Surfaces

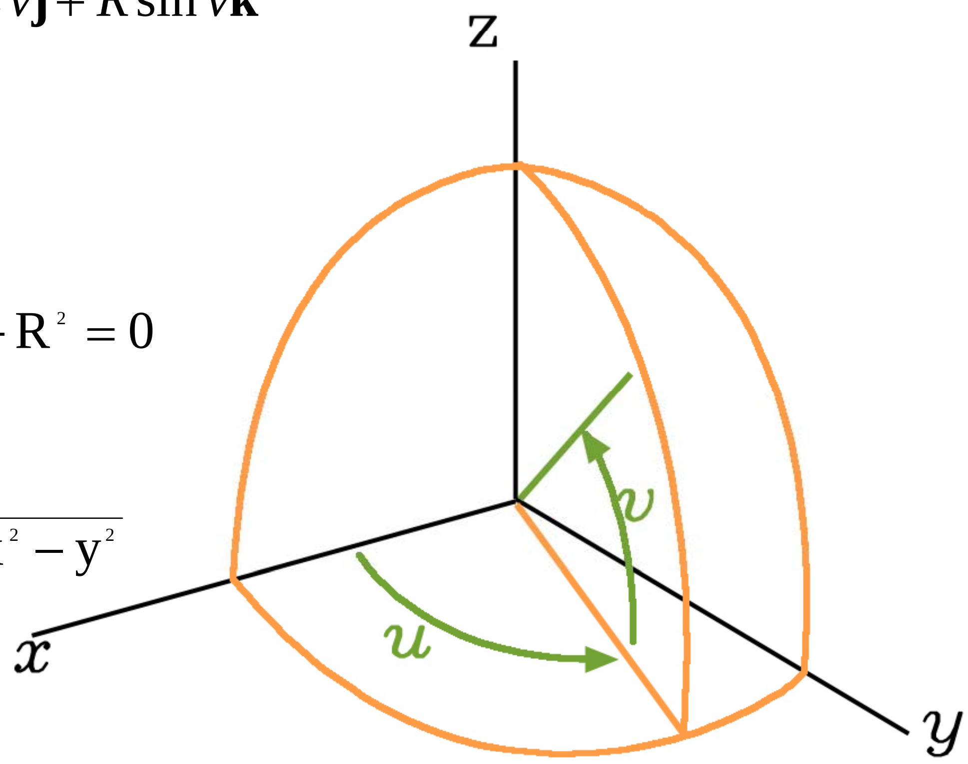
CAD Lab.

Surfaces

- Parametric eq.

$$\mathbf{P}(u,v) = R\cos u\cos v\mathbf{i} + R\sin u\cos v\mathbf{j} + R\sin v\mathbf{k}$$

