Introduction to Heat Transfer (Week3, 14 & 16 Sept)

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Content of last week's lecture - largely qualitative

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- What is Geothermal Energy? And the origin.
- Types – indirect/direct uses
- The elements – what is needed
- History and status
- Types of Geothermal Power Plants
- Enhanced Geothermal System (EGS)
- Geothermal Heat Pump
- Advantages/disadvantages
- Environmental Impacts

Q & A (1)
Flow Short-circuiting

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- "Flow short circuiting" in EGS???
  - What does that exactly mean?
  - Short circuit: A preferential pathway that allows cool injected water to return too rapidly to the production well without being heated by the hot rock in the reservoir (Tester et al., 2006).
  - Two problems:
    - Not sufficiently high temperature at the production well
    - Rapid temperature drop at production well

Short circuiting

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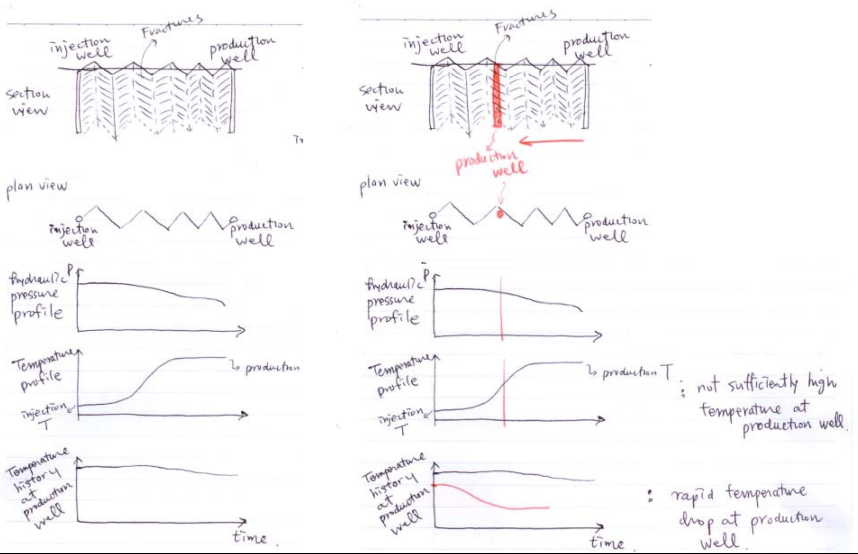

Short circuiting

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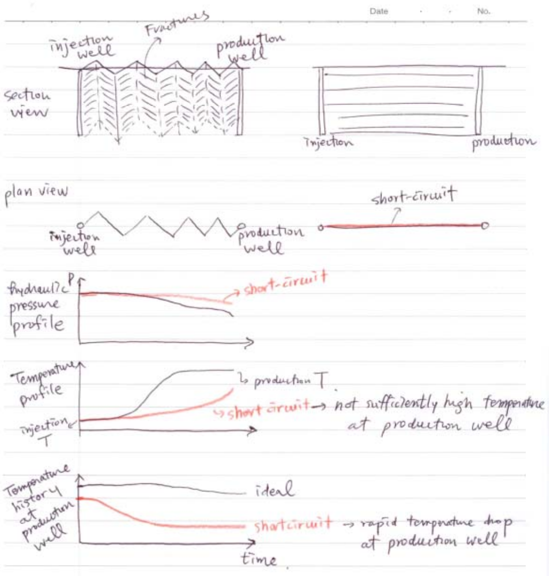

Q & A (2)
Hydraulic Fracturing

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- Open for discussion.

Term project

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- Instructions are available at eTL.

Milestone	Length	Due date	Mark
Proposal	~1 page	25 Sept	10%
Progress Report	~5 pages	30 Oct	20%
Final Report	~20 pages	4 Dec	35%
Presentation	20 minutes (including questions)	7 & 9 Dec	35%

- Updated example topics are available at eTL.
- You need to submit the 'topics submission form' via e-mail. One form per group is sufficient.

Heat Transfer
Scope of contents in this course

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- We will spend four lectures to cover;
  - three modes of heat transfer (conduction, convection, radiation)
  - Heat diffusion equation and its application
- References
  - Incropera et al., 2007, Introduction to heat transfer (Asia student edition), Wiley (Main text for week 2 & 3)
  - Carslaw and Jaeger, 1959, Conduction of heat in solids, Oxford Univ Press (Conduction only, classic but very mathematical)
  - Beardsmore and Cull, 2001, Crustal Heat Flow – A guide to measurement and modelling, Cambridge Univ Press (Geological relevance)

Heat Transfer
What is it?

