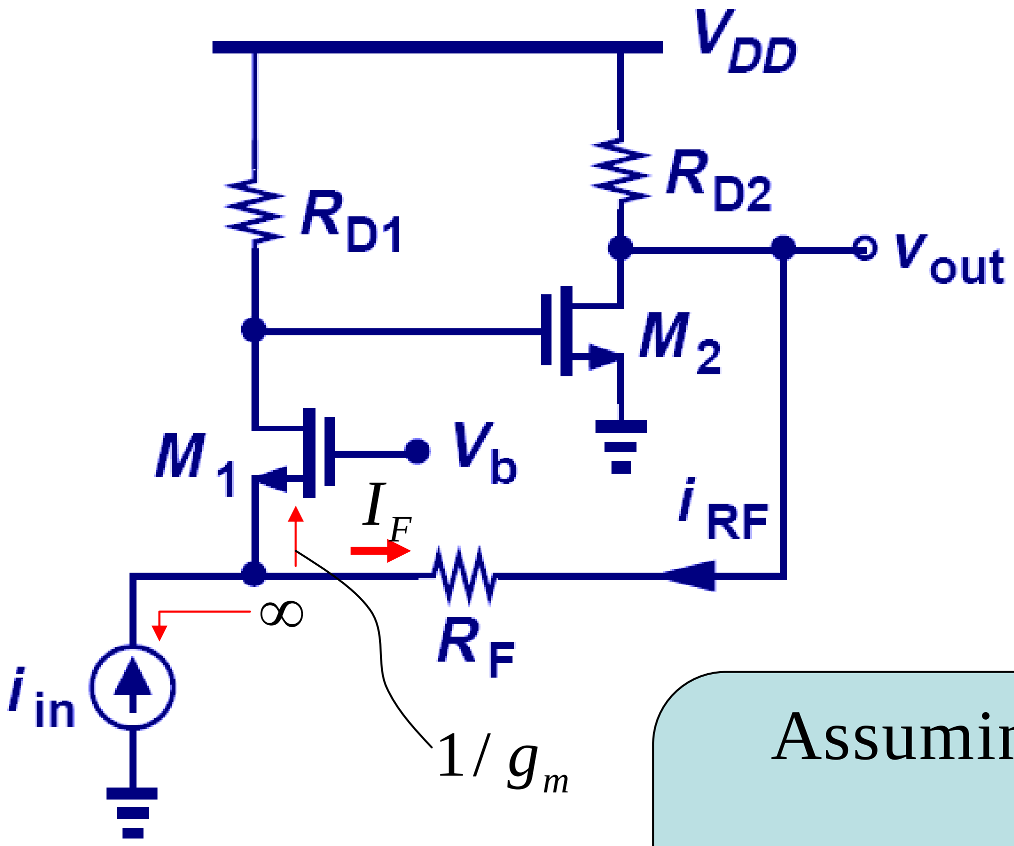


$$\frac{V_{out}}{i_{in}} = R_0 = R_{D1}\left(-g_{m2}R_{D2}\right)$$

Assuming R_F is very large, open loop gain (V_{out}/I_{in}):

$$R_0 = R_{D1}(-g_{m2} \cdot R_{D2})$$

Example 12.18: Voltage-Current Feedback



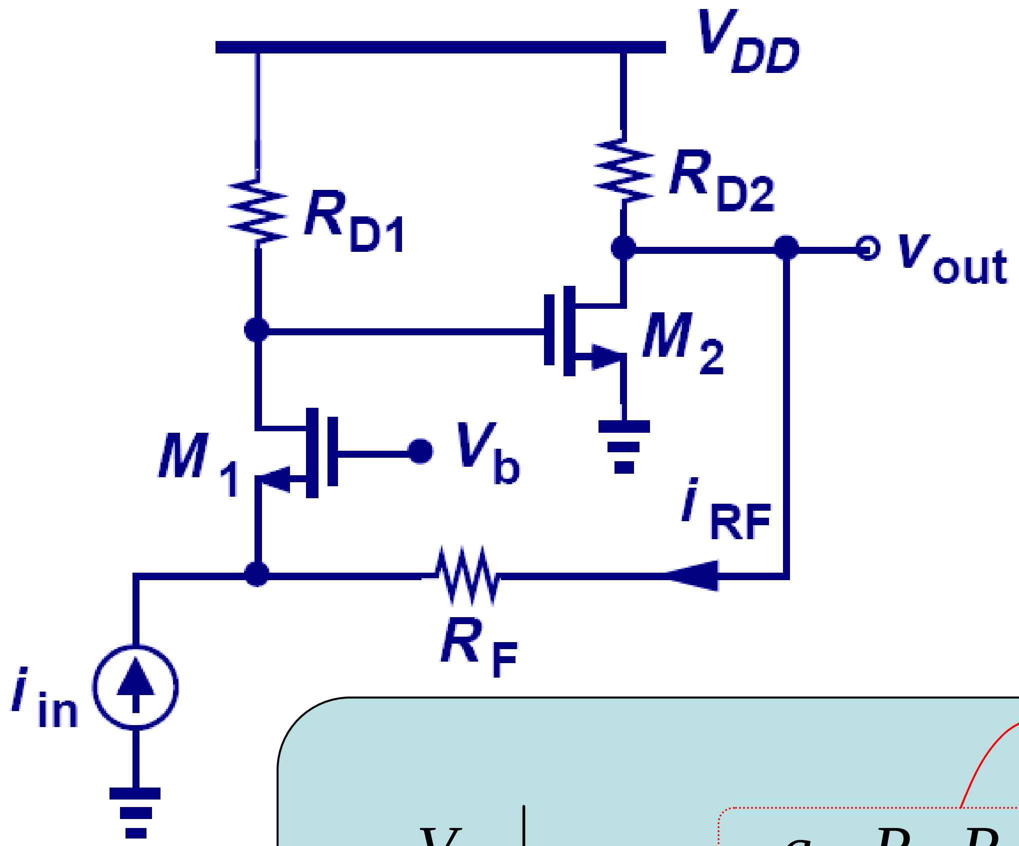
$$\because I_F = KV_{out}$$

Assuming R_F is very large,

$$I_{RF} = \frac{V_{out}}{R_F + 1/g_{m1}} \approx \frac{V_{out}}{R_F}$$

$$K = -1/R_F$$

Example 12.18: Voltage-Current Feedback



$$\left.\frac{V_{out}}{I_{in}}\right|_{closed} \approx R_F$$

if $g_{m2}R_{D1}R_{D2} >> R_F$

$$\left.\frac{V_{out}}{I_{in}}\right|_{closed} = \frac{\boxed{-g_{m2}R_{D1}R_{D2}}}{1+\boxed{\dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}}$$

R_0

KR_0

Input Impedance of a V-I Feedback

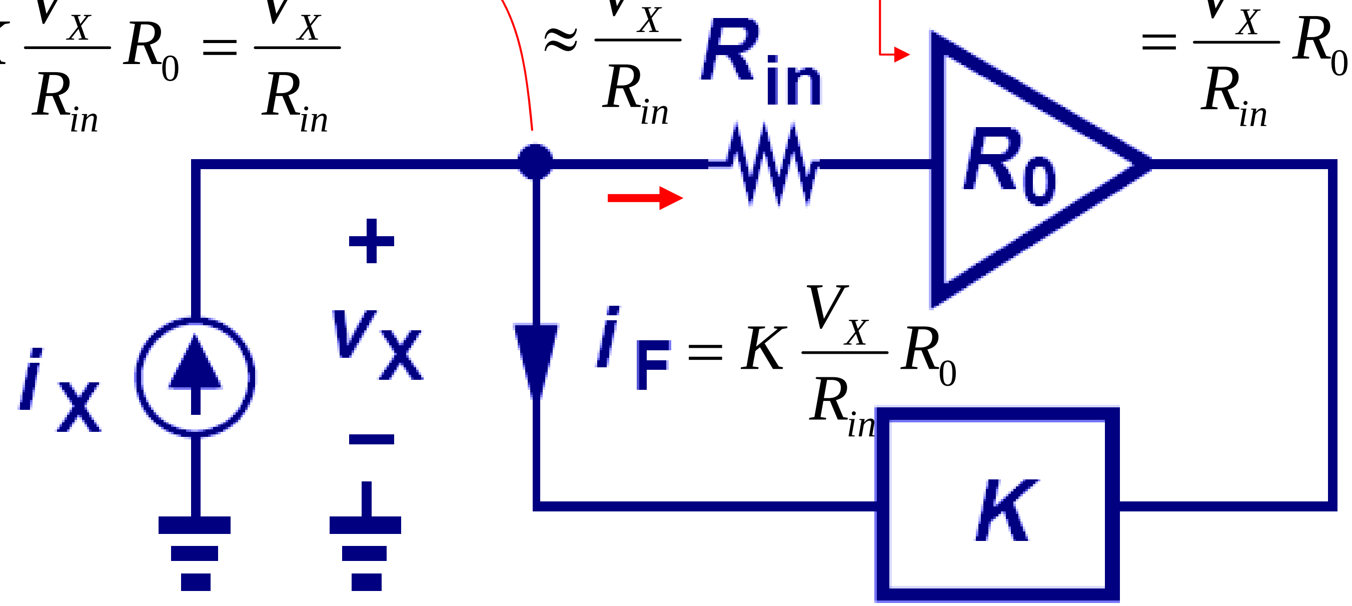
By KCL

$$I_X - K\frac{V_X}{R_{in}}R_0 = \frac{V_X}{R_{in}}$$

Current sensor or ammeter

$$\approx \frac{V_X}{R_{in}}$$

$$= \frac{V_X}{R_{in}}R_0$$

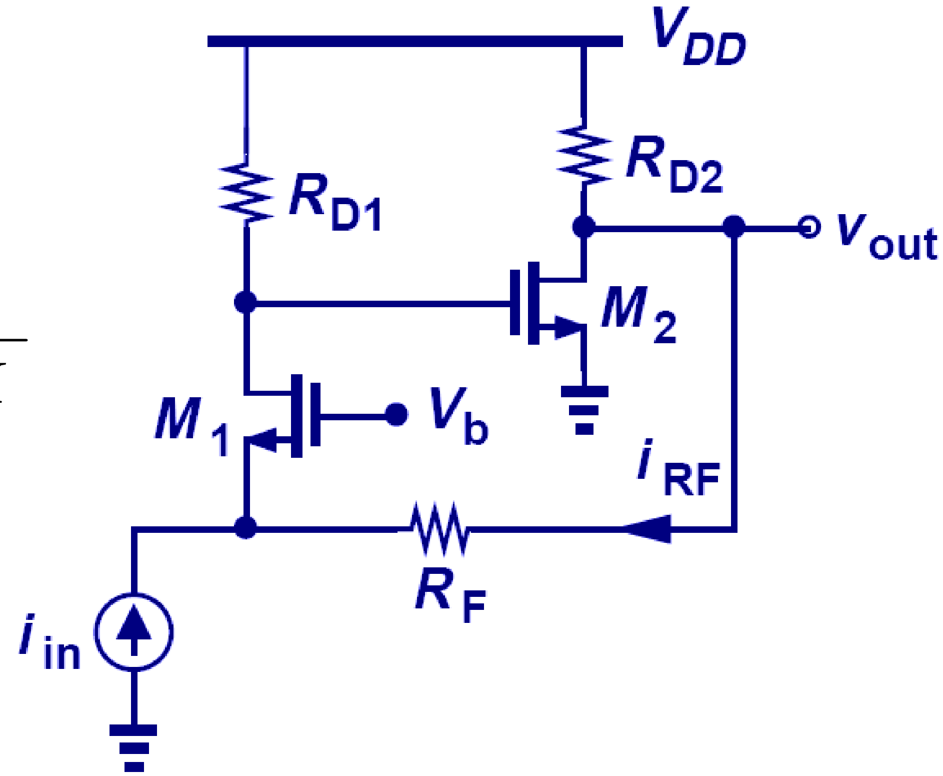


$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + R_0 K}$$

- **A better current sensor.**

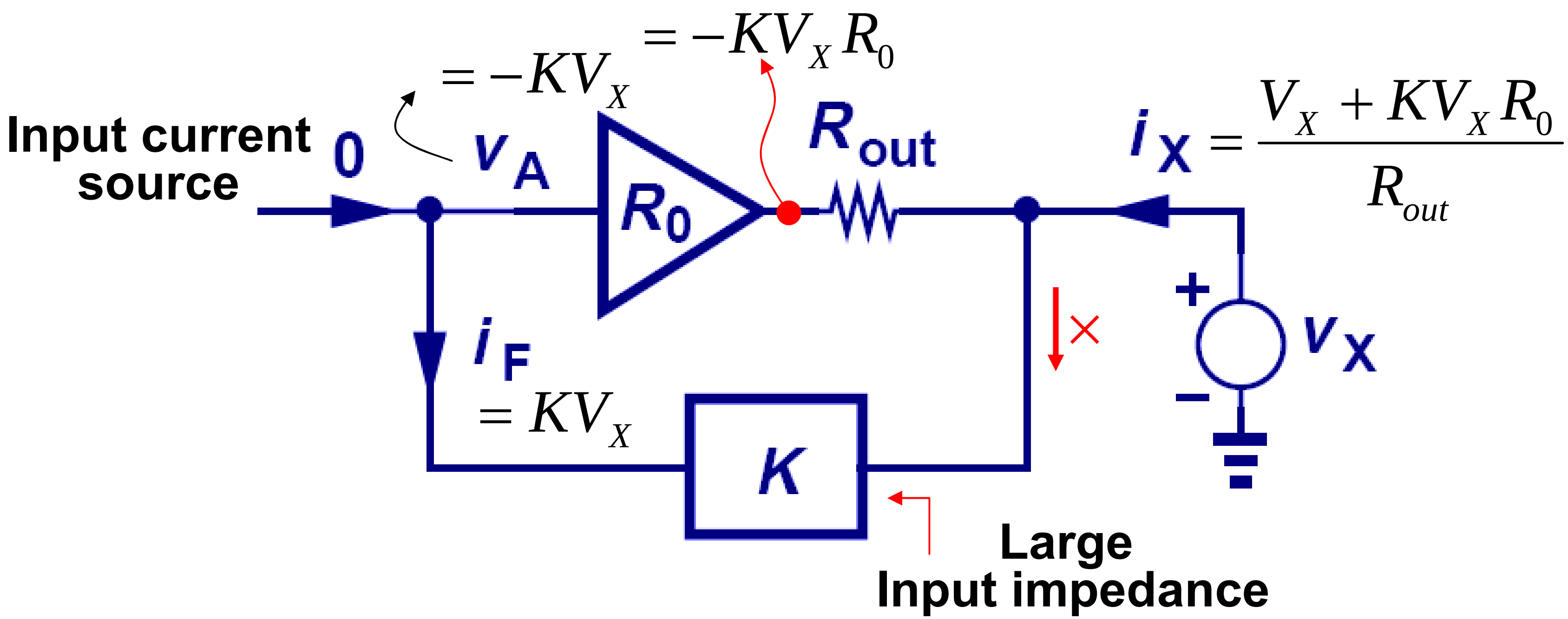
Example 12.19: V-I Feedback Input Impedance

$$\because \frac{V_X}{I_X} = \frac{R_{in}}{1 + R_0 K}$$



$$R_{in,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + \dfrac{g_{m2} R_{D1} R_{D2}}{R_F}}$$

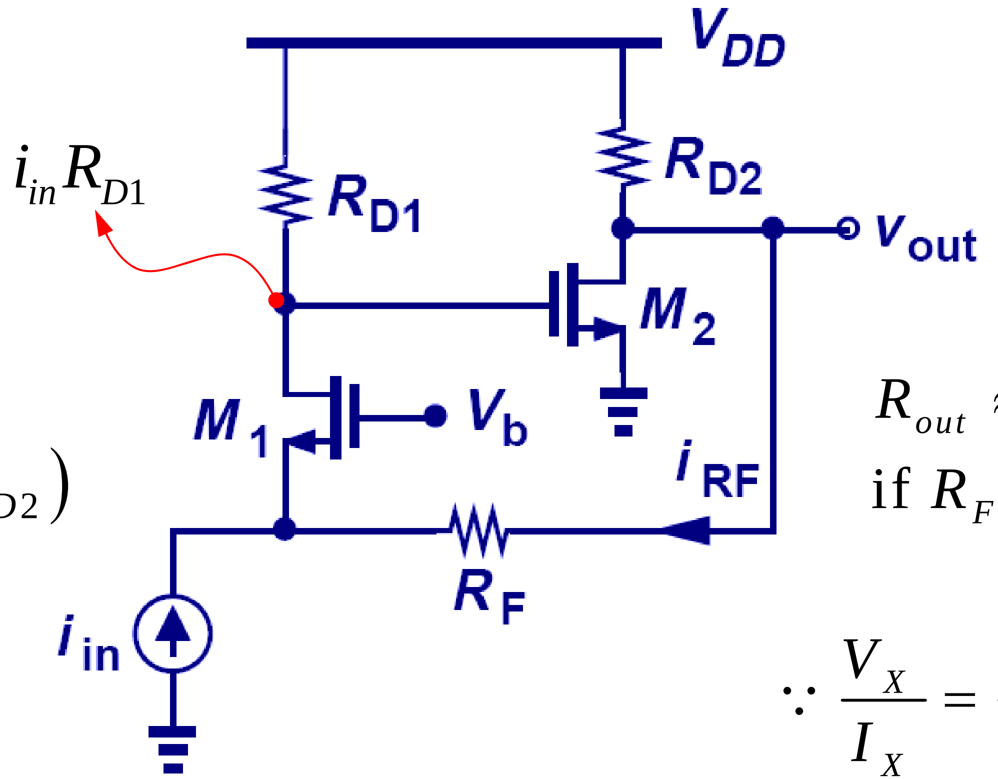
Output Impedance of a V-I Feedback



$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + R_0 K}$$

- **A better voltage source.**

Example12.20: V-I Feedback Output Impedance



$$\because \frac{V_{out}}{i_{in}} = R_0$$

$$= R_{D1}\left(-g_{m2}R_{D2}\right)$$

$R_{out} \approx R_{D2}$

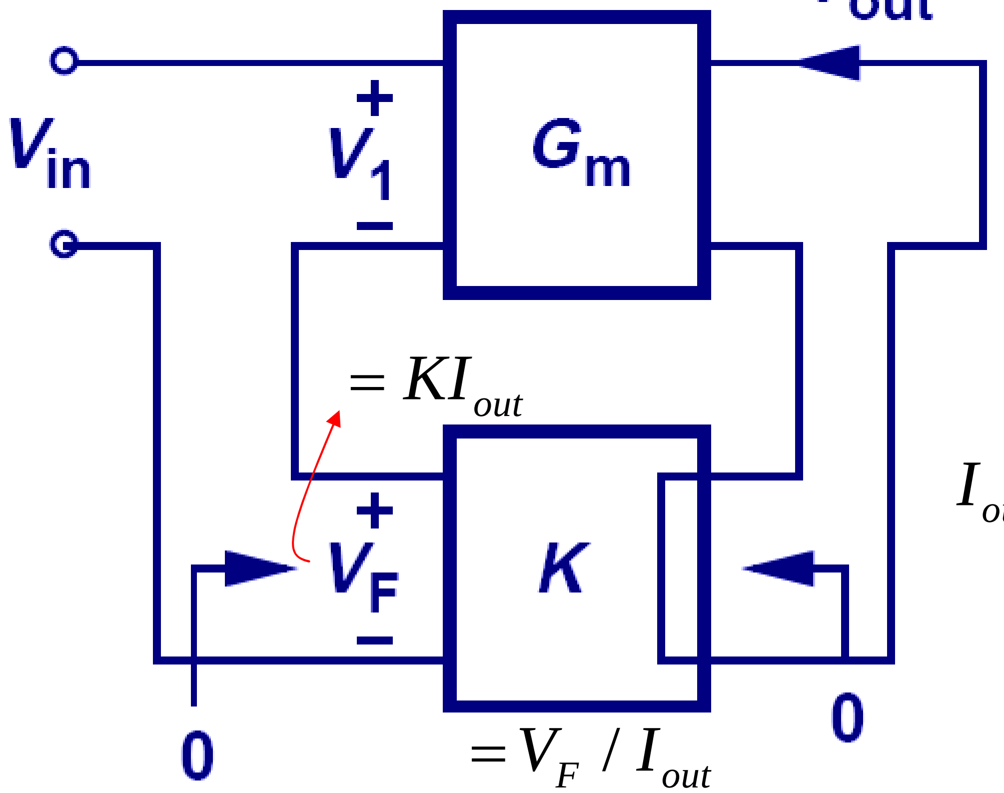
if R_F is very large

$$\because \frac{V_X}{I_X} = \frac{R_{out}}{1 + R_0 K}$$

$$R_{out,closed} = \frac{R_{D2}}{1 + \dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}$$

