Geothermal Energy (Week 8, 19 Oct)
- Reservoir Geomechanics

민기복

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Mid-term exam

- Average: 85
- Maximum: 98

(1) Typical range of thermal conductivity of rock is around 1.5 ~4.0 W/m·K. (T, F)

(2) Specific heat capacity of water is larger than average rock, e.g., unweathered granite. (T, F)

(3) Linear thermal expansion coefficient of rock (for example, sandstone) is in the order of 10^{-5} /°C. (T, F)

(4) Hydraulic conductivity of shale is typically a few orders larger than that of sandstone (T,F)

(5) Peclet number expresses the transport of energy by bulk fluid motion to the energy transport by conduction. (T, F)

Introduction
1st half of the course

- Week 1: Introduction to the course
- Week 2: Overview of Geothermal Energy
- Week 3: Heat Transfer (1) – conduction, convection, radiation
- Week 4: Heat Transfer (2) – Heat diffusion equation
- Week 5: Fluid flow in porous media
- Week 6: Fluid flow in fractured media
  Exploration techniques (invited lecture)
- Week 7: Fluid flow in porous media (conduction-convection problem)
  Mid-term exam

Introduction
2nd half of the course

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- Week 8: Reservoir Geomechanics
- Week 9: Reservoir Geomechanics — Progress report (24:00 30 Oct)
- Week 10: Environmental Impact/Enhanced Geothermal System (EGS)
  Geothermal Power Generation
- Week 11: Video (direct and indirect use of geothermal energy)
- Week 12: Field Visit (석모도, Friday, 20 Nov)
- Week 13: Geothermal Energy in Korea (invited lecture)
  Heat Pump applications in Korea (invited lecture)
  Final Exam - take-home exam (?)
- Week 14: Report writing guide — Final report (24:00 4 Dec)
- Week 15: Student conference (7 & 9 Dec)

Questions

- What if I didn't take 3rd year 'rock mechanics', 'petroleum engineering' or 'hydrogeology'?

- Method of evaluation?
  - absolute evaluation (절대평가)

Reservoir Geomechanics
outline

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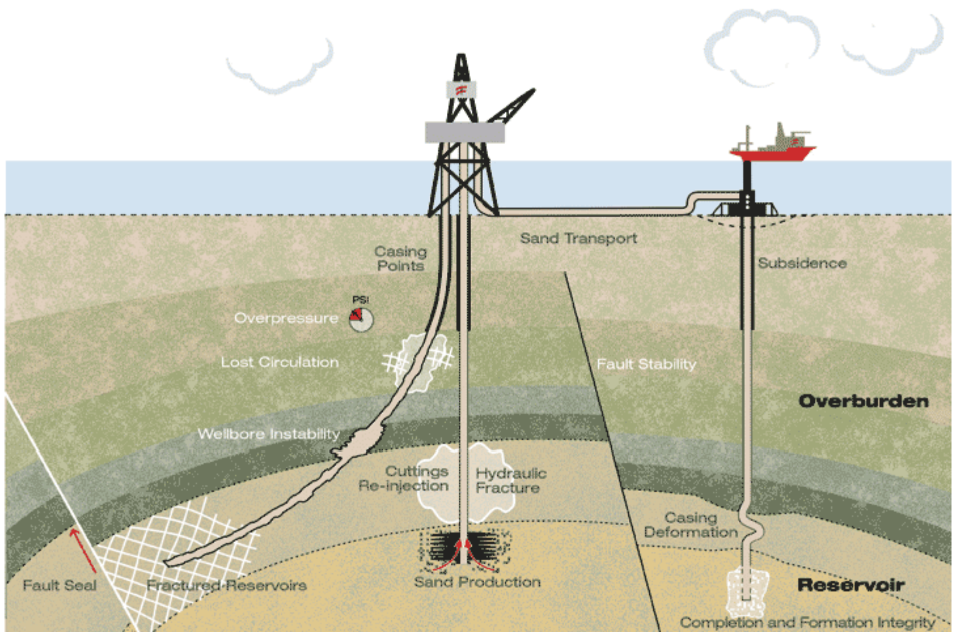
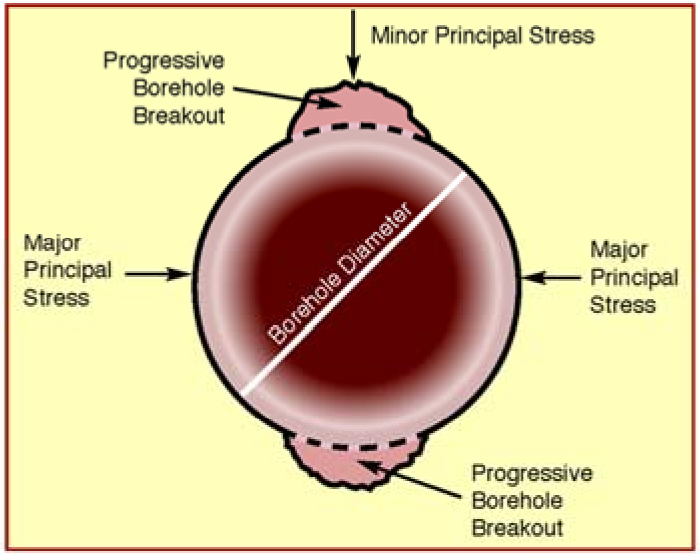
- Fundamentals of rock mechanics
- Borehole stability – stability of geothermal wellbore
- Mechanics of Hydraulic fracturing
- Reservoir Geomechanics





http://www.swri.edu/3PUBS/BROCHURE/D20/geotech/geotech.HTM

http://www.helix-rds.com/EnergyServices/HelixRDS/Capabilities/Geomechanics/tabid/178/Default.aspx

Reservoir Geomechanics

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- Fundamentals of rock mechanics
  - Concept of stress & Mohr Circle
  - Mechanical properties (Elastic modulus and Poisson's ratio)
  - Strength (compressive, tensile)
  - In situ stress
  - Equilibrium Equation

Reservoir Geomechanics
What is it?