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- Heat transfer is thermal energy in transit due to a temperature difference – 열전달은 온도차에 의해 이동하는 열에너지.
- Relevance to Energy Resources and Geo-environmental Engineering
  - Geothermal Energy (up to more than 200°C)
  - Geological disposal of nuclear waste (up to 100°C)
  - LNG underground storage cavern (< -162°C)

Heat Transfer
Relevance to Energy Resources Engineering

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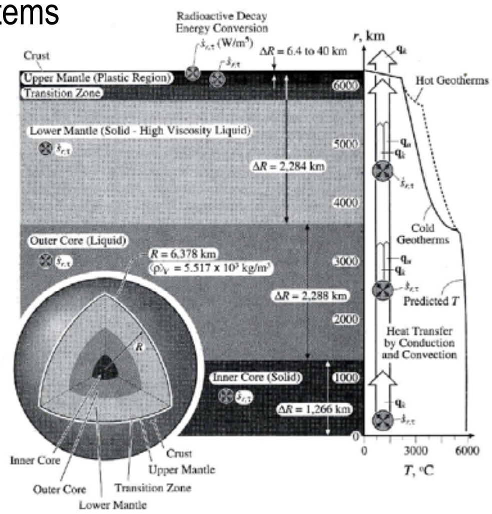
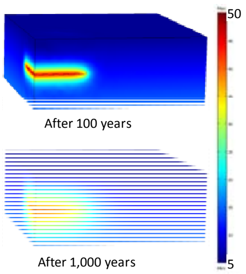
- Earth Systems

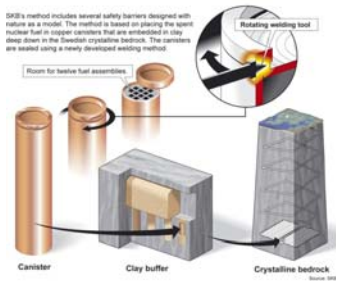


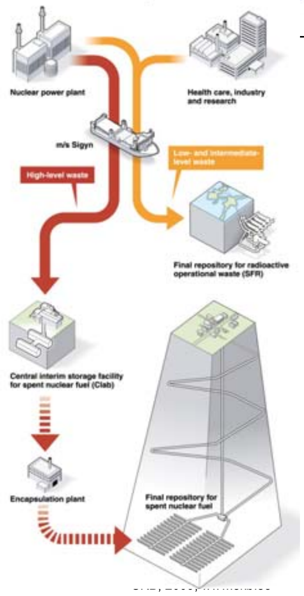
JS Lee, lecture note, SNU, 2008

Heat Transfer
Relevance to Energy Resources Engineering

- Geological Disposal of High-Level Radioactive Waste







Min & Stephansson, 2009

Nomenclature

Quantity	Meaning	symbol	unit
Temperature	A means of indirectly assessing the amount of thermal energy stored in matter	T	K or °C
Heat Transfer	Thermal energy transport due to temperature gradients		
Heat	Amount of thermal energy transferred over a time interval $\Delta t > 0$	Q	J
Heat rate	Thermal energy transfer per unit time	q	W
Heat flux	Thermal energy transfer per unit time and surface area	q"	W/m2

Three modes of heat transfer

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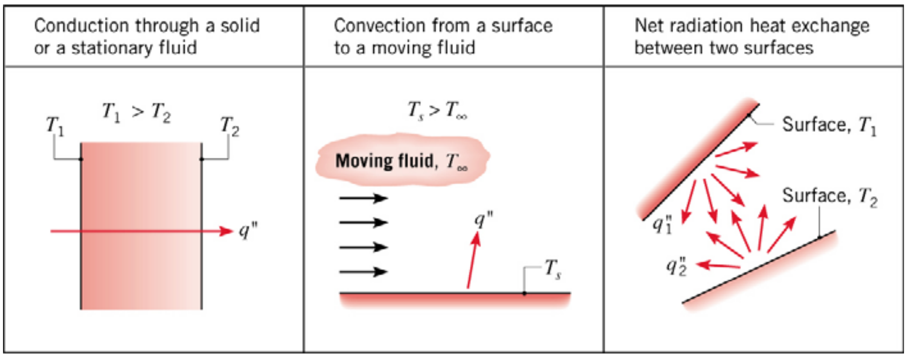
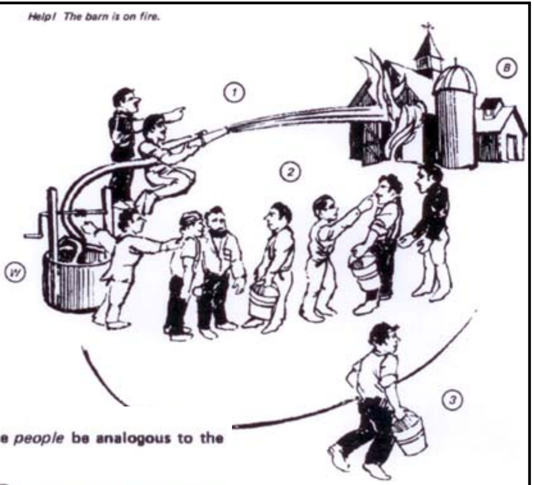


