Introduction to fluid flow in rock (Week 6, 5 Oct)

Ki-Bok Min, PhD

Assistant Professor
Department of Energy Resources Engineering
Seoul National University

SEOUL NATIONAL UNIVERSITY

Content of last week and this week's lecture

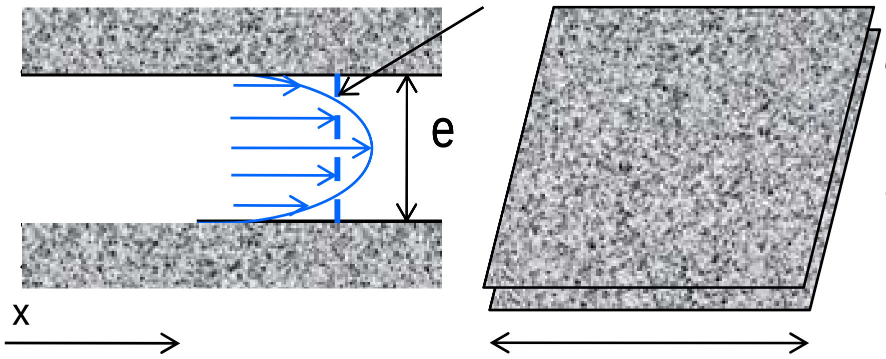
SEOUL NATIONAL UNIVERSITY

- Fluid flow in fractured media – last week
  - Cubic law
  - Permeability defined in fractured rock
  - Characterisation and Discrete Fracture Network (DFN)
- Some useful steady state and transient solutions – this week
  - Flow to a well in confined aquifer
    - Steady state solution
    - Transient Theis solution
  - Method of measuring hydraulic conductivity and specific storage
    - Curve matching method, Time drawdown method & Distance drawdown method

Fractured rock fluid flow
Cubic Law

SEOUL NATIONAL UNIVERSITY

We take average velocity. $V_{av}=2/3\ V_{max}$



$$Q=\int_{-\frac{e}{2}}^{\frac{e}{2}} v(wdy)=-\frac{we^3}{12\mu}\frac{d}{dx}(p+\rho_w gz)$$

$$average\ velocity, v=Q/ew=-\frac{e^2}{12\mu}\frac{d}{dx}(p+\rho_w gz)=-\frac{e^2}{12\mu}\frac{d}{dx}\rho_w gh$$

Hydraulic conductivity (K) of parallel plate model

$$q=-\frac{\rho_w ge^2}{12\mu}\frac{\partial h}{\partial x} \qquad Q=qA=-\frac{\rho_w ge^2}{12\mu}A\frac{\partial h}{\partial x}=-\frac{\rho_w ge^2}{12\mu}(e\times 1)\frac{\partial h}{\partial x}$$

ρ_w: density of fluid
g: acceleration of gravity
μ: viscosity

$$\boxed{Q=-\frac{\rho_w ge^3}{12\mu}\frac{\partial h}{\partial x}} \qquad \boxed{Q=-\frac{e^3}{12\mu}\frac{\partial p}{\partial x}} \leftarrow \text{with zero elevation}$$

– Cubic law: for a given gradient in head and unit width (w), flow rate through a fracture is proportional to the <u>cube</u> of the fracture aperture.

Terminology

- Aquifer (대수층): a rock unit that is sufficiently permeable so as to supply water to wells.
- Aquitard (준대수층): beds of lower permeability in the stratigraphic sequence that contain water but do not readily yield water to pumping wells.
- Artesian well (자분정): a well that flows at the surface without pumping – hydraulic head lies above the ground level*
- Reservoir :1.저수지, <u>2. A subsurface body of rock having sufficient porosity and permeability to store and transmit fluids</u> (Schlumberger oilfield glossary, 2009).
- Geothermal reservoir: reservoir suitable for geothermal energy utilization.