Current-Voltage Feedback

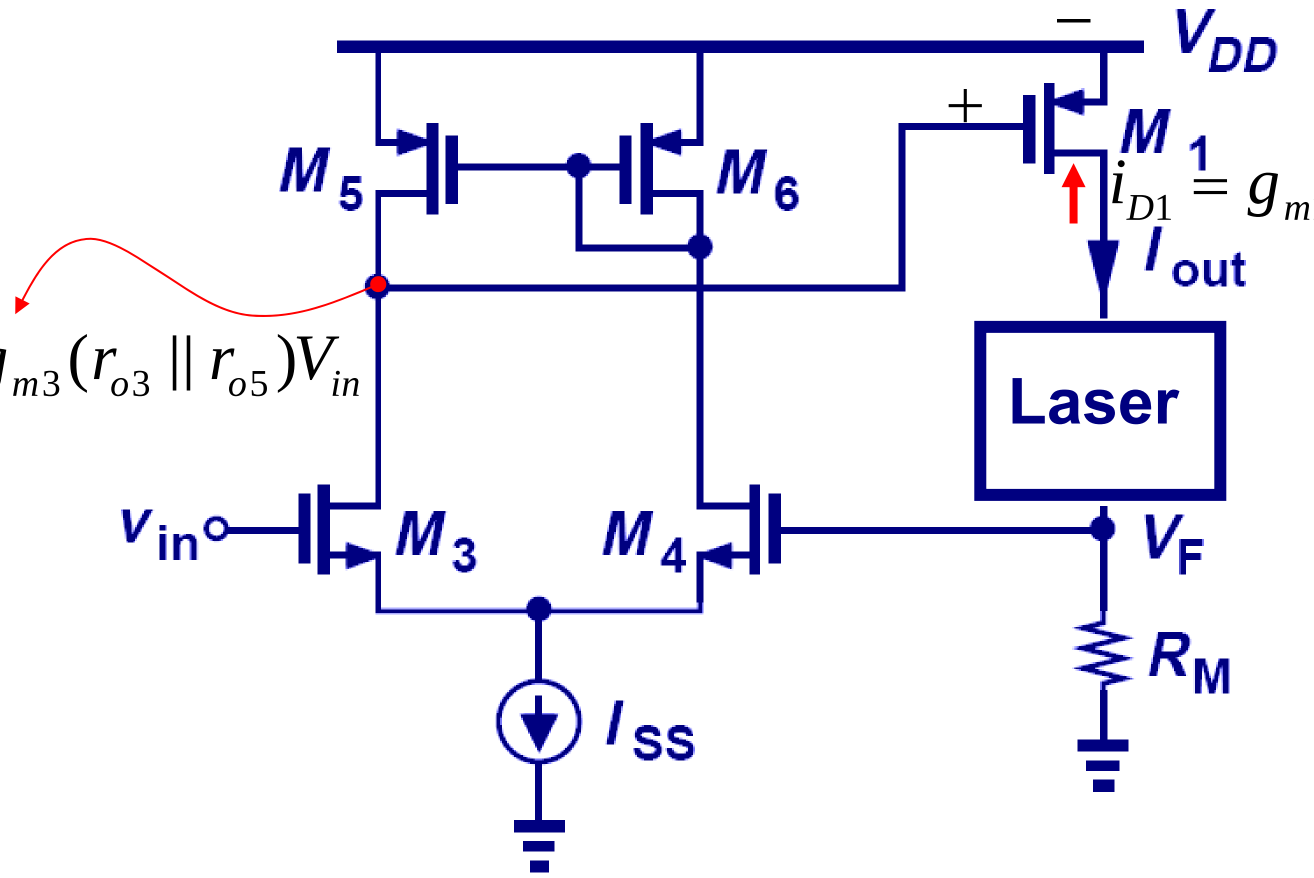
$$I_{out} = G_m (V_{in} - V_F)$$



$$I_{out} = G_m (V_{in} - KI_{out})$$

$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + KG_m}$$

Example12.21: Current-Voltage Feedback



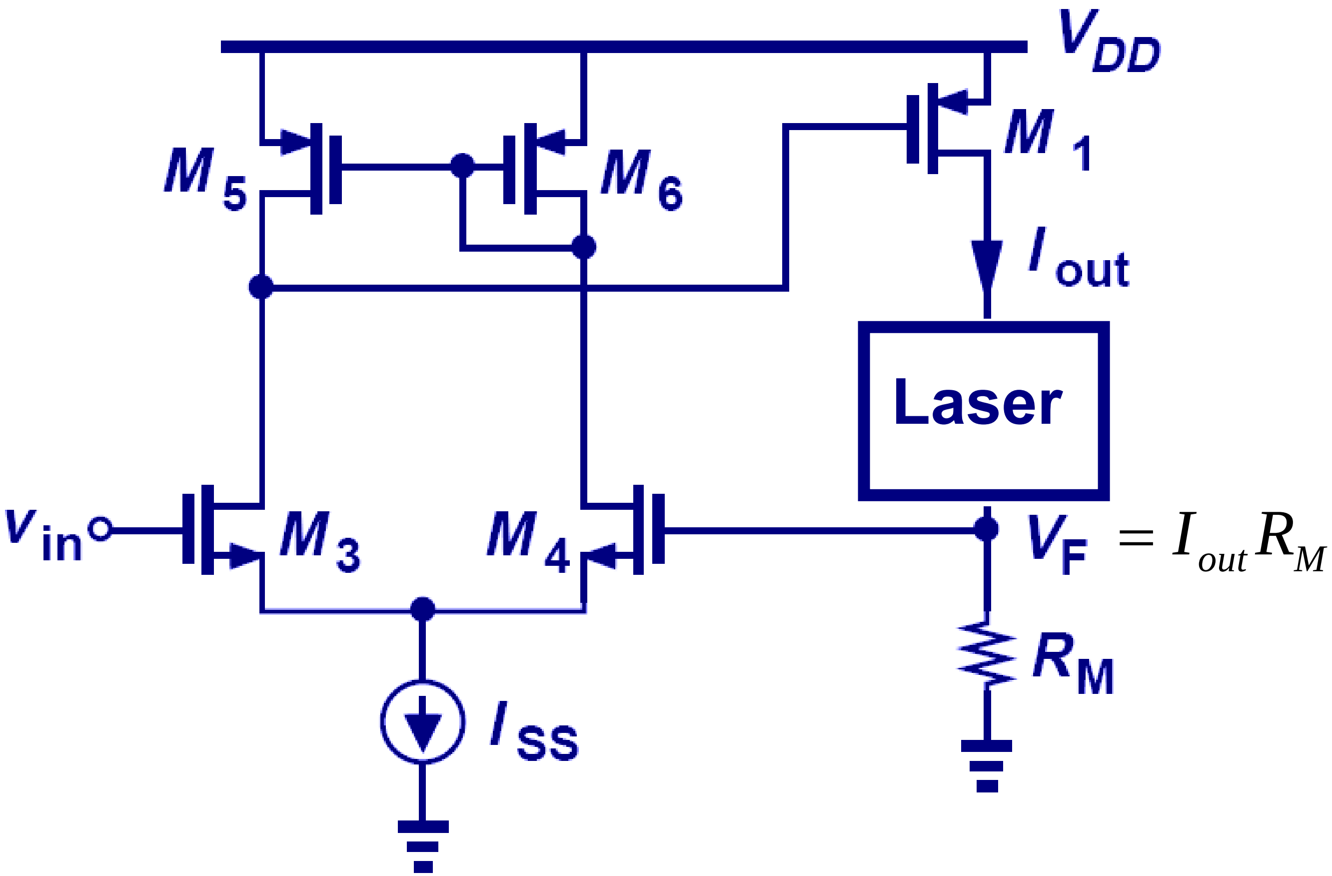
$$i_{D1} = g_{m1}\left[-g_{m3}(r_{o3} \parallel r_{o5})V_{in}\right]$$

$$-g_{m3}(r_{o3} \parallel r_{o5})V_{in}$$

$$I_{out} = -i_{D1}$$

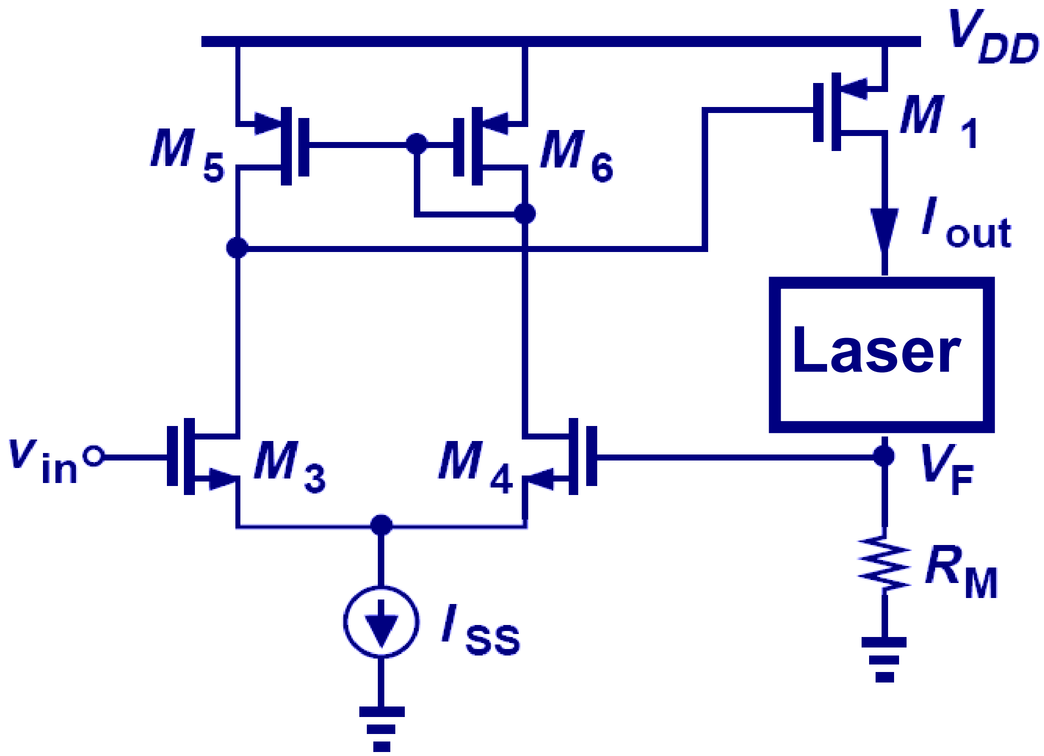
$$G_m = \frac{I_{out}}{V_{in}}\bigg|_{open} = g_{m3} \cdot \left(r_{O3} \parallel r_{O5}\right) \cdot g_{m1}$$

Example12.21: Current-Voltage Feedback



$$K = \frac{V_F}{I_{out}} = R_M$$

Example12.21: Current-Voltage Feedback

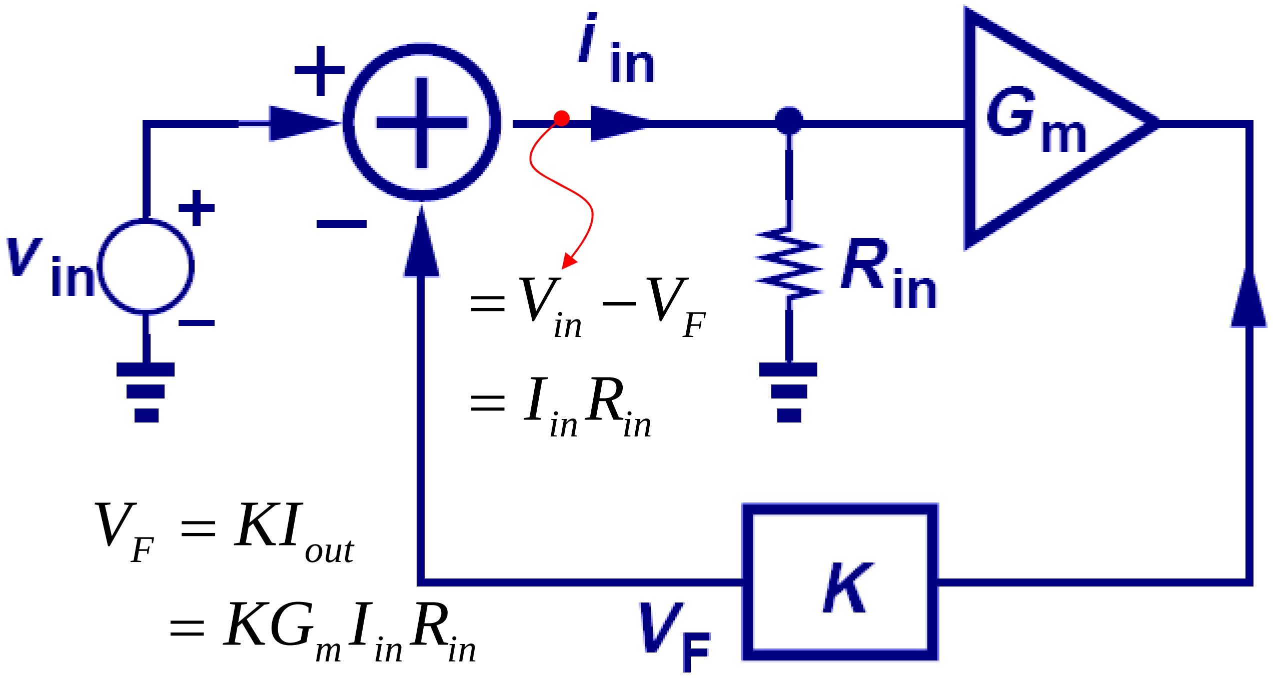


$$\frac{I_{out}}{V_{in}}\Big|_{closed} = \frac{G_m}{1+K\cdot G_m} = \frac{g_{m1}g_{m3}\left(r_{O3}\,\|\,r_{O5}\right)}{1+g_{m1}g_{m3}\left(r_{O3}\,\|\,r_{O5}\right)R_M} \approx \frac{1}{R_M}$$

Topics in Last and Today's Lectures

- **12.5 Polarity of Feedback**
- **12.5 Feedback Topologies**
  - Voltage-Voltage Feedback
  - Voltage-Current Feedback
  - Current-Voltage Feedback
  - Current-Current Feedback
- **12.6 Effect of Finite I/O Impedances**
  - Inclusion of I/O Effects
- **12.7 Stability in Feedback Systems**

Input Impedance of a I-V Feedback



$$= V_{in} - V_F$$
$$= I_{in} R_{in}$$

$$I_{out} = G_m I_{in} R_{in}$$

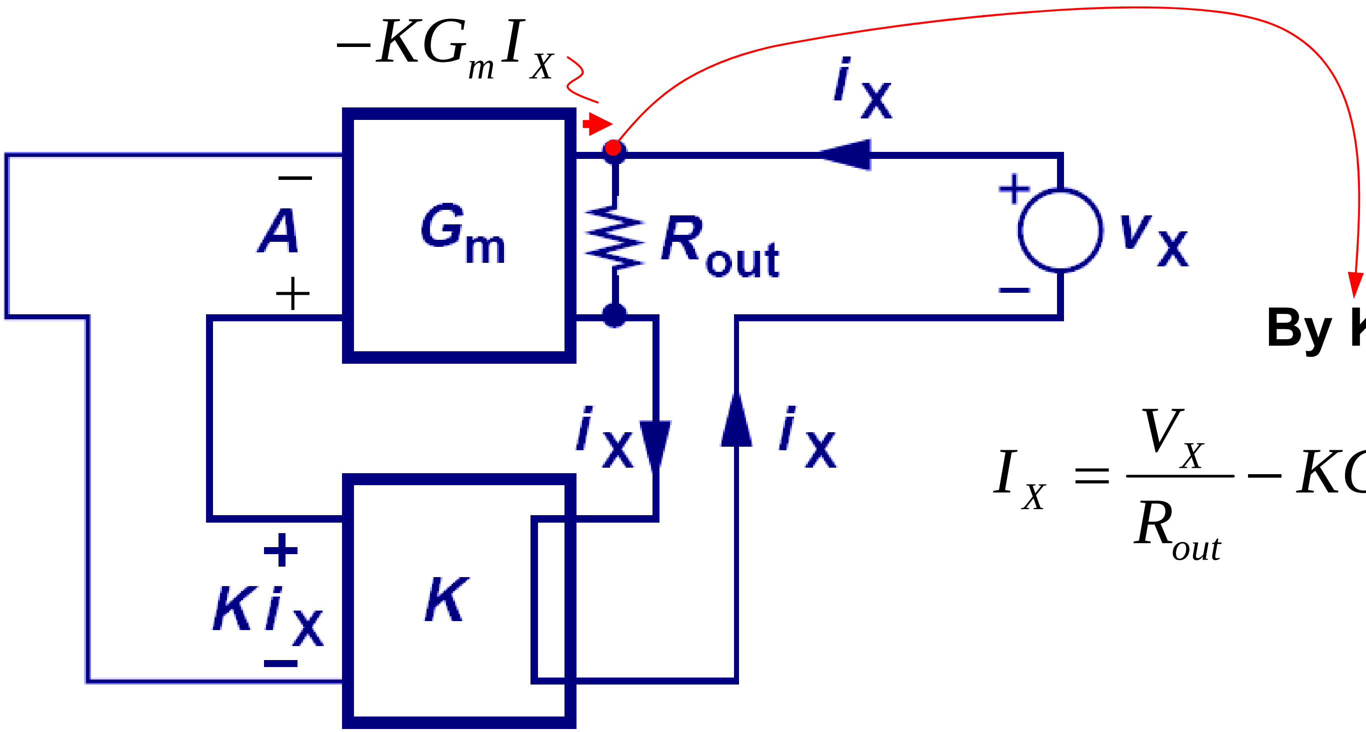
$$V_F = K I_{out}$$
$$= K G_m I_{in} R_{in}$$

$$I_{in} R_{in} = V_{in} - V_F$$
$$= V_{in} - K G_m I_m R_{in}$$

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + K G_m)$$

- A better voltage sensor.

Output Impedance of a I-V Feedback



$$I_X = \frac{V_X}{R_{out}} - KG_m I_X$$

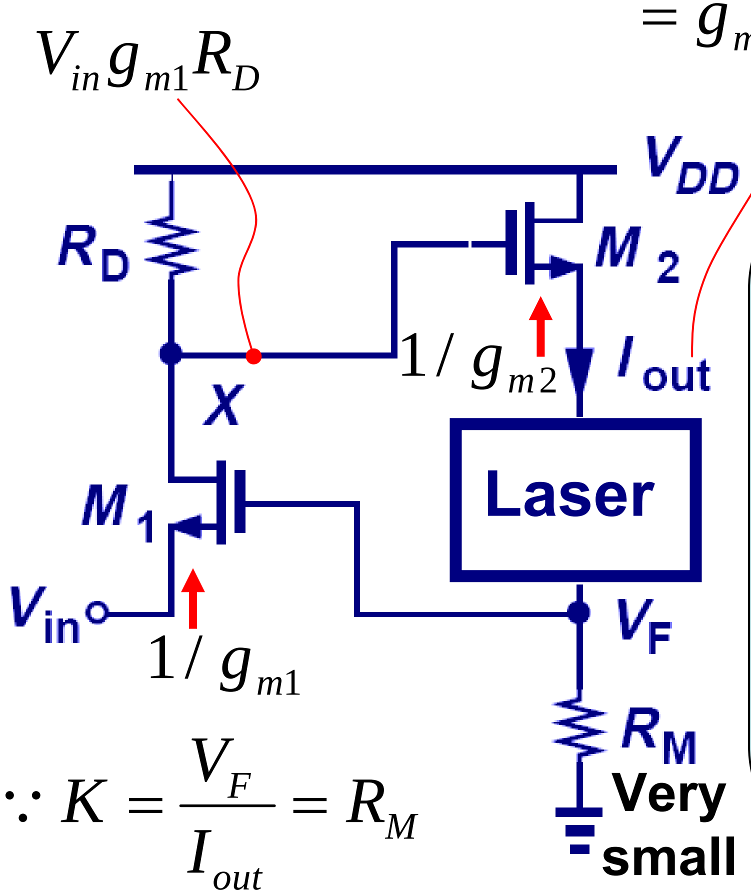
$$V_F = KI_X$$

$$K = \frac{V_F}{I_X}$$

$$\frac{V_X}{I_X} = R_{out}(1 + KG_m)$$

- A better current source.

Example: Current-Voltage Feedback



$$= g_{m2}\left(V_{in}g_{m1}R_D\right) \text{ for open loop circuit}$$

$$\therefore G_m = g_{m1}R_D \cdot g_{m2}$$

$$\frac{I_{out}}{V_{in}}\bigg|_{closed} = \frac{g_{m1}g_{m2}R_D}{1+g_{m1}g_{m2}R_DR_M}$$

$$R_{in}\big|_{closed} = \frac{1}{g_{m1}}(1+g_{m1}g_{m2}R_DR_M)$$

$$R_{out}\big|_{closed} = \frac{1}{g_{m2}}(1+g_{m1}g_{m2}R_DR_M)$$

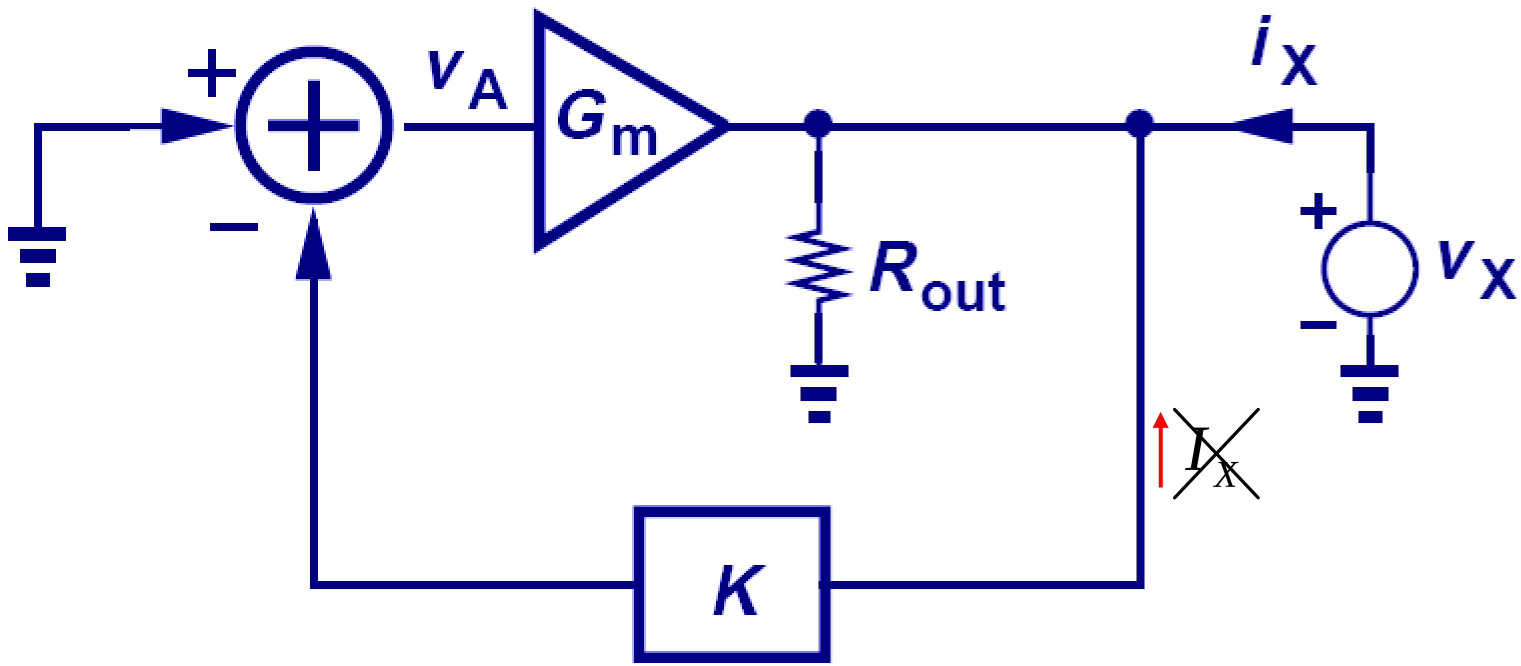
$$\because K = \frac{V_F}{I_{out}} = R_M$$

$$\because \frac{I_{out}}{V_{in}} = \frac{G_m}{1+KG_m}$$

$$\because \frac{V_{in}}{I_{in}} = R_{in}(1+KG_m)$$

$$\because \frac{V_X}{I_X} = R_{out}(1+KG_m)$$

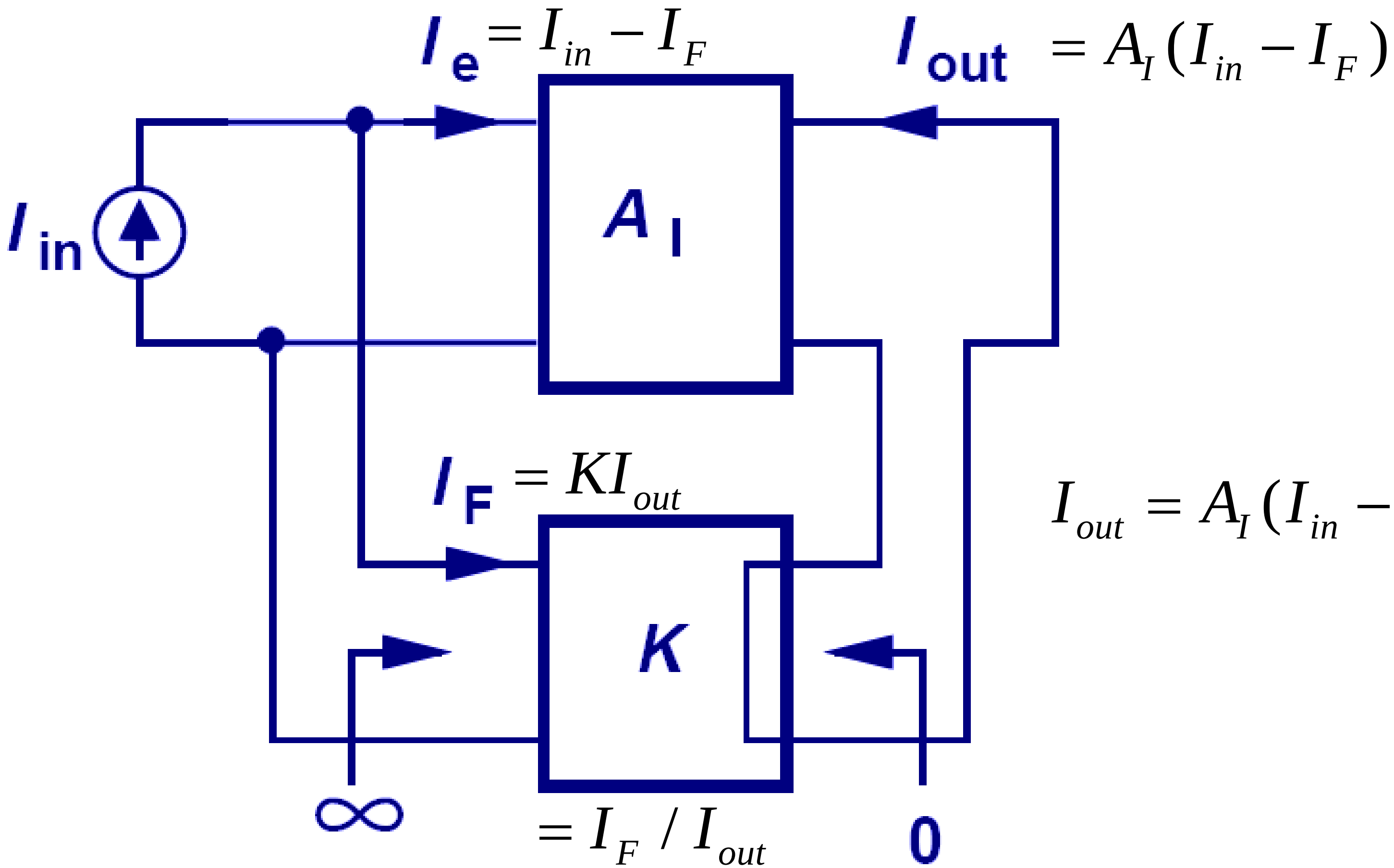
Wrong Technique for Measuring Output Impedance