Conduction (전도) Convection (대류) Radiation (복사)

Heat Transfer Analogy



Let the *water* be analogous to *heat*, and let the *people* be analogous to the *heat transfer medium*. Then:

Case 1 The hose directs water from (W) to (B) independently of the medium. This is analogous to *thermal radiation* in a vacuum or in most gases.

Case 2 In the bucket brigade, water goes from (W) to (B) through the medium. This is analogous to *conduction*.

Case 3 A single runner, representing the medium, carries water from (W) to (B). This is analogous to *convection*.

Lienhard JH, 2008, A Heat Transfer Textbook

Conduction

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- Atomic and molecular activities are responsible for conduction (physical explanation).

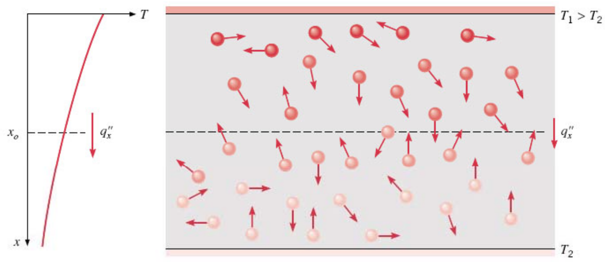
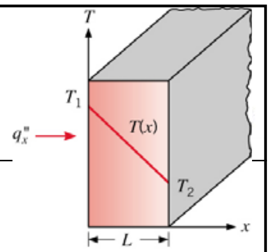


FIGURE 1.2 Association of conduction heat transfer with diffusion of energy due to molecular activity.

Conduction
Constitutive Equation(rate equation)



- Fourier's Law
  - The heat flux, q'' (W/m^2) resulting from thermal conduction is proportional to the magnitude of the temperature gradient and opposite to it in sign.

$$q''_x = -k\frac{dT}{dx} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

  - q'' : heat flux (W/m^2), rate of heat transfer per unit area (in the x direction, perpendicular to the direction of transfer)
  - k: thermal conductivity (W/m·K)
  - q: heat rate (W): q = q''·A

Conduction
Example 1

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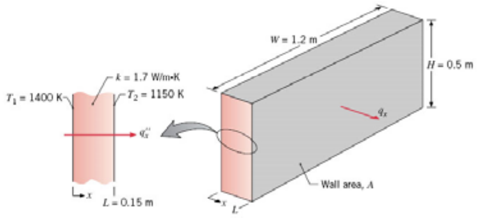
- The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m·K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?



Convection

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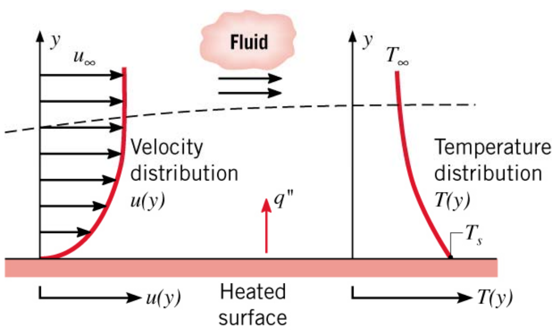
- Energy transfer occurring within a fluid due to the combined effects of conduction and bulk fluid motion.



Convection
Forced convection and free(natural) convection

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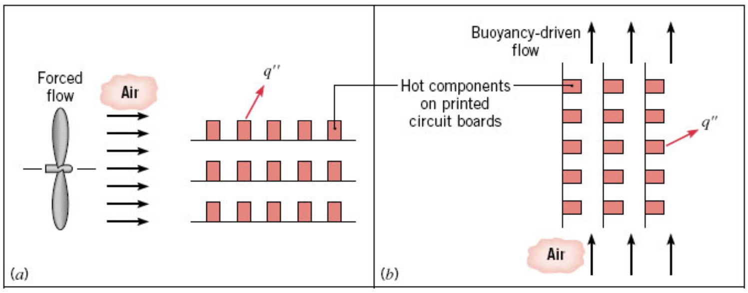
- Forced convection
- Free (natural) convection