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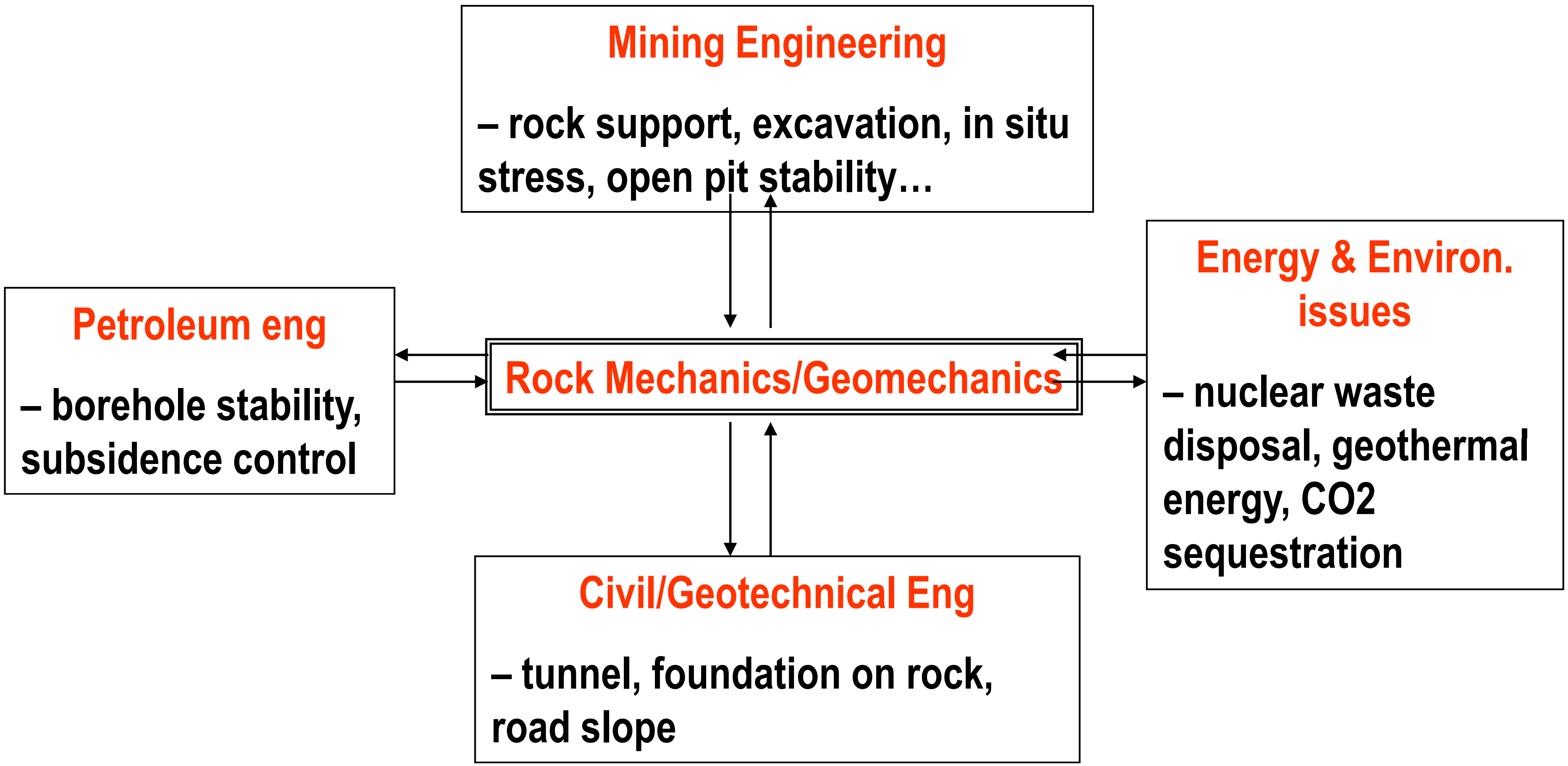

Reservoir Geomechanics
What is it?

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- Rock mechanics: discipline concerned with the stressing, deformation and failure of rock
- Rock Engineering: Rock mechanics + application to engineering
- Geomechanics: Rock mechanics + Soil Mechanics← used more by petroleum industry
- Geotechnical Engineering: (Rock mechanics + soil Mechanics) + application to engineering ← used more by civil engineering industry
- Reservoir Geomechanics: application of rock mechanics or geomechanics to (petroleum/geothermal/???) reservoirs
  - Borehole stability, hydraulic fracturing, subsidence, …

Stress (응력)
Definition

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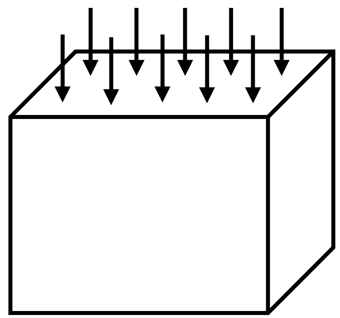
- Stress
  - : a force acting over a given area, F/A ← simple definition
  - the internal distribution of force per unit area that balances and reacts to external loads applied to a body← exact definition
- Normal stress: Normal force/Area

$$\sigma = \frac{F_n}{A}$$

- Shear stress: Shear force/Area

$$\tau = \frac{F_s}{A}$$



- Unit: N/m^2=Pa, 10^6Pa=MPa, 10^9Pa=GPa
  145 psi = 1 MPa = 10 bar = 10 kg중/cm^2

Stress
2D

– Stress in 2D

2D: $\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$

τ_{xy}

Direction of surface normal upon which the stress acts (first subscript, x)

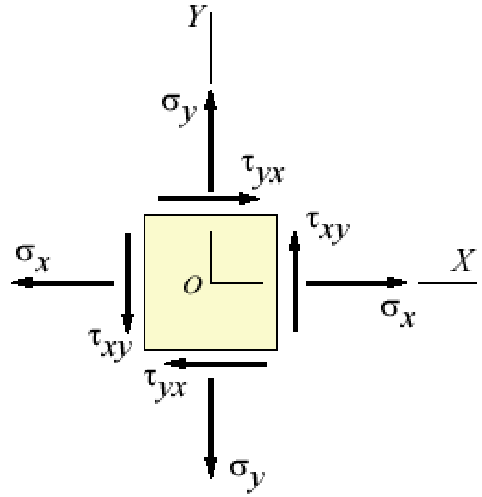
Direction of the stress component (second subscript, y)

– Normal stress: acting perpendicular to the plane

– Shear stress: acting tangent to the plane

– Stress is a 2nd order tensor

- Force is 1st order tensor (=vector)
- Can be defined according to the reference axis
- Principal stresses are defined