Output current sensing

- **If we want to measure the output impedance of a I-V closed-loop feedback topology directly, we have to place V_X in series with K and R_{out}. Otherwise, the feedback will be disturbed.**

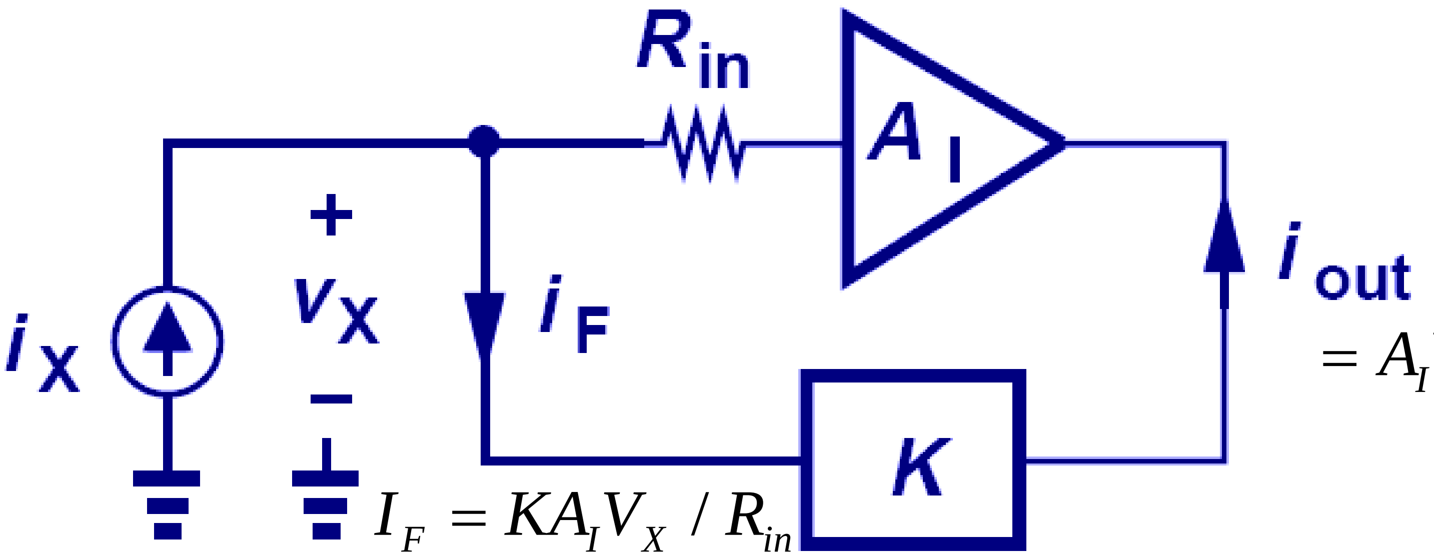
Current-Current Feedback



$$I_{out} = A_I(I_{in} - KI_{out})$$

$$\frac{I_{out}}{I_{in}} = \frac{A_I}{1 + KA_I}$$

Input Impedance of I-I Feedback



$$i_{out} = A_I V_X / R_{in}$$

$$I_F = K A_I V_X / R_{in}$$

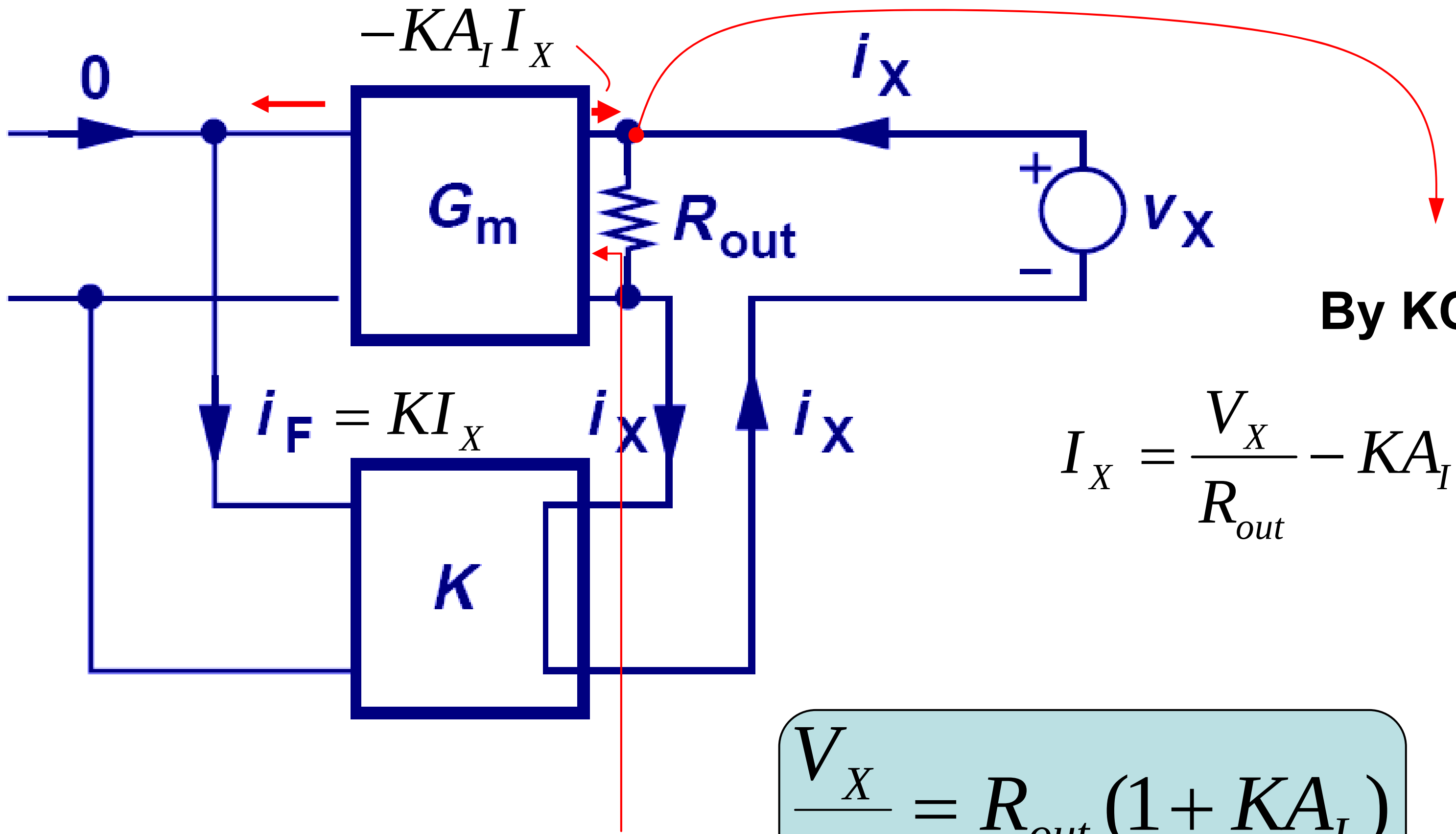
$$I_X = \frac{V_X}{R_{in}} + I_F$$

$$= \frac{V_X}{R_{in}} + K A_I \frac{V_X}{R_{in}}$$

$$\boxed{\frac{V_X}{I_X} = \frac{R_{in}}{1 + K A_I}}$$

- A better current sensor.

Output Impedance of I-I Feedback



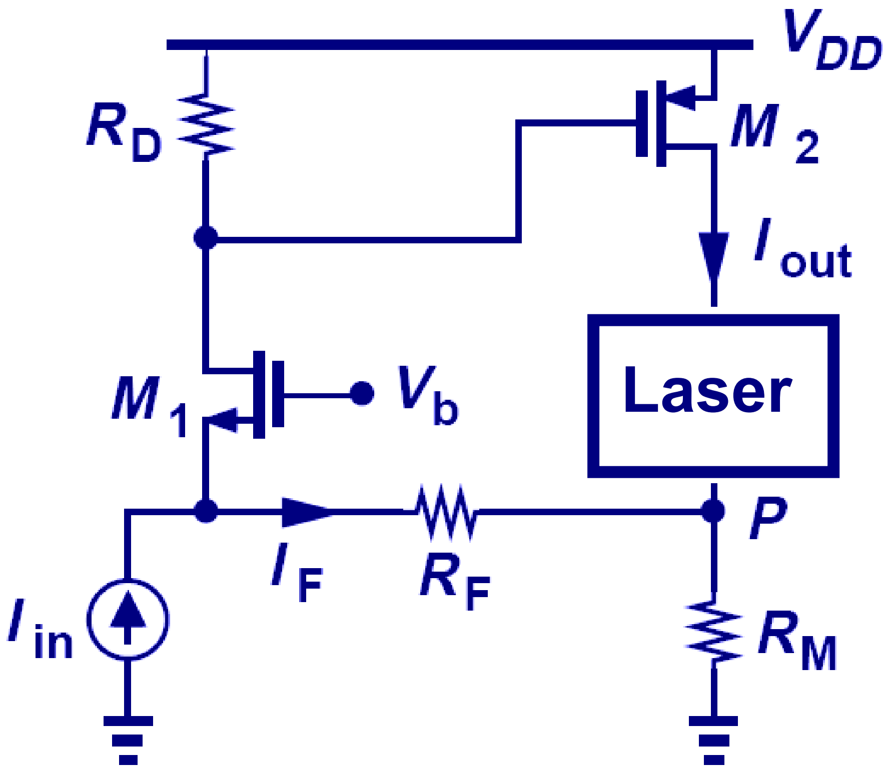
By KCL

$$I_X = \frac{V_X}{R_{out}} - KA_I I_X$$

$$\frac{V_X}{I_X} = R_{out}(1 + KA_I)$$

- **A better current source.**

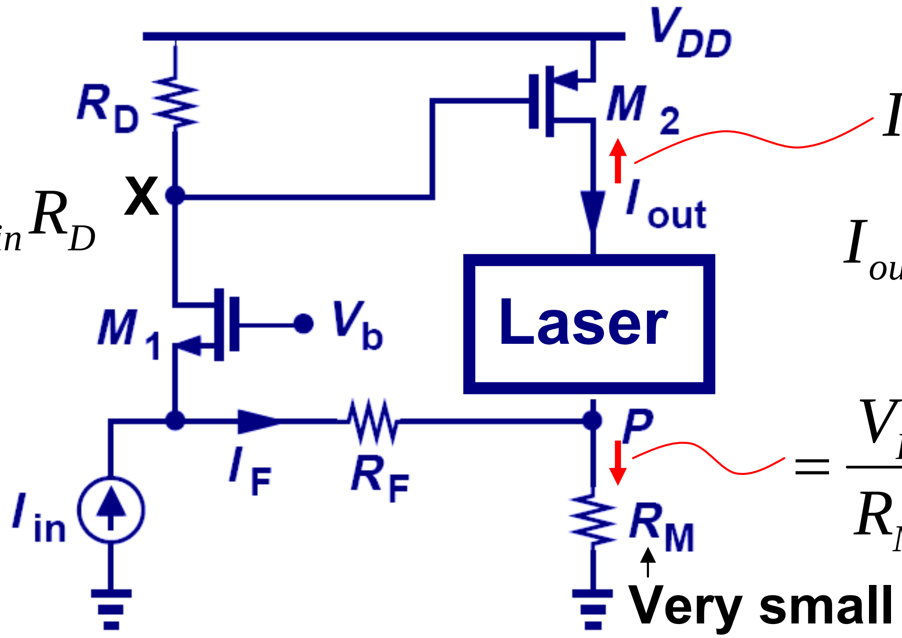
Example 12.24: Test of Negative Feedback



$$I_{in}\uparrow \Rightarrow V_{D1}\uparrow, I_{out}\downarrow \Rightarrow V_P\downarrow, I_F\uparrow \Rightarrow V_{D1}\downarrow, I_{out}\uparrow$$

Negative Feedback

Example 12.24: I-I Negative Feedback



$$V_X = I_{in} R_D$$

$$I_{D2} = g_{m2} V_X$$

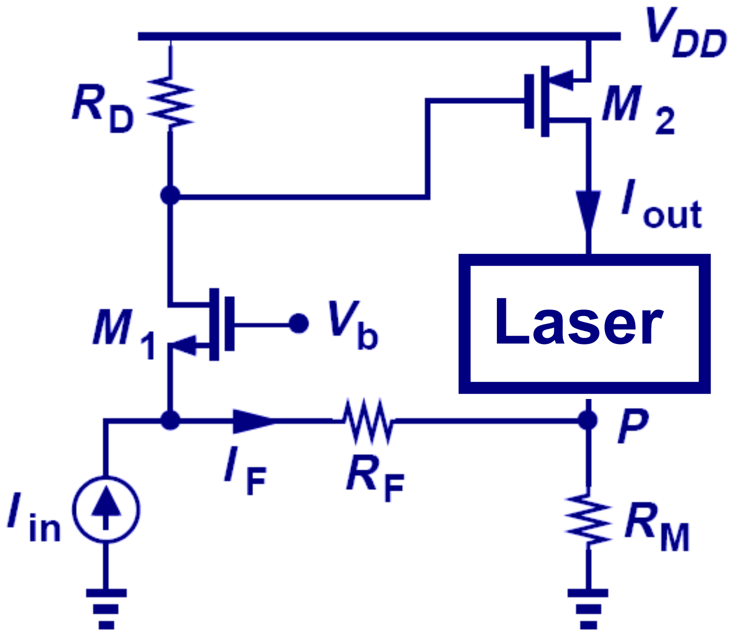
$$I_{out} = -g_{m2} V_X = -g_{m2} R_D I_{in}$$

$$= \frac{V_P}{R_M} \qquad V_P = I_{out} R_M$$

$$A_I \big|_{open} = \frac{I_{out}}{I_{in}} = \frac{-g_{m2} V_X}{I_{in}} = \frac{-g_{m2} R_D I_{in}}{I_{in}} = -g_{m2} R_D$$

$$K = \frac{I_F}{I_{out}} = \frac{-V_P}{R_F} \cdot \frac{1}{I_{out}} = -\frac{R_M}{R_F}$$

Example: I-I Negative Feedback



$$\because \frac{I_{out}}{I_{in}} = \frac{A_I}{1+KA_I}$$

$$\because \frac{V_X}{I_X} = \frac{R_{in}}{1+KA_I}$$

$$\because \frac{V_X}{I_X} = R_{out}(1+KA_I)$$

$$A_I \mid_{closed} = \frac{-g_{m2}R_D}{1+g_{m2}R_D(R_M / R_F)}$$

$$R_{in} \mid_{closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1+g_{m2}R_D(R_M / R_F)}$$

$$R_{out} \mid_{closed} = r_{O2}[1+g_{m2}R_D(R_M / R_F)]$$

Summary

- Determine whether the returned quantity enhances or opposes the original change.

Negative feedback — **Positive feedback**

Voltage-Voltage

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{A_0}{1+KA_0}} \qquad \frac{V_{in}}{I_{in}} = R_{in}(1+A_0K)$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{(1+KA_0)}$$

Voltage-Current

$$\boxed{\frac{V_{out}}{I_{in}} = \frac{R_O}{1+KR_O}} \qquad \frac{V_X}{I_X} = \frac{R_{in}}{1+R_0K}$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{1+R_0K}$$

Current-Voltage

$$\boxed{\frac{I_{out}}{V_{in}} = \frac{G_m}{1+KG_m}} \qquad \frac{V_{in}}{I_{in}} = R_{in}(1+KG_m)$$

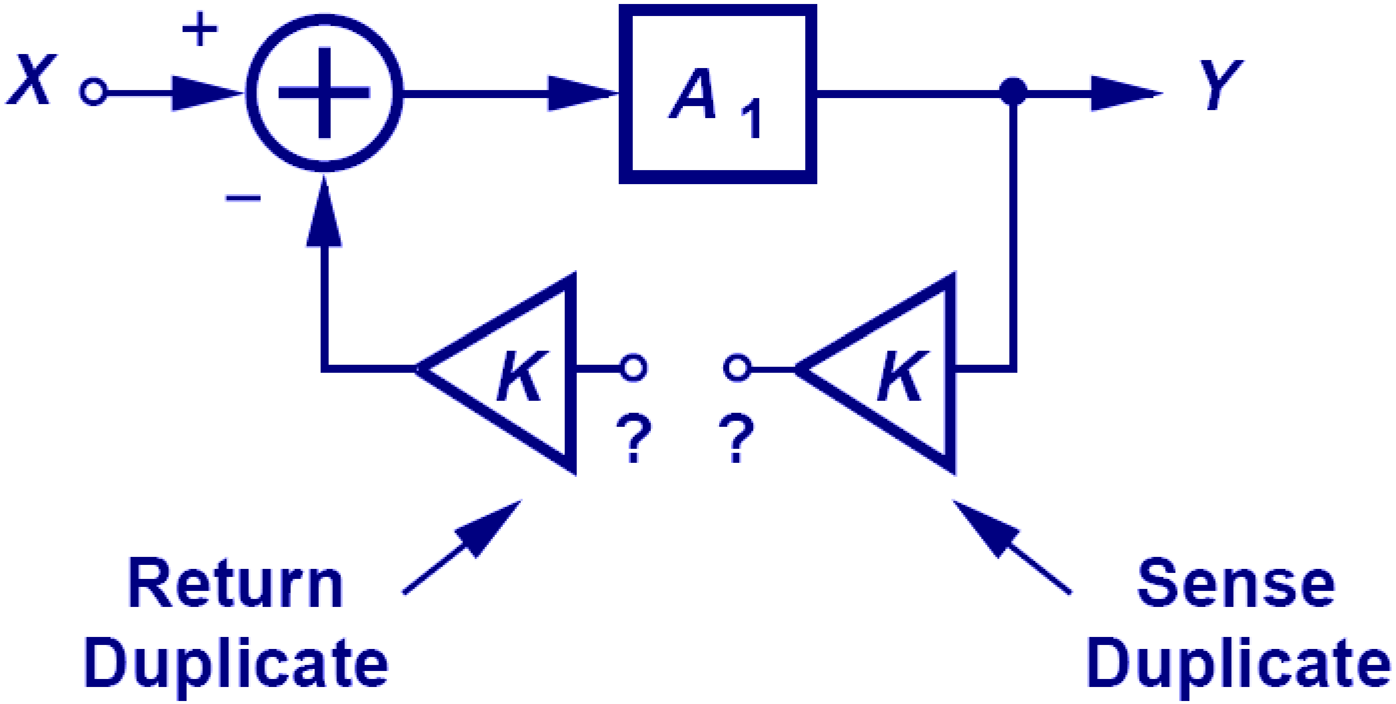
$$\frac{V_X}{I_X} = R_{out}(1+KG_m)$$

Current-Current

$$\boxed{\frac{I_{out}}{I_{in}} = \frac{A_I}{1+KA_I}} \qquad \frac{V_X}{I_X} = \frac{R_{in}}{1+KA_I}$$

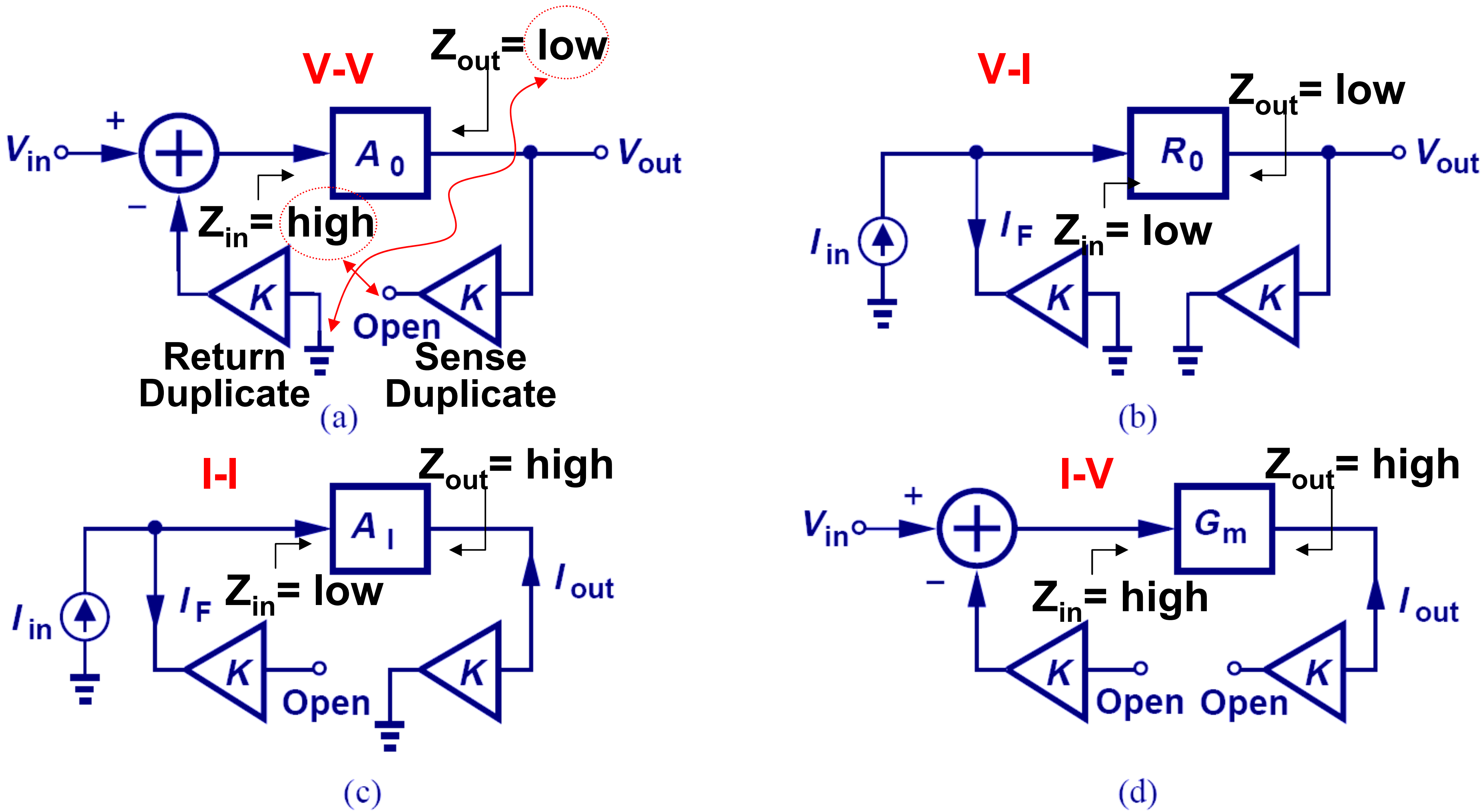
$$\frac{V_X}{I_X} = R_{out}(1+KA_I)$$

How to Break a Loop



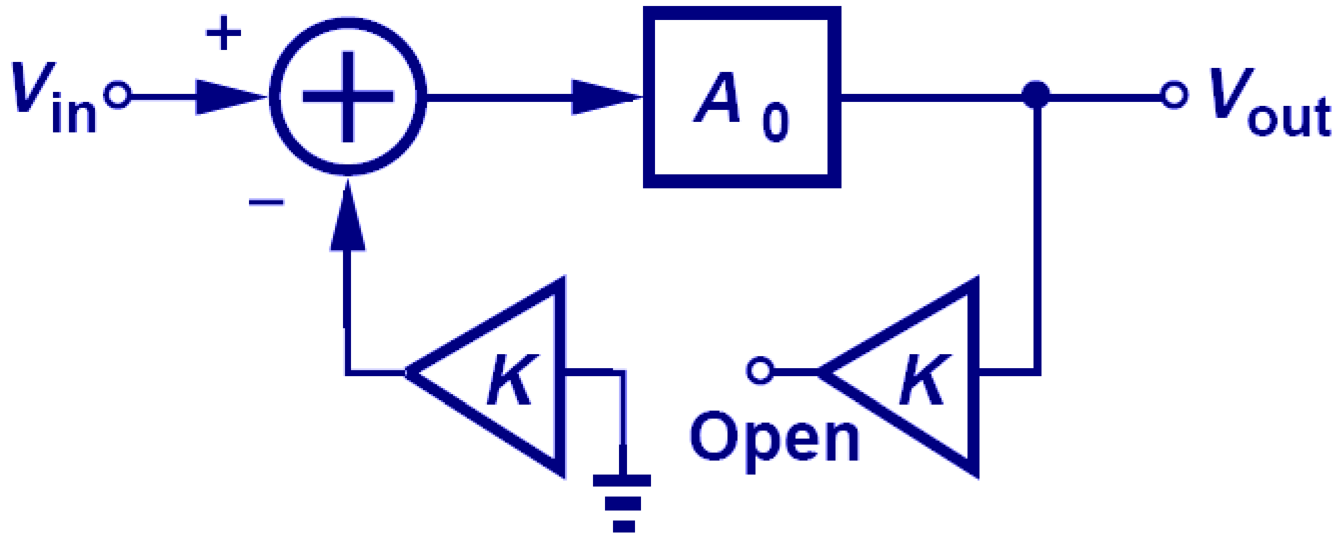
- The correct way of breaking a loop is such that the loop does not know it has been broken. Therefore, we need to present the feedback network to both the input and the output of the feedforward amplifier.