Convection
Constitutive Equation

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- Newton's law of cooling
  - Convective heat flux is proportional to the difference between the surface and fluid temperature.

$$q'' = h(T_s - T_\infty)$$

  - (+) if heat is transferred *from* the surface, (-) *to* the surface
  - q'': convective heat flux ($W/m^2 \cdot K$)
  - h: convection heat transfer coefficient ($W/m^2 \cdot K$).
  - T_s: surface Temperature
  - T_∞: fluid temperature

Convection
convection heat transfer coefficient, h

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- Unit: $W/m^2 \cdot K$
- NOT a property
- Depends on conditions in the boundary layer
  - Surface geometry
  - Nature of the fluid motion
  - Fluid thermodynamics and transport properties
- Typical values

TABLE 1.1 Typical values of the convection heat transfer coefficient

Process	h ($W/m^2 \cdot K$)
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

Radiation

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- Energy emitted by matter (solids, liquids and gases) that is at a nonzero temperature.
  - Energy is transported by electromagnetic waves (전자기파) or Photons(광자).
  - Doesn't require the presence of a material medium (unlike conduction or convection)

Radiation
Stefan-Boltzmann law

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- Upper limit of emissive power, Applicable for an ideal radiator or *blackbody (흑체)*

$$E_b = \sigma T_s^4$$

- E_b: emissive power of blackbody (W/m^2)
- T_s: Absolute temperature (K) of the surface
- σ: Stefan-Boltzmann constant (5.67×10^{-8} W/m$^2 \cdot$K^4)
- Heat flux emitted by real surface:

$$E = \varepsilon \sigma T_s^4$$

- ε : emissivity (dimensionless), a measure of how efficiently a surface emits energy relative to a blackbody, $0 \le \varepsilon \le 1$

Radiation
irradiation (조사, 照射)

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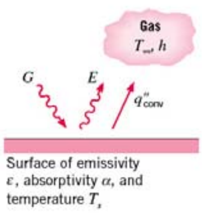
- Radiation may also be incident (입사) on a surface from its surroundings.
- Irradiation (조사, G): the rate at which all radiation is incident on a unit area of the surface



Radiation

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- A portion, or all, of the irradiation may be absorbed by the surface → increase of thermal energy of the material.

$$G_{abs} = \alpha G$$

  - α: absorptivity, $0 \le \alpha \le 1$

Radiation
A special case: a body in an enclosure

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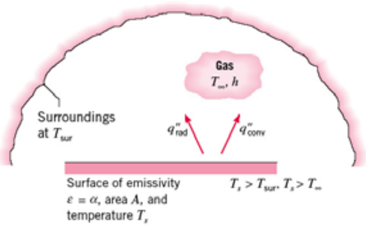
- radiation exchange between a small surface and a much large surface that completely surrounds the smaller one.
- The irradiation can be approximated by emission from a blackbody at T_{sur} → $G = \sigma T_{sur}^4$
- Net rate of radiatin heat transfer from the surface

$$q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G = \varepsilon \sigma T_s^4 - \alpha \sigma T_{sur}^4$$

- With the assumption of $\alpha = \varepsilon$

$$q''_{rad} = \varepsilon \sigma \left(T_s^4 - T_{sur}^4 \right)$$

The difference between thermal energy that is released due to radiation emissin and that which is gained due to radiatin absorption



Total rate of heat transfer
A special case: a body in an enclosure

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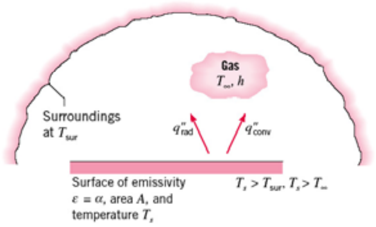

- Total rate of heat transfer

$$q = q_{conv} + q_{rad} = hA\left(T_s - T_\infty\right) + \varepsilon A\sigma\left(T_s^4 - T_{sur}^4\right)$$

Convection and Radiation
Example 1.2

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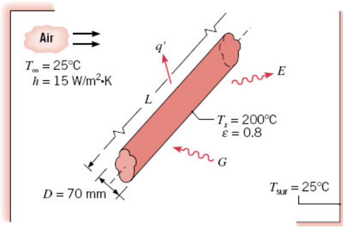
- Known:
  - Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperature
- Find
  - Surface emissive power and irradiation
  - Pipe heat loss per unit length, q'



- Assumption
  - Steady-state conditions
  - Radiation exchange between the pipe and the room is between a small surface and a much larger enclosure
  - Surface emissivity = absorptivity

