Introduction of Heat Transfer (Week4, 21 & 23 Sept)

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Content of last lecture

- Heat Conduction
  - Thermal conductivity

$$q'' = iq_x'' + jq_y'' + kq_z'' = -k\nabla T$$

  - Thermal diffusivity

$$\alpha = \frac{k}{\rho c_p}$$

  - Derivation of Heat Diffusion equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

  - Solution methods

Today

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- Steady state one-dimensional conduction
  - A plane wall
  - Radial conduction in cylindrical coordinate
- Thermal conductivity measurement method
  - Steady state method
  - Transient method

Feed back on homework #1

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- Distinction of formal and informal words.
  - Like, a little bit, …
- Avoid using subjective, emotional words
  - Very
- Proper tense (시제)
- Try to be more specific when you explain
  - Use numbers if possible, take examples
- Maintain a critical attitude

  Because of many problems of fossil fuel energy, <u>the importance of renewable energy is increased.</u>
- 한국말을 영어로 직역하려고 하지말고, 이에 적합한 영어 표현이 무얼까라고 생각할 것.
- Run the spelling check – MS Word

Feed back on homework #1

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- Why is the earth hot?
- Why is there a thermal gradient?

- Decay of radioactive element.
- Hot interior of the earth

Feed back on homework #1

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- What about the disadvantage of geothermal energy?
- Why don't we use it now? Economic feasibility???
- What about the safety of power plant in tectonic boundaries?
- Why doesn't Korea build a geothermal power plant?

용한다고 설명되어진다. 매개체로 물을 사용하는 것은 물의 비열이나 기타 성질을 고려해봤을 때 더없이 탁월한 선택이다. 하지만 내 생각에 이러한 '물'을 매개로 한 것에는 한계가 있고 미래에는 무언가 더욱 혁명적인 에너지 활용이 필요한 것 같다.

Homework #2

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Thermal Resistance (열저항)

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- Thermal resistance for conduction in a plane wall
  - From Fourier's law

$$q_x = q_x'' A = -kA\frac{dT}{dx} = kA\frac{T_{s,1} - T_{s,2}}{L}$$

  - Through rearrangement,

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA}$$

Thermal resistance for conduction
전도열저항

Analogy with electrical resistance

$$R_e = \frac{E_{s,1} - E_{s,2}}{I} = \frac{L}{\sigma A}$$

  - From Newton's law of cooling (convection),

$$q = q'' A = hA\left(T_s - T_\infty\right)$$

$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

Thermal resistance for conduction
대류열저항

1D steady state solutions
Plane wall

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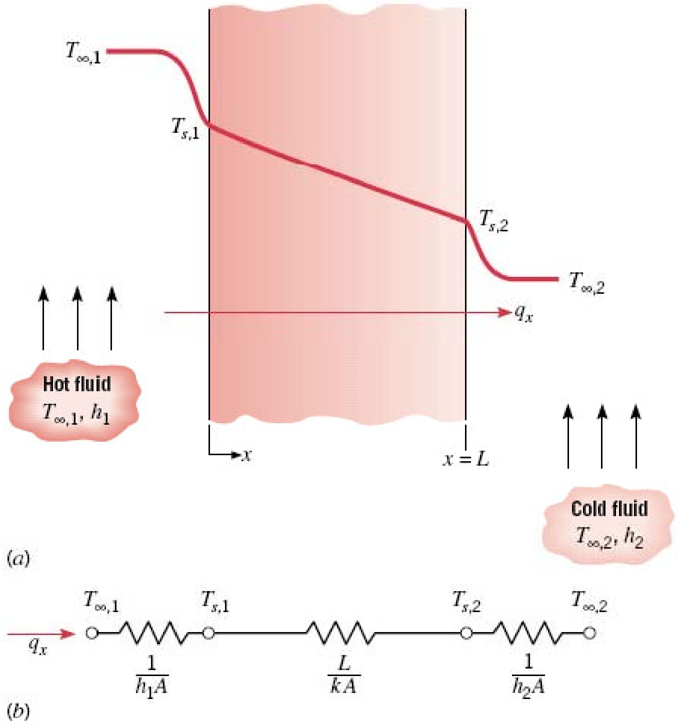


FIGURE 3.1 Heat transfer through a plane wall. (a) Temperature distribution. (b) Equivalent thermal circuit.

– 1D, steady state, no heat generation,

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

– From constant heat flux

$$T(x) = C_1x + C_2$$

– Applying BC,

$$T(0) = T_{s,1} \;\; and \;\; T(L) = T_{s,2}$$

$$C_2 = T_{s,2} \;\; and \;\; C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$