$$(0 \le u \le 2\pi,\ -\pi/2 \le v \le \pi/2)$$



- Implicit eq $\quad x^2 + y^2 + z^2 - R^2 = 0$

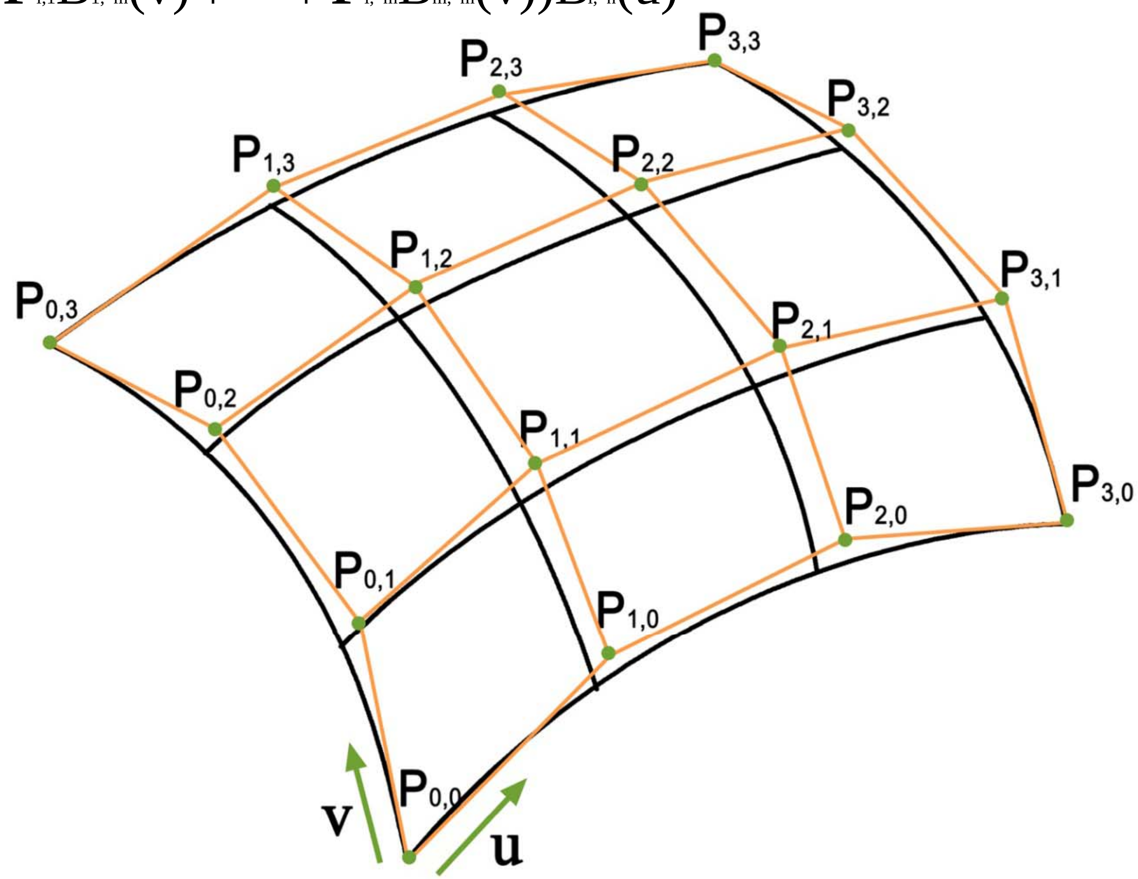
- Explicit eq $\quad z = \pm\sqrt{R^2 - x^2 - y^2}$

Bezier surface

$$\mathbf{P}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}\mathbf{P}_{i,j}B_{i,n}(u)B_{j,m}(v)\qquad 0\le u\le 1,\qquad 0\le v\le 1$$

$$=\sum_{i=0}^{n}(\mathbf{P}_{i,0}B_{0,m}(v)+\mathbf{P}_{i,1}B_{1,m}(v)+\cdots+\mathbf{P}_{i,m}B_{m,m}(v))B_{i,n}(u)$$

Bezier curve



Bezier surface – cont'

- Surface obtained by blending (n+1) Bezier curves
  - (or by blending (m+1) Bezier curves)
- Four corner points on control polyhedron lie on surface

Bezier surface equation

$$\mathbf{P}(0,0) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{i,j} B_{i,n}(0) B_{j,m}(0)$$

$$= \sum_{i=0}^{n} \underbrace{\left[\sum_{j=0}^{m} \mathbf{P}_{i,j} B_{j,m}(0) \right]}_{P_{i,0}} B_{i,n}(0)$$

$$= \sum_{i=0}^{n} \mathbf{P}_{i,0} B_{i,n}(0) = \mathbf{P}_{0,0}$$

Bezier surface – cont'