Convection and Radiation Example 1.2

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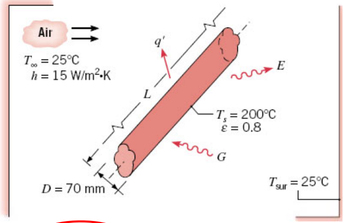
- Surface emissive power and irradiation

$$E = \varepsilon\sigma T_s^4 = 0.8\left(5.67\times10^{-8} W/m^2\cdot K^4\right)\left(473K\right)^4 = 2270 W/m^2$$

$$G = \sigma T_{sur}^4 = 5.67\times10^{-8} W/m^2\cdot K^4\left(298K\right)^4$$
$$= 447 W/m^2$$



- Heat loss from the pipe

Can be °C or K

$$q = q_{conv} + q_{rad} = h\left(\pi DL\right)\left(T_s - T_\infty\right) + \varepsilon\left(\pi DL\right)\sigma\left(T_s^4 - T_{sur}^4\right)$$

Must be in kelvins (K)

$$q' = 15W/m^2\cdot K\left(\pi\times0.07m\right)\left(200-25\right) + 0.8\left(\pi\times0.07m\right)5.67\times10^{-8}W/m^2\cdot K^4\left(473^4 - 298^4\right)$$

$$q' = 557W/m + 421W/m = 998W/m$$

Summary of heat transfer processes

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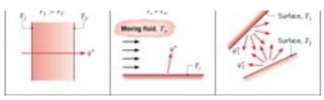
TABLE 1.5 Summary of heat transfer processes

Mode	Mechanism(s)	Rate Equation	Transport Property or Coefficient
Conduction	Diffusion of energy due to random molecular motion	$q''_x\ (W/m^2) = -k\frac{dT}{dx}$	k (W/m · K)
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(W/m^2) = h(T_s - T_\infty)$	h (W/m^2 · K)
Radiation	Energy transfer by electromagnetic waves	$q''(W/m^2) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ or $q(W) = h_r A(T_s - T_{sur})$	ε h_r (W/m^2 · K)



Content of today's lecture

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- Heat Conduction
  - Thermal conductivity and diffusivity
  - Derivation of Heat Diffusion equation
  - Solution methods

Thermal conductivity
typical value

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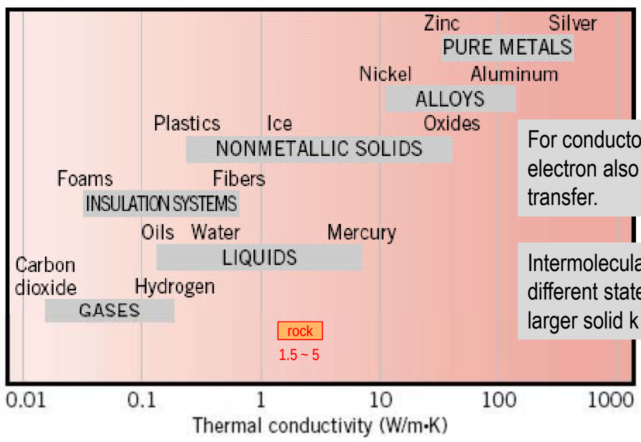

For conductor, motion of the free electron also contribute to energy transfer.

Intermolecular spacing for the different states can explain the larger solid k than liquid.

FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

Thermal conductivity
water, air and Rock

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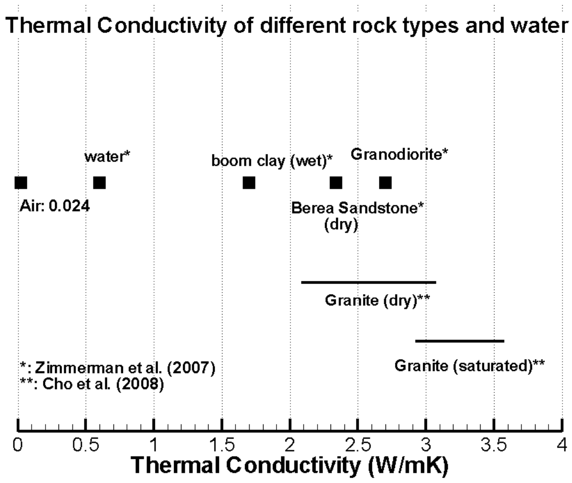