Rules for Breaking the Loop of Amplifier Types



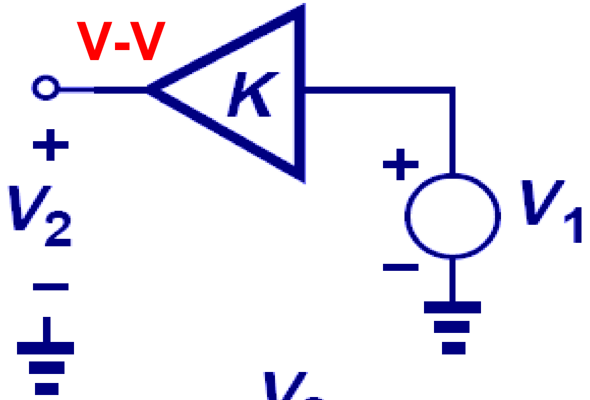
Intuitive Understanding of these Rules

Voltage-Voltage Feedback

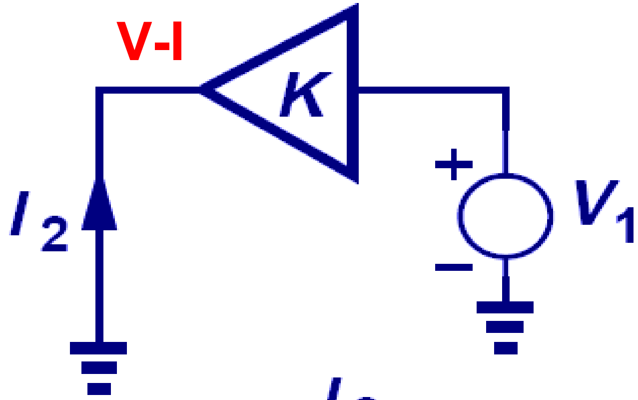


- Since ideally, the input of the feedback network sees zero impedance (Z_{out} of an ideal voltage source), the return replicate needs to be grounded. Similarly, the output of the feedback network sees an infinite impedance (Z_{in} of an ideal voltage sensor), the sense replicate needs to be open.
- Similar ideas apply to the other types.

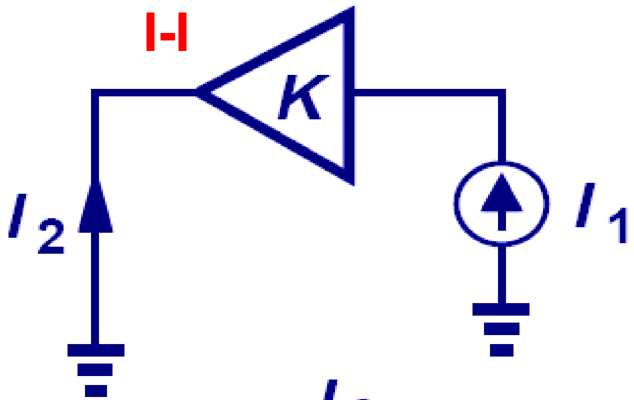
Rules for Calculating Feedback Factor



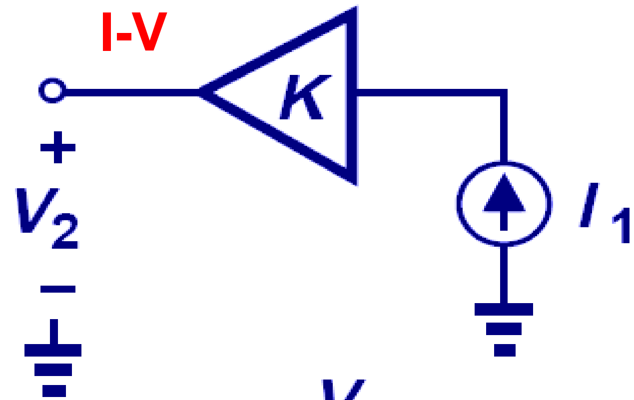
$$K = \frac{V_2}{V_1}$$



$$K = \frac{I_2}{V_1}$$



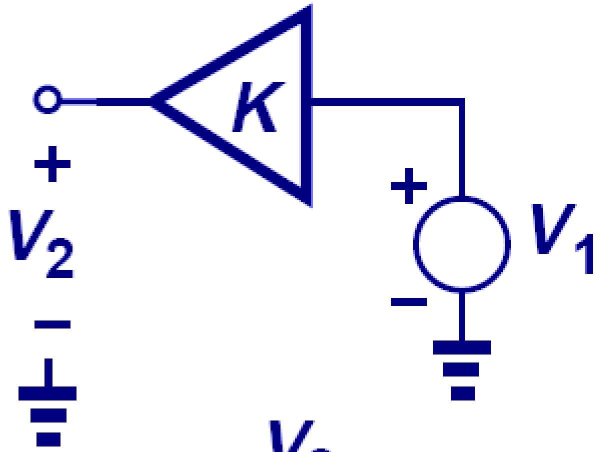
$$K = \frac{I_2}{I_1}$$



$$K = \frac{V_2}{I_1}$$

Intuitive Understanding of these Rules

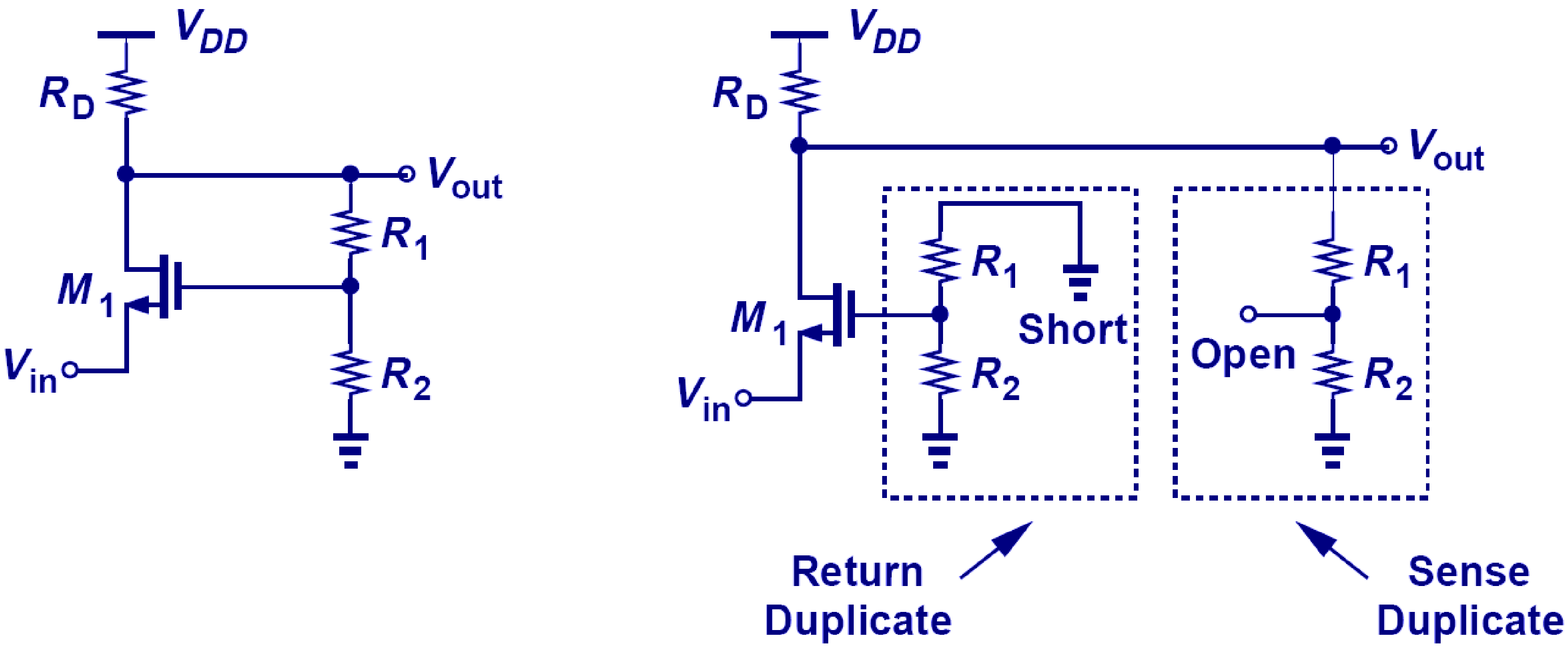
Voltage-Voltage Feedback



$$K = \frac{V_2}{V_1}$$

- Since the feedback senses voltage, the input of the feedback is a voltage source. Moreover, since the return quantity is also voltage, the output of the feedback is left open (a short means the output is always zero).
- Similar ideas apply to the other types.

Example 12.26: Breaking the Loop

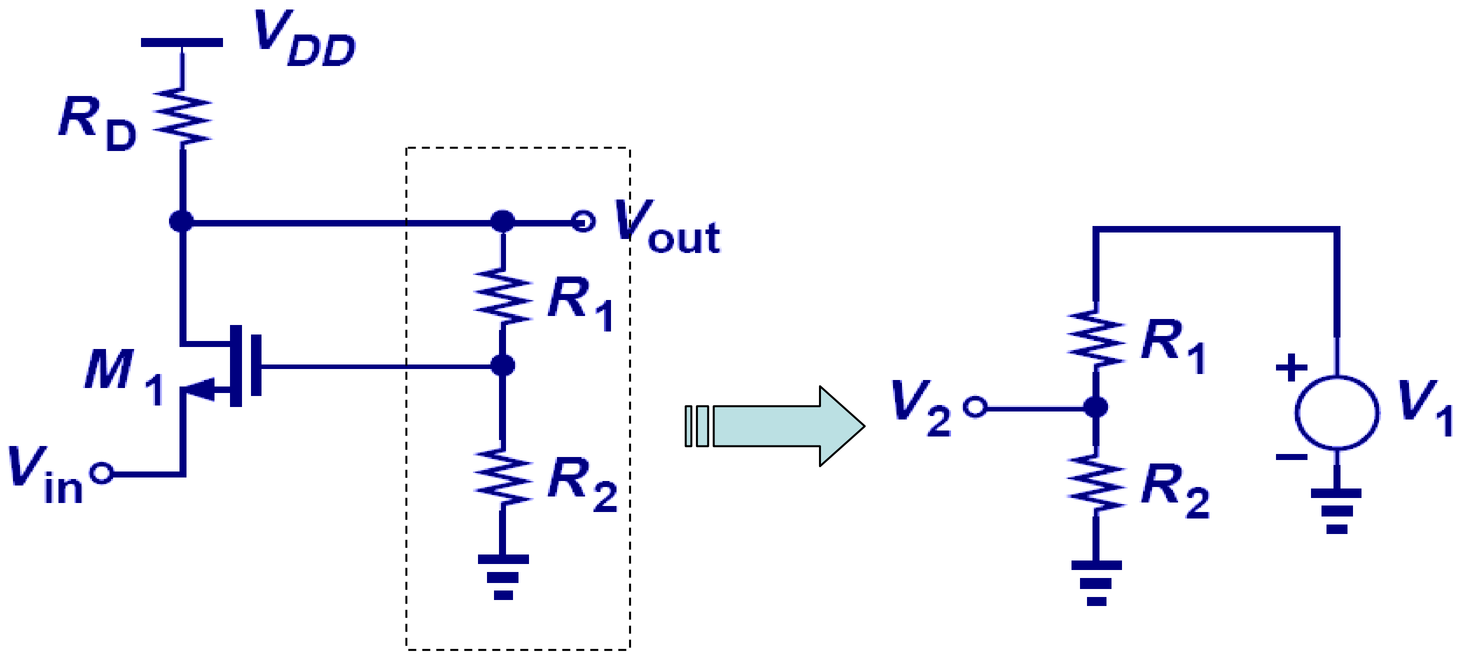


$$A_{v,open} = g_{m1}\left[R_D \parallel (R_1 + R_2)\right]$$

$$R_{in,open} = 1/g_{m1}$$

$$R_{out,open} = R_D \parallel (R_1 + R_2)$$

Example 12.26: Feedback Factor



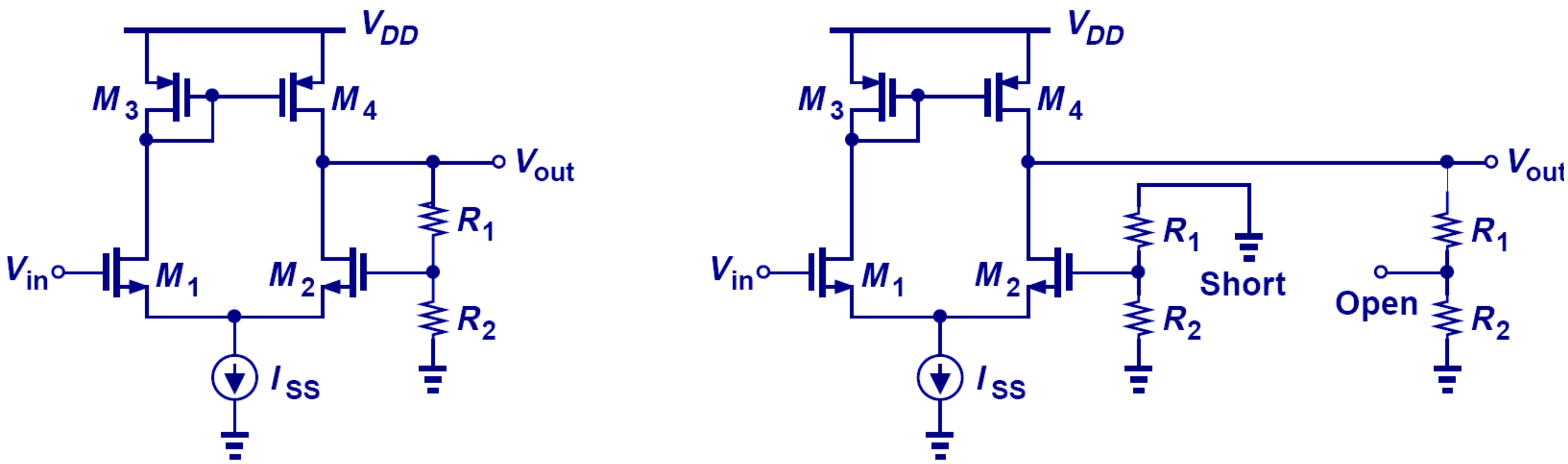
$$K = R_2/(R_1 + R_2)$$

$$A_{v,closed} = A_{v,open}/(1 + KA_{v,open})$$

$$R_{in,closed} = R_{in,open}(1 + KA_{v,open})$$

$$R_{out,closed} = R_{out,closed}/(1 + KA_{v,open})$$

Example 12.27: Breaking the Loop

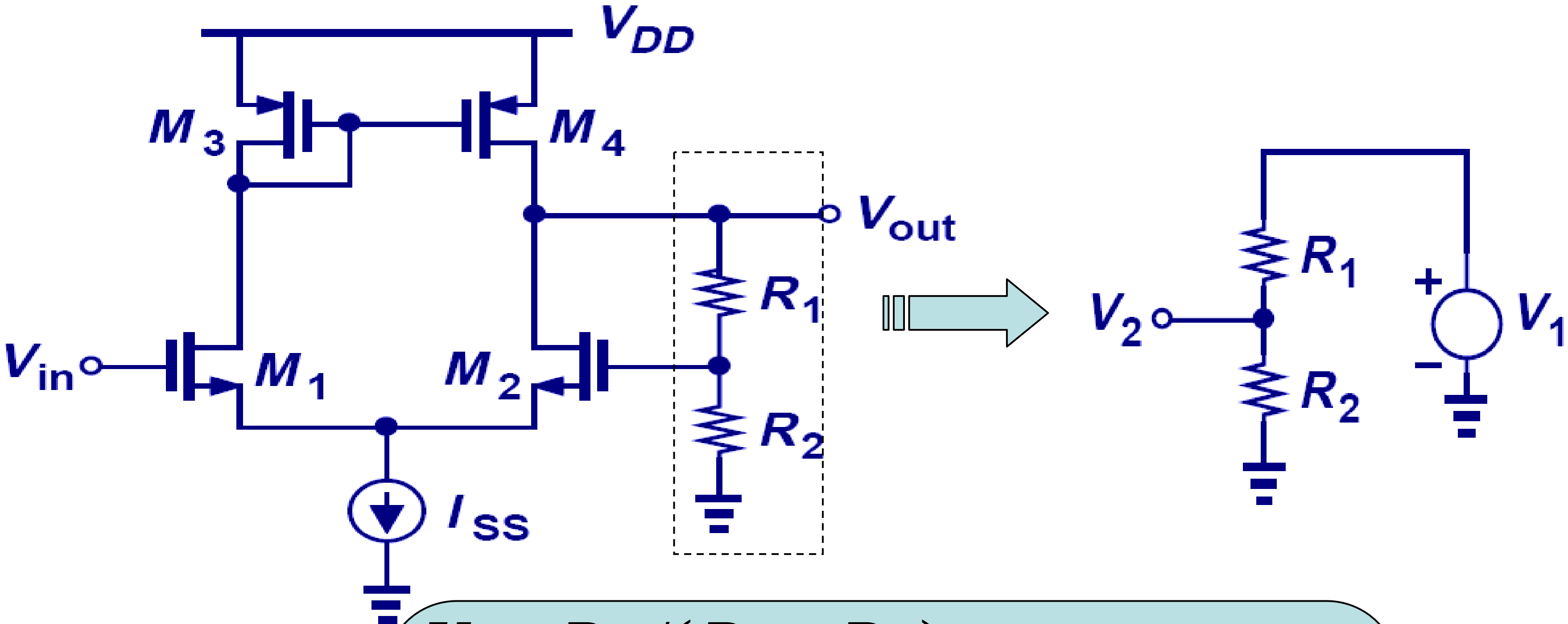


$$A_{v,open} = g_{mN}\left[r_{ON} \,||\, r_{OP} \,||\, \left(R_1 + R_2\right)\right]$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{ON} \,||\, r_{OP} \,||\, \left(R_1 + R_2\right)$$

Example 12.27: Feedback Factor



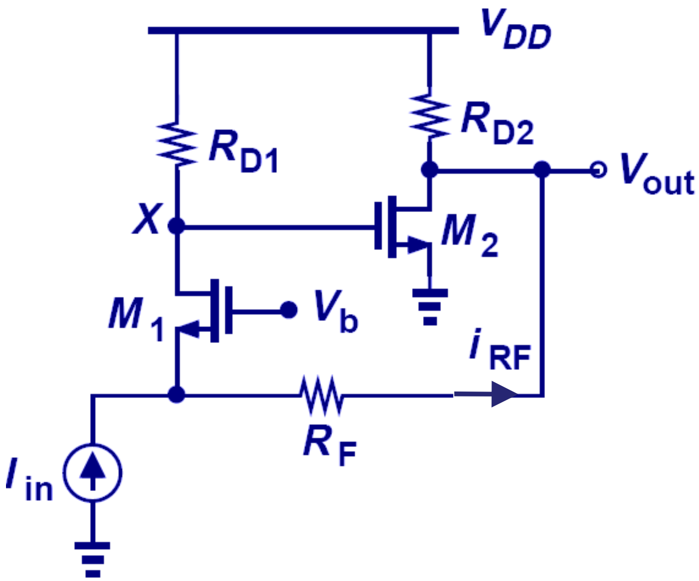
$$K = R_2/(R_1 + R_2)$$

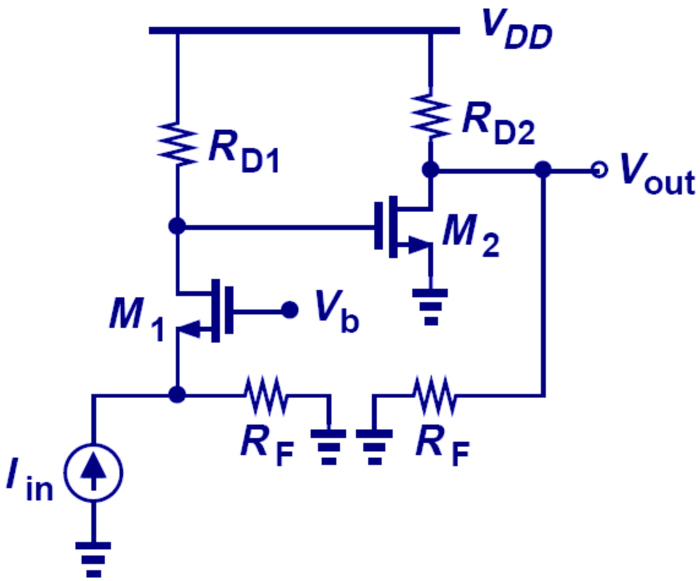
$$A_{v,closed} = A_{v,open}/(1 + KA_{v,open})$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = R_{out,open}/(1 + KA_{v,open})$$