*Oxford Dictionary of earth sciences, 2003

Diffusion equation

SEOUL NATIONAL UNIVERSITY

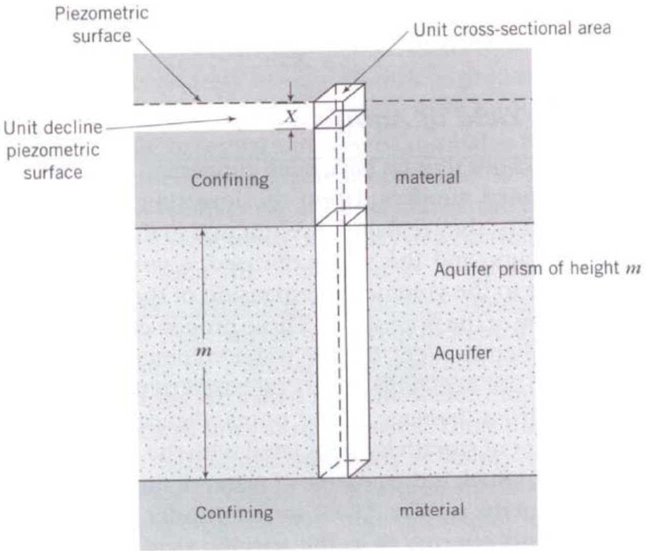
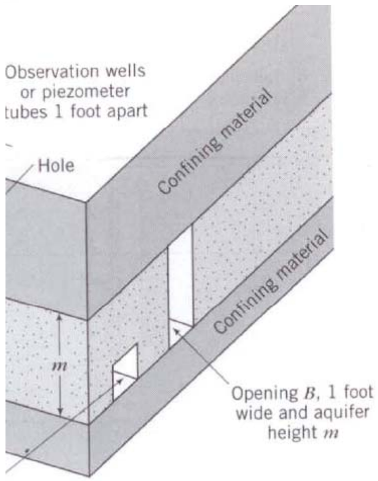


Figure 4.6 Diagram illustrating the storativity for confined conditions (from Ferris and others, 1962).



Domenico & Schwartz, 1998

$$\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right)+\dot{q}=S_s\frac{\partial h}{\partial t}$$

– For a confined aquifer of thickness m. Transmissivity, T is defined as (hydraulic conductivity defined in a unit width);

$$T = Km$$

$$\nabla^2 h = \frac{S_s}{K}\frac{\partial h}{\partial t} = \frac{S_s m}{Km}\frac{\partial h}{\partial t} = \frac{S}{T}\frac{\partial h}{\partial t}$$

- Storativity S is the product of S_s and m.

Steady state solution
Flow to a well in a confined aquifer

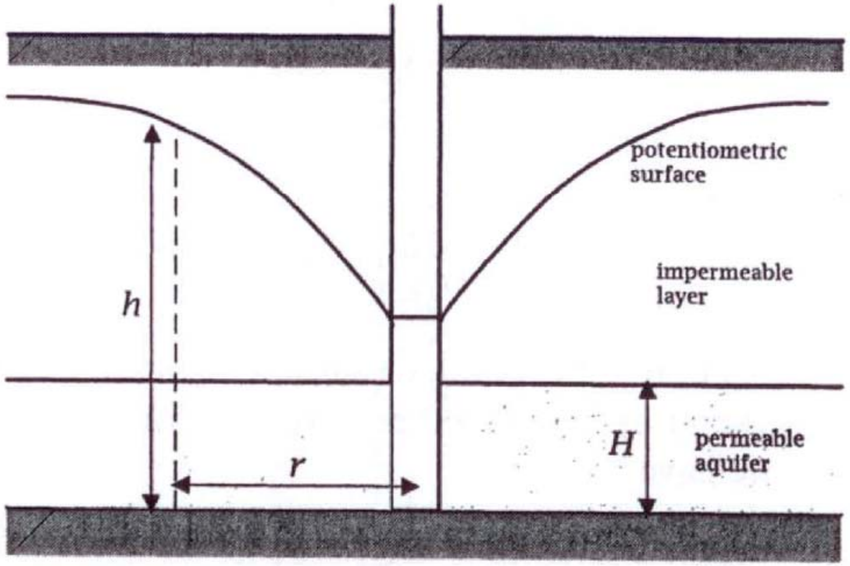


Fig. 6.5. Flow to a well in a confined aquifer.

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}+\frac{1}{r^2}\frac{\partial^2 h}{\partial \phi^2}=\frac{S}{T}\frac{\partial h}{\partial t}$$

$$\frac{1}{r}\frac{\partial h}{\partial r}\left(r\frac{\partial h}{\partial r}\right)=0$$

$$r\frac{\partial h}{\partial r}=C_1 \qquad h=C_1\ln r+C_2$$

– Applying BC (known h_1 and h_2 at r_1 and r_2, respectively)