Thermal conductivity
temperature dependency

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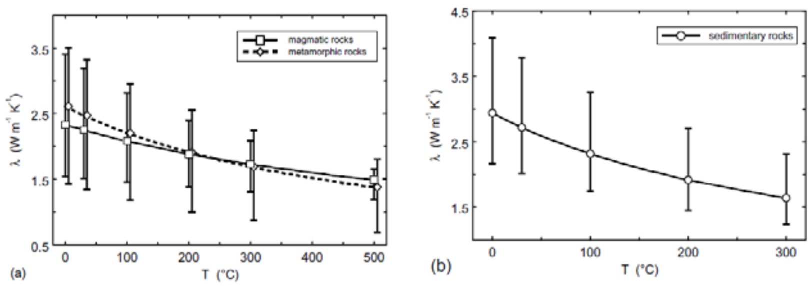

Fig. 3. Mean values (symbols) and ranges of variation (vertical bars) of thermal conductivity λ with temperature for (a) magmatic and metamorphic and (b) sedimentary rocks.

- Thermal conductivity of rock ↓ with T ↑ - why?

Vosteen, 2003

Specific heat

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- Specific heat (capacity): the measure of the heat energy required to increase the temperature of a unit mass of a substance by a 1°C – ability to store thermal energy.

$$Q = mc_p \Delta T$$

  - Q: heat (J)
  - m: mass (kg)
  - c_p: specific heat capacity (J/kg·K)
  - ΔT: temperature difference
- Volumetric heat capacity = ρc_p $\quad unit:(J/m^3 \cdot K)$
  - Defined on a volume

Thermal diffusivity

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- Thermal diffusivity: the ratio of thermal conductivity to the heat capacity

$$\alpha = \frac{k}{\rho c_p}$$

- Materials of large *α* will respond quickly to changes in their thermal environment, while materials of small *α* will respond more sluggishly, taking longer to reach equilibrium.
- For granite from Forsmark, Sweden,
  - k = 3.58 W/mK, ρ: 1000 kg/m^3, cp: 796 (J/kg·K) → $\alpha = 4.5 \times 10^{-6}$ m^2/sec

Heat Diffusion Equation
Rate Equation - Fourier's Law

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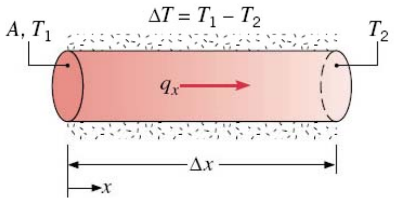
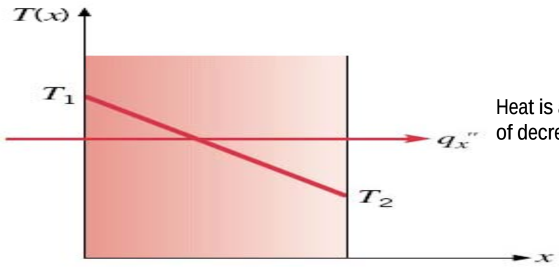


$$q_x = -kA\frac{dT}{dx}$$

$$q_x'' = \frac{q_x}{A} = -k\frac{dT}{dx}$$



Heat is always transferred in the direction of decreasing T
--> (-) sign is necessary

Heat Diffusion Equation
Rate Equation - Fourier's Law

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- Heat flux is a vector quantity

$$q_x'' = -k\frac{\partial T}{\partial x},\ q_y'' = -k\frac{\partial T}{\partial y},\ q_z'' = -k\frac{\partial T}{\partial z}$$

$$q'' = iq_x'' + jq_y'' + kq_z'' = -k\left(i\frac{\partial T}{\partial x} + i\frac{\partial T}{\partial y} + i\frac{\partial T}{\partial z}\right) = -k\nabla T$$

- Heat flux is normal to isothermal surface

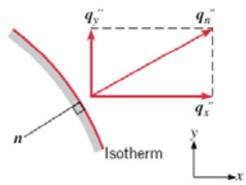

FIGURE 2.3 The heat flux vector normal to an isotherm in a two-dimensional coordinate system.

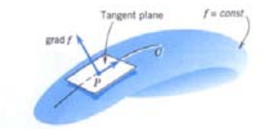

Fig. 197. Gradient as surface normal vector

Kreyszig, 1999

Heat Diffusion Equation

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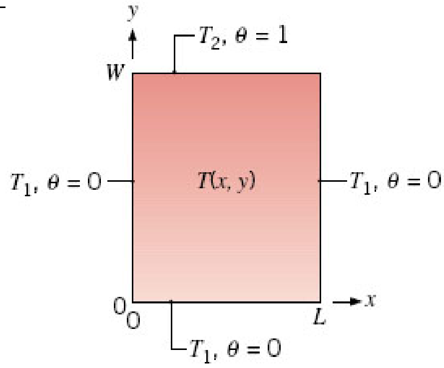