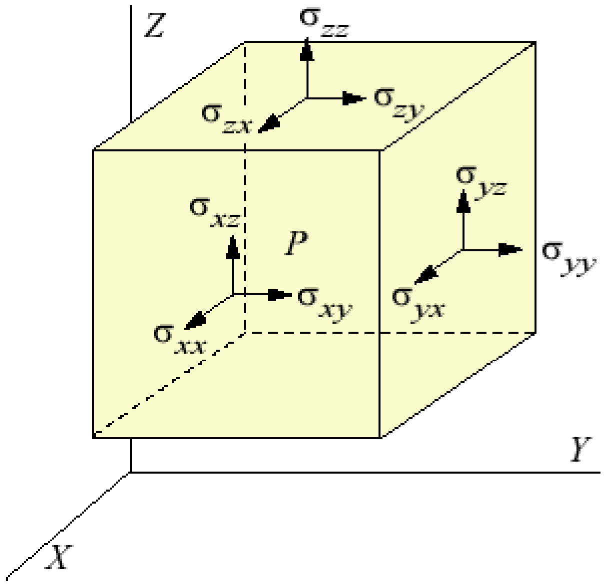
Stress
3D

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

Tensor form

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}$$

matrix form



Stress Transformation

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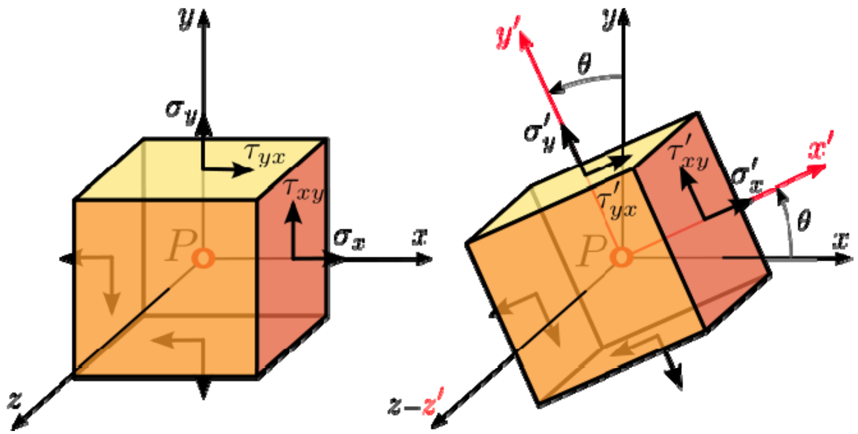
- Transformation of stress
  - When we choose axis rotated by θ, normal and shear stress becomes

$$\sigma_x{}' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \tau_{xy}{}' = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



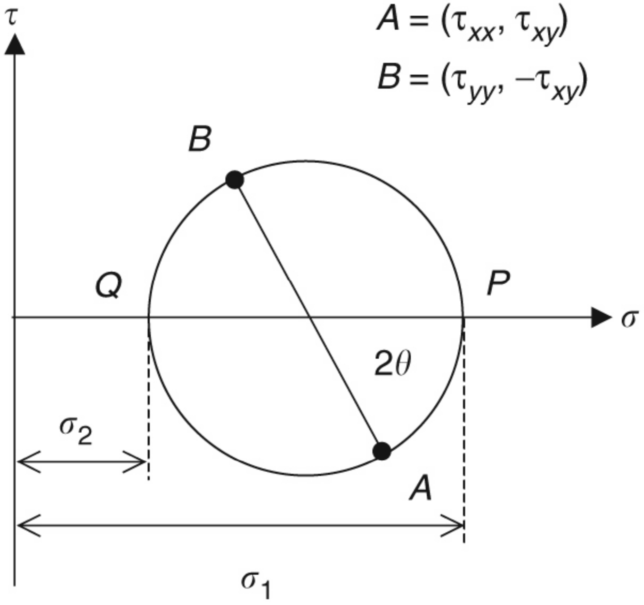
Stress
Mohr's Circle

- One can always find an axis where shear stress goes zero → principal axsi & principal stress
- Using principal stresses (σ_1 and σ_2),

$$\sigma_x' = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$ (0)

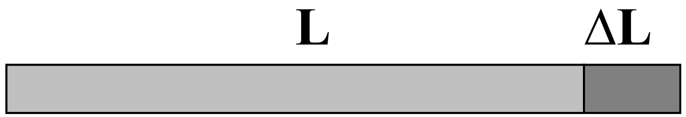
$$\tau_{xy}' = -\frac{\sigma_1 - \sigma_2}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$ (0)

- Radius: $\frac{\sigma_1 - \sigma_2}{2}$
- Center: $\frac{\sigma_1 + \sigma_2}{2}$
- Maximum shear stress: $\frac{\sigma_1 - \sigma_2}{2}$

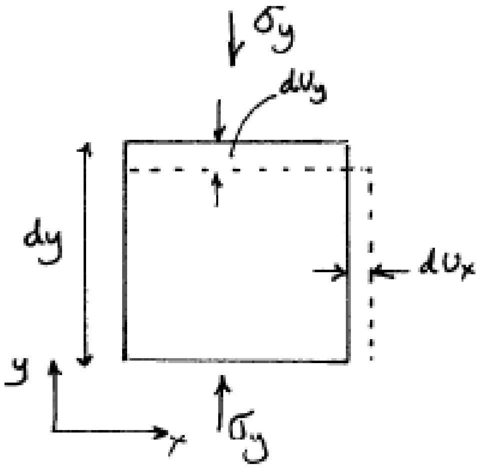


Strain – 1D & 2D

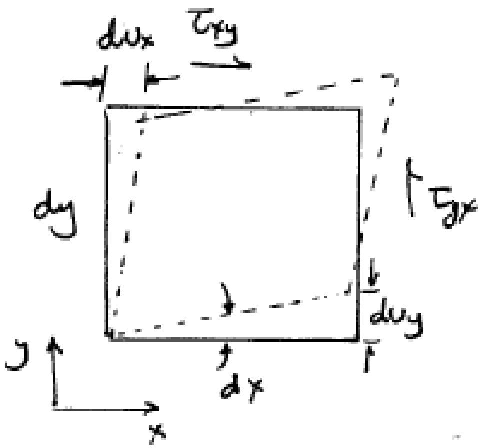
- Geometric expression of deformation caused by stress (dimensionless)



1D $$\varepsilon = \frac{\Delta L}{L} = \frac{du}{dx}$$



$$\varepsilon_y = \frac{\partial u_y}{\partial y} \simeq \frac{\Delta u_y}{\Delta y}$$



$$\gamma_{xy} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \varepsilon_{yy} = \frac{\partial u_y}{\partial y},$$

$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

Strain – 3D

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$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z},$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)$$

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

Tensor form

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix}$$

Matrix form

Strain is also a 2nd order tensor and symmetric by definition.