$$h_1=C_1\ln r_1+C_2 \qquad h_2=C_1\ln r_2+C_2$$

$$C_1=\frac{(h_1-h_2)}{\ln(r_1/r_2)}$$

$$\frac{\partial h}{\partial r}=\frac{(h_1-h_2)}{\ln(r_1/r_2)}\frac{1}{r}$$

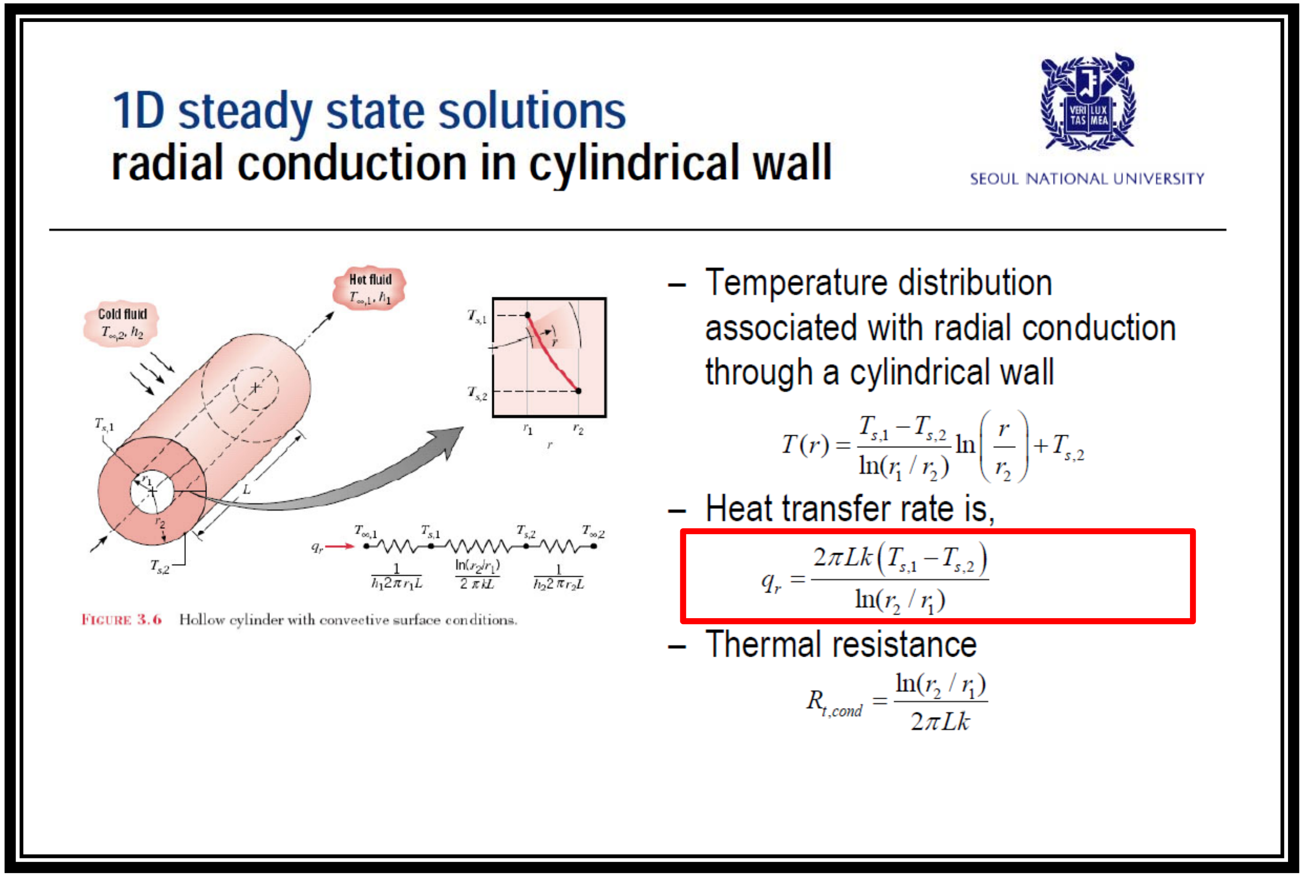
$$Q=-KA\frac{\partial h}{\partial r}=-K(2\pi rH)\frac{\partial h}{\partial r}$$