- Is there a equation that shows the distribution of temperature?
  - Yes!
  - $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$

Heat Diffusion Equation
Derivation (1)

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$$\dot{E}_{in}+\dot{E}_g-\dot{E}_{out}=\dot{E}_{st}$$

$\dot{E}_{in}$: *energy inflow*

$\dot{E}_{out}$: *energy outflow*

$\dot{E}_g$: *energy generation*

$\dot{E}_{st}$: *energy storage*

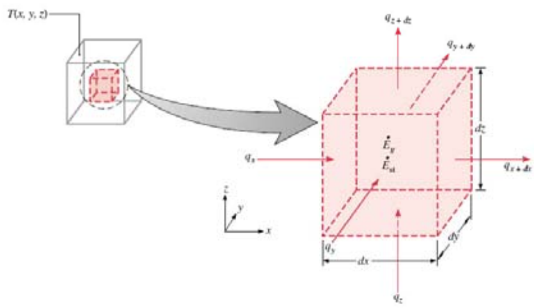

FIGURE 2.11 Differential control volume, dx dy dz, for conduction analysis in Cartesian coordinates.

- E_g: some energy conversion process, e.g., chemical or nuclear reaction
- E_{st}: rate of change of thermal energy stored by the matter

Heat Diffusion Equation
Derivation (2)

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- q - Through Taylor Expansion,

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx \qquad q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy \qquad q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

  - x component of the *q* at *x+dx* = the value of *q* at *x* + gradient x *dx*

- Energy source term

$$\dot{E}_g = \dot{q} dxdydz$$

  - $\dot{q}$ is the rate at which energy is generated per unit volume of the medium (W/m^3)

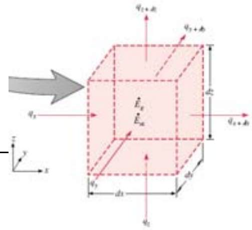
- Energy Storage

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

Taylor Expansion

$$f(b) = f(a) + \frac{f'(a)}{1!}(b-a) + \frac{f''(a)}{2!}(b-a)^2 + \ldots$$

Heat Diffusion Equation
Derivation (3)



$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$q_x + q_y + q_z + \dot{q} dxdydz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

Through rearrangement

$$-\frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz + \dot{q} dxdydz = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

Heat rates may be evaluated from Fourier's law,

$$q_x = -kdydz \frac{\partial T}{\partial x}, \; q_y = -kdxdz \frac{\partial T}{\partial y}, \; q_z = -kdxdy \frac{\partial T}{\partial z}$$

Heat Diffusion Equation
Derivation (4)

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- Putting Fourier's Law and dividing by control volume

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

  - Above are called "Heat Diffusion Equation"
  - We can obtain the temperature distribution T(x,y,z,t) as a function of time.
  - It simply embody the 'conservation of energy'.
  - 'at any point in the medium the net rate of energy transfer by conduction into a unit volume plus the volumetric rate of thermal energy generation must equal the rate of change of thermal energy stored within the volume'.

Heat Diffusion Equation
Derivation (5)

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$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

↓ For constant k

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

↓ Under steady-state condition, no change in the energy storage

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

↓ 1D & no energy generation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0$$

Heat Diffusion Equation
boundary condition

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- Heat diffusion equation is 2nd order in space → two boundary conditions needed
  - Fixed temperature (Dirichlet condition)
  - Fixed heat flux (Neumann condition)
  - Convection surface condition
- Heat diffusion equation is 1st order in time → one initial condition needed

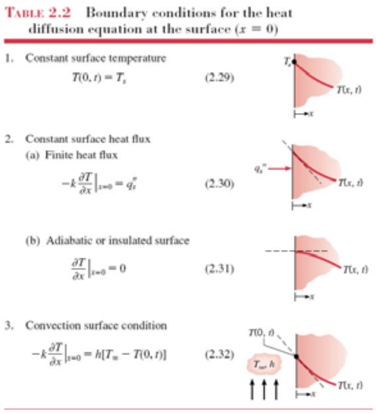

TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

$$T(0, t) = T_s \qquad (2.29)$$

2. Constant surface heat flux

(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\Big|_{x=0} = q_s'' \qquad (2.30)$$

(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\Big|_{x=0} = 0 \qquad (2.31)$$

3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\Big|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.32)$$

Heat Diffusion Equation
In cylindrical coordinate

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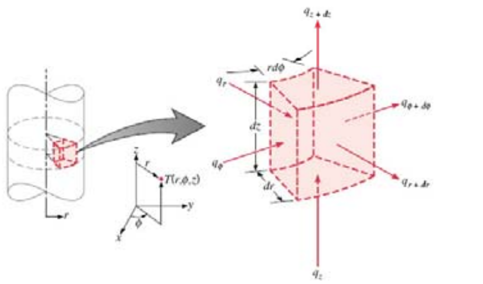