Mechanical Properties
elastic properties

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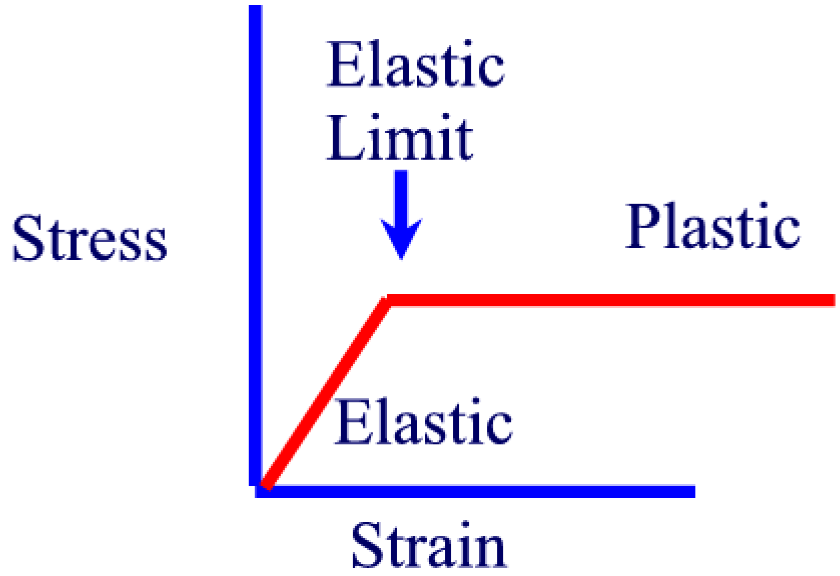
- Elastic
  - Loaded deformation recovers when unloaded
- Plastic
  - Not recoverable



Mechanical Properties
- Rate equation/Constitute Equation

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- Hooke's law

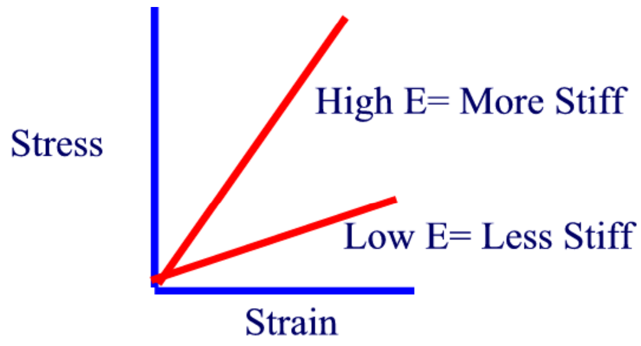
$$q''_x = -k\frac{dT}{dx}$$

$$q_x = -K_x\frac{\partial h}{\partial x}$$

$$\sigma = E\varepsilon = E\frac{du}{dx}$$

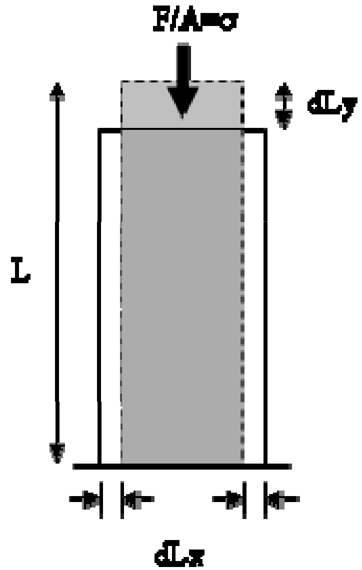
- Elastic (Young's) modulus, E (N/m^2=Pa)

$$E = \frac{\sigma_y}{\varepsilon_y}$$



- Poisson's ratio, v (dimensionless)

$$\nu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_x}{\varepsilon_y}$$



$$\varepsilon = \frac{\Delta L}{L}$$

- Typical range of properties
  - Concrete σ_c = 20-50 MPa E ~ 25 GPa
  - Granite σ_c = 100-200 MPa E ~ 60 GPa
  - Alloy steel σ_c = >500 MPa E ~ 200 GPa

Mechanical Properties
Hooke's law in 3D

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- Hooke's Law

1D $\sigma = E\varepsilon$

- Shear modulus G

$$\tau_{xy} = G\gamma_{xy}$$

- Generalized Hooke's law (isotropy)
  - 2 independent parameters (E, v) for isotropic material

$$G = \frac{E}{2(1+\nu)}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}$$

Mechanical Properties
Uniaxial compressive strength (단축압축강도)

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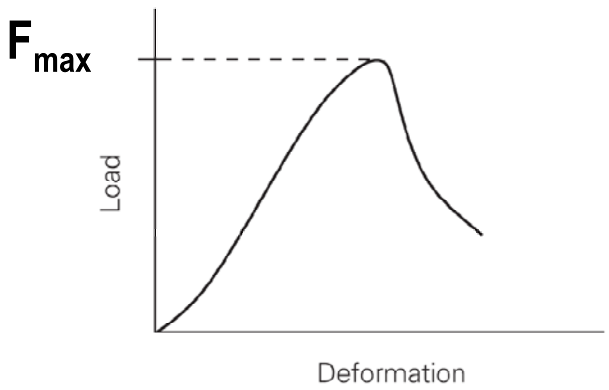
- Uniaxial Compressive Strength: maximum stress that the specimen can sustain under uniaxial stress condition
- Same unit as stress (Pa), usually in MPa.

$$\sigma_c = \frac{F_{\max}}{A}$$



Properties
Uniaxial compressive strength (단축압축강도)

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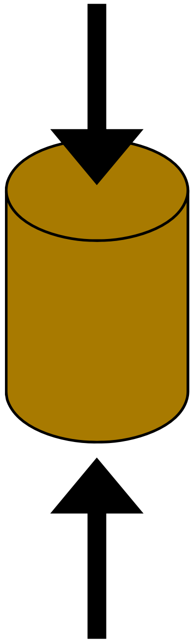
Gold medal in Beijing Olympic, 186 kg