$$Q=-2\pi KH\frac{(h_1-h_2)}{\ln(r_1/r_2)}$$

Analogy with heat transfer

SEOUL NATIONAL UNIVERSITY

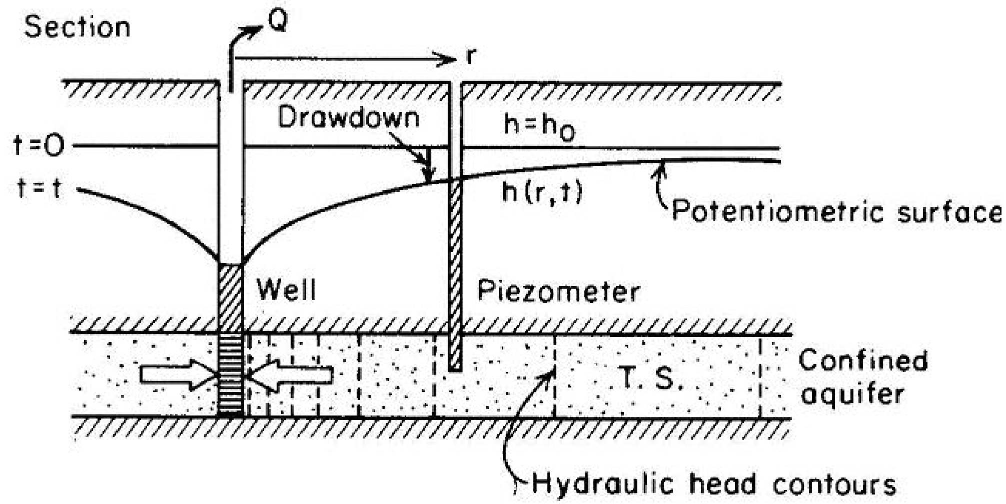
- Much of solutions for porous media fluid flow can be taken from heat conduction.



Transient Theis solution

Flow to a well in a confined aquifer

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}+\frac{1}{r^2}\frac{\partial^2 h}{\partial \phi^2}=\frac{S}{T}\frac{\partial h}{\partial t}$$



$$h_0-h=s=\frac{Q}{4\pi T}\int_{r^2S/4Tt}^{\infty}\frac{e^{-z}}{z}dz$$

h_0: original heat at any distance *r* from a fully penetrating well at time *t* equals zero

h: heat at some later time *t*

s: drawdown, difference between h_0 and h

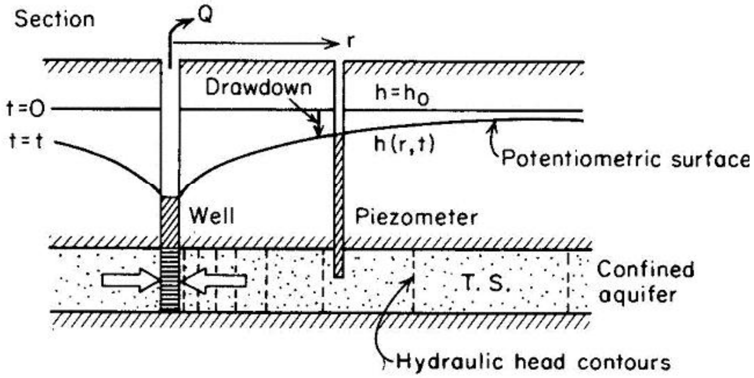
Q: steady pumping rate

T: transmissivity

S: storativity

Transient Theis solution
Flow to a well in a confined aquifer

SEOUL NATIONAL UNIVERSITY



- Based on the work by Theis (1935)
- Used the analogy with heat transfer
- When a steadying pumping is conducted in a well, the head difference at any given radius is expressed as follows.

$$h_0 - h = s = \frac{Q}{4\pi T}\int_{r^2S/4Tt}^{\infty}\frac{e^{-z}}{z}dz = \frac{Q}{4\pi T}W(u) \qquad u = \frac{r^2 S}{4Tt} = \frac{r^2 S_s}{4Kt}$$

h_0: original heat at any distance *r* from a fully penetrating well at time *t* equals zero
h: heat at some later time *t*
s: drawdown, difference between h_0 and h
Q: steady pumping rate (m^3/sec)
T: transmissivity (hydraulic conductivity x thickness), m^2/sec
S: storativity (specific storage x thickness), dimensionless

Transient Theis solution
Flow to a well in a confined aquifer

SEOUL NATIONAL UNIVERSITY

- Expressed in terms of <u>exponential integral,</u>

$$\int_u^\infty \frac{e^{-z}}{z} dz = -0.577216 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots = Ei(x)$$

$$where\ u = \frac{r^2 S}{4Tt} = \frac{r^2 S_s}{4Kt}$$

- Initial and boundary conditions are expressed as follows

$$h(r,0) = h_0,\ h(\infty,t) = h_0$$

$$\lim_{r\to 0}\left(r\frac{\partial h}{\partial r}\right) = \frac{Q}{2\pi T} \quad for\ \ t > 0 \longleftarrow K(2\pi rm)\frac{\partial h}{\partial r} = Q = 2\pi Tr\frac{\partial h}{\partial r}$$

