Engineering Economic Analysis

2019 SPRING

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Chap. 6

DEMAND

Properties of Demand Functions

- Comparative statics analysis of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,m)$ and $x_2^*(p_1,p_2,m)$ change as prices p_1, p_2 and income m change.

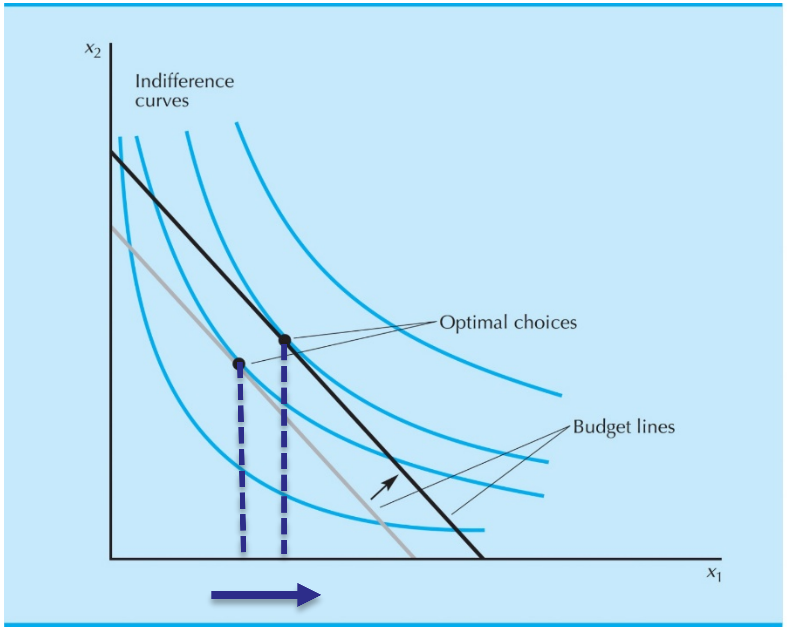
Income changes

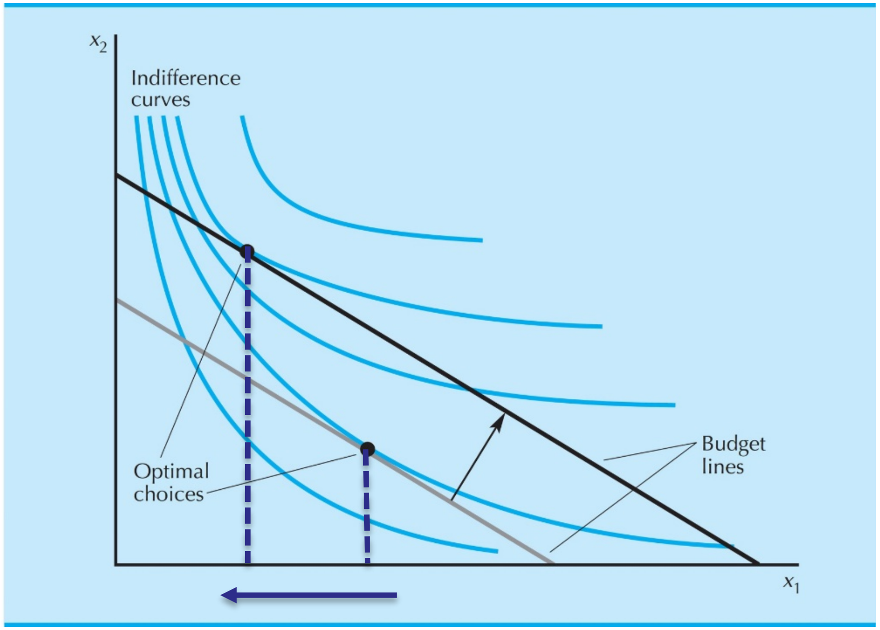
- How a consumer's demand for a good changes as his income changes with prices unchanged
- A good is called as normal good if

$$\frac{\partial x_i(\tilde{p}, m)}{\partial m} > 0$$

- Otherwise, inferior good

Income changes





Normal good

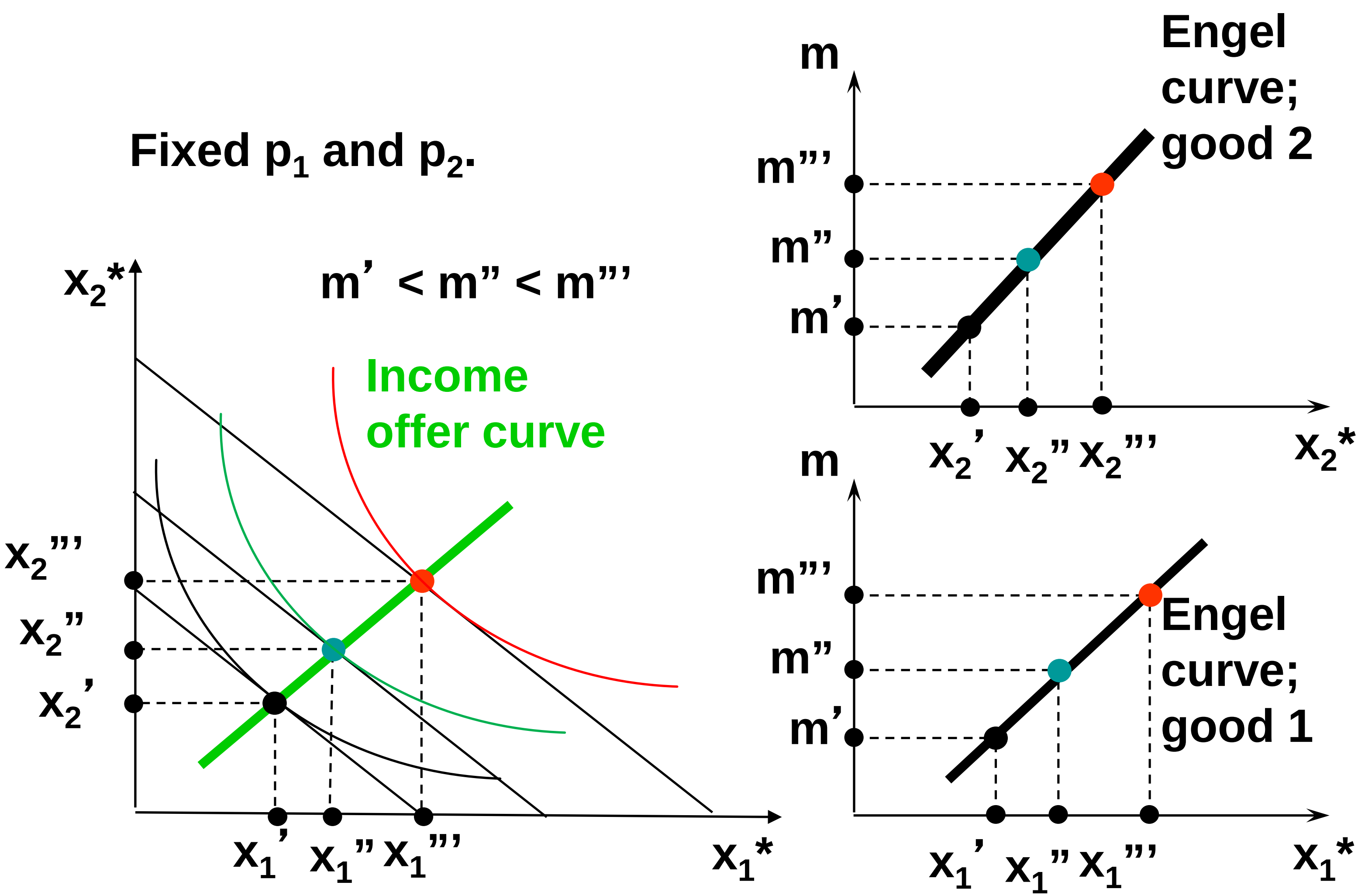
Inferior good

Income changes

- Income offer curves (income expansion path)
  - Illustrates the bundles of goods that are demanded at the different levels of income
- Engel curves
  - A graph of the demand for one good as a function of income, with all prices being held constant

Income changes

Fixed p_1 and p_2.



Examples: Cobb-Douglas

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$x_1^*(p_1, p_2, m) = \frac{am}{p_1}, \quad x_2^*(p_1, p_2, m) = \frac{(1-a)m}{p_2}$$

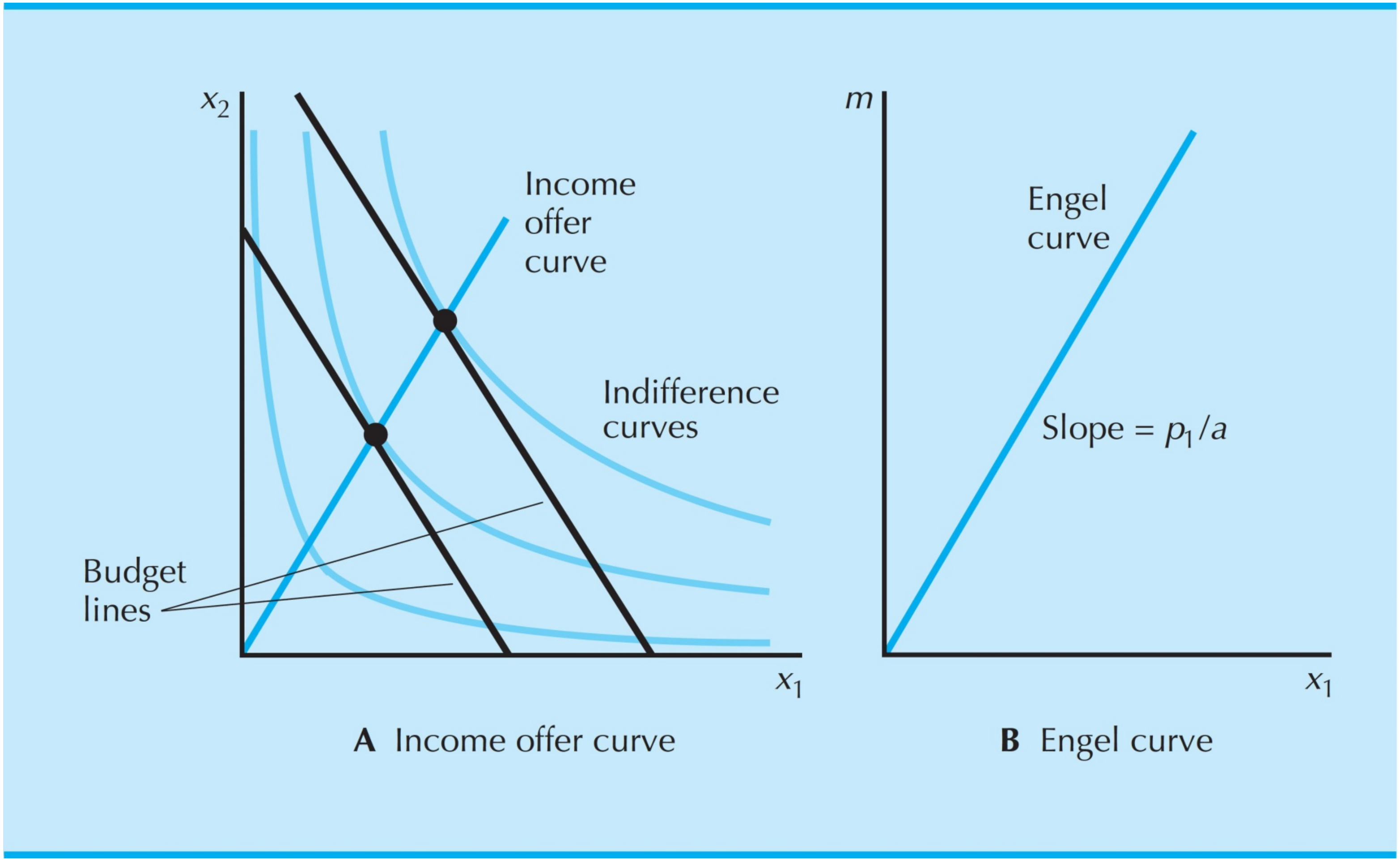
- Since x_i is linear function of m, doubling m will double demand, tripling m will triple demand, and so on.

$$x_1^*(p_1, p_2, tm) = \frac{a(tm)}{p_1} = t\frac{am}{p_1} = tx_1^*(p_1, p_2, tm), \quad x_2^*(p_1, p_2, tm) = tx_2^*(p_1, p_2, m)$$

- Thus income expansion curve will be straight line
- Engel curves

$$m = \frac{p_1}{a}x_1, \quad m = \frac{p_2}{1-a}x_2$$

Examples: Cobb-Douglas



Homothetic preference

- Homothetic utility function:
  - A function $f: R^n \to R$ is **homogeneous of degree 1** (H(1)) if

    $$f(t\tilde{x}) = tf(\tilde{x}) \text{ for all } t > 0$$

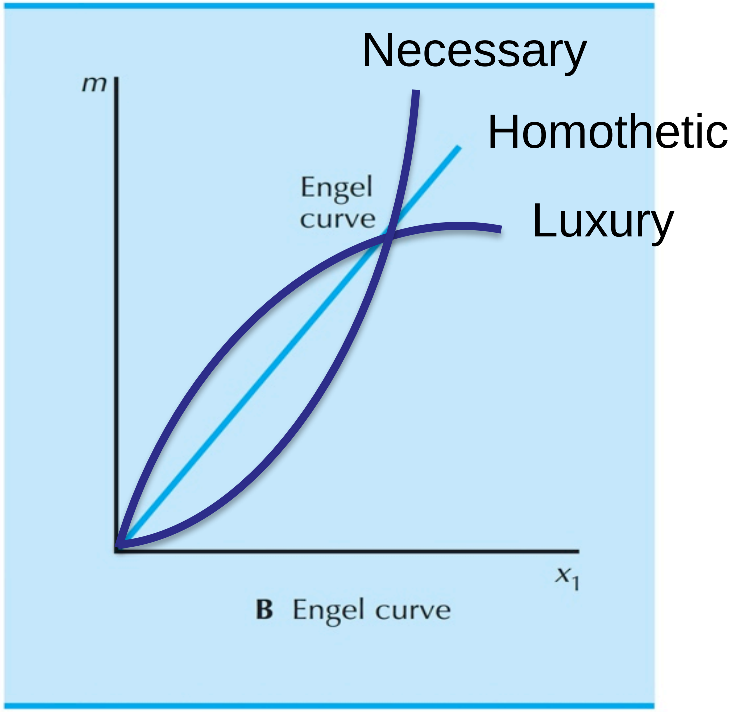
  - A **homothetic function** is a (positive) monotonic transformation of a homogeneous function
- If a consumer has a homothetic utility function, the consumer is said to have a homothetic preference

  $$If \ (x_1, x_2) \succ (y_1, y_2), \text{ then } (tx_1, tx_2) \succ (ty_1, ty_2)$$

  - Consumer's preferences only depend on the ratio of both goods
- If homothetic preference, then the income offer curves and also Engel curves are all straight lines through the origin

Homothetic preference

- Luxury good: demand for a good goes up by a greater proportion than income
- Necessary good: demand for a good increases by a lesser greater proportion than income



Quasilinear preference

- Quasilinear utility function

$$u(x_0, x_1, \ldots, x_k) = x_0 + v(x_1, \ldots, x_k)$$

  - x_0 can be interpreted as money for other goods

- Quasilinear utility maximization with two goods

$$\max \quad x_0 + v(x_1)$$
$$s.t. \quad p_0 x_0 + p_1 x_1 = m$$

$$\Downarrow$$

$$\max \quad v(x_1) + m / p_0 - p_1 x_1 / p_0$$

Quasilinear preference

- F.O.C.

$$v'(x_1) = p_1 / p_0$$

  - Demand function of x_1 is independent of income (and only a function of p_1 if p_0 is unity)
  - Zero income effect
  - Inverse demand function

$$p_1(x_1) = v'(x_1)\, p_0$$

Quasilinear preference

- Example

$$u(x_1, x_0) = \ln x_1 + x_0$$

  - F.O.C.

$$\frac{1}{x_1} = \frac{p_1}{p_0}$$

  - Demand function $x_1 = \frac{p_0}{p_1}$
    - Regardless of income amount, the consumer purchase good 1 by the amount of demand function
    - Then use all the remaining income to buy good 0

$$x_0 = \frac{m}{p_0} - 1$$

Quasilinear preference

- When income is too small such that $m<p_0$
  - Impossible to buy good 0
  - Thus use all income to purchase only good 1
  - Demand function

$$x_1 = \frac{m}{p_1} \qquad x_0 = 0$$

- Thus, a better way to write the demand for good 0 is:

$$x_0 = \begin{cases} 0 & \text{when } m \le p_0 \\ \dfrac{m}{p_0} - 1 & \text{when } m > p_0 \end{cases} \qquad x_1 = \begin{cases} \dfrac{m}{p_1} & \text{when } m \le p_0 \\ \dfrac{p_0}{p_1} & \text{when } m > p_0 \end{cases}$$

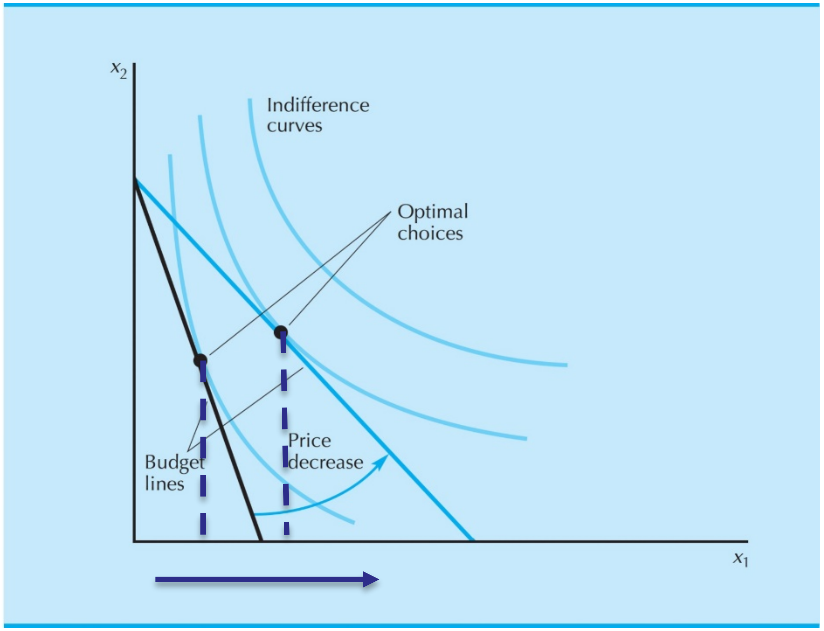
Own price changes

- How a consumer's demand for a good changes as its own price changes with other prices and income unchanged
- A good is called as ordinary good if

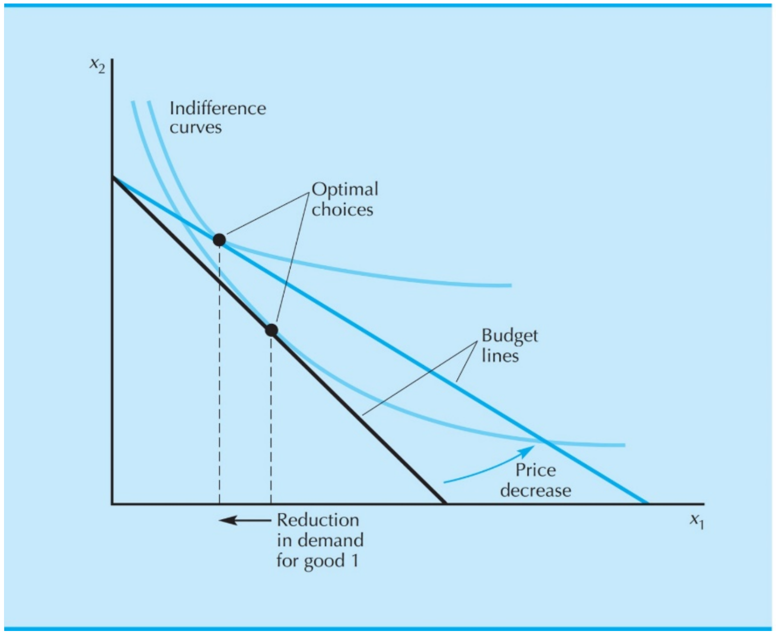
$$\frac{\partial x_i(\tilde{p}, m)}{\partial p_i} < 0$$

- Otherwise, Giffen good

Own price changes



Ordinary good