Conduction
Steady-state one dimensional conduction

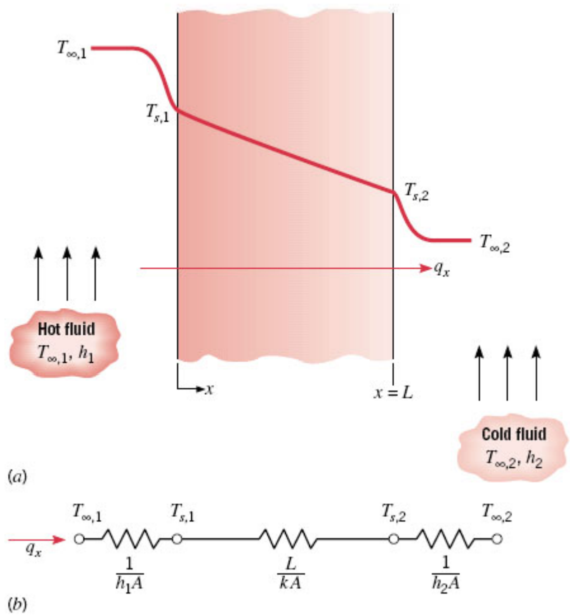


FIGURE 3.1 Heat transfer through a plane wall. (a) Temperature distribution. (b) Equivalent thermal circuit.

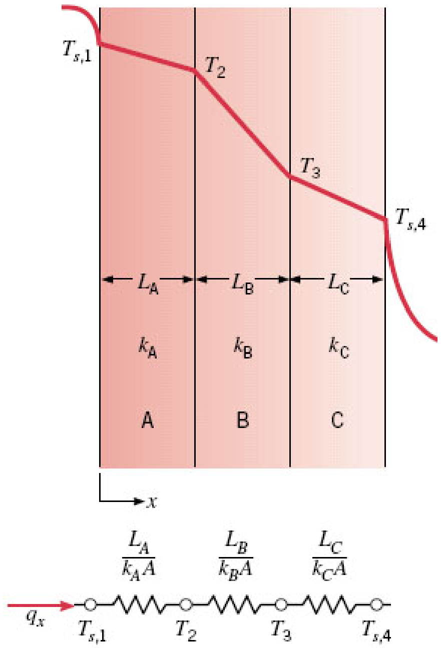
– Temperature distribution,

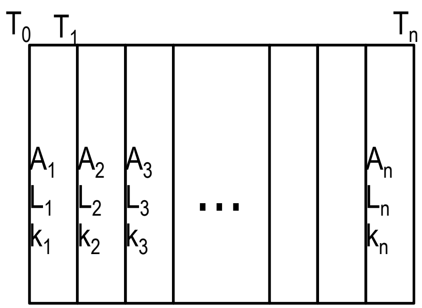
$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1}$$

– Conduction heat transfer rate,

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}(T_{s,1} - T_{s,2})$$

1D steady state solutions
Composite wall (layered rock)





– Equivalent thermal conductivity for a series composite wall (layered rock).

$$\frac{1}{k_{eq}} = \frac{1}{L}\sum_{i=1}^{n}\frac{L_i}{k_i} \qquad L = \sum_{i=1}^{n} L_i$$

$$\frac{1}{k_{eq}} = \frac{1}{L}\left(\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right)$$

1D steady state solutions
radial conduction in cylindrical wall

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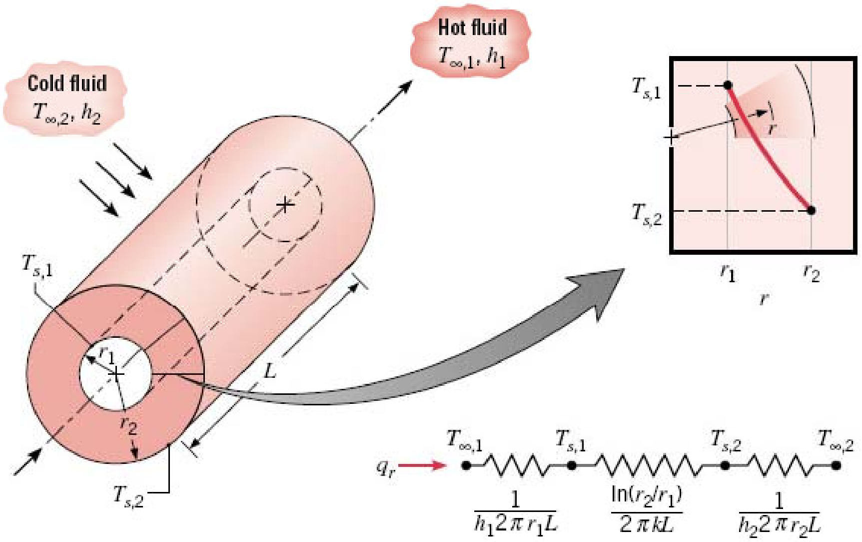

FIGURE 3.6 Hollow cylinder with convective surface conditions.

– 1D, Steady state, no heat generation,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = 0$$

$$q_r = -kA\frac{\partial T}{\partial r} = -k(2\pi rL)\frac{\partial T}{\partial r}$$

– Heat transfer rate is a constant in the radial direction.

– Assuming constant k,

$$T(r) = C_1 \ln r + C_2$$

$$T_{s,1} = C_1 \ln r_1 + C_2 \;\; and \;\; T_{s,2} = C_1 \ln r_2 + C_2$$

1D steady state solutions
radial conduction in cylindrical wall

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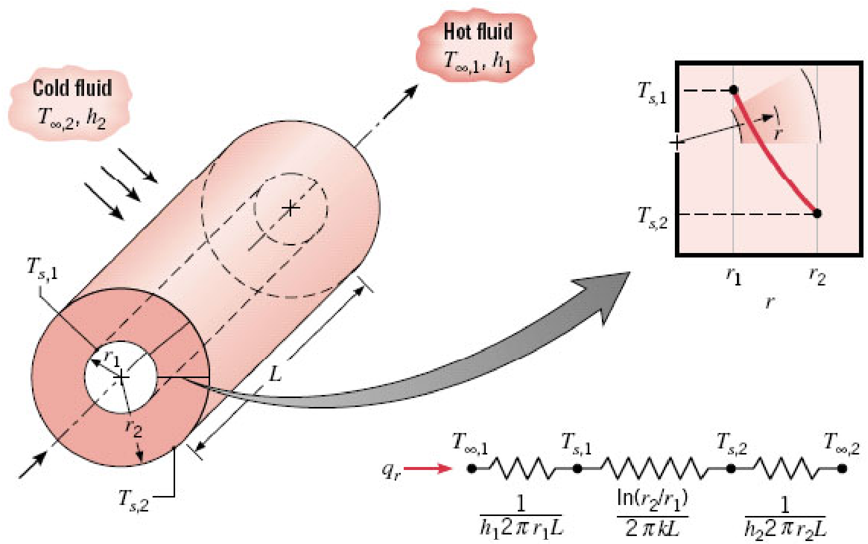

FIGURE 3.6 Hollow cylinder with convective surface conditions.

- Temperature distribution associated with radial conduction through a cylindrical wall

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

- Heat transfer rate is,

$$q_r = \frac{2\pi Lk\left(T_{s,1} - T_{s,2}\right)}{\ln(r_2/r_1)}$$

- Thermal resistance

$$R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi Lk}$$

1D steady state solutions
Summary

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TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

Hydraulic conductivity measurement

Two groups of methods (Beardsmore & Cull, 2001)

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- Steady-state method
  - Divided-bar apparatus
- Transient method
  - Needle probe

Hydraulic conductivity measurement
Steady State Method

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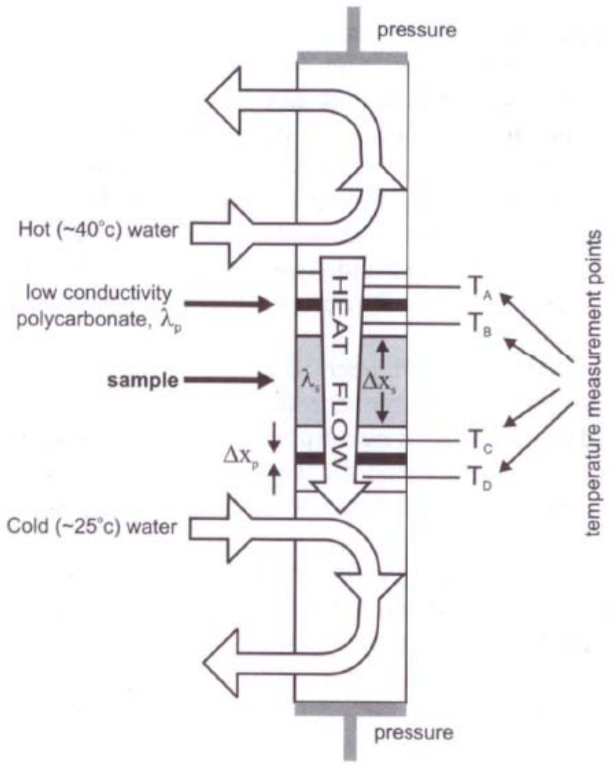

Figure 4.6. A typical divided-bar apparatus.

- Use a ‘divided-bar apparatus’
- Measure the ‘k’ directly $$q''_x = k\frac{T_2 - T_1}{L}$$
- Takes long time to achieve thermal equilibrium
- More accurate than ‘transient method’
- Rock sample in discs or cylindrical shape
- Top and bottom sections of the bar maintained at constant but different temperatures (warm end at the top! Why?).

Beardsmore and Cull, 2001

Hydraulic conductivity measurement
Transient Method

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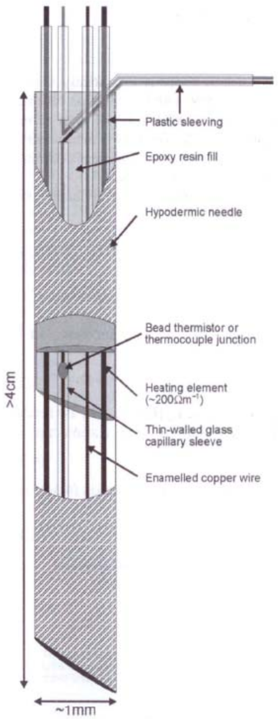

Single needle probe

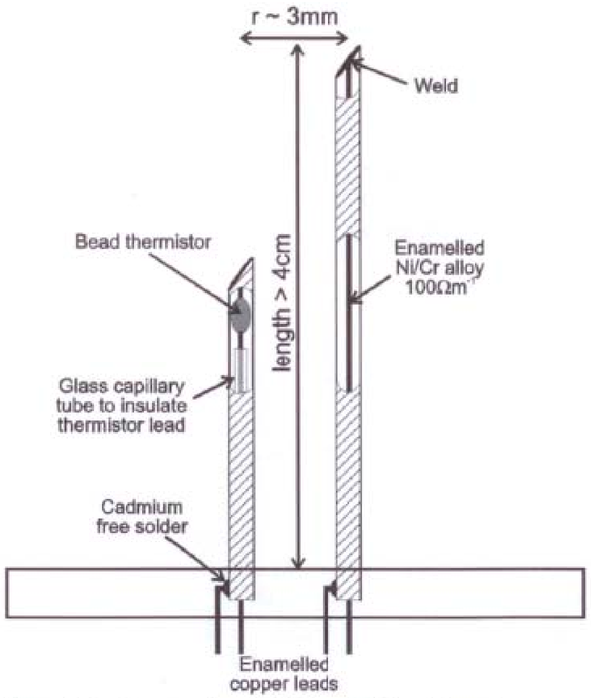

Figure 4.10. Construction details for a dual-needle probe.

Dual-needle probe

Beardsmore and Cull, 2001

- "Needle probe method" is the best known method.
- k can be deducted from the rate at which its T changed in response to an applied heat source.
- Measure the thermal diffusivity (α)and thermal conductivity is calculated ← need to know ρ and c_p.

$$\alpha = \frac{k}{\rho c_p}$$

– Less accurate than 'steady state' method

Hydraulic conductivity measurement

Transient Method (2) - an example

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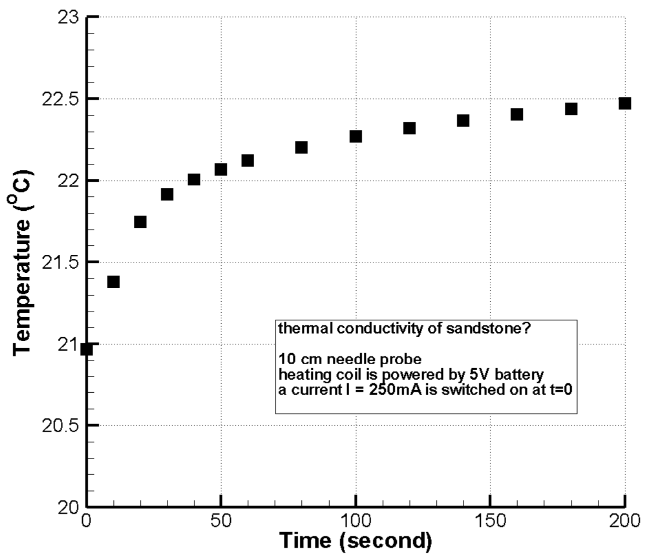

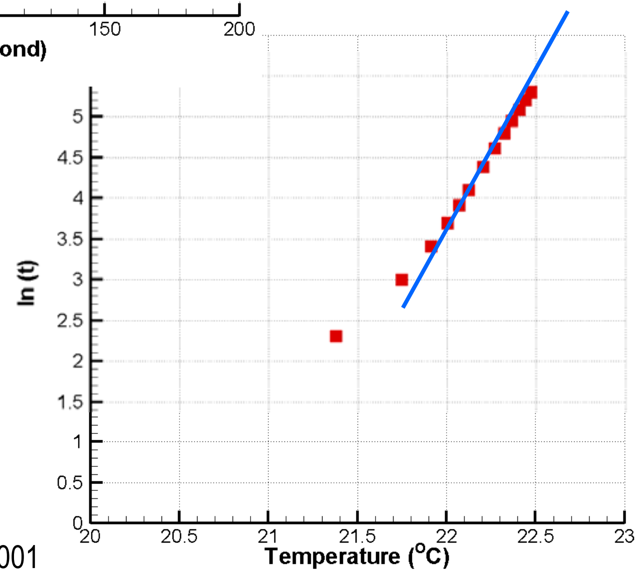

Data from Beardsmore and Cull, 2001

- With a line source of heat and a temperature sensor packed closely,

$$k = \left(Q_l / 4\pi\right)\left(\partial \ln(t) / \partial T\right)$$

- Q_l : applied heat per unit length(W/m)
  t : time (second)
  T: Temperature (K)

- Find a linearity and obtain k

P= V x I = 5 x 0.25 = 1.25W
Q=P/0.1(cm)=12.5W/m
Gradient between 60 & 200 sec is ~3.449
K = 12.5/4π x 3.449 = 3.43 W/mK

Heat Diffusion Equation

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- Verbal description of heat diffusion equation
  - The rate at which the temperature at a point is changing with time is proportional to the rate at which the temperature gradient at that point is changing in the direction of heat flow. (Middleton and wilcock, 1999)

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Last lecture

- Steady state one-dimensional conduction
  - A plane wall
  - Radial conduction in cylindrical coordinate
- Thermal conductivity measurement method
  - Steady state method
  - Transient method
- Time dependent (transient) conduction
- Convective heat transfer, thermal expansion & thermal stress (very briefly)

Conduction
Transient state

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- If a solid body is suddenly subjected to a change in environment, some time must elapse before an equilibrium temperature condition will prevail in the body. Transient heating/cooling process takes place before equilibrium (steady state).
- Transient (or unsteady) problem:
  - Arise when boundary conditions are changed
  - E.g.) if surface temperature is altered, temperature at each point in the system will also begin to change → The change will continue until a steady state temperature distribution is reached.

Time dependent conduction
semi-infinite half space problem

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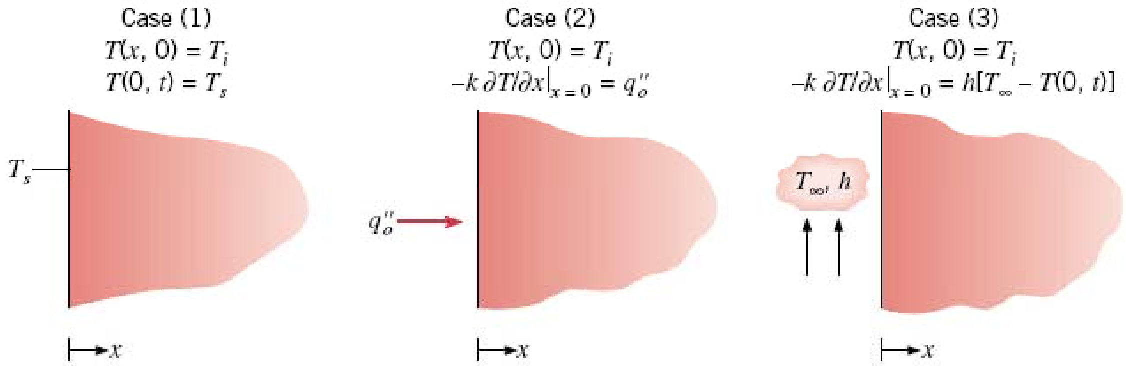
- A single identifiable surface + a solid extends to infinity
- A sudden change of conditions is imposed at this surface → transient, 1D conduction occur within the solid.
- Semi-infinite solid (half-space) provides a useful idealization for many practical problems.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



Time dependent conduction
semi-infinite half space problem

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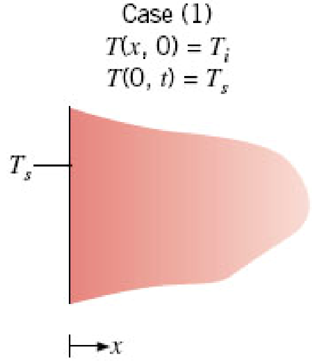
- Derivation for case (1)



$$\frac{T(x,t)-T_s}{T_i-T_s}=\left(\frac{2}{\sqrt{\pi}}\right)\int_0^{\eta}\exp(-u^2)du=erf\left(\eta\right)=erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

$$q_s''=\frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}$$

Semi-infinite half space problem
Derivation (1)

- Existence of similarity variable, *η*
- Partial differential equation with x, t → ordinary differential equation with *η*

$$\eta = \frac{x}{\sqrt{4\alpha t}}$$

- We first transform the pertinent differential operator,

$$\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}\left(\frac{\partial T}{\partial x}\right)\frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t}\frac{d^2 T}{d\eta^2}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} = -\frac{x}{2t\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

Semi-infinite half space problem
Derivation (2)

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- Substituting into 1D diffusion equation,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \longrightarrow \frac{\partial^2 T}{\partial \eta^2} = -2\eta \frac{\partial T}{\partial \eta}$$

- Boundary conditions become;

$$T(0,t) = T_s \longrightarrow T(\eta = 0) = T_s$$

$$\left.\begin{array}{l} T(x \to \infty, t) = T_i \\ T(x,0) = T_i \end{array}\right\} \longrightarrow T(\eta \to \infty) = T_i$$

Both the IC and the interior BC correspond to the single requirement

Semi-infinite half space problem
Derivation (3)

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– Now T is uniquely defined by η. Let's solve T now. T(η) may be obtained by separating variables, such that

$$\frac{d(\partial T/\partial\eta)}{(\partial T/\partial\eta)} = -2\eta d\eta$$

– By integration,

$$\ln(dT/d\eta) = -\eta^2 + C_1' \quad or \quad \frac{dT}{d\eta} = C_1 \exp(-\eta^2)$$

– Integrating a second time,

– $$T = C_1 \int_0^{\eta} \exp(-u^2)du + C_2$$ where u is dummy.

$$T(\eta = 0) = T_s \longrightarrow C_2 = T_s$$

$$T = C_1 \int_0^{\eta} \exp(-u^2)du + T_s$$

Semi-infinite half space problem
Derivation (4)

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$$T(\eta \to \infty) = T_i \longrightarrow T_i = C_1 \int_0^{\infty} \exp(-u^2)du + T_s \longrightarrow C_1 = \frac{2(T_i - T_s)}{\sqrt{\pi}}$$

$$T(x,t) = \left(\frac{2}{\sqrt{\pi}}\right)(T_i - T_s)\int_0^{\eta} \exp(-u^2)du + T_s$$

– Note that error function is defined as;

– $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^{x} \exp(-u^2)du$ For example, $erf(0) = 0, \quad erf(\infty) = 1$

– The final solution can be also described as;

$$\frac{T(x,t) - T_s}{T_i - T_s} = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^{\eta} \exp(-u^2)du = erf(\eta) = erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

Semi-infinite half space problem
Derivation (5)

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– Surface heat flux may be obtained by applying Fourier's law,

$$q_s'' = -k \left.\frac{\partial T}{\partial x}\right|_{x=0} = -k(T_i - T_s) \left.\frac{d(erf\,\eta)}{d\eta}\frac{d\eta}{dx}\right|_{\eta=0}$$

$$q_s'' = -k(T_s - T_i) \left.\frac{2}{\sqrt{\pi}}\exp(-\eta^2)\frac{1}{\sqrt{4\alpha t}}\right|_{\eta=0}$$

$$q_s'' = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

Time dependent conduction
semi-infinite half space problem

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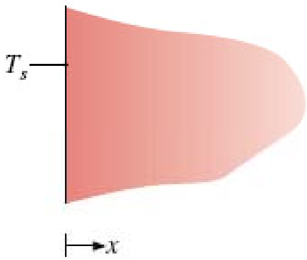
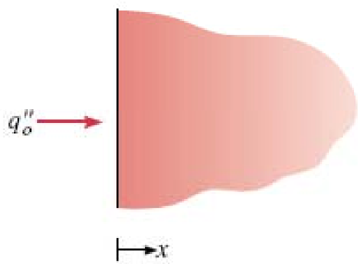
Case (1)
$T(x, 0) = T_i$
$T(0, t) = T_s$



$$\frac{T(x,t)-T_s}{T_i-T_s} = erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

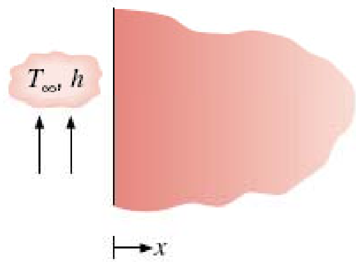
$$q_s'' = \frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}$$

Case (2)
$T(x, 0) = T_i$
$-k\,\partial T/\partial x|_{x=0} = q_o''$



$$T(x,t)-T_i = \frac{2q_0''(\alpha t/\pi)^{1/2}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_0'' x}{k}\, erfc\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

Case (3)
$T(x, 0) = T_i$
$-k\,\partial T/\partial x|_{x=0} = h[T_\infty - T(0, t)]$



$$\frac{T(x,t)-T_s}{T_i-T_s} = erf\left(\frac{x}{\sqrt{4\alpha t}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

Time dependent conduction
semi-infinite half space problem

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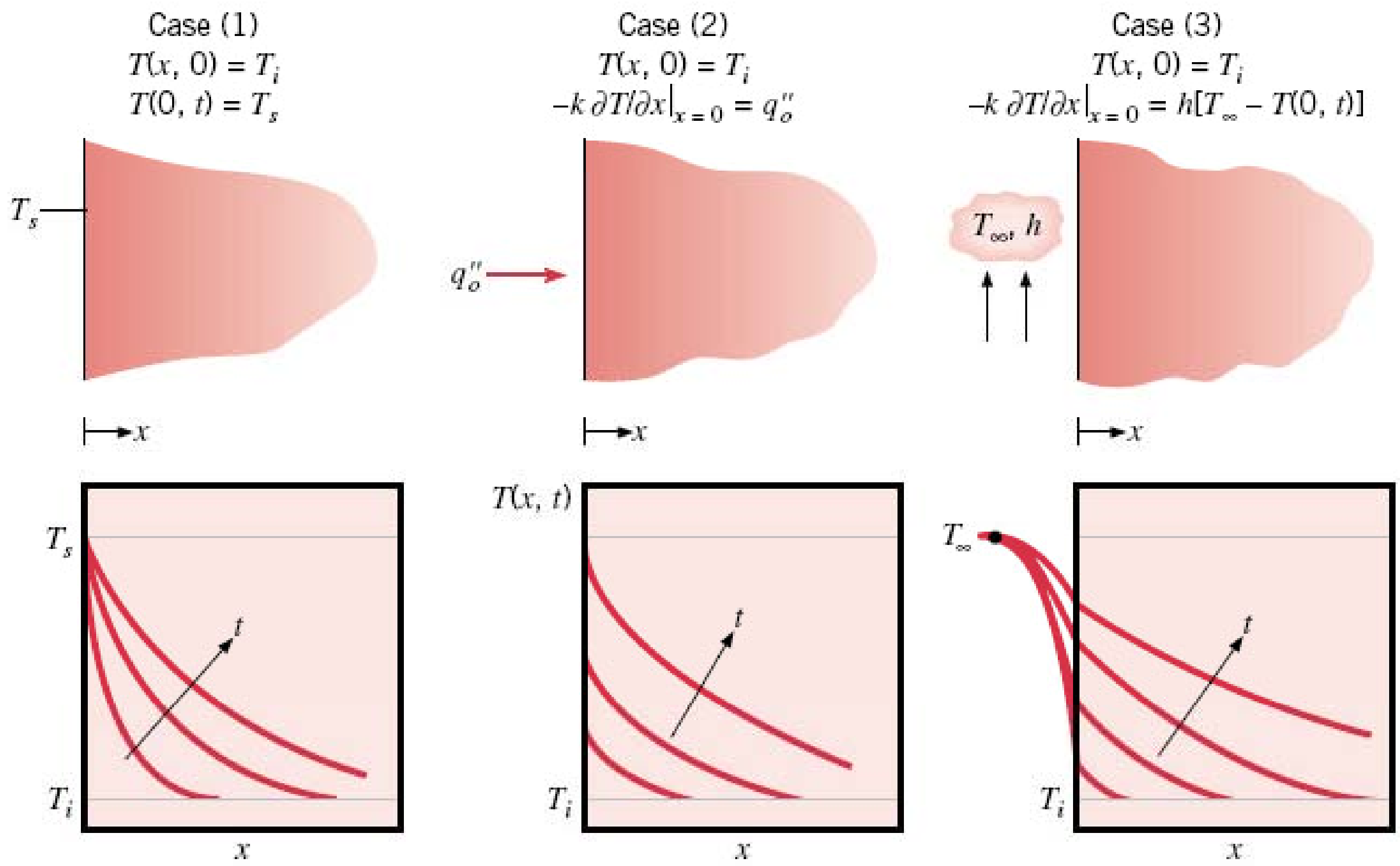

FIGURE 5.7 Transient temperature distributions in a semi-infinite solid for three surface conditions: constant surface temperature, constant surface heat flux, and surface convection.

Time dependent conduction
semi-infinite half space problem

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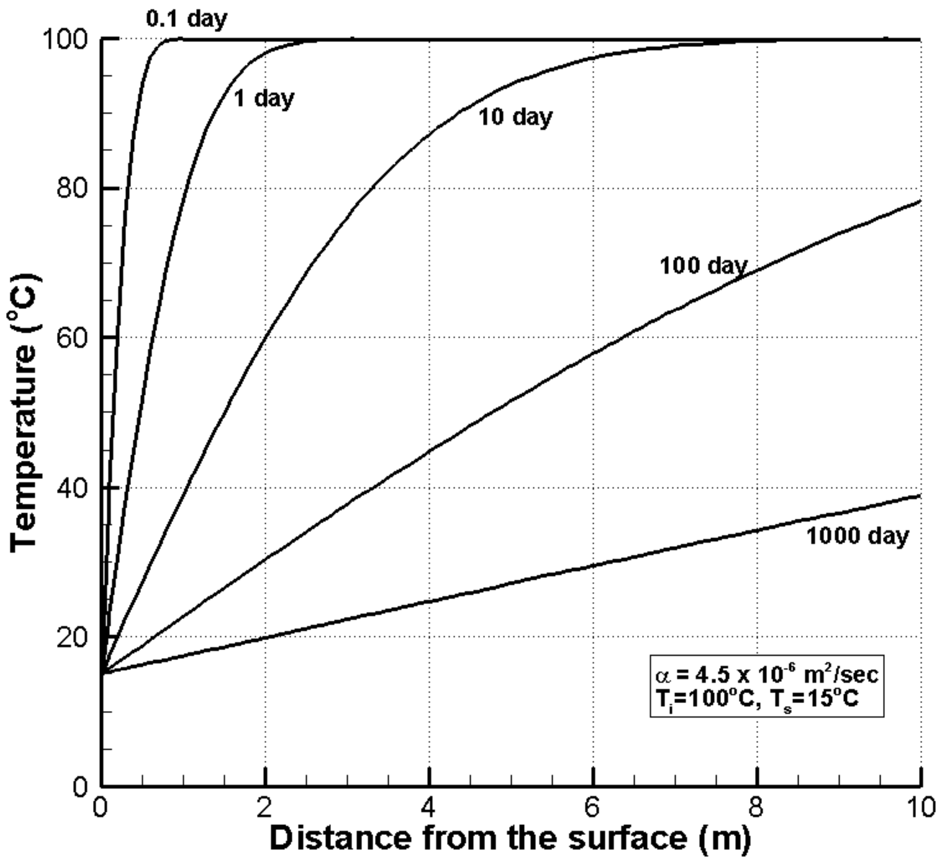

- For granite from Forsmark, Sweden,
- k = 3.58 W/mK, ρ: 1000 kg/m^3, cp: 796 (J/kg·K) → α = 4.5x10^{-6} m^2/sec
- Homework#2 Q3. Reproduce this graph.

Time dependent conduction
a cases with fixed T at both ends

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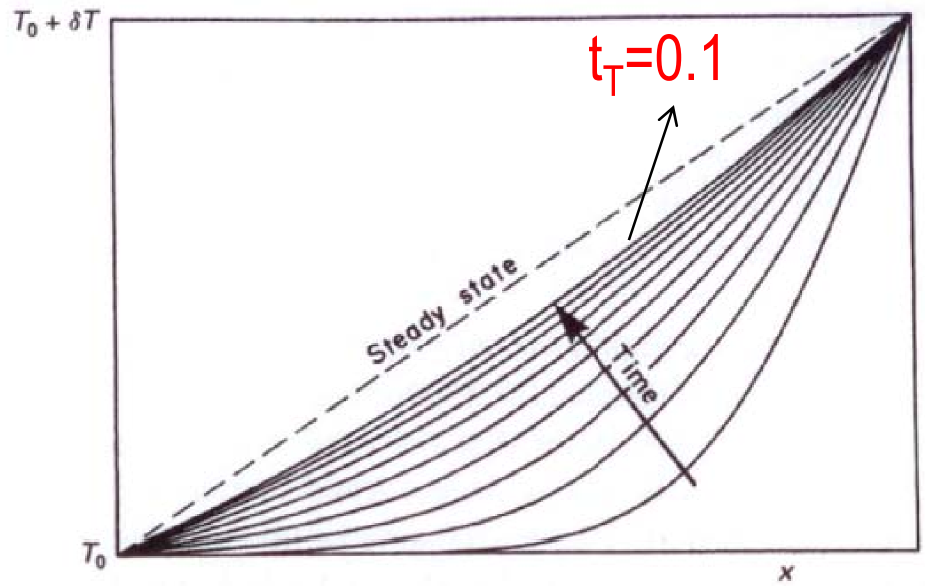


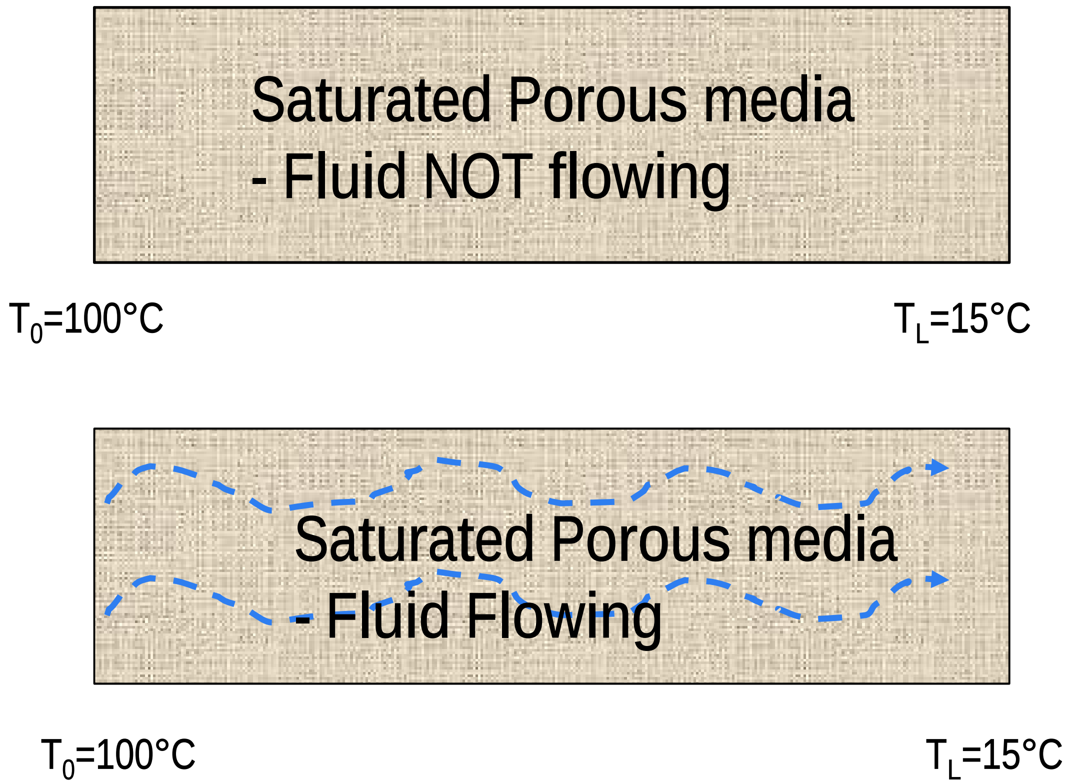
Fig. 12.1. Conduction of heat into a region of lower temperature (on the left) from a boundary (on the right) suddenly raised in temperature. Distance is normalized by the total thickness of the conducting region, and temperature by the temperature increment δT.

Middleton and Wilcock, 1999

– Thermal relaxation time: a characteristic time for heat to diffuse through the layer(Middleton and Wilcock,1999). *d*: thickness of layer, *α*: diffusivity. $t_T = d^2 / \alpha$

Convective heat transfer

- What will be the factors that make these two cases different?



- These will be covered after we deal with fluid flow in rock

Summary

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- Transient conduction problem
  - Temperature changes with time
- Coupled process associated with thermal transfer
  - Convective heat transfer
  - Thermal expansion and thermal stress
- Fluid flow in rock   - next week
  - Porous rock
  - Fractured rock

References used for this lecture

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- Somerton WH, 1992, Thermal properties and temperature-related behavior of rock/fluid systems, Elsevier
- Middleton GV and Wilcock PR, 1999, Mechanics in the Earth and Environmental Sciences, Cambridge Univ Press