Example 12.29: Breaking the Loop



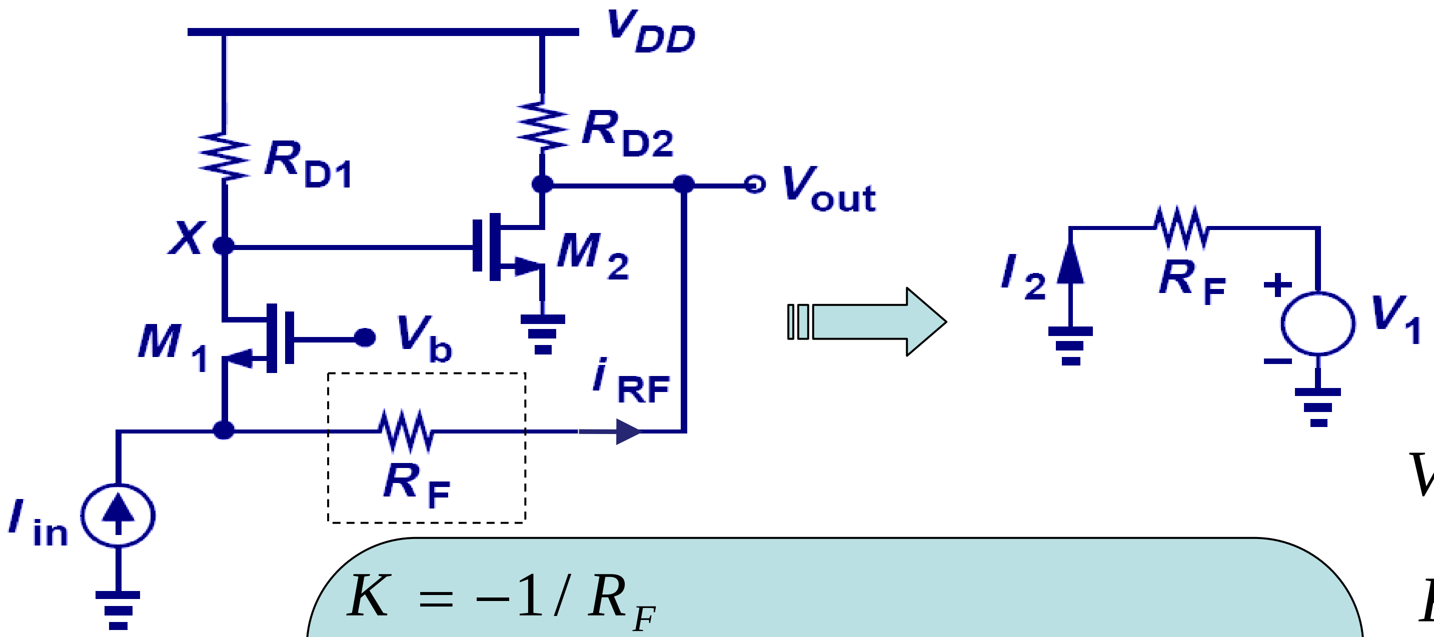


$$\frac{V_{out}}{I_{in}}\bigg|_{open} = \frac{R_F R_{D1}}{R_F + \dfrac{1}{g_{m1}}} \cdot \left[-g_{m2}\left(R_{D2} \parallel R_F\right)\right]$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel R_F$$

$$R_{out,open} = R_{D2} \parallel R_F$$

Example 12.29: Feedback Factor



$$V_1 = -I_2 R_F$$

$$K = \frac{I_2}{V_1}$$

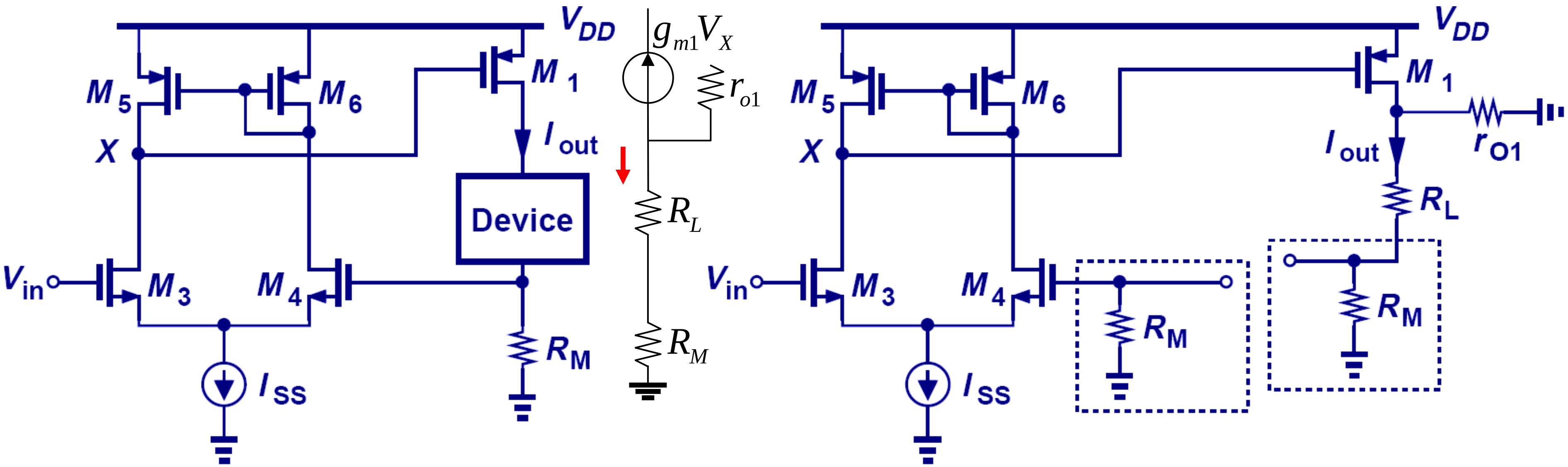
$$K = -1/R_F$$

$$\frac{V_{out}}{I_{in}}\Big|_{closed} = \frac{V_{out}}{I_{in}}\Big|_{open} \Big/ \left(1 + K\frac{V_{out}}{I_{in}}\Big|_{open}\right)$$

$$R_{in,closed} = R_{in,open} \Big/ \left(1 + K\frac{V_{out}}{I_{in}}\Big|_{open}\right)$$

$$R_{out,closed} = R_{out,open} \Big/ \left(1 + K\frac{V_{out}}{I_{in}}\Big|_{open}\right)$$

Example 12.30: Breaking the Loop



$$\frac{V_X}{V_{in}} = -g_{m3}\left(r_{o3} \,||\, r_{o5}\right)$$

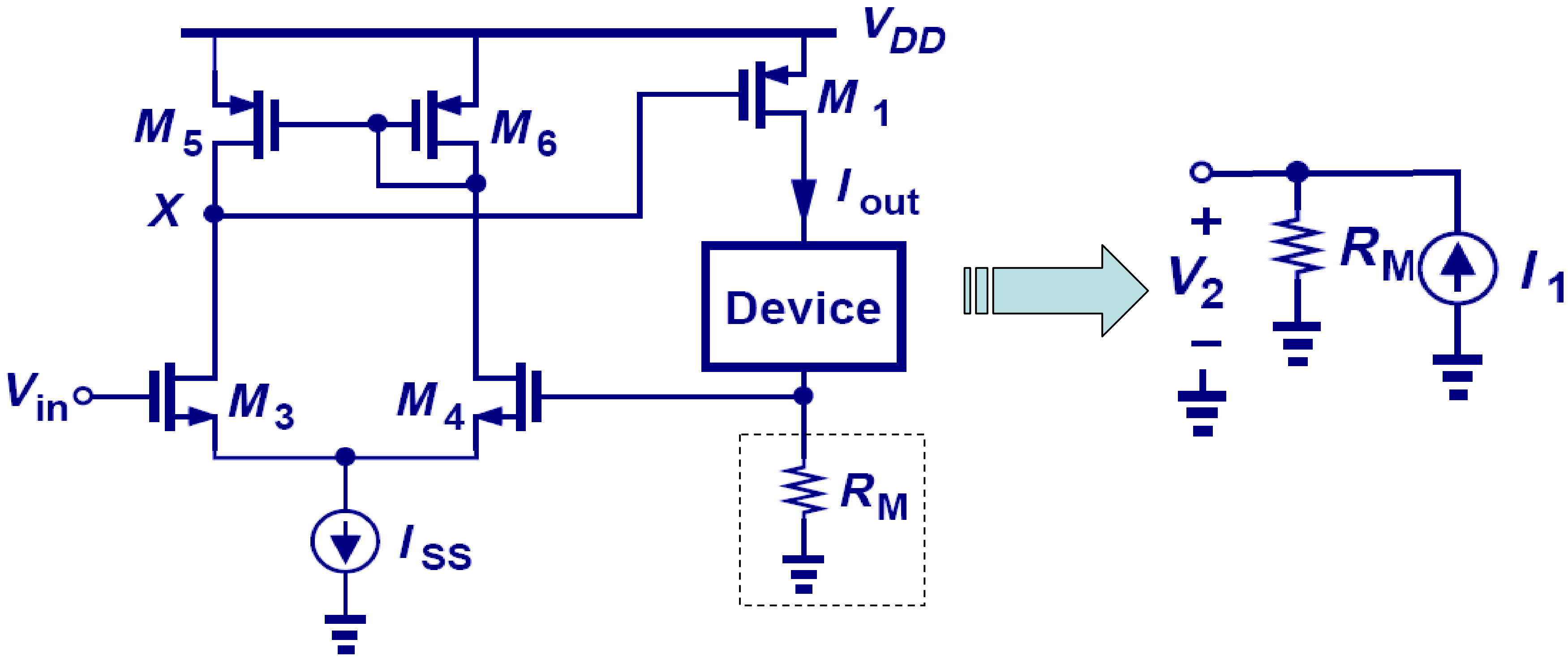
$$I_{out} = -\frac{r_{o1}\left(g_{m1}V_X\right)}{r_{o1} + R_L + R_M}$$

$$\frac{I_{out}}{V_{in}}\Big|_{open} = \frac{g_{m3}\left(r_{O3} \,||\, r_{O5}\right)g_{m1}r_{O1}}{r_{O1} + R_L + R_M}$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{O1} + R_M$$

Example 12.30: Feedback Factor



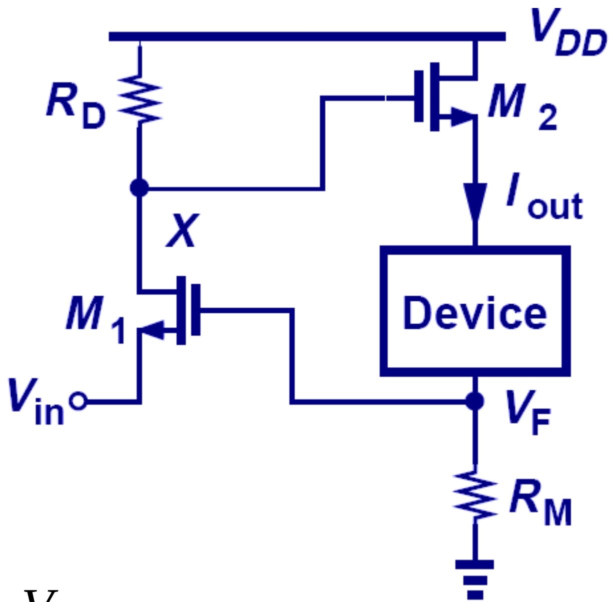
$$K = R_M = V_2 / I_1$$

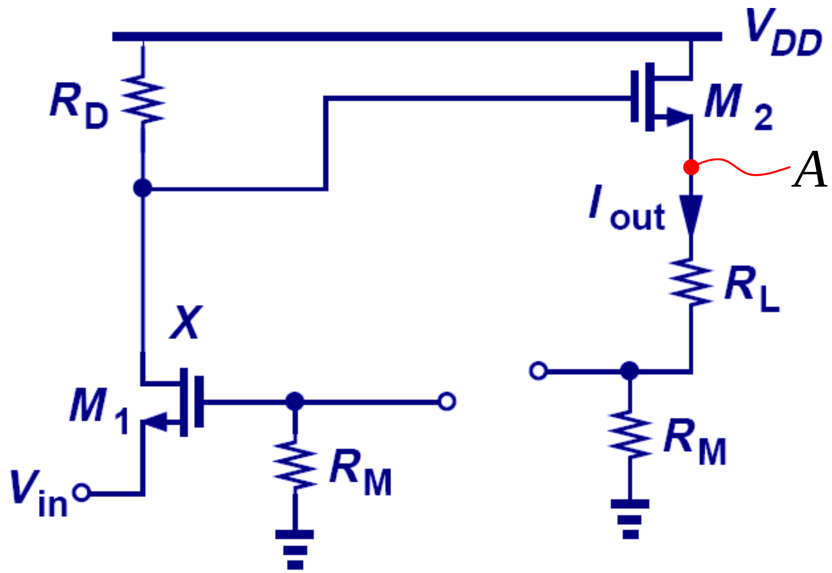
$$(I_{out} / V_{in}|_{closed}) = (I_{out} / V_{in}|_{open}) / [1 + K(I_{out} / V_{in})|_{open}]$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = R_{out,open}[1 + K(I_{out} / V_{in})|_{open}]$$

Example 12.31: Breaking the Loop



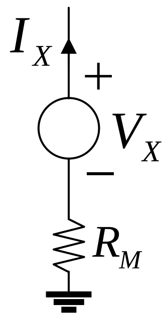


$$\frac{V_X}{V_{in}} = g_{m1}R_D$$

$$\frac{V_A}{V_X} = \frac{R_L + R_M}{R_L + R_M + 1/g_{m2}}$$

$$I_{out} = \frac{V_A}{R_L + R_M}$$

$$R_{out}\big|_{open} = ?$$

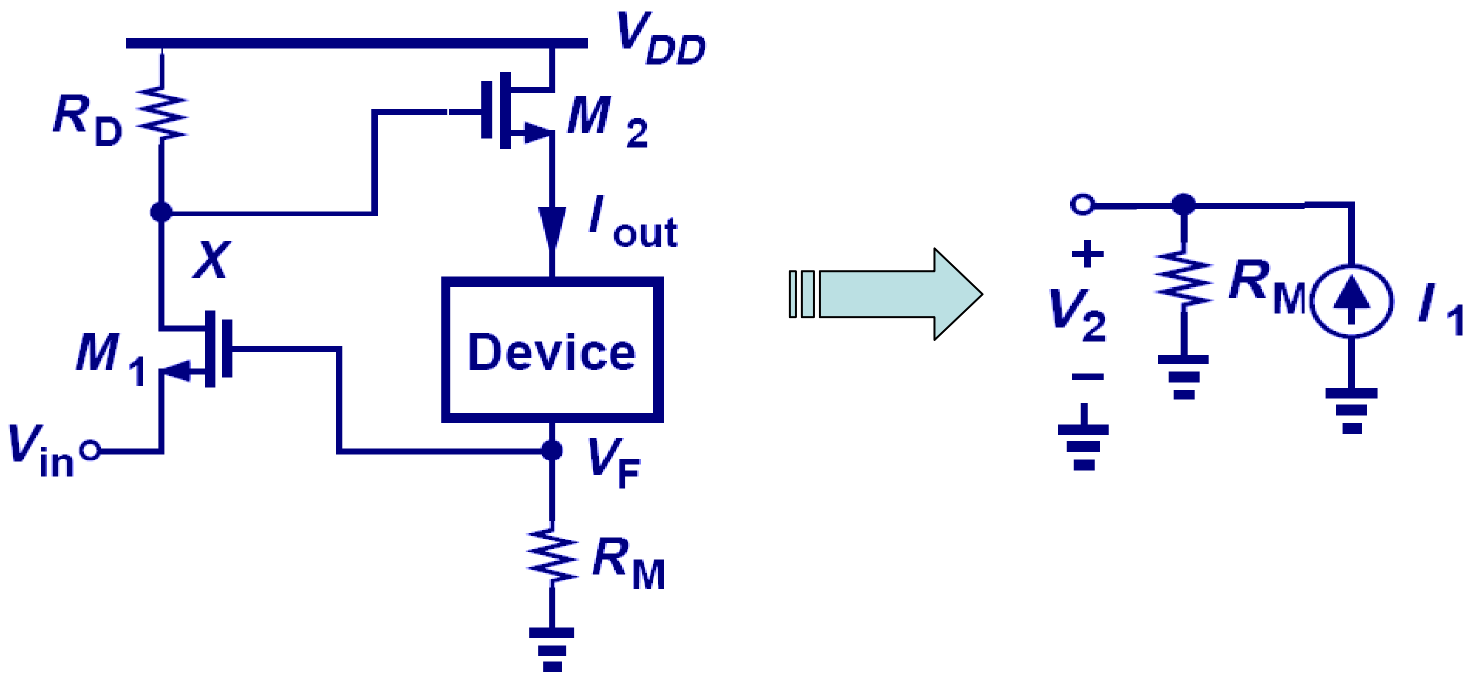


$$\frac{I_{out}}{V_{in}}\bigg|_{open} = \frac{g_{m1}R_D}{R_L + R_M + 1/g_{m2}}$$

$$R_{in,open} = 1/g_{m1}$$

$$R_{out,open} = (1/g_{m2}) + R_M$$

Example 12.31: Feedback Factor



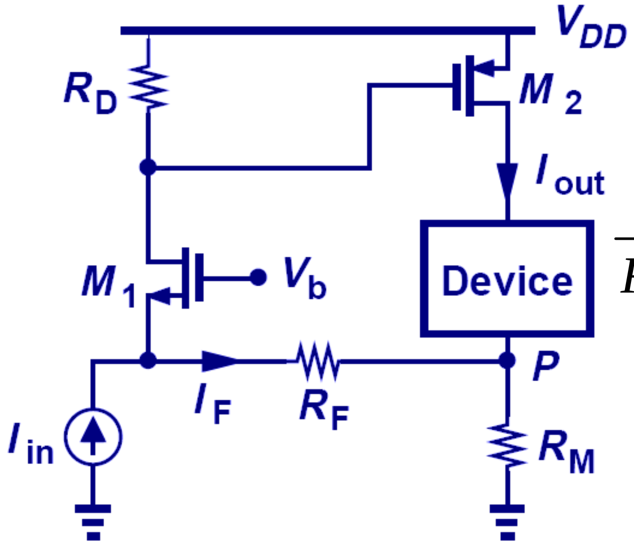
$$K = R_M$$

$$(I_{out}/V_{in}|_{closed}) = (I_{out}/V_{in}|_{open})/[1 + K(I_{out}/V_{in})|_{open}]$$

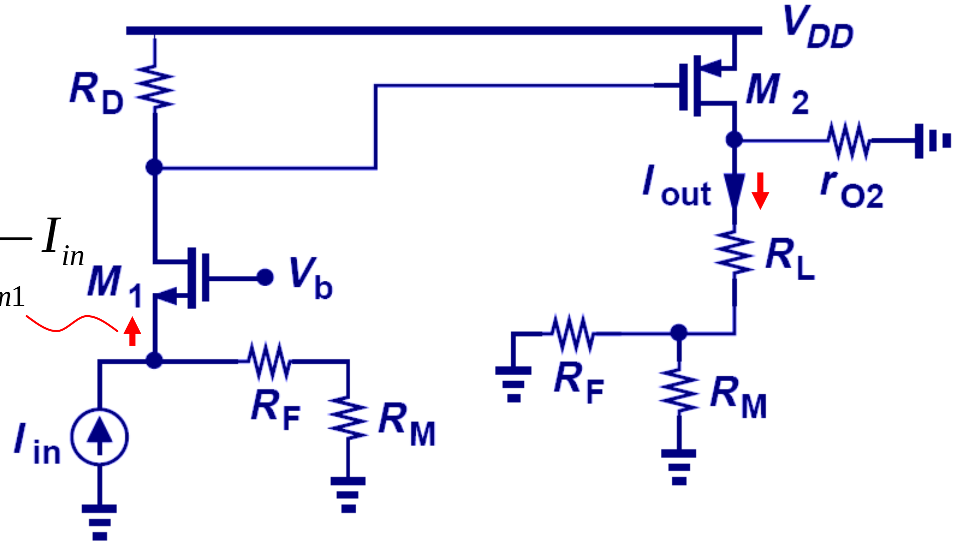
$$R_{in,closed} = R_{in,open}[1 + K(I_{out}/V_{in})|_{open}]$$

$$R_{out,closed} = R_{out,open}[1 + K(I_{out}/V_{in})|_{open}]$$

Example 12.32: Breaking the Loop



$$\frac{R_F + R_M}{R_F + R_M + 1/g_{m1}} I_{in}$$

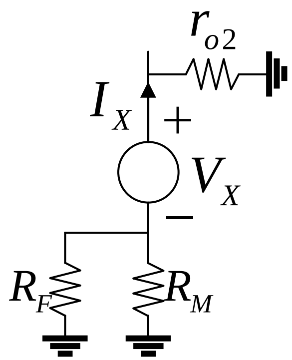


$$A_{I,open} = \frac{(R_F + R_M)R_D}{R_F + R_M + \dfrac{1}{g_{m1}}} \cdot \frac{-g_{m2} r_{O2}}{r_{O2} + R_L + R_M \parallel R_F}$$