Force (F) = mg = 186 kg x 9.8 m/s^2 = 1860 N

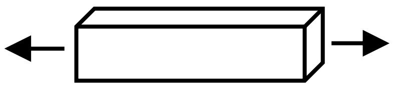
Are of two palms (A) = 0.1 (m) x 0.1 (m) x 2 = 0.02 m^2

σ_c (maximum stress = Strength) = F/A = 1860 / 0.02 = 93000 N/m^2 = 93 kPa

Properties
Tensile strength

- Tensile strength: Maximum tensile stress that the specimen can sustain

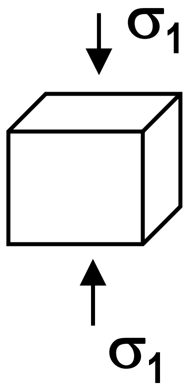
$$\sigma_t = \frac{T_{\max}}{A}$$



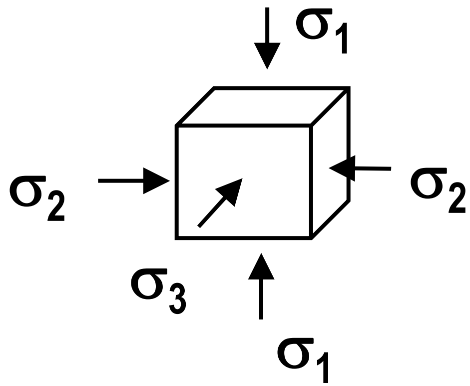
Tensile loading

- Tensile strength of rock is usually 1/10 ~ 1/20 of UCS

State of Stress



Uniaxial stress



Triaxial stress

Close to the reality in deep underground

In situ Stress (초기응력 혹은 현지응력)

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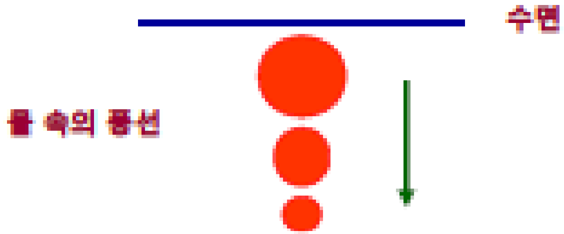
- Water pressure with respect to depth (z)

$$p = \rho g z = density \times acceleration(due\ to\ gravity) \times depth$$

$$p = \rho g z = 0.01 \times z(MPa)$$



전석원, 2008

In situ Stress (초기응력 혹은 현지응력)

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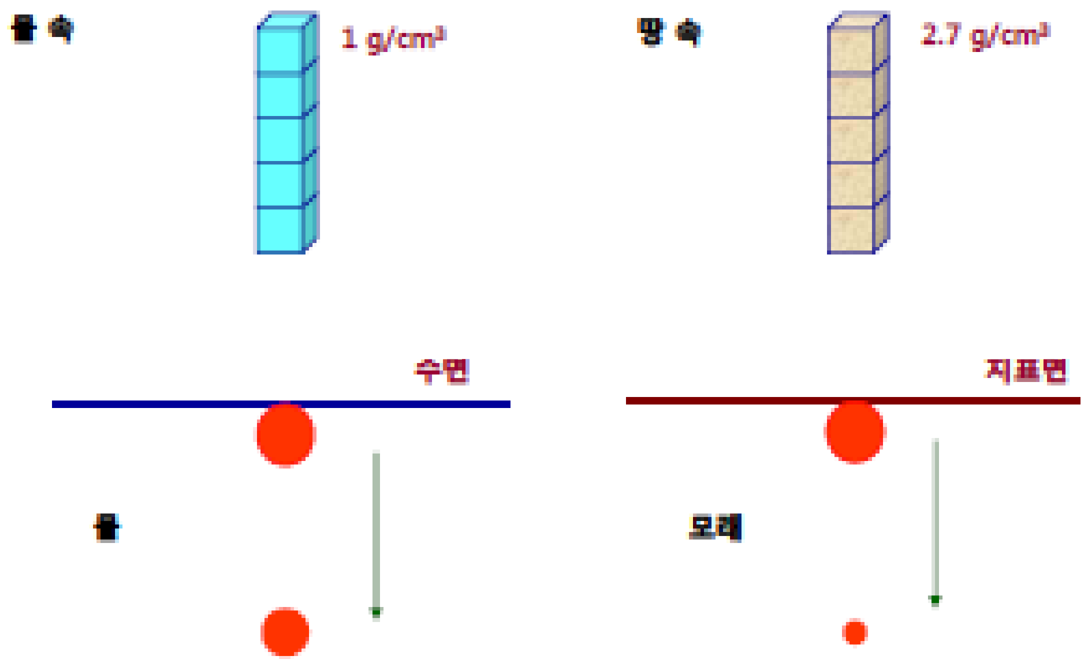
$$p = \rho g z = 밀도 \times 중력가속도 \times 심도$$