- Boundary curves are Bezier curves defined by associated control points

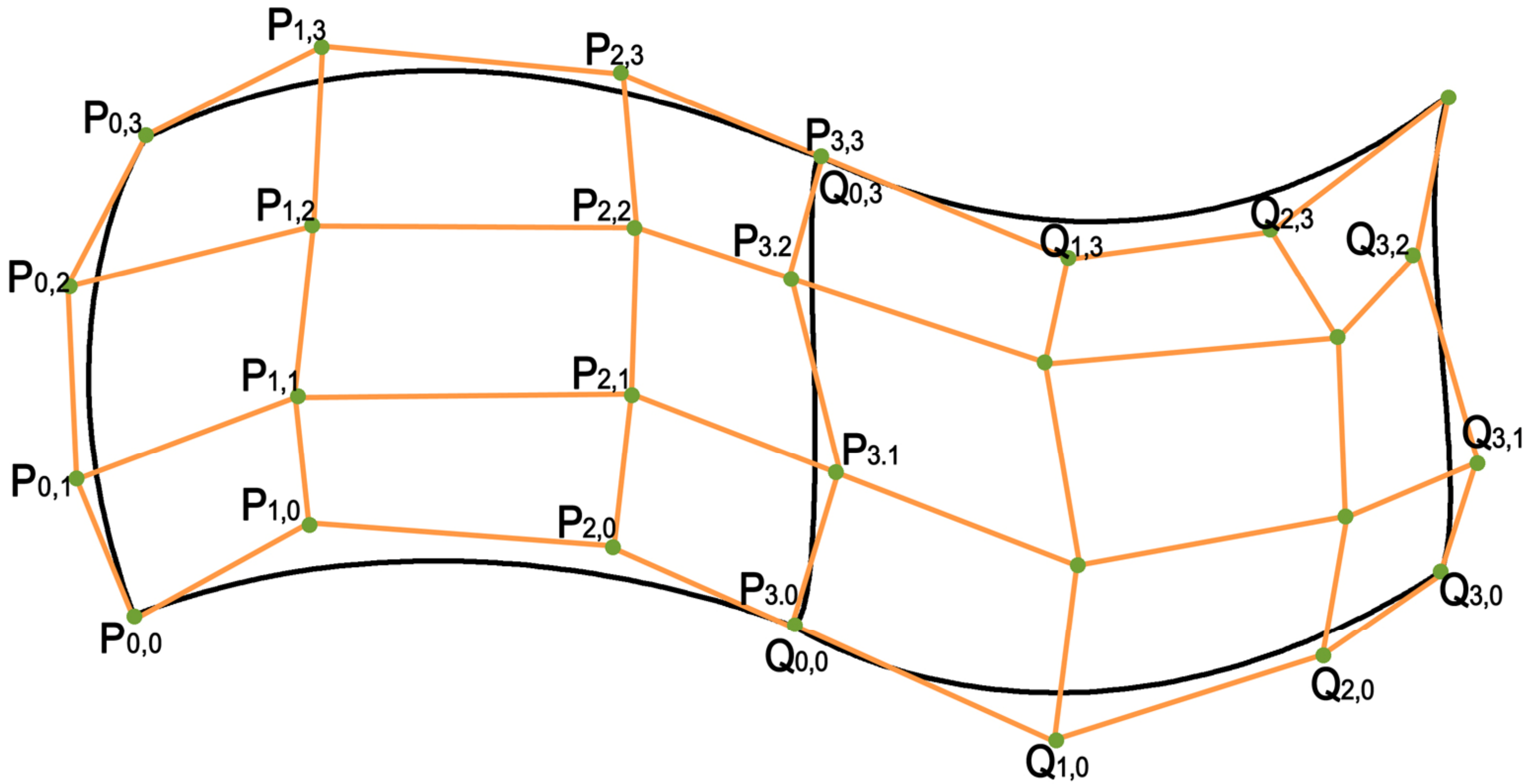
$$\mathbf{P}(0, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{i,j} B_{i,n}(0) B_{j,m}(v)$$

$$= \sum_{j=0}^{m} \underbrace{\left[\sum_{i=0}^{n} \mathbf{P}_{i,j} B_{i,n} \right]_{u=0}}_{P_{0,j}} B_{j,m}(v)$$

$$= \underbrace{\sum_{j=0}^{m} \mathbf{P}_{0,j} B_{j,m}(v)}$$

- Bezier curve defined by $\mathbf{P}_{0,0}$, $\mathbf{P}_{0,1}$, …, $\mathbf{P}_{0,m}$

Bezier surface – cont'



- When two Bezier surfaces are connected, control points before and after connection should form straight lines to guarantee G1 continuity

B-spline surface

$$\mathbf{P}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \mathbf{P}_{i,j} N_{i,k}(v) N_{j,l}(v) \qquad \begin{array}{c} s_{k+1} \le u \le s_{n+1} \\ t_{l-1} \le v \le t_{m+1} \end{array}$$

- $N_{i,k}(u)$ is defined by $s_0, s_1, \ldots, s_{n+k}$
- $N_{j,l}(v)$ is defined by $t_0, t_1, \ldots, t_{l+m}$
- If $k=(n+1)$, $l=m+1$ and non-periodic knots are used, the resulting surface will become Bezier surface

B-spline surface – cont’

- Bezier surface is a special case of B-spline surface.
- Boundary curves are B-spline curves defined by associated control points.
- Four corner points of control polyhedron lie one the surface (when non-periodic knots are used)

NURBS surface

$$\mathbf{P}(\mathrm{u},\mathrm{v}) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{i,j}\mathbf{P}_{i,j}N_{i,k}(\mathrm{u})N_{j,l}(\mathrm{v})}{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{i,j}N_{i,k}(\mathrm{u})N_{j,l}(\mathrm{v})}$$

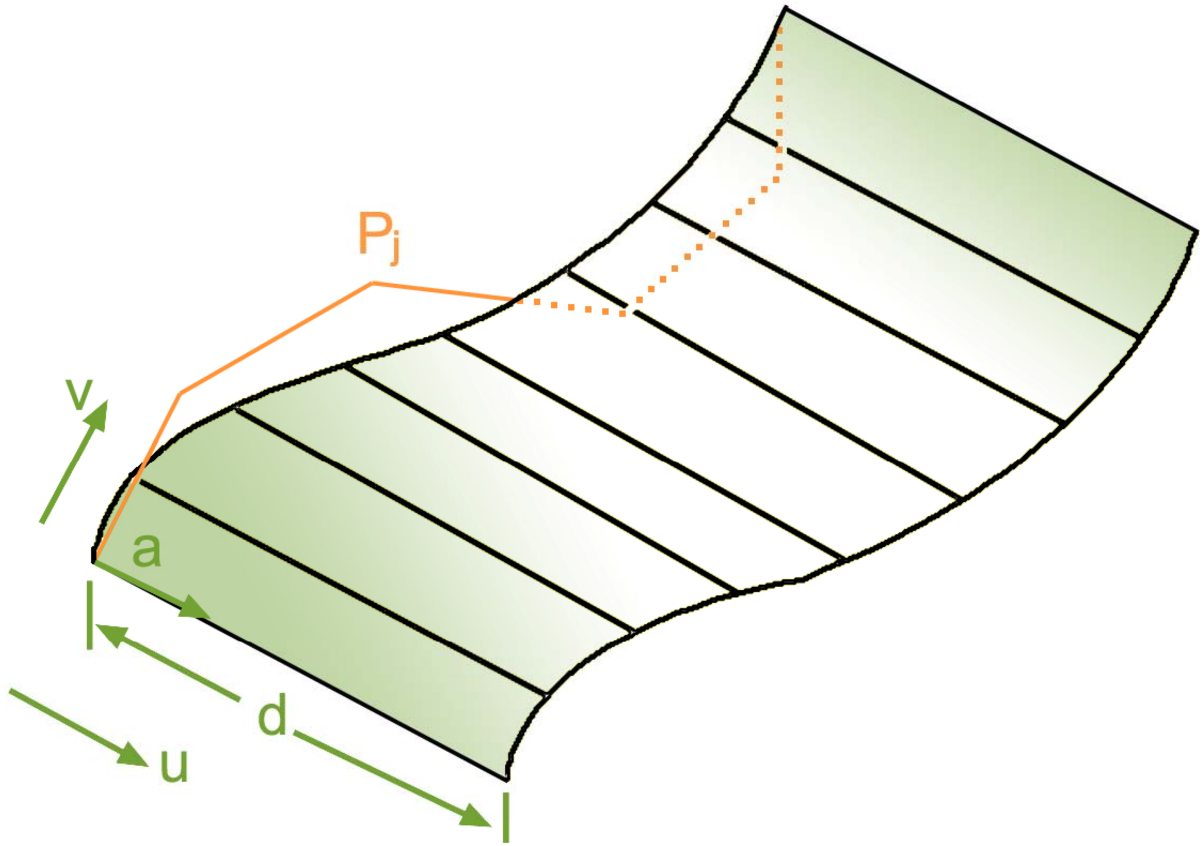
$$\mathrm{s}_{\mathrm{k}-1} \le \mathrm{u} \le \mathrm{s}_{\mathrm{n}+1}$$

$$\mathrm{t}_{-1} \le \mathrm{v} \le \mathrm{t}_{\mathrm{m}+1}$$

- If $\mathrm{h}_{\mathrm{i,j}} = 1$, B-spline surface is obtained
- Represent quadric surface(cylindrical, conical, spherical, paraboloidal, hyperboloidal) exactly

NURBS surface – cont'

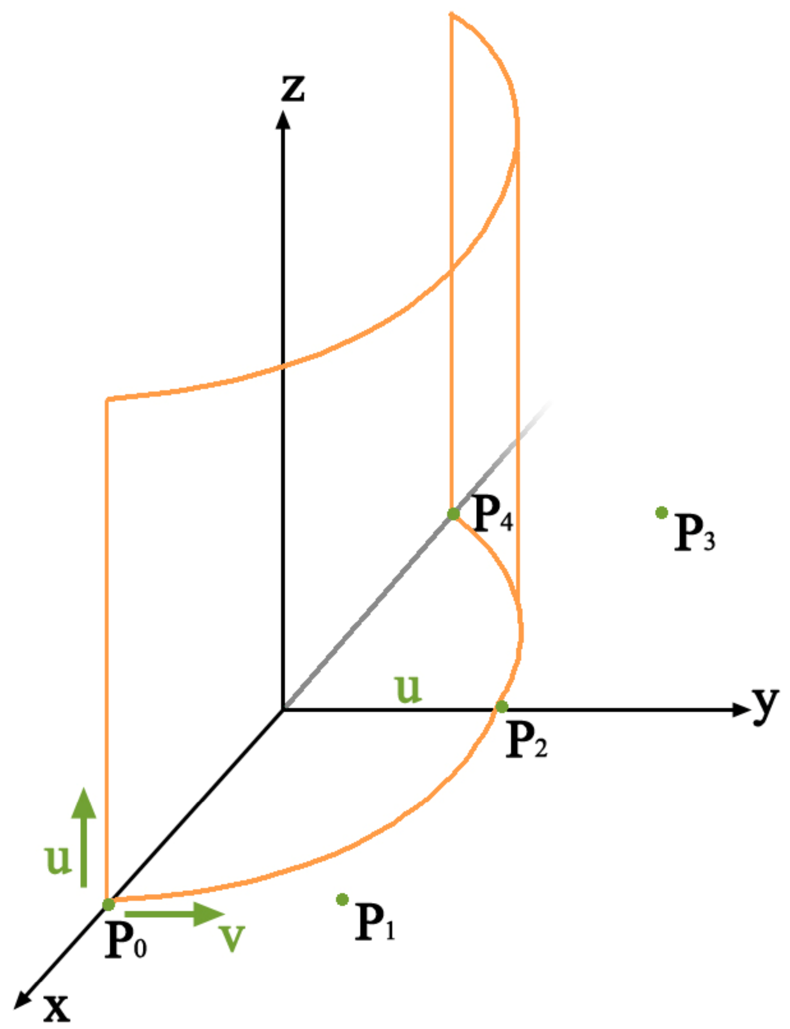
- Represent a surface obtained by sweeping a curve



NURBS surface – cont'

- Assume that v-direction of surface is a given $\mathbf{P}_j$
- v-direction knot & order is the same as the NURBS Curve's (order: l, knot: t_p)
- u-direction order is 2
- control point: 2
  - u direction knot: 0 0 1 1
  - $\mathbf{P}_{0,j} = \mathbf{P}_j$
  - $\mathbf{P}_{1,j} = \mathbf{P}_j + d\,\mathbf{a}$
  - $h_{0,i} = h_{1,i} = h_i$ from the given curve

Ex) Translate half circle to make cylinder

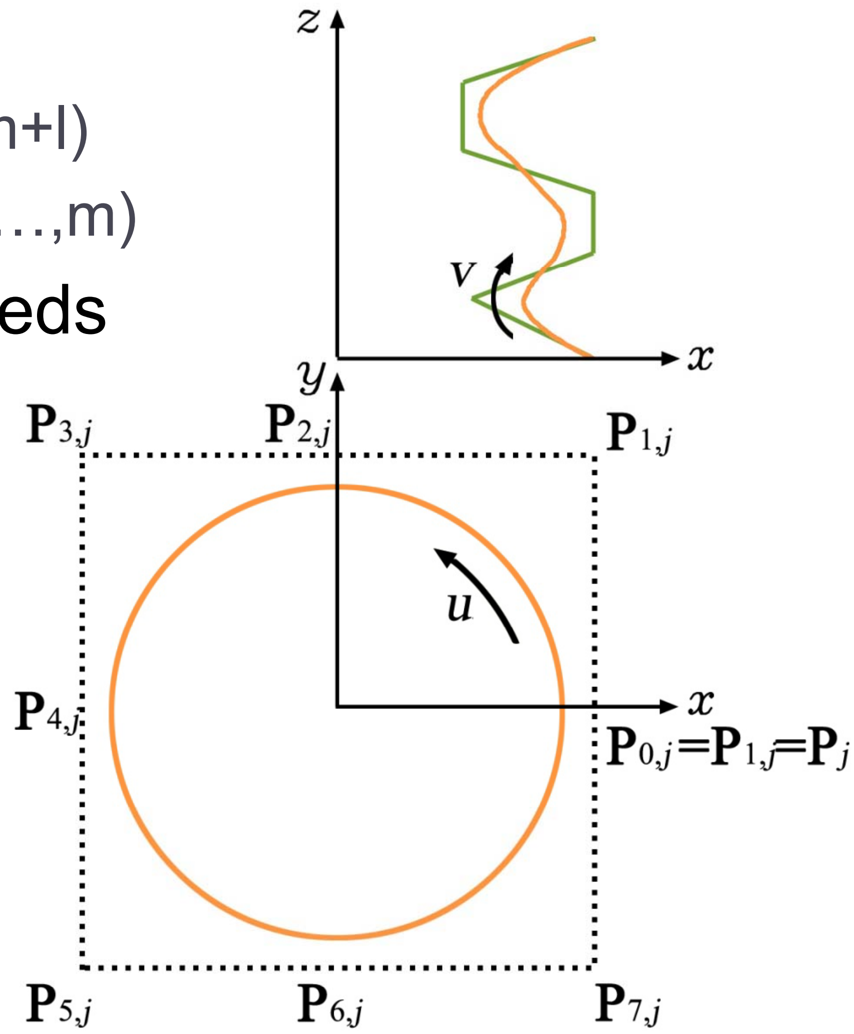


Ex) Translate half circle to make cylinder

- $\mathbf{P}_0=(1, 0, 0)$ $h_0=1$ $\mathbf{P}_1=(1, 1, 0)$ $h_1=1/\sqrt{2}$
- $\mathbf{P}_2=(0, 1, 0)$ $h_2=1$ $\mathbf{P}_3=(-1, 1, 0)$ $h_3= 1/\sqrt{2}$
- $\mathbf{P}_4=(-1, 0, 0)$ $h_4=1$
- $\mathbf{P}_{0,0} = P_0,$ $\mathbf{P}_{1,0} = \mathbf{P}0+H\mathbf{k}$ $h_{0,0} = h_{1,0}=1$
- $\mathbf{P}_{0,1} = P_1,$ $\mathbf{P}_{1,1} = \mathbf{P}1+H\mathbf{k}$ $h_{0,1} = h_{1,1}= 1/\sqrt{2}$
- $\mathbf{P}_{0,2} = P_2,$ $\mathbf{P}_{1,2} = \mathbf{P}_2+H\mathbf{k}$ $h_{0,2} = \mathrm{h}_{1,2}=1$
- $\mathbf{P}_{0,3} = P_3,$ $\mathbf{P}_{1,3} = \mathbf{P}_3+H\mathbf{k}$ $h_{0,3} = \mathrm{h}_{1,3}= 1/\sqrt{2}$
- $\mathbf{P}_{0,4} = P_4,$ $\mathbf{P}_{1,4} = \mathbf{P}_4+H\mathbf{k}$ $h_{0,4} = h_{1,4}=1$
- Knots for v: 0 0 0 1 1 2 2 2
- Knots for u: 0 0 1 1

Ex) Surface obtained by revolution

- Curve
  - order l, knot t_p (p=0,1,…,m+l)
  - control points P_j, h_j (j=0,1,…,m)
- Original control points needs to be split into 9.



Ex) Surface obtained by revolution – cont'

- $P_{0,j} = P_j$ $\quad h_{0,j} = h_j$
- $P_{1,j} = P_{0,j} + x_j\,\mathbf{j} = P_j + x_j\,\mathbf{j}$ $\quad h_{1,j} = h_j \cdot 1/\sqrt{2}$
- $P_{2,j} = P_{1,j} - x_j\,\mathbf{i} = P_j - x_j\,(\mathbf{i}\text{-}\mathbf{j})$ $\quad h_{2,j} = h_j$
- $P_{3,j} = P_{2,j} - x_j\,\mathbf{i} = P_j - x_j\,(2\,\mathbf{i}\text{-}\mathbf{j})$ $\quad h_{3,j} = h_j \cdot 1/\sqrt{2}$
- $P_{4,j} = P_{3,j} - x_j\,\mathbf{j} = P_j - 2x_j\,\mathbf{i}$ $\quad h_{4,j} = h_j$
- $P_{5,j} = P_{4,j} - x_j\,\mathbf{j} = P_j - x_j\,(2\,\mathbf{i}+\mathbf{j})$ $\quad h_{5,j} = h_j \cdot 1/\sqrt{2}$
- $P_{6,j} = P_{5,j} + x_j\,\mathbf{i} = P_j - x_j\,(\mathbf{i}+\mathbf{j})$ $\quad h_{6,j} = h_j$
- $P_{7,j} = P_{6,j} + x_j\,\mathbf{i} = P_j - x_j\,\mathbf{j}$ $\quad h_{7,j} = h_j \cdot 1/\sqrt{2}$
- $P_{8,j} = P_{0,j} = P_j$ $\quad h_{8,j} = h_j$
- u-direction order: 3
- u-direction knot: 0 0 0 1 1 2 2 3 3 4 4 4
- Synthesize four quarter circles