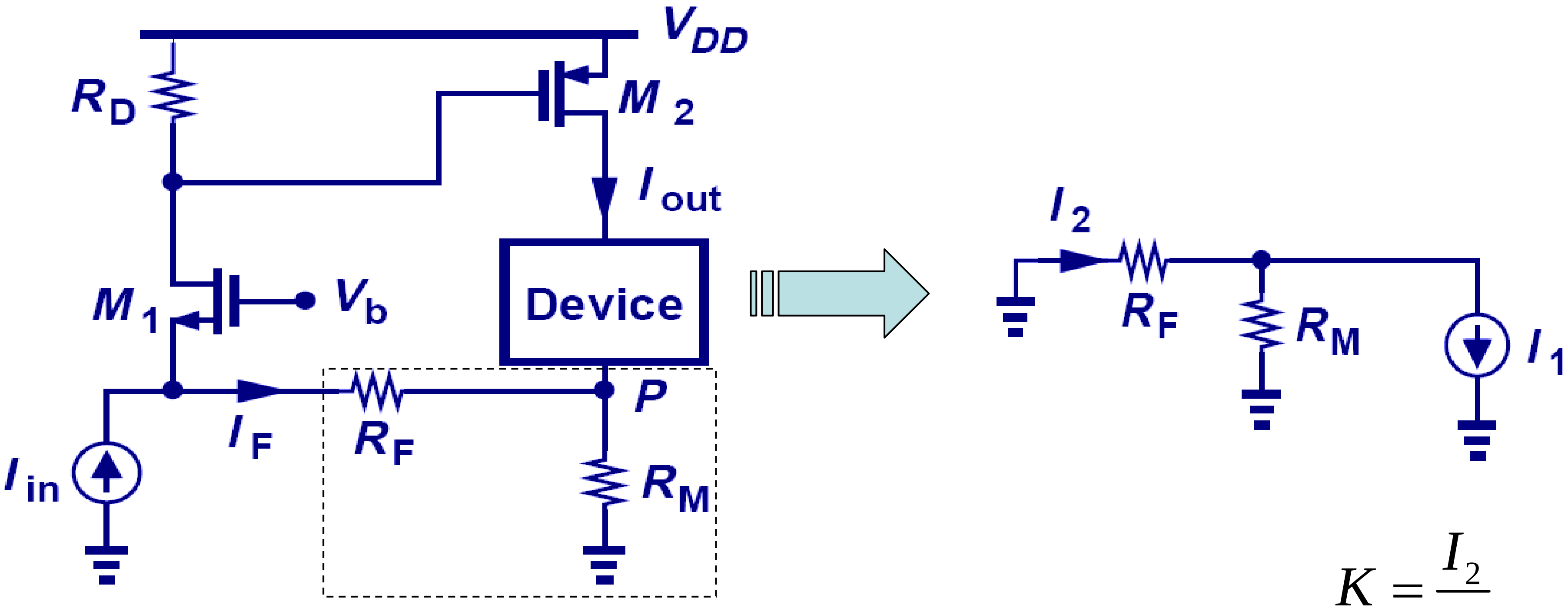
$$R_{in,open} = \frac{1}{g_{m1}} \parallel (R_F + R_M)$$

$$R_{out,open} = r_{O2} + R_F \parallel R_M$$

$$R_{out}\big|_{open} = ?$$



Example 12.32: Feedback Factor



$$K = \frac{I_2}{I_1}$$

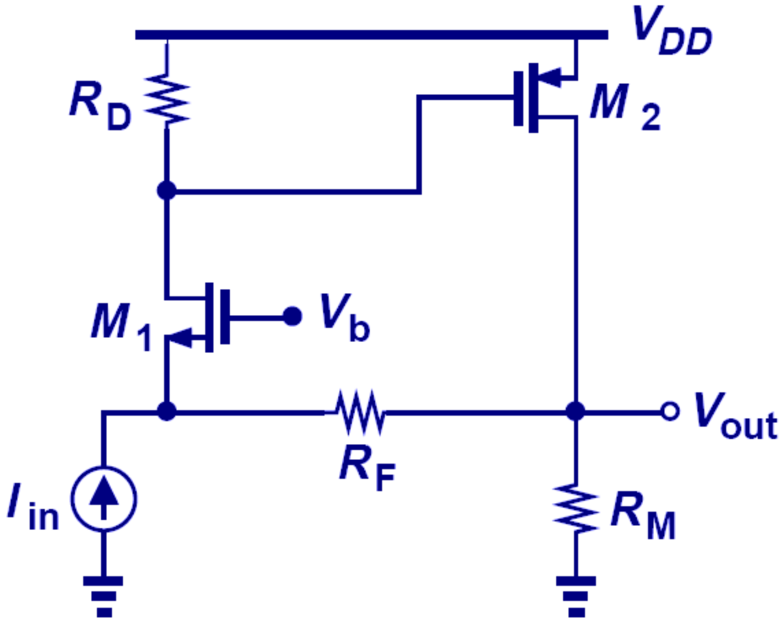
$$K = -R_M / (R_F + R_M)$$

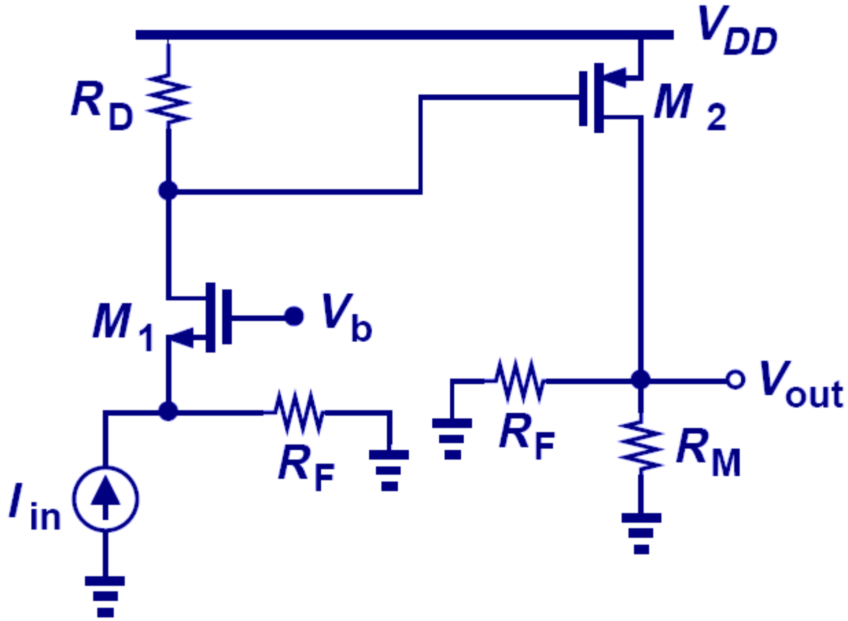
$$A_{I,closed} = A_{I,open} / (1 + KA_{I,open})$$

$$R_{in,closed} = R_{in,open} / (1 + KA_{I,open})$$

$$R_{out,closed} = R_{out,open} (1 + KA_{I,open})$$

Example 12.33: Breaking the Loop



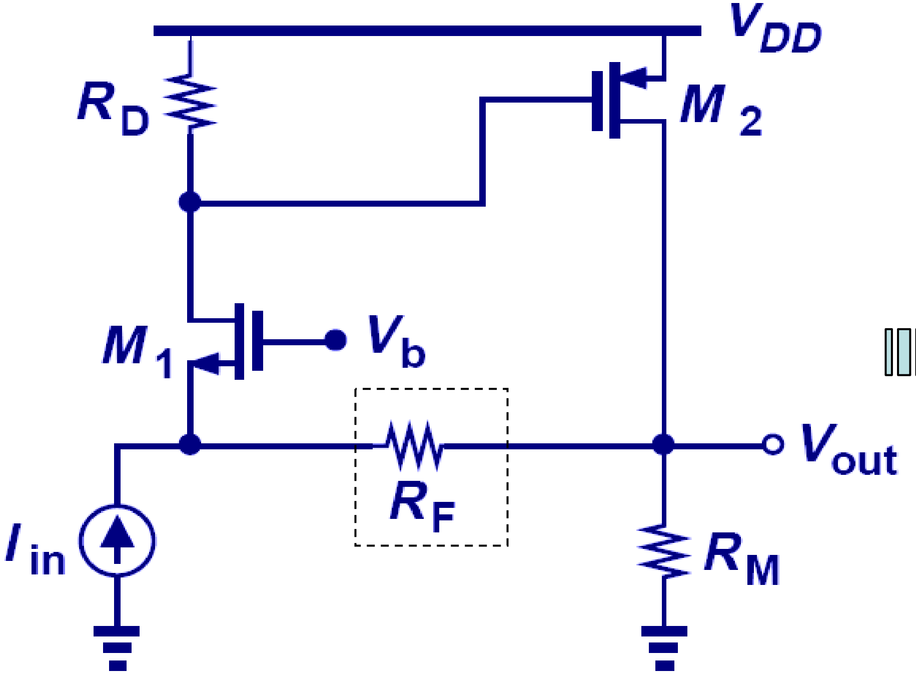


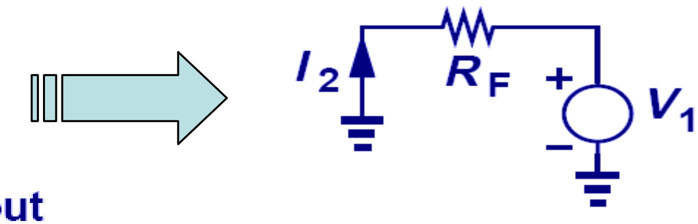
$$\frac{V_{out}}{I_{in}}\Big|_{open} = \frac{R_F R_D}{R_F + 1/g_{m1}}[-g_{m2}(R_F \parallel R_M)]$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel R_F$$

$$R_{out,open} = R_F \parallel R_M$$

Example 12.33: Feedback Factor





$$K = \frac{I_2}{V_1}$$

$$K = -1/R_F$$

$$(V_{out}/I_{in})|_{closed} = (V_{out}/I_{in})|_{open} /[1+K(V_{out}/I_{in})|_{open}]$$

$$R_{in,closed} = R_{in,open} /[1+K(V_{out}/I_{in})|_{open}]$$

$$R_{out,closed} = R_{out,open} /[1+K(V_{out}/I_{in})|_{open}]$$

Summary

Current-Voltage

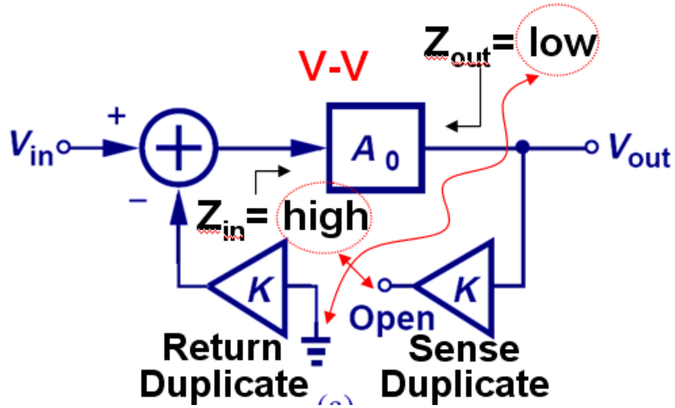
$$\boxed{\frac{I_{out}}{V_{in}} = \frac{G_m}{1+KG_m}} \qquad \frac{V_{in}}{I_{in}} = R_{in}(1+KG_m)$$

$$\frac{V_X}{I_X} = R_{out}(1+KG_m)$$

Current-Current

$$\boxed{\frac{I_{out}}{I_{in}} = \frac{A_I}{1+KA_I}} \qquad \frac{V_X}{I_X} = \frac{R_{in}}{1+KA_I}$$

$$\frac{V_X}{I_X} = R_{out}(1+KA_I)$$



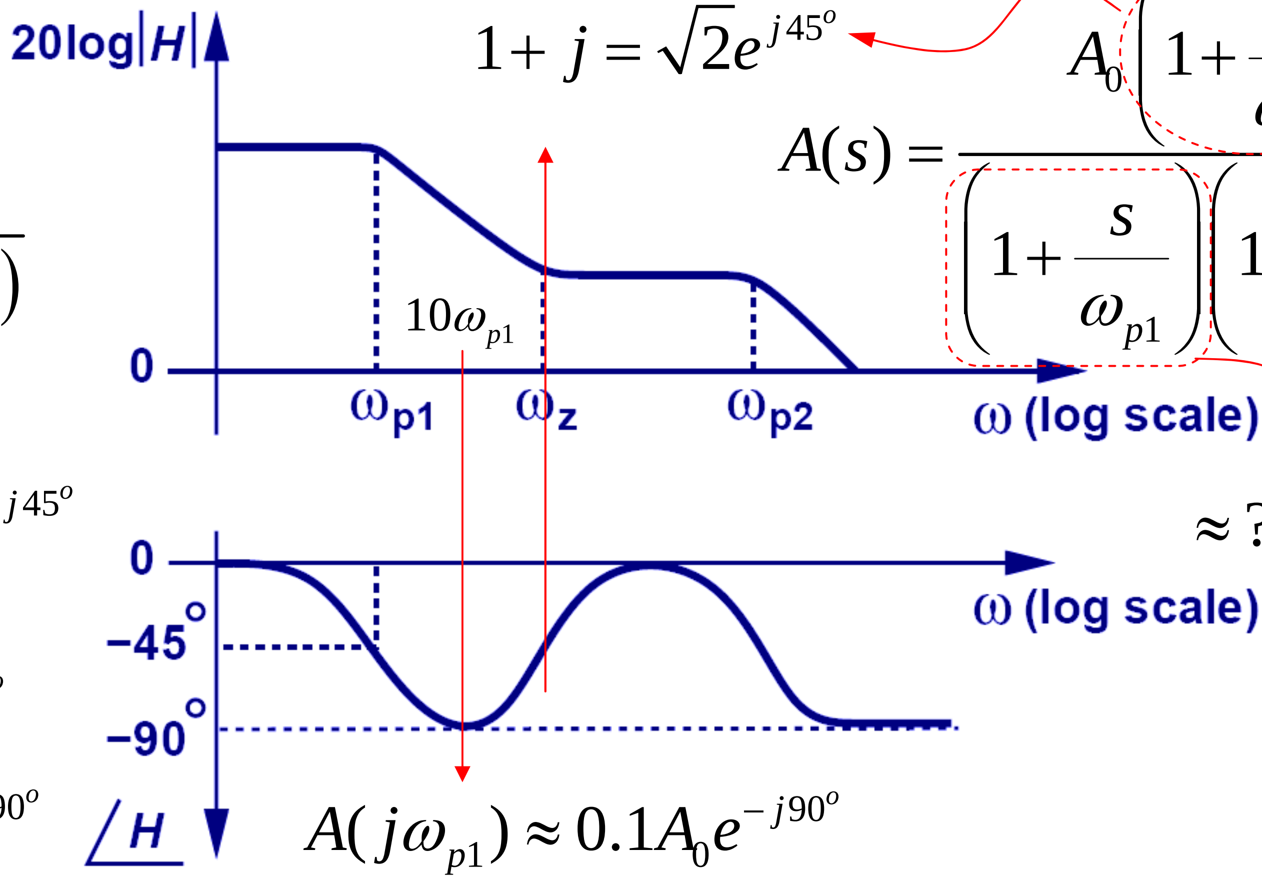
- Since ideally, the input of the feedback network sees zero impedance (Z_{out} of an ideal voltage source), the return replicate needs to be grounded. Similarly, the output of the feedback network sees an infinite impedance (Z_{in} of an ideal voltage sensor), the sense replicate needs to be open.
- Similar ideas apply to the other types.

Example 12.34: Phase Response

$$1+j=\sqrt{2}e^{j45^o}$$

$$A(s)=\frac{A_0\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

$$A(s)\big|_{\omega_{p1}}\approx\frac{A_0}{(1+j)}$$



$$\approx ?\,A_0e^{-j90^o}$$

$$A(j\omega_{p1})\approx\frac{A_0}{\sqrt{2}}e^{-j45^o}$$

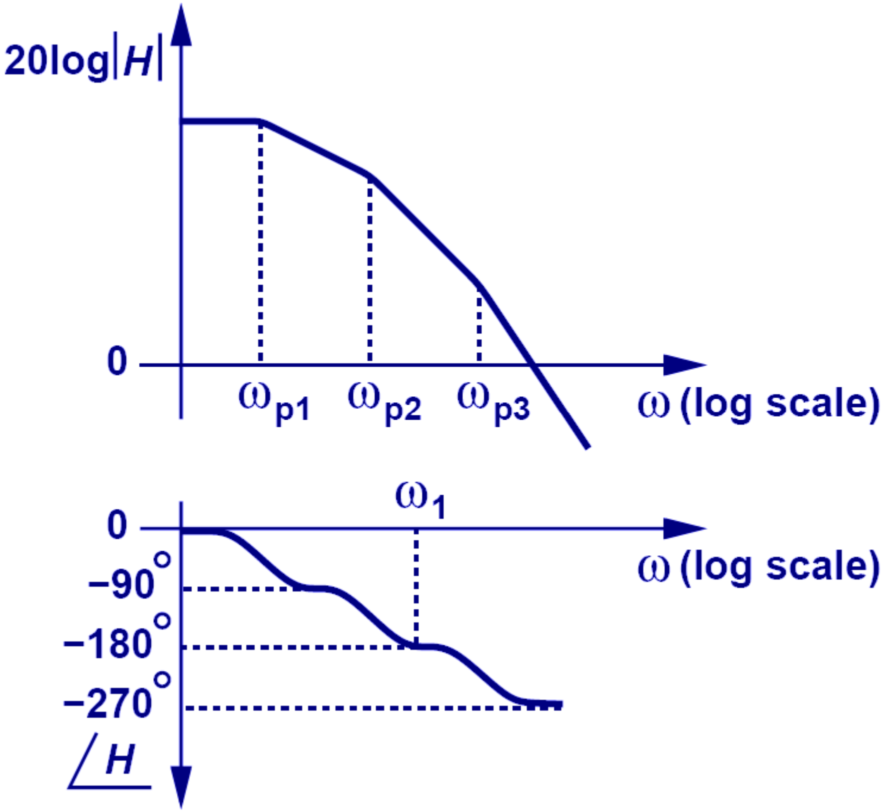
$$\because 1+j=\sqrt{2}e^{j45^o}$$

$$\because 1+j10\approx 10e^{j90^o}$$

$$A(j\omega_{p1})\approx 0.1A_0e^{-j90^o}$$

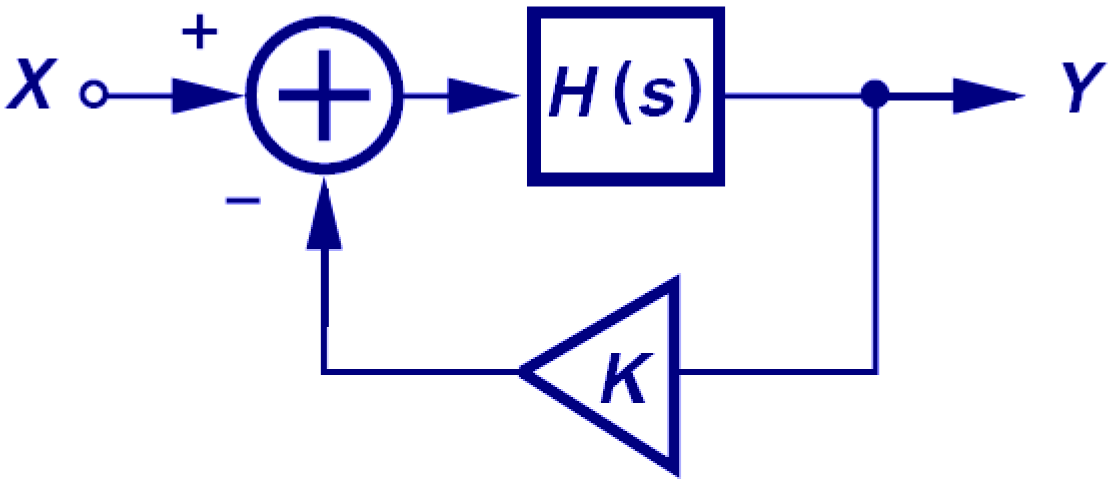
- As it can be seen, the phase of H(jω) starts to drop at 1/10 of the pole, hits -45° at the pole, and approaches -90° at 10 times the pole.

Example 12.35: Three-Pole System



- **For a three-pole system, a finite frequency produces a phase of -180°, which means an input signal that operates at this frequency will have its output inverted.**

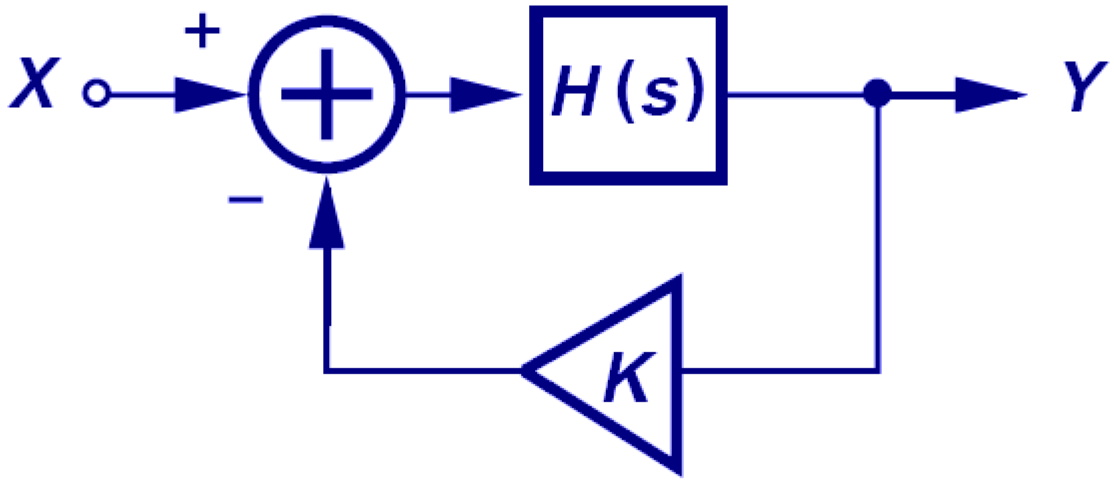
Instability of a Negative Feedback Loop



$$\frac{Y}{X}(s) = \frac{H(s)}{1 + KH(s)}$$

- Substitute $j\omega$ for s. If for a certain ω_1, $KH(j\omega_1)$ reaches -1, the closed loop gain becomes infinite. This implies for a very small input signal at ω_1, the output can be very large. Thus the system becomes unstable.