$$p = \rho g z = 0.01 \times z(MPa) \qquad p = \rho g z = 0.027 \times z(MPa)$$

전석원, 2008

In situ stress
Magnitude of Vertical stress

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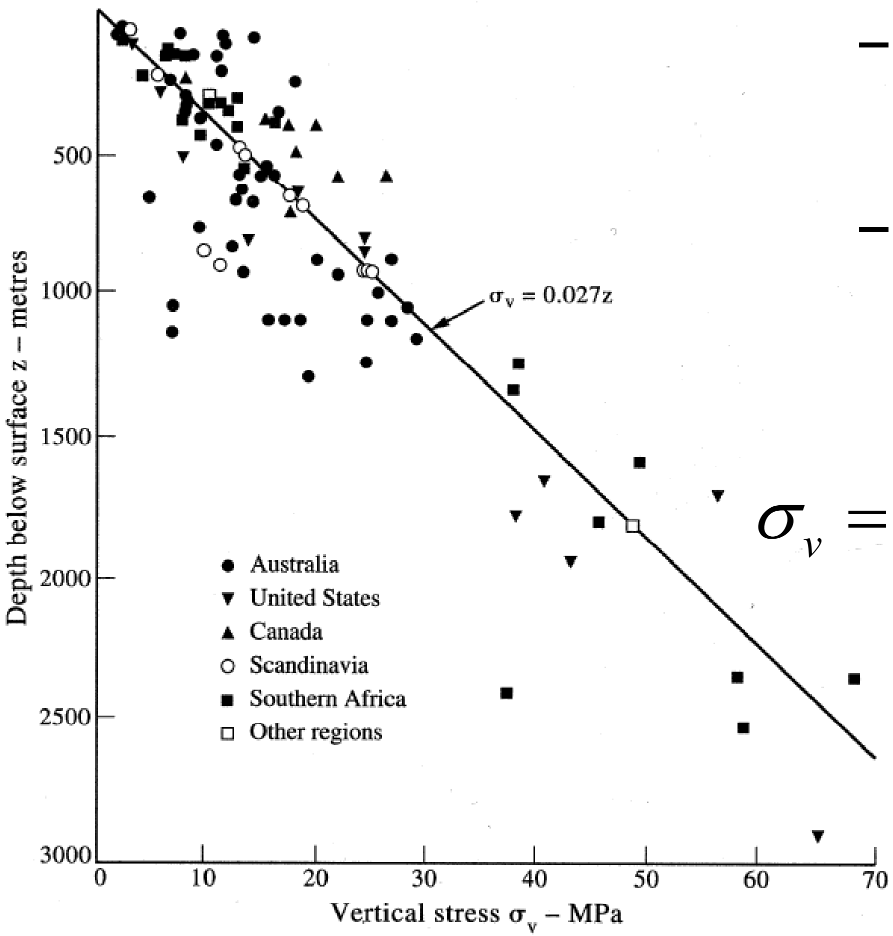

Vertical stress vs depth

- Vertical component of in situ stress
- More or less similar to predicted stress

$$\sigma_v = \rho g h = 0.027 \times h (MPa)$$

In situ stress
Magnitude of Horizontal stress

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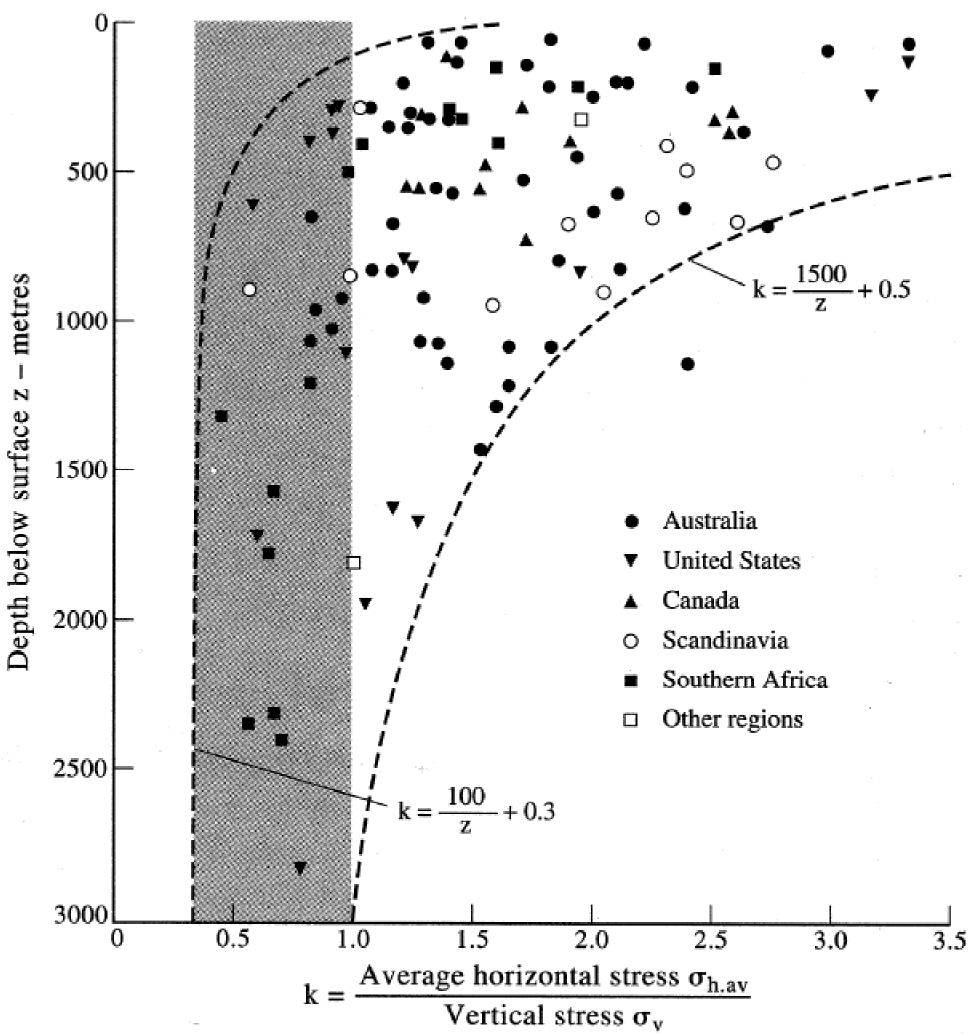
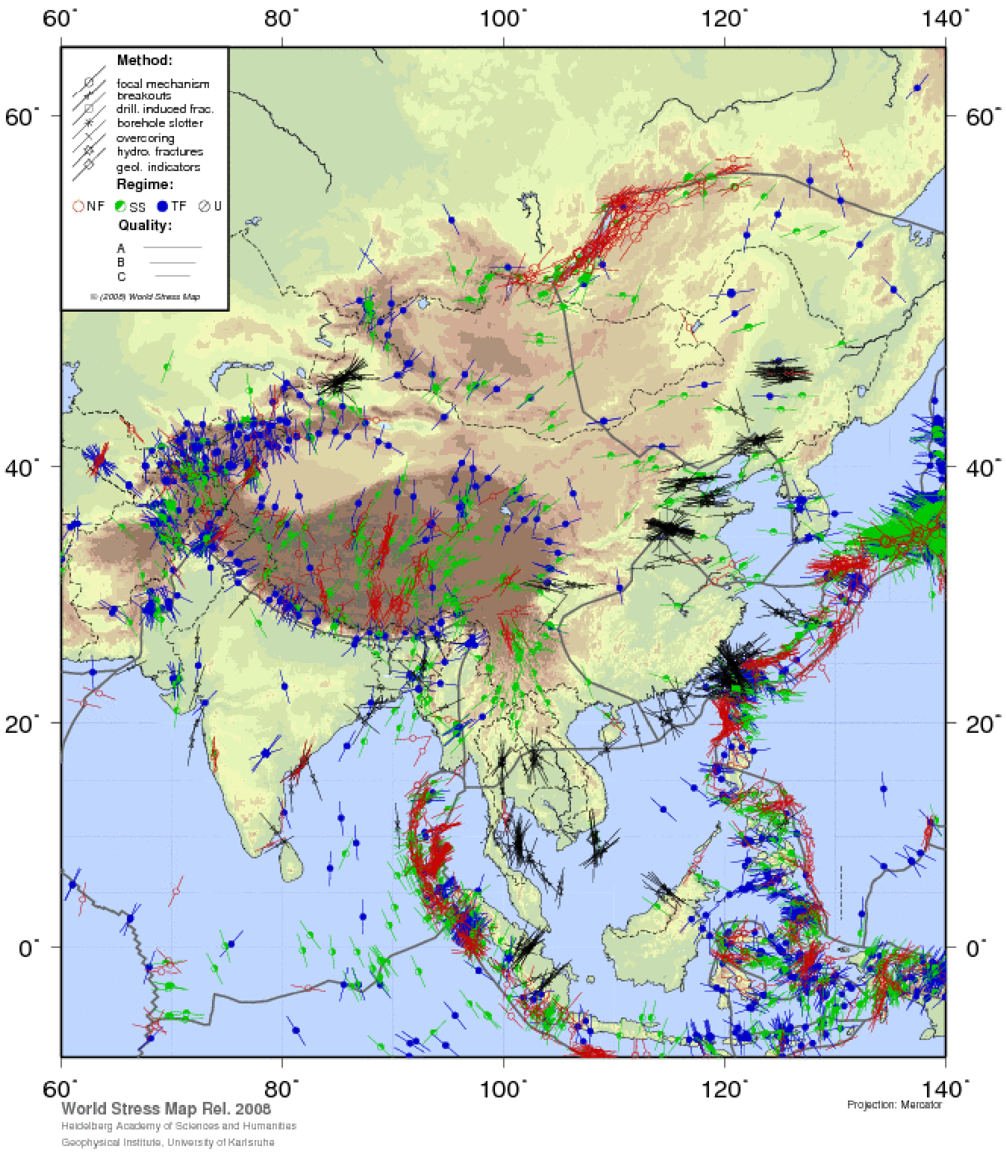


- Horizontal components of insitu stress
- Average horizontal stress is usually 0.3 ~ 4.0 times of vertical stress
- High horizontal stress: tectonic stress, erosion, topography

In situ stress
World stress map

- http://dc-app3-14.gfz-potsdam.de/



Heidbach, O., Tingay, M., Barth, A., Reinecker, J., Kurfeß, D. and Müller, B., The World Stress Map database release 2008 doi:10.1594/GFZ.WSM.Rel2008, 2008.

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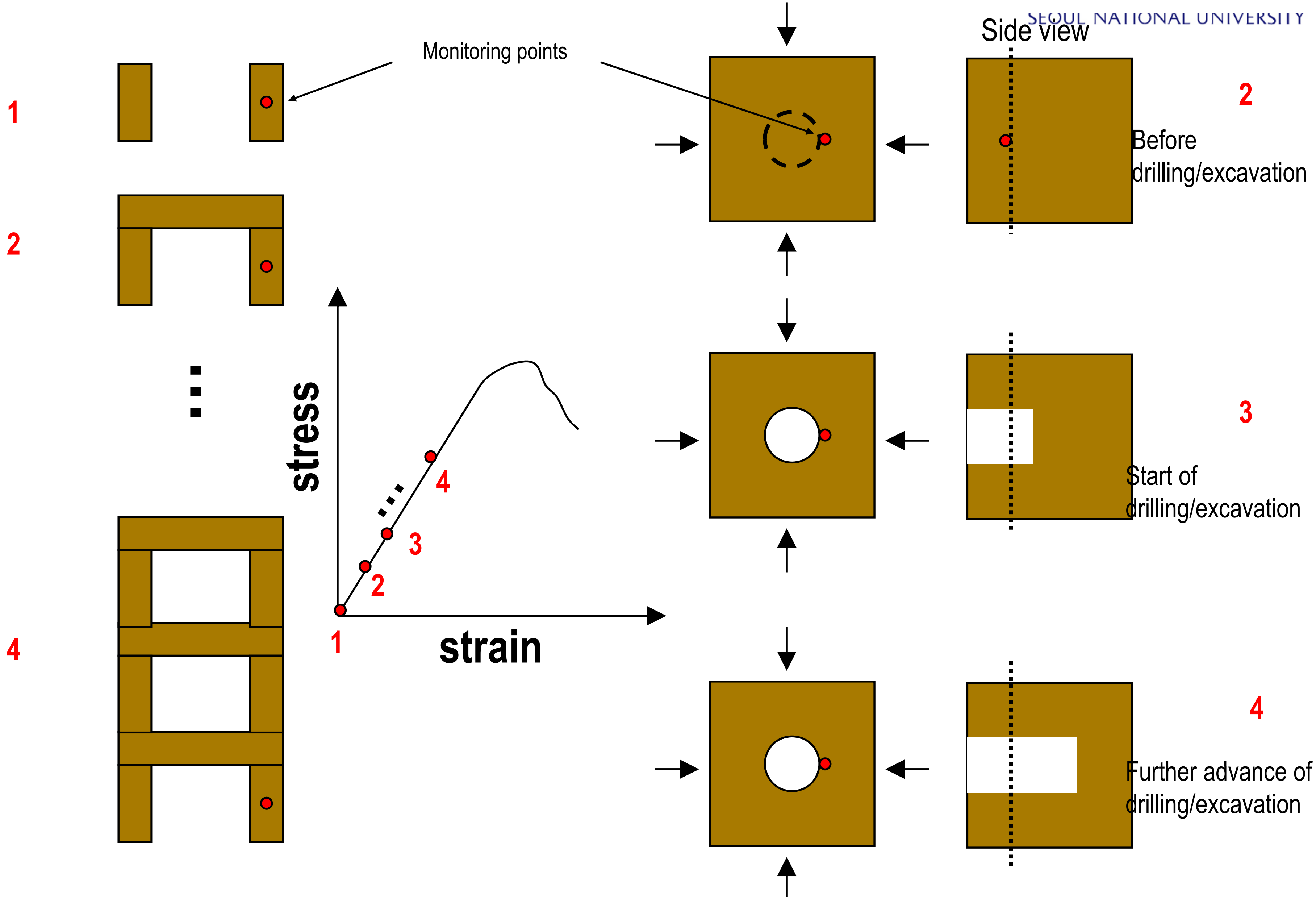
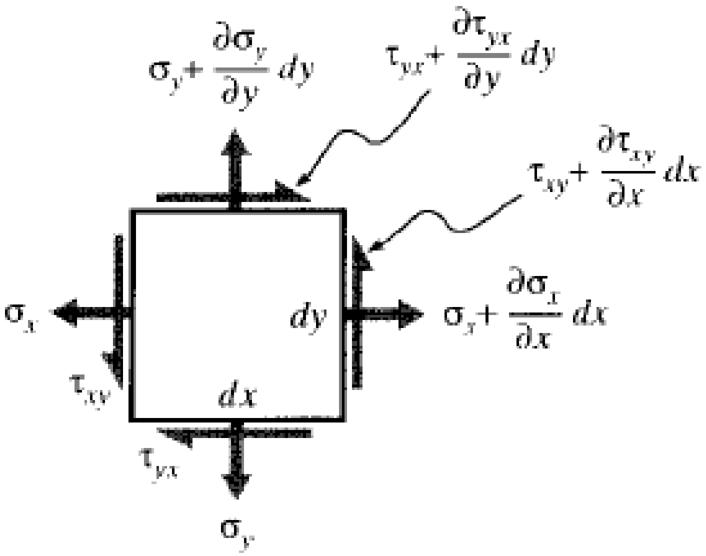
Civil structural problem: Mechanics of "Addition"

Petroleum/ Mining Geomechanics problem: Mechanics of "Removal"



Equilibrium equation

– Sum of traction, body forces (and moment) are zero (static case)



$$\sum F_i = 0 \longrightarrow$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho b_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho b_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho b_z = 0$$

– b_x, b_y, b_z are components of acceleration due to gravity.

$$\sum M_i = 0 \longrightarrow \tau_{xy} = \tau_{yx}$$

Equilibrium equation

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- Strain-displacement relationship $$\varepsilon = \frac{du}{dx}$$
- Stress-strain relationship $$\varepsilon = \frac{1}{E}\sigma$$
- Static Equillibrium Equation $$\frac{\partial \sigma_{xx}}{\partial x} + \rho b_x = 0$$
- Final equation for elasticity

$$E\frac{\partial^2 u_x}{\partial x^2} + \rho b_x = 0$$

Equation of motion (Equilibrium equation) – A 1D example

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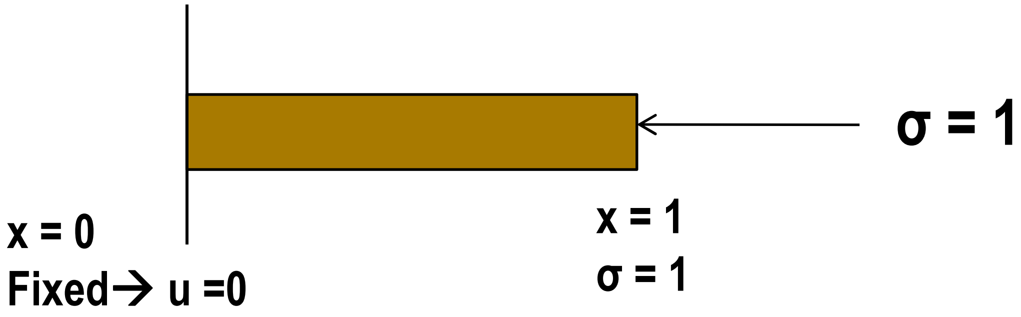
– With no body force, and static case

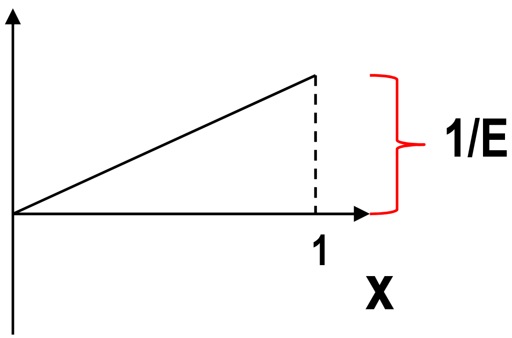
$$E\frac{\partial^2 u_x}{\partial x^2} + \rho b_x = \rho \frac{\partial^2 u_x}{\partial t^2} \qquad E\frac{\partial^2 u_x}{\partial x^2} = 0$$



$$E\frac{\partial u_x}{\partial x} = C_1$$ → **From σ = 1, C_1=1**

$$Eu_x = x + C_2$$ → **From u = 0 at x=0, C_2=0**

$$u_x = \frac{x}{E}$$



Formulation for elasticity (2) – 3D

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- In 3D, it is slightly more complicated - Navier's equation

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}+\frac{\partial^2 u_z}{\partial x\partial z}\right)+\rho b_x=0$$

$$G\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x\partial y}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_z}{\partial y\partial z}\right)+\rho b_y=0$$

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x\partial z}+\frac{\partial^2 u_y}{\partial y\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)+\rho b_z=0$$

$$\lambda=\frac{E\nu}{(1+\nu)(1-2\nu)}$$

Physical variables for various physical problems

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Physical problem	Conservation Principle $\nabla \cdot q = 0$	State Variable u	Flux σ	Material properties k	Source f	Constitutive equation $\sigma = ku'$
Elasticity	Conservation of linear momentum (equilibrium)	Displacement u	Stress σ	Young's modulus & Poisson's ratio	Body forces	Hooke's law
Heat conduction	Conservation of energy	Temperature T	Heat flux Q	Thermal conductivity k	Heat sources	Fourier's law
Porous media flow	Conservation of mass	Hydraulic head h	Flow rate Q	Permeability k	Fluid source	Darcy's law
Mass transport	Conservation of mass	Concentration C	Diffusive flux q	Diffusion coefficient D	Chemical source	Fick's law

Structure of state variables and fluxes are mathematically similar – ***a convenient truth!***

Today

- Fundamentals of rock mechanics
  - Concept of stress & Mohr Circle
  - Mechanical properties (Elastic modulus and Poisson's ratio)
  - Strength (compressive, tensile)
  - In situ stress
  - Equilibrium Equation

Wednesday

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- Stability of geothermal wellbore
  - Failure criteria
  - Basic solutions for stress distribution around wellbore