- Assumption:
  - Homogeneous and isotropic medium
  - The aquifer is infinite in areal extent and infinite amount of water is stored in the aquifer.
  - Well is of intinitesimal diameter and fully penetrates the aquifer.
  - 2D approximation ← hydraulic head does not vary in the third dimension

Transient Theis solution

Flow to a well in a confined aquifer

SEOUL NATIONAL UNIVERSITY

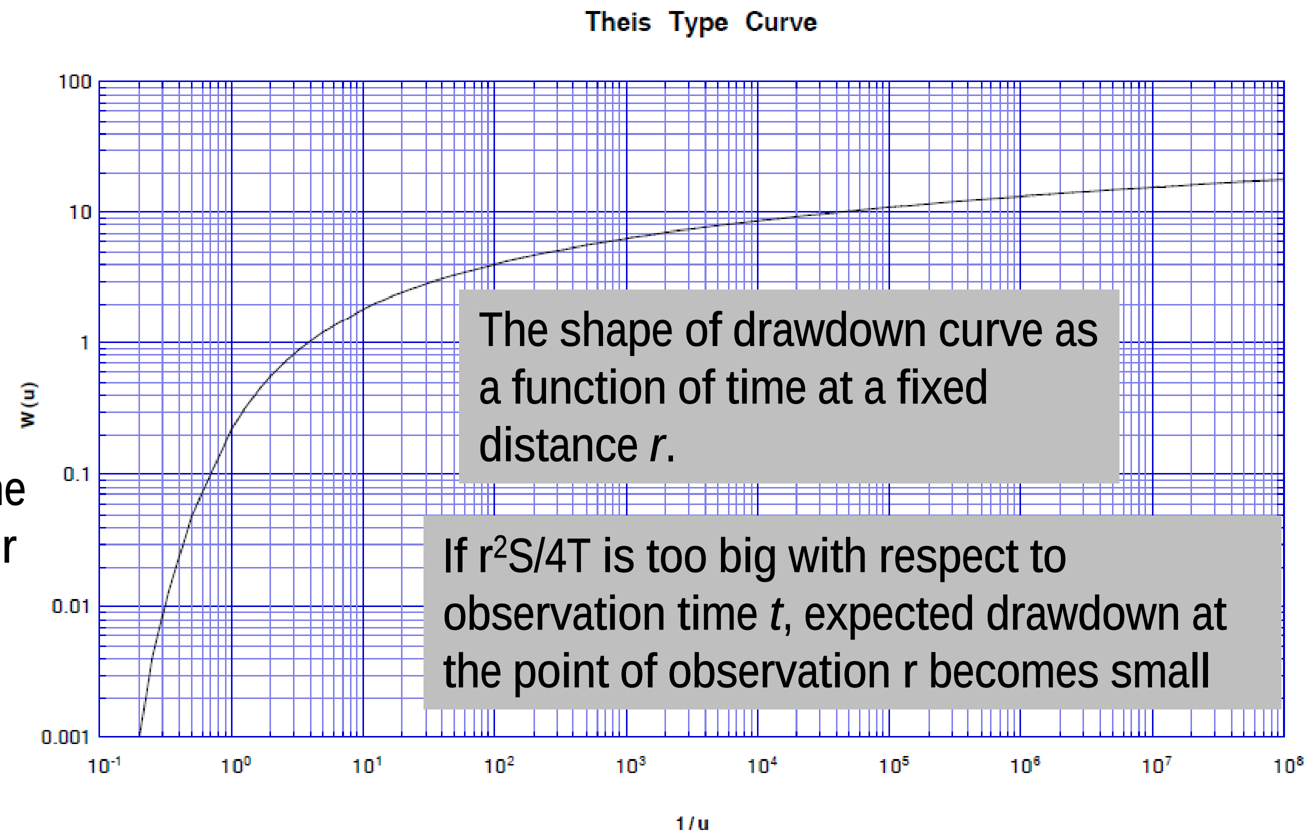
– Well function W(u)

$$\int_u^\infty \frac{e^{-z}}{z}dz = -0.577216 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots = W(u)$$

$$s = \frac{Q}{4\pi T}W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

Type curve: curves showing the relation between W(u) and u (or 1/u more precisely)



Transient Theis equation
Curve matching procedure

SEOUL NATIONAL UNIVERSITY

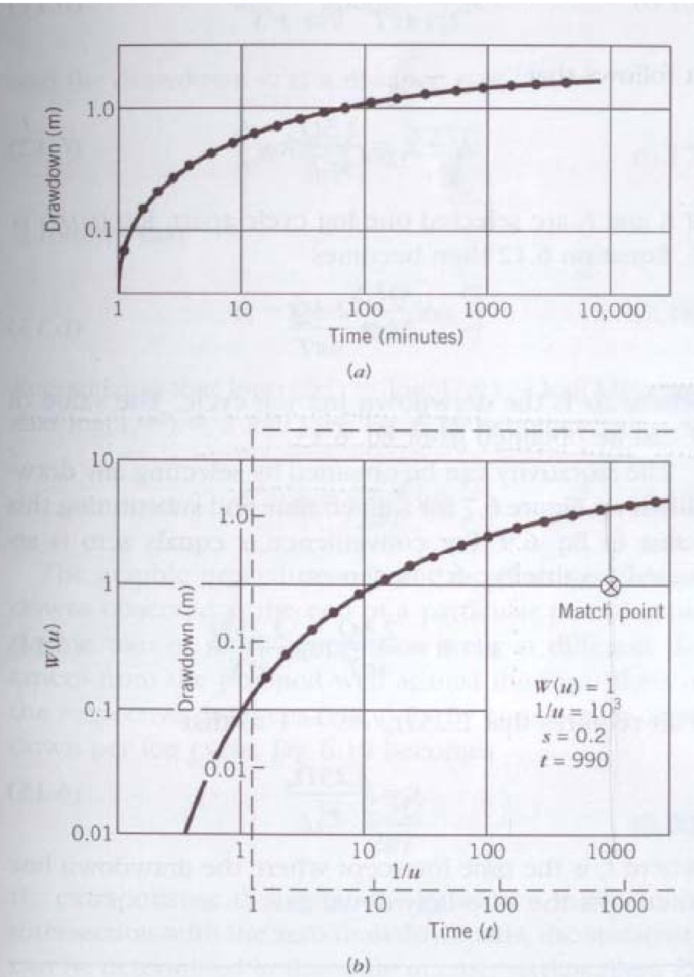


Figure 6.6 Graphs of (*a*) field data and (*b*) superposition of the field data on the type curve to obtain the formation parameters. Domenico & Schwartz, 1998

- Data collected at a distance of 150 m from a well pumped at a rate of 5.43×10^3 m^3/day.
- $W(u) = 1$, $s = 0.2$ m, $1/u = 10^3$, t=990 min (0.7 day)
- Transmissivity:

$$T = \frac{Q}{4\pi s} W(u) = \frac{5.43 \times 10^3 m^3 / day \times 1}{4 \times 3.14 \times 0.1 m} = 2.2 \times 10^3 m^2 / day$$

- Storativity;

$$S = \frac{4uTt}{r^2} = \frac{4 \times 1 \times 10^{-3} \times 2.2 \times 10^3 m^3 / day \times 7 \times 10^{-1} day}{225 \times 10^2 m^2} = 2.7 \times 10^{-4}$$

Modification of transient equation

SEOUL NATIONAL UNIVERSITY

0 for small u

$$\int_u^\infty \frac{e^{-z}}{z}dz = -0.577216 - \ln u + \boxed{u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots = W(u)}$$

$$h_0 - h = s = \frac{Q}{4\pi T}\left(-0.5772 - \ln\frac{r^2 S}{4Tt}\right)$$

$$h_0 - h = s = \frac{Q}{4\pi T}\left(\ln\frac{4Tt}{r^2 S} - 0.5772\right) = \frac{Q}{4\pi T}\left(\ln\frac{4Tt}{r^2 S} - \ln 1.78\right)$$

$$h_0 - h = s = \frac{Q}{4\pi T}\ln\frac{2.25Tt}{r^2 S} = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r^2 S} \longleftarrow \ln x = 2.3\log x$$