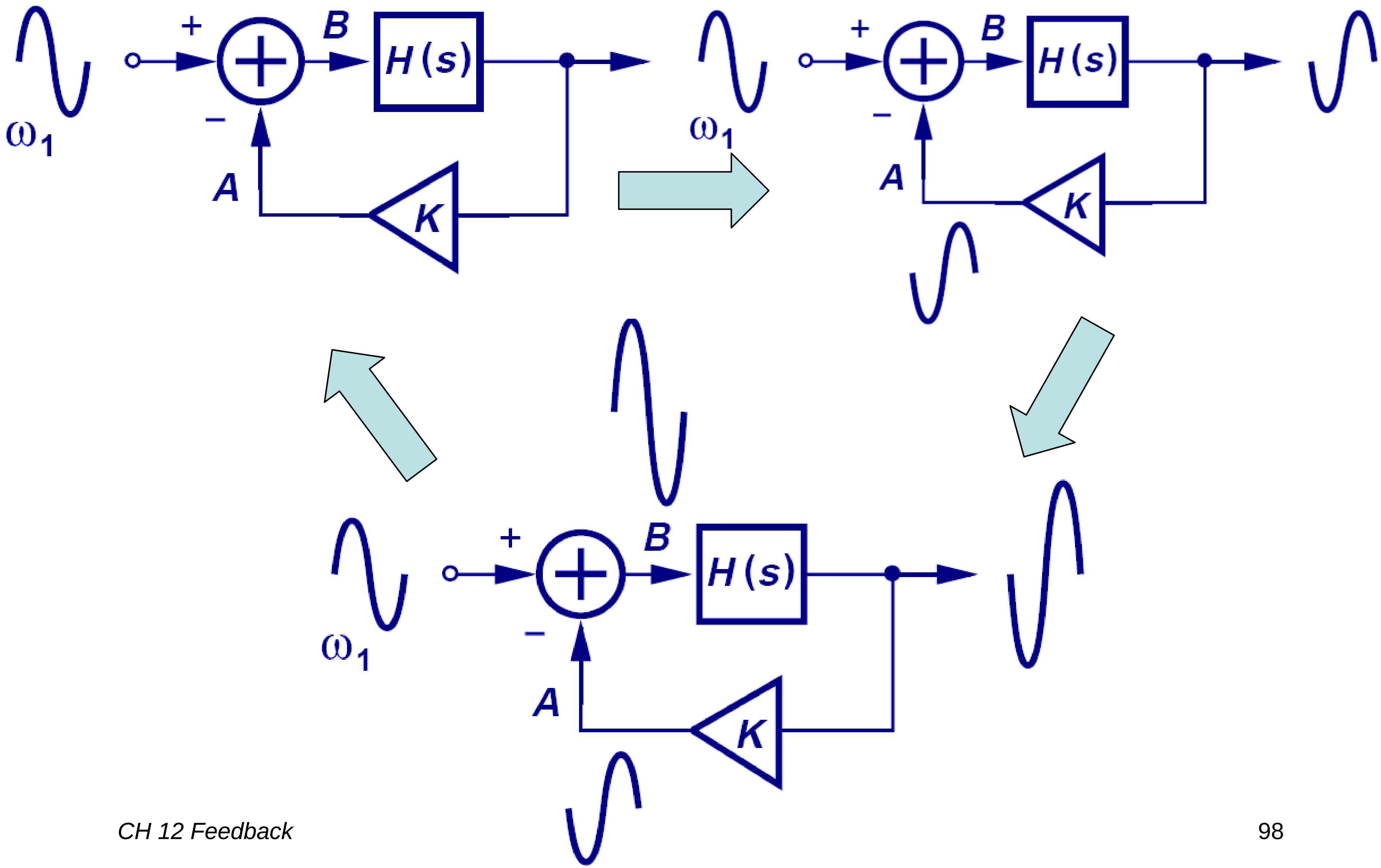
“Barkhausen’s Criteria” for Oscillation



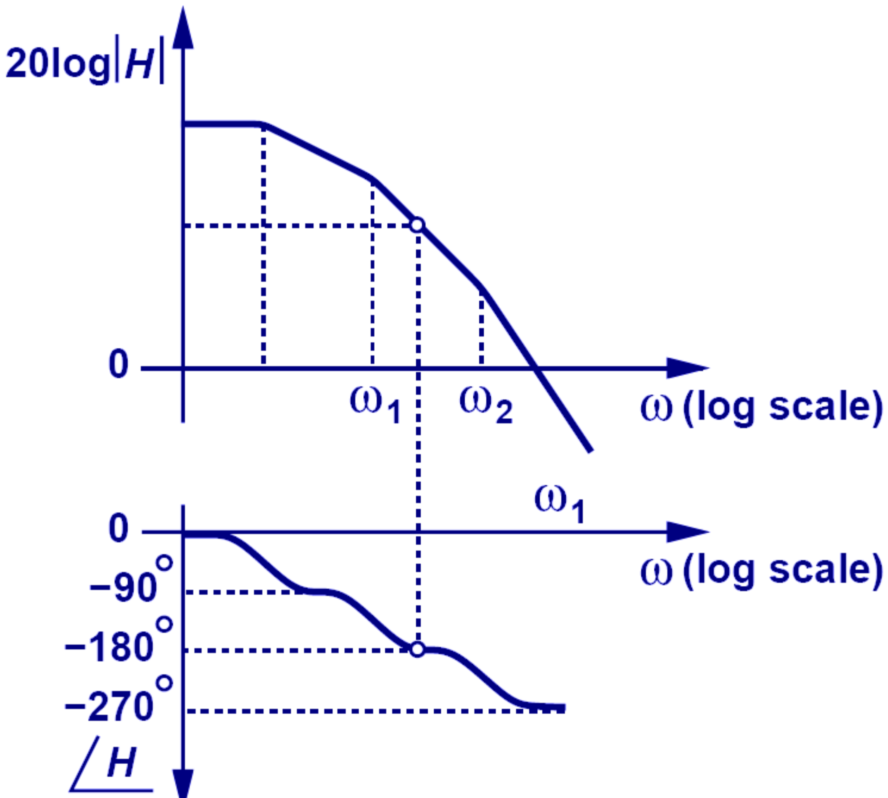
$$|KH(j\omega_1)| = 1$$

$$\angle KH(j\omega_1) = -180$$

Time Evolution of Instability



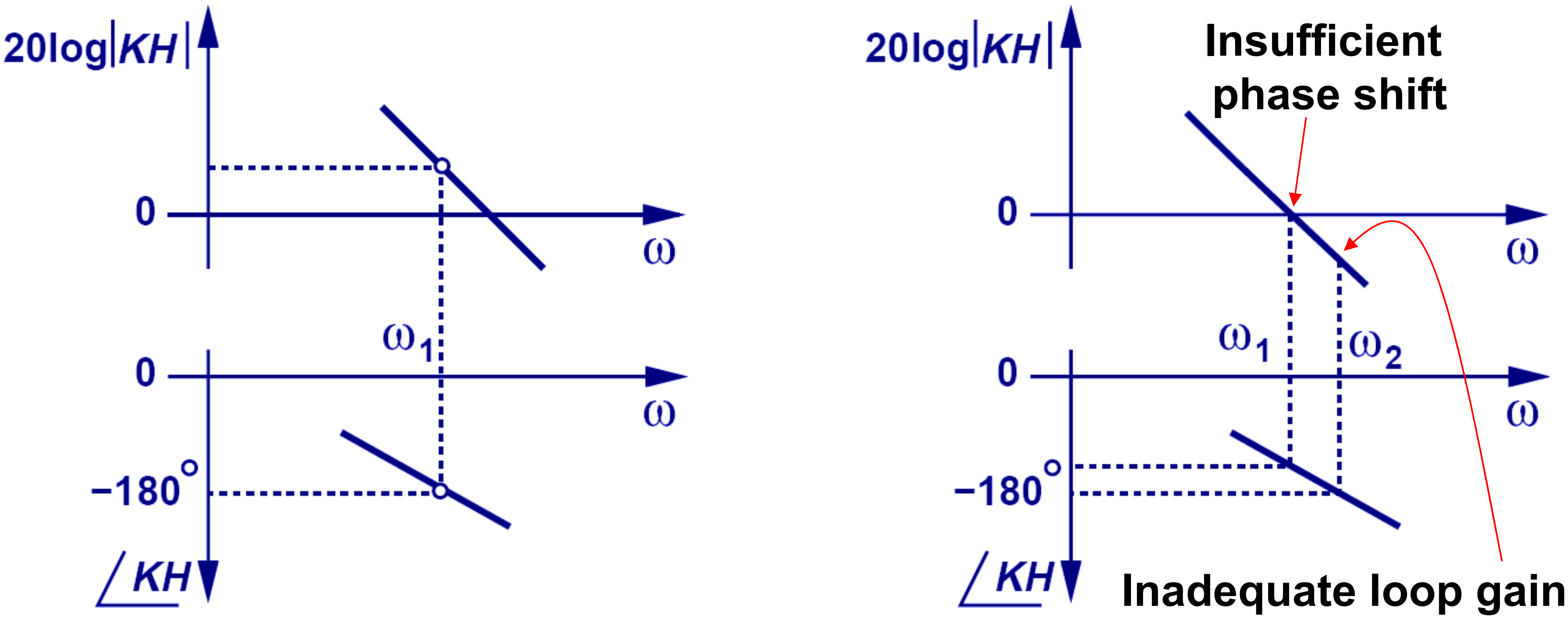
Oscillation Example



(c)

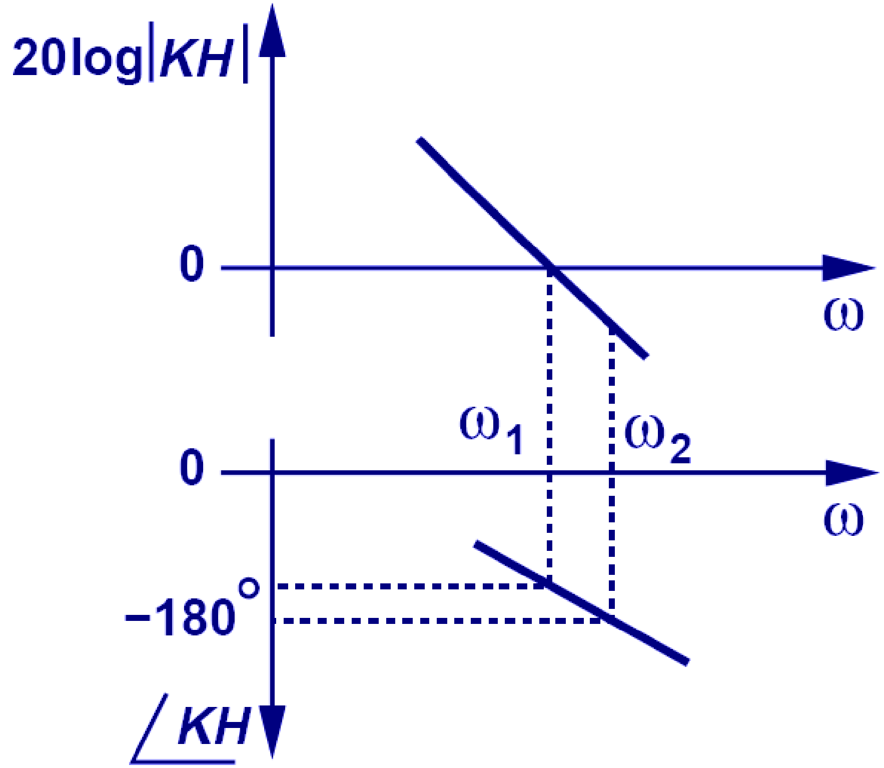
- This system oscillates, since there's a finite frequency at which the phase is -180° and the gain is greater than unity. In fact, this system exceeds the minimum oscillation requirement.

Condition for Oscillation



- **Although for both systems above, the frequencies at which |KH|=1 and ∠KH=-180° are different, the system on the left is still unstable because at ∠KH=-180°, |KH|>1. Whereas the system on the right is stable because at ∠KH=-180°, |KH|<1.**

Condition for Stability

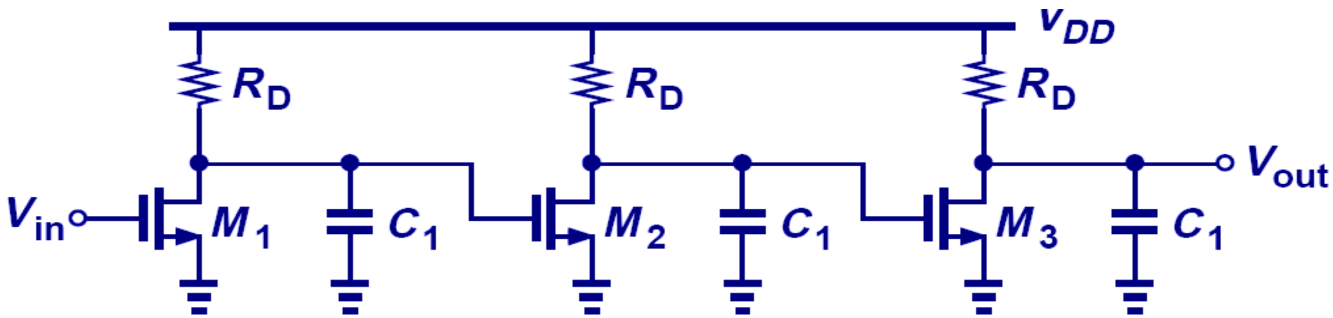


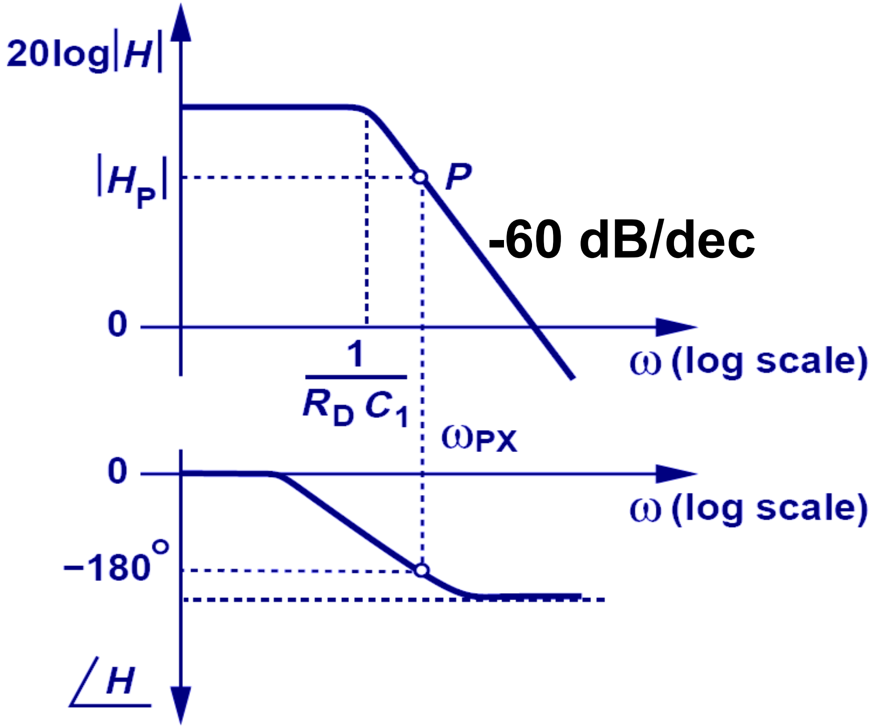
$$\omega_{GX} < \omega_{PX}$$

The loop gain falls to unity before the phase shift reaches -180° so that Barkhausen's criteria do not hold at the same frequency

- **ω_{PX}, ("phase crossover"), is the frequency at which $\angle KH=-180°$.**
- **ω_{GX}, ("gain crossover"), is the frequency at which $|KH|=1$.**

Example 12.38: Stability



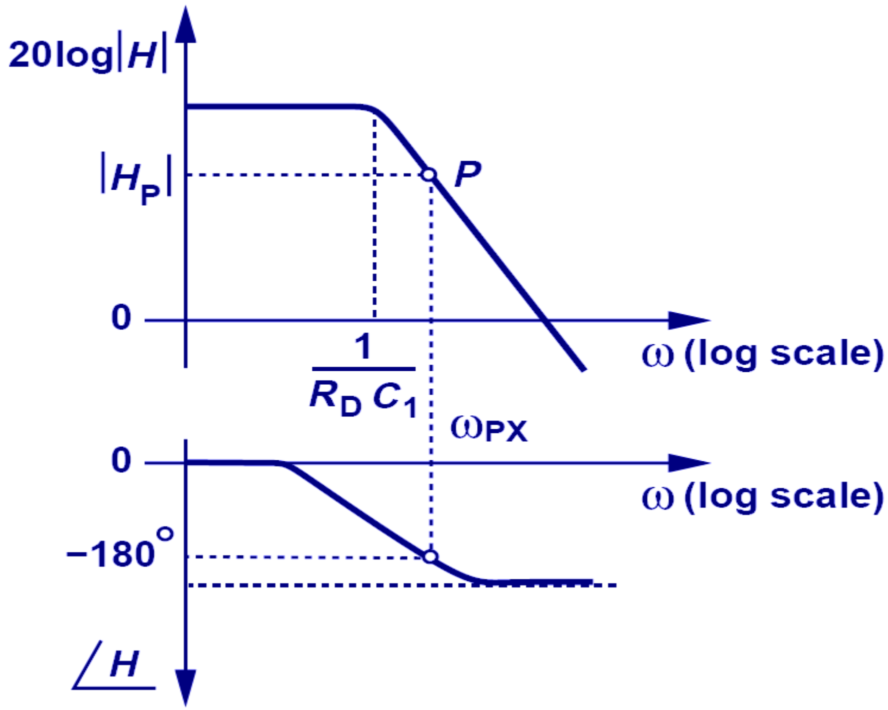


$$A_0 = -(g_m R_D)^3$$

Three poles at $\omega_p = (R_D C_1)^{-1}$

$$H(s) = -\frac{(g_m R_D)^3}{\left(1 + s/\omega_p\right)^3}$$

Example 12.38: Stability



For the unity-gain feedback system (K=1) to remain stable,

$$|H_p| < 1$$

Example 12.38: Stability (Analytical Approach)

$$H(s) = -\frac{(g_m R_D)^3}{\left(1+s/\omega_p\right)^3}$$

$$\text{Hence, } \angle H(j\omega) = -3\cdot\tan^{-1}\left(\frac{\omega}{\omega_P}\right)$$

$$\text{Since } \angle H(j\omega_{PX}) = -180°$$

$$\omega_{PX} = \sqrt{3}\cdot\omega_P$$

$$\text{For } \frac{(g_m R_D)^3}{\left[\sqrt{1+\left(\frac{\omega_{PX}}{\omega_P}\right)^2}\,\right]^3} < 1$$

$$g_m R_D < 2$$

$$H(s) = r + j\omega$$

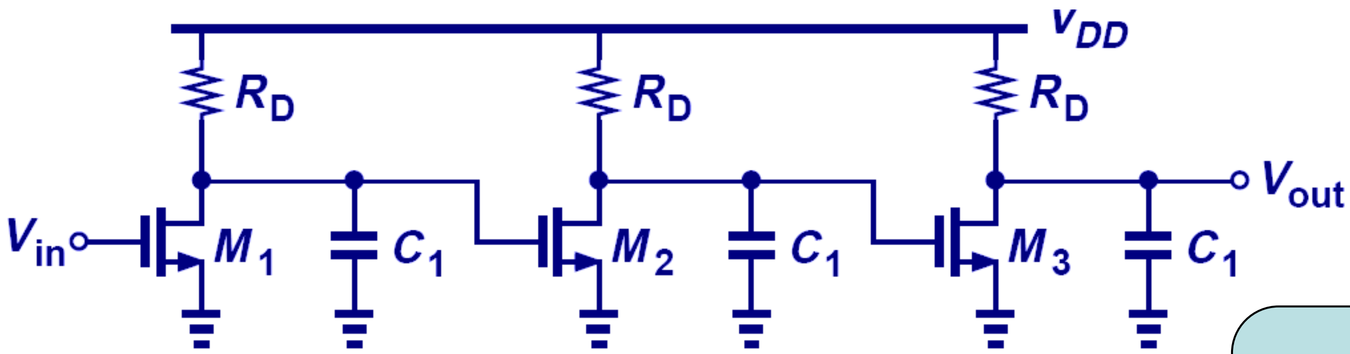
$$|H(s)| = \sqrt{r^2+\omega^2}$$

$$\theta = \tan^{-1}\left(\frac{\omega}{r}\right)$$

$$H(s) = |H(s)|e^{j\theta}$$

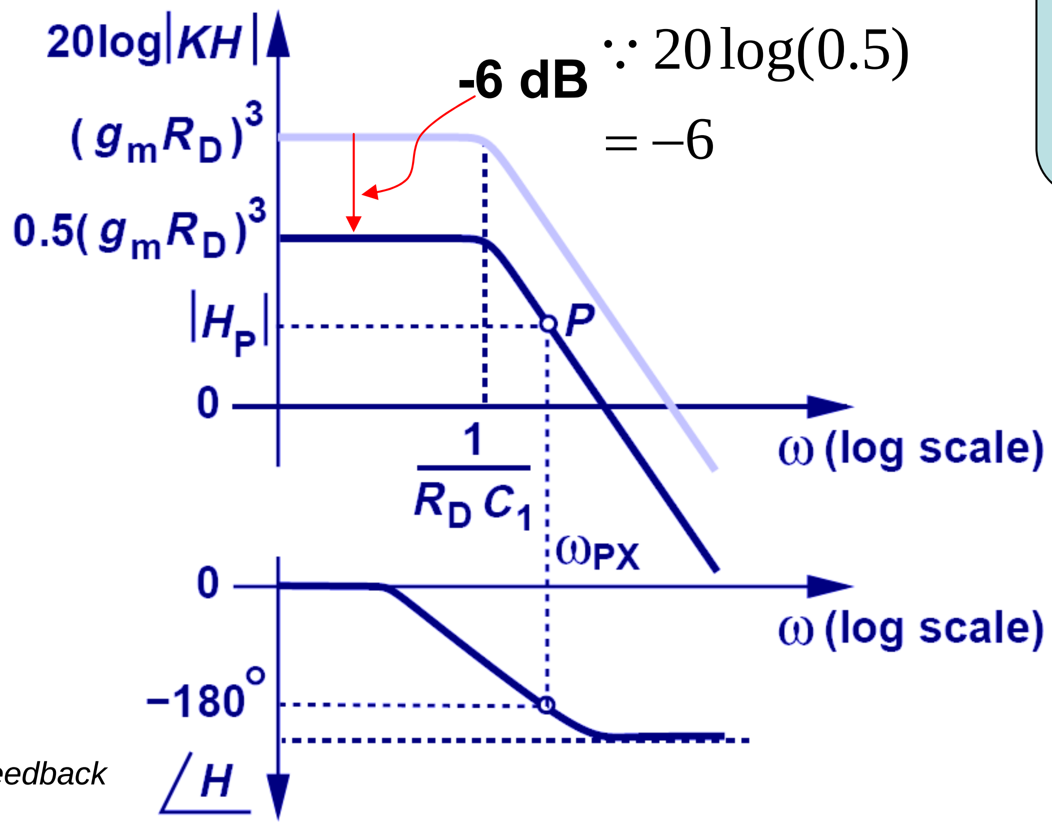
The phase crossover occurs if $\tan^{-1}(\omega/\omega_p) = 60°$

Stability Example II



What if $K = 0.5$?

$0.5|H_p| < 1$

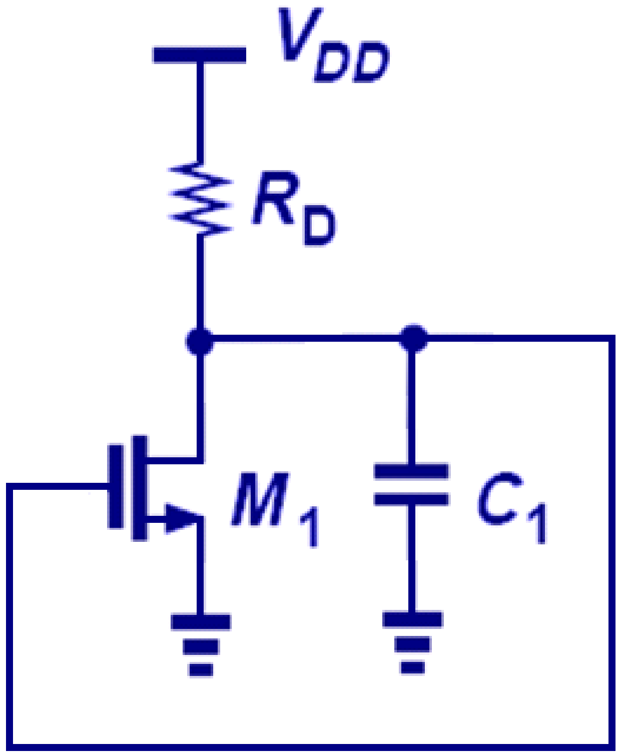


$$\because 20\log(0.5) = -6$$

$$\frac{0.5(g_m R_D)^3}{\left[\sqrt{1+\left(\frac{\omega_{PX}}{\omega_P}\right)^2}\right]^3} < 1$$

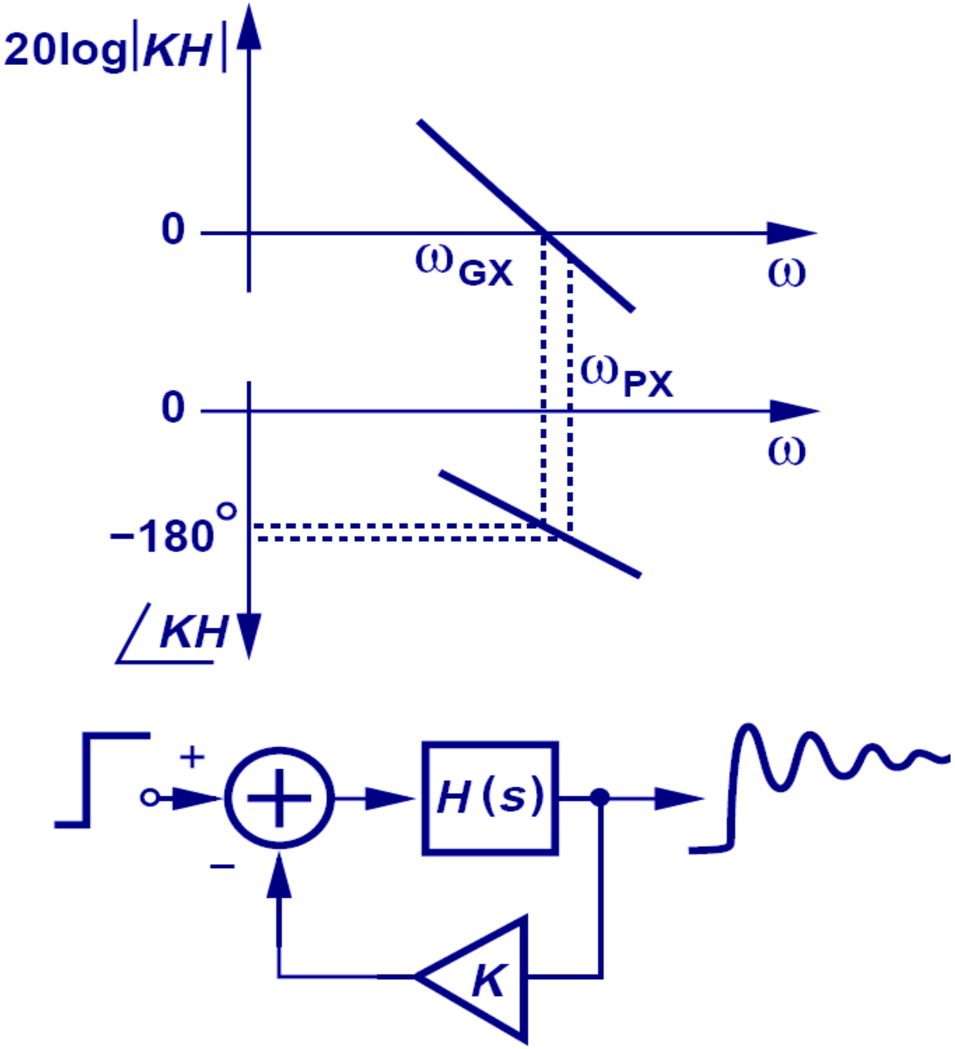
$$\therefore (g_m R_D)^3 < \frac{2^3}{0.5}$$