FIGURE 2.12 Differential control volume, $dr \cdot r\,d\phi \cdot dz$, for conduction analysis in cylindrical coordinates (r, ϕ, z).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Analogy with fluid flow in porous rock

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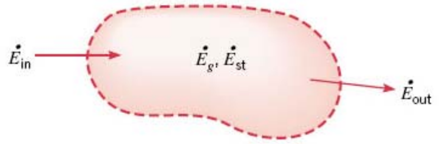

Heat transfer Energy Conservation	Fluid flow in porous rock Mass Conservation
$\Delta E_{st} = E_{in} - E_{out} + E_g$	$\Delta m_{st} = m_{in} - m_{out} + m_g$
E_{st} : *Energy stored in control volume*	m_{st} : *mass stored in control volume*
E_{in} : *Energy entering the control volume*	m_{in} : *mass entering the control volume*
E_{out} : *Energy leaving the control volume*	m_{out} : *mass leaving the control volume*
E_g : *Thermal Energy Generation*	m_g : *mass Generation*

Solution methods
Analytical method

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- Analytical method
  - Obtaining exact mathematical solution to equation
  - Provide exact results at any continuous points

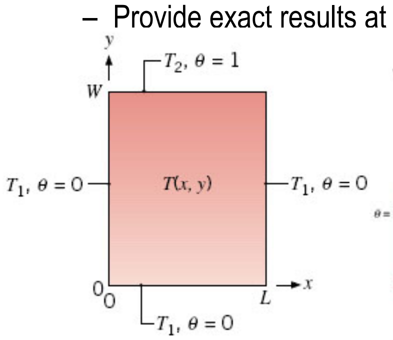
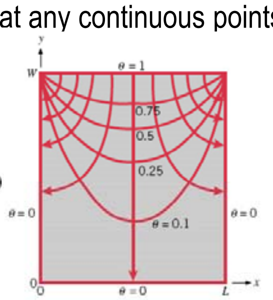


FIGURE 4.2
Two-dimensional conduction in a thin rectangular plate or a long rectangular rod.



$$\theta(x,y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\frac{n\pi x}{L}\frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$$

FIGURE 4.3
Isotherms and heat flow lines for two-dimensional conduction in a rectangular plate.

Solution methods
Analytical method

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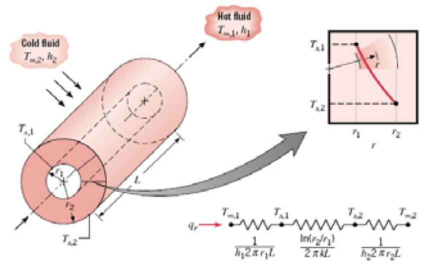

FIGURE 3.6 Hollow cylinder with convective surface conditions.

$$T(r)=\frac{T_{s,1}-T_{s,2}}{\ln(r_1/r_2)}\ln\left(\frac{r}{r_2}\right)+T_{s,2}$$

$$q_r=\frac{2\pi Lk\left(T_{s,1}-T_{s,2}\right)}{\ln(r_2/r_1)}$$

Solution methods
Numerical method

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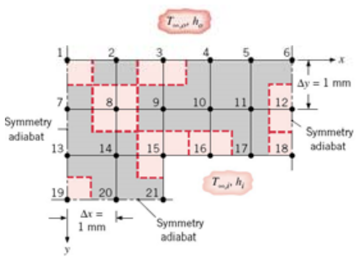
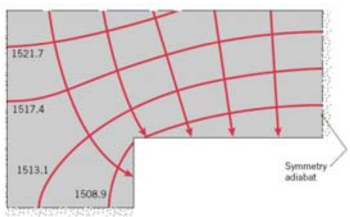
- Numerical method
  - Approximate results at discrete points
  - Finite Element Method, Finite Difference Method
  - Suitable for complex geometry and boundary conditions





Content of today's lecture

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- Heat Conduction
  - Thermal conductivity and diffusivity $\alpha = \dfrac{k}{\rho c_p}$
  - Derivation of Heat Diffusion equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

  - Solution methods

Next week

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- Method of thermal conductivity measurement
- Steady state and transient problem of heat conduction
- Heat conduction and convection problem relevant to geothermal energy

References used for this lecture

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- Kreyszig E, 1999, Advanced Engineering Mathematics, 8th ed., John Wiley & Sons Inc,