Modification of transient equation
Time-drawdown Method

SEOUL NATIONAL UNIVERSITY

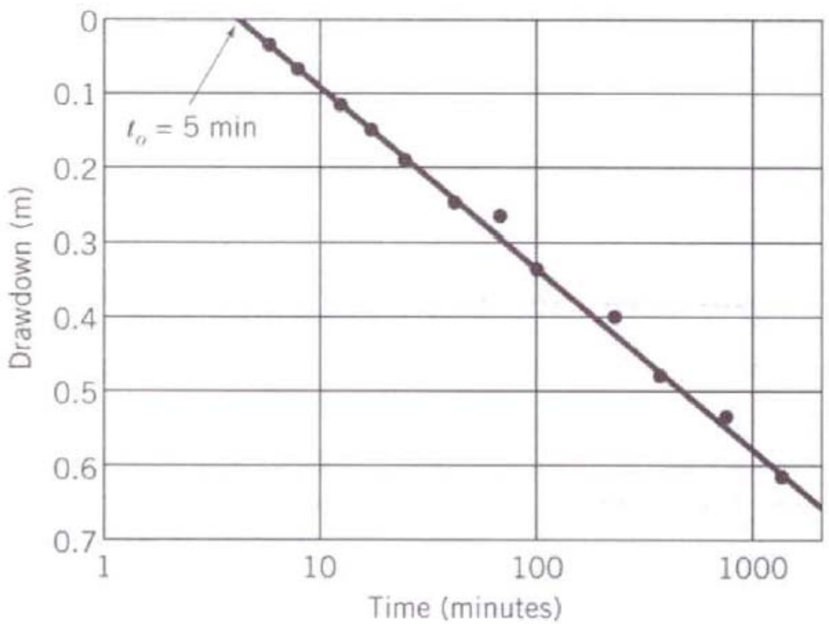


Figure 6.7 Semilogarithmic plot of drawdown versus time in an observation well.

- One observation well for various times
- If a drawdown observation is made in a single well for various times, a plot of drawdown versus the logarithm of time will yield a straight line.

$$h_0 - h_1 = s_1 = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt_1}{r^2 S}$$

$$h_0 - h_2 = s_2 = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt_2}{r^2 S}$$

$$s_1 - s_2 = \frac{2.3Q}{4\pi T}\log\frac{t_2}{t_1} = \frac{Q}{4\pi T}\ln\frac{t_2}{t_1}$$

Domenico & Schwartz, 1998

Modification of transient equation
Time-drawdown Method (example)

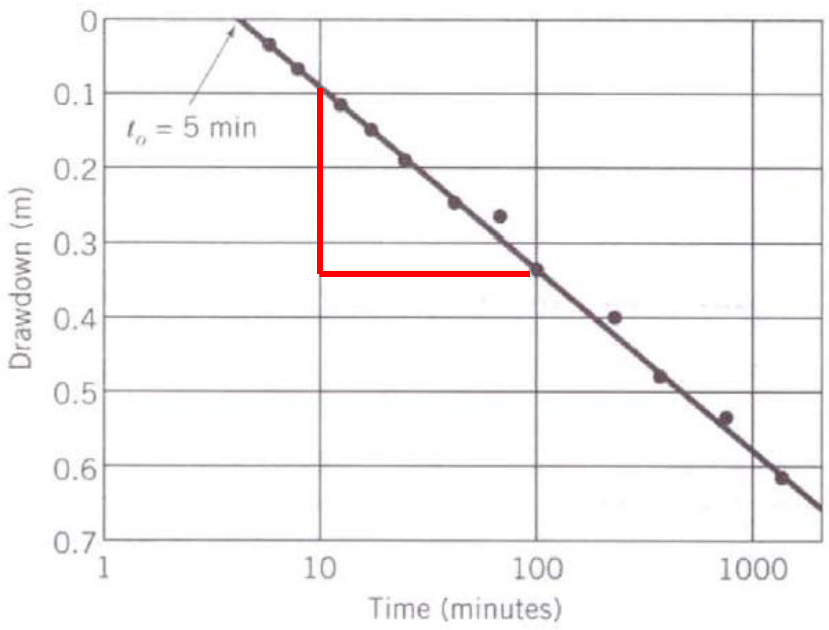


Figure 6.7 Semilogarithmic plot of drawdown versus time in an observation well.

- 305 m from a well pumping at a rate of 5.43 x 10^3 m^3/day
- Drawdown from 10 to 100 sec is 0.24 m.

$$T = \frac{2.3Q}{4\pi s}\log\frac{t_2}{t_1} = \frac{2.3\times 5.43\times 10^3 m^3/day}{4\times 3.14\times 0.24m}$$

$$= 4.1\times 10^3 m^2/day$$

- We can select any point in the graph. When s=0 is selected for convenience, $s = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r^2S}$

1

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25\times 4.1\times 10^3 m^2/day\times 5\min}{93025m^2\times 1440\min/day}$$

$$= 4.1\times 10^3 m^2/day$$

Domenico & Schwartz, 1998

Analogy with heat transfer

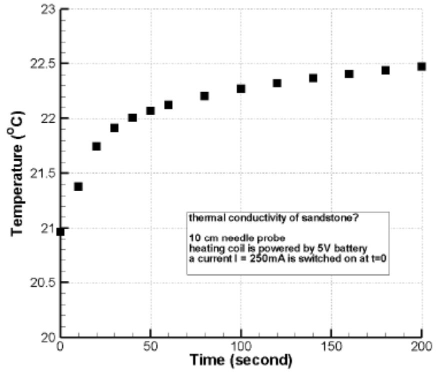
Time-drawdown Method – analogy with thermal transfer

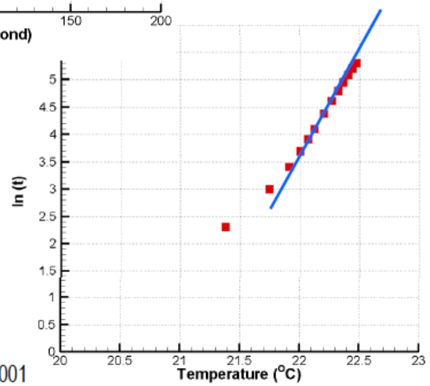
SEOUL NATIONAL UNIVERSITY

Hydraulic conductivity measurement

Transient Method (2) - an example

SEOUL NATIONAL UNIVERSITY





Data from Beardsmore and Cull, 2001

- With a line source of heat and a temperature sensor packed closely,

$$k = (Q_l / 4\pi)(\partial \ln(t) / \partial T)$$

- Q_l : applied heat per unit length(W/m)
  t : time (second)
  T: Temperature (K)

- Find a linearity and obtain k

P= V x I = 5 x 0.25 = 1.25W
Q=P/0.1(cm)=12.5W/m
Gradient between 60 & 200 sec is ~3.449
K = 12.5/4π x 3.449 = 3.43 W/mK

Modification of transient equation
Distance-drawdown Method

SEOUL NATIONAL UNIVERSITY

- Observation at two or more points at one instant of time

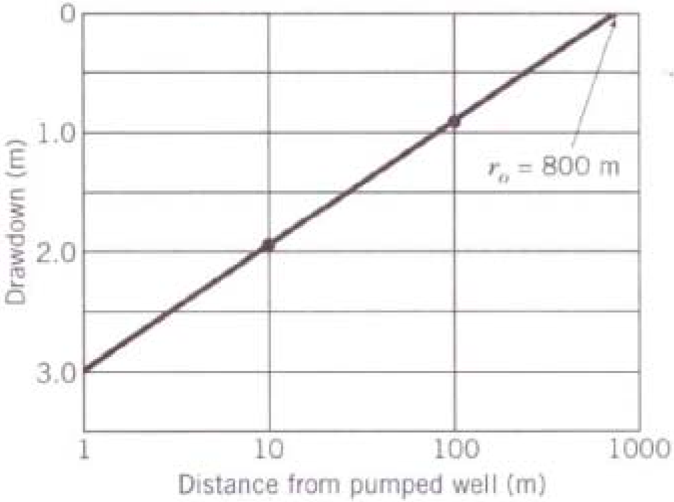


Figure 6.8 Semilogarithmic plot of drawdown versus distance.

$$s_1 = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r_1^2 S}$$

$$s_2 = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r_2^2 S}$$

$$s_1 - s_2 = \frac{2.3Q}{2\pi T} \log \frac{r_1^2}{r_1^2} = \frac{Q}{2\pi T} \ln \frac{r_1^2}{r_1^2}$$

Domenico & Schwartz, 1998

Content of today's lecture

SEOUL NATIONAL UNIVERSITY

- Some useful steady state and transient solutions
  - Terminology
  - Flow to a well in confined aquifer
    - Steady state solution
    - Transient Theis solution
  - Method of measuring hydraulic conductivity and specific storage
    - Curve matching method, Time drawdown method & Distance drawdown method
- Wednesday
  - Exploration techniques
    By Tae Jong Lee from KIGAM (Korea Institute of Geosciences and Mineral Resources)
  - One question gives 2 points. (attendance of one lecture = 2 points)

References

SEOUL NATIONAL UNIVERSITY

- Domenico PA, Schwartz FW, 1998, Physical and Chemical Hydrogeology, John Wiley & Sons, Inc.
- Schlumberger oilfield dictionary, http://www.glossary.oilfield.slb.com/default.cfm
- de Marsily G, 1986, Quantitative Hydrogeology, Academic press, Inc.
- Oxford Dictionary of Earth Sciences, 2003, Oxford University Press