Example 12.39: Single-Stage Amplifier

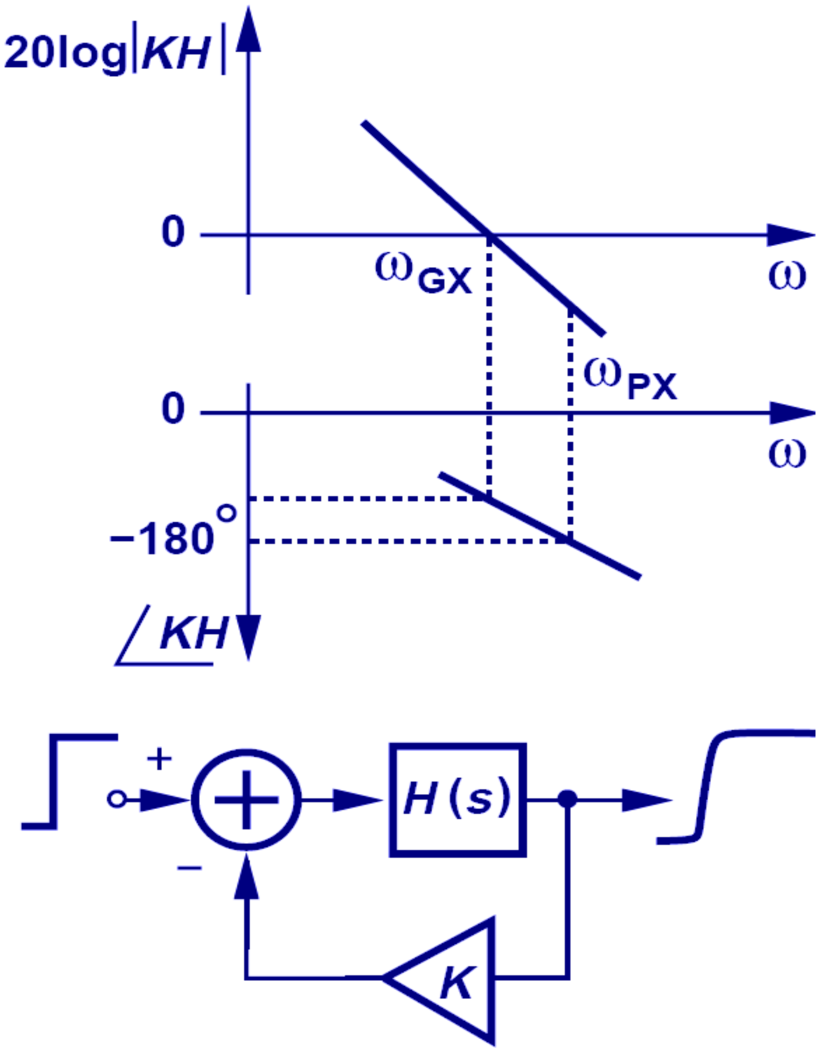


- A common-source stage in a unity-gain feedback loop does not oscillate. Since the circuit contains only one pole, the phase shift cannot reach 180˚ at any frequency. The circuit is thus stable.

Marginally Stable vs. Stable



Marginally Stable



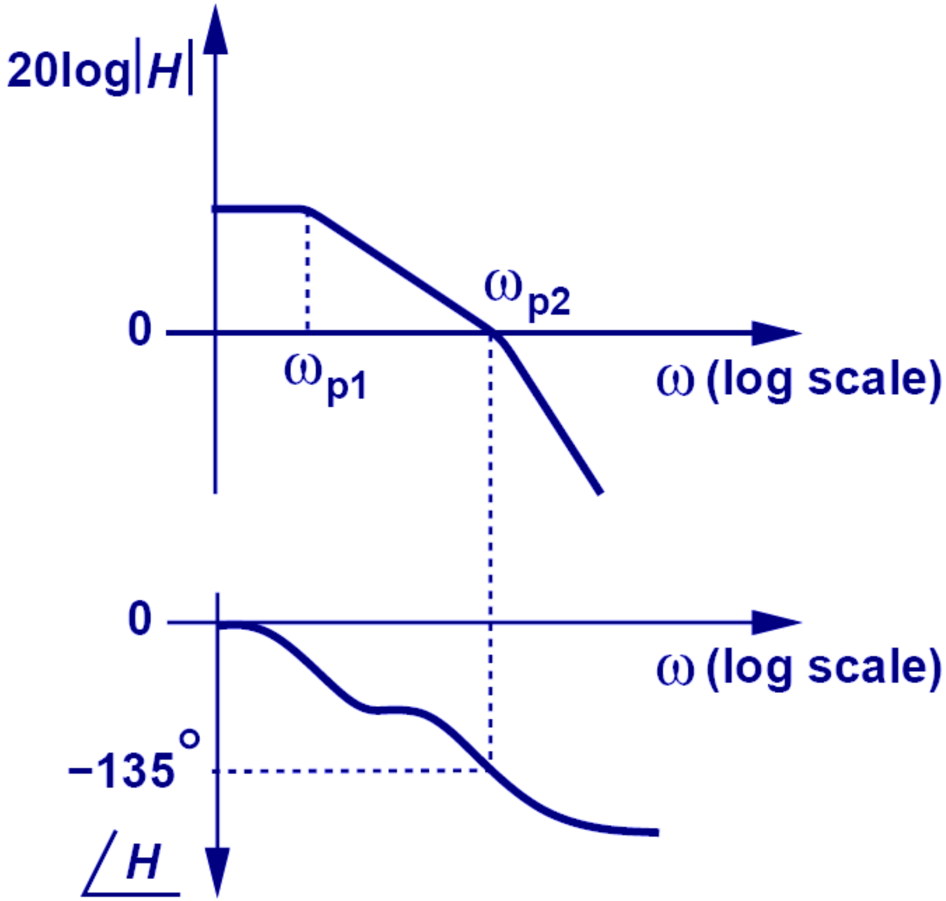
Stable

Phase Margin

The difference between $\angle H(\omega_{GX})$ and -180°

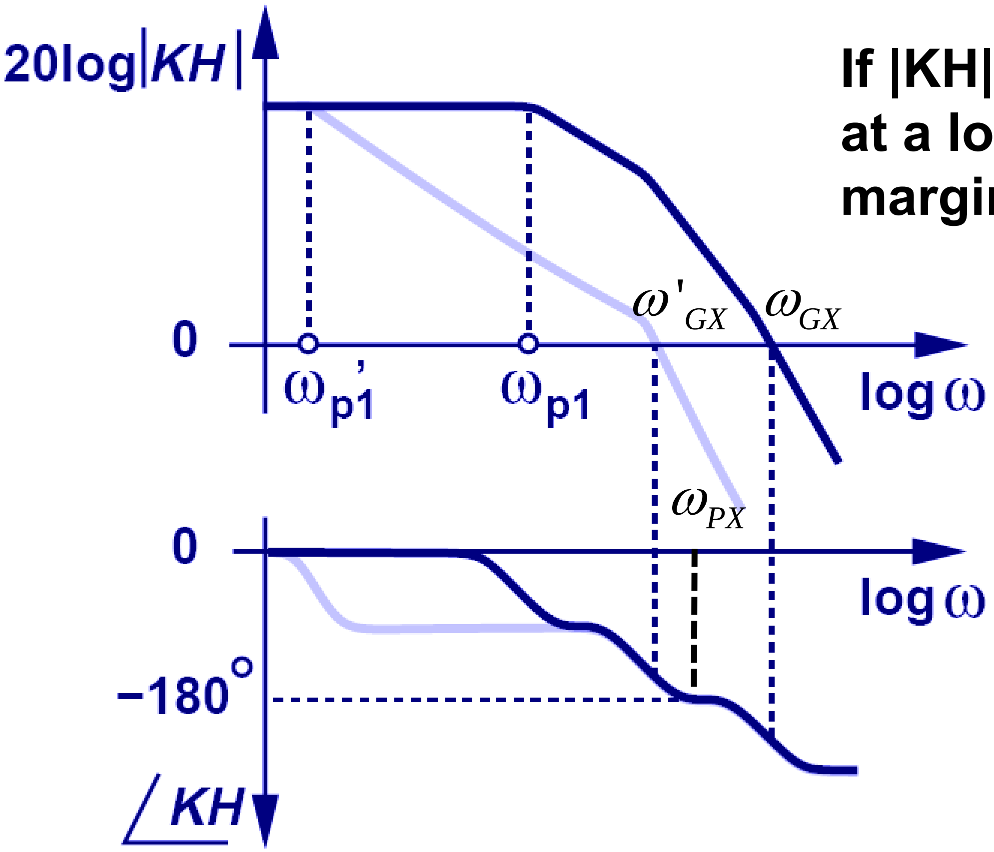
- **Phase Margin = $\angle H(\omega_{GX})+180$**
- **The larger the phase margin,
  the more stable the negative feedback becomes**

Example 12.41: Phase Margin



$$PM = 45^\circ$$

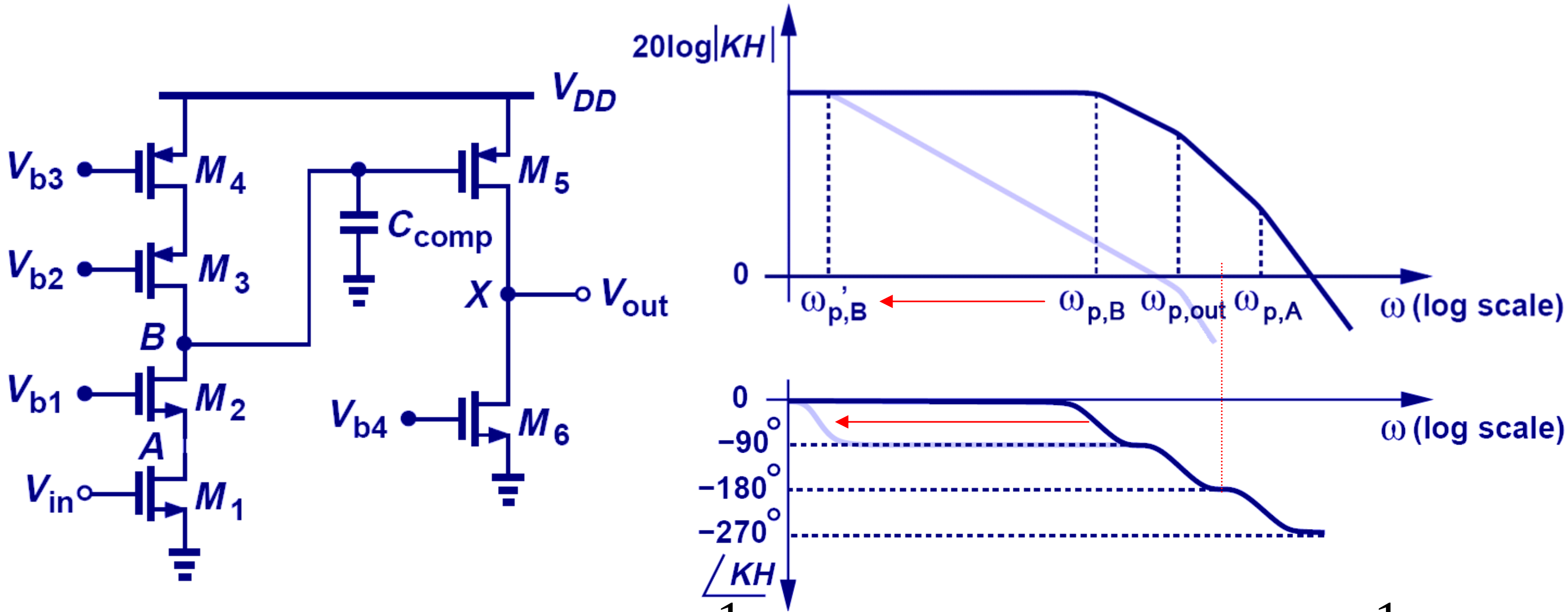
Frequency Compensation



If |KH| is forced to drop to unity at a lower frequency, then the phase margin increases

- **Phase margin can be improved by moving ω_{GX} closer to origin while maintaining ω_{PX} unchanged.**

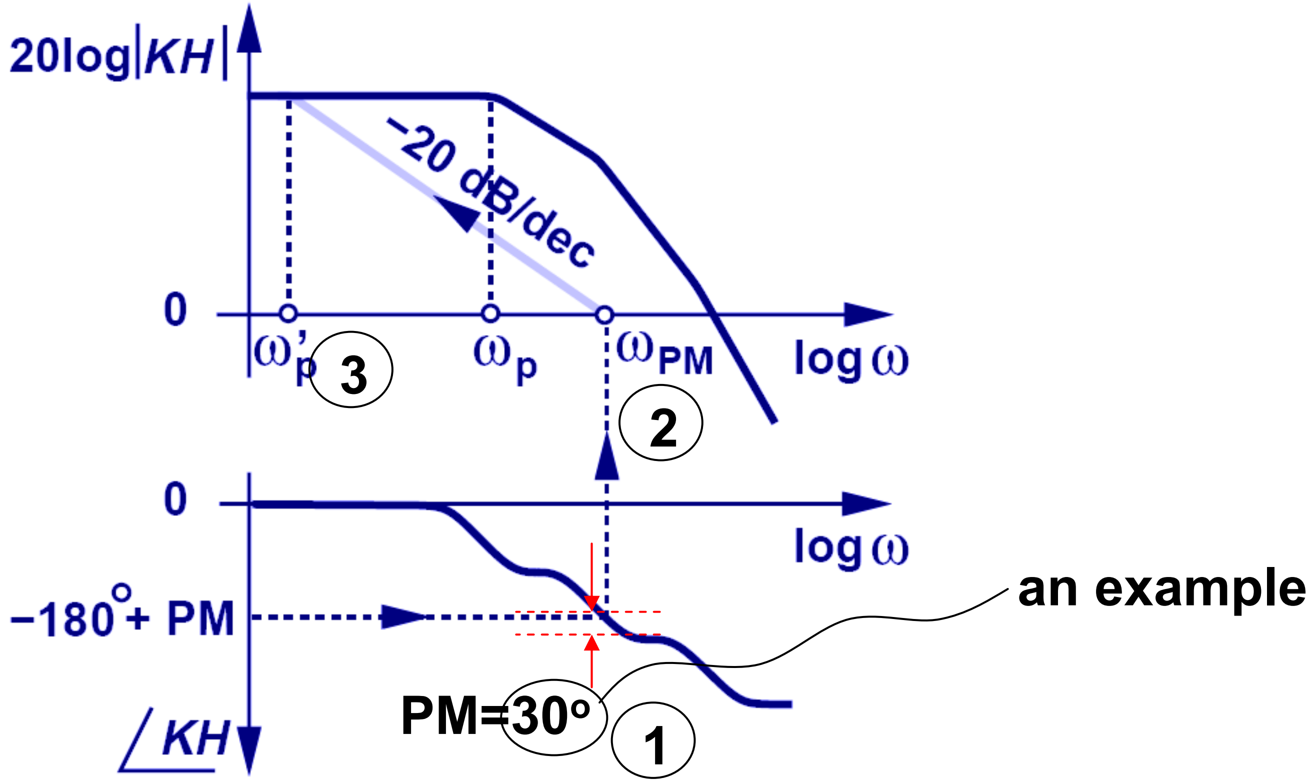
Example 12.42: Frequency Compensation



$$\omega_{p,A} \approx \frac{g_{m2}}{C_A} \quad \omega_{p,B} \approx \frac{1}{[(g_{m2}r_{o2}r_{o1}) \| (g_{m3}r_{o3}r_{o4})]C_A} \quad \omega_{p,out} \approx \frac{1}{(r_{o5} \| r_{o6})C_{out}}$$

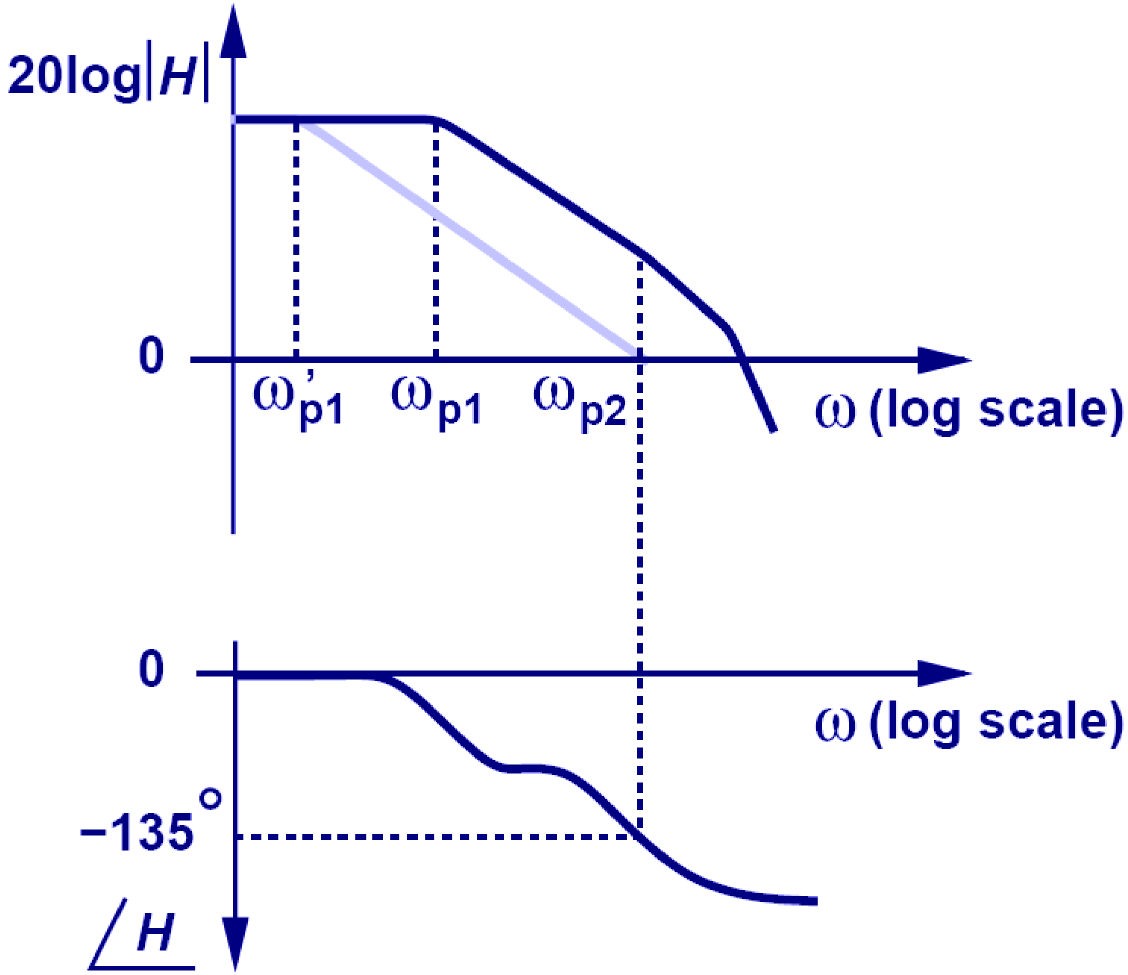
- C_{comp} is added to lower the dominant pole so that ω_{GX} occurs at a lower frequency than before, which means phase margin increases.

Frequency Compensation Procedure



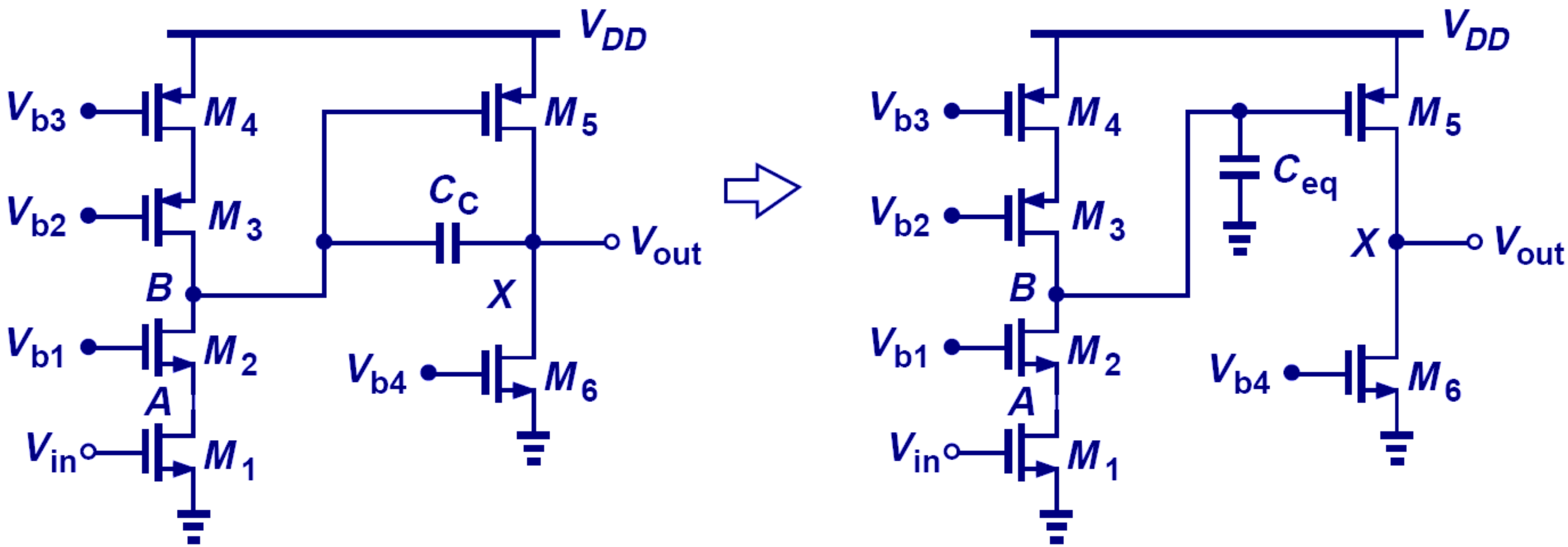
- 1) We identify a PM, then -180°+PM gives us the new ω_{GX}, or ω_{PM}.
- 2) On the magnitude plot at ω_{PM}, we extrapolate up with a slope of -20dB/dec until we hit the low frequency gain then we look "down" and the frequency we see is our new dominant pole, ω_P'.

Example 12.43: 45° Phase Margin Compensation



$$\omega_{PM} = \omega_{p2}$$

Miller Compensation



$$C_{eq} = [1 + g_{m5}(r_{O5} \parallel r_{O6})]C_c$$

- To save chip area, Miller multiplication of a smaller capacitance creates an equivalent effect.
- Miller compensation shifts not only the dominant pole but also the output pole.

Summary

- Concrete understanding phase response using Bode plot.
- "Barkhausen's Criteria" for Oscillation.

$$|KH(j\omega_1)| = 1$$
$$\angle KH(j\omega_1) = -180$$

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + KH(s)}$$

- Condition for stability

$$\omega_{GX} < \omega_{PX}$$

- Phase Margin = $\angle H(\omega_{GX})+180$

- Frequency Compensation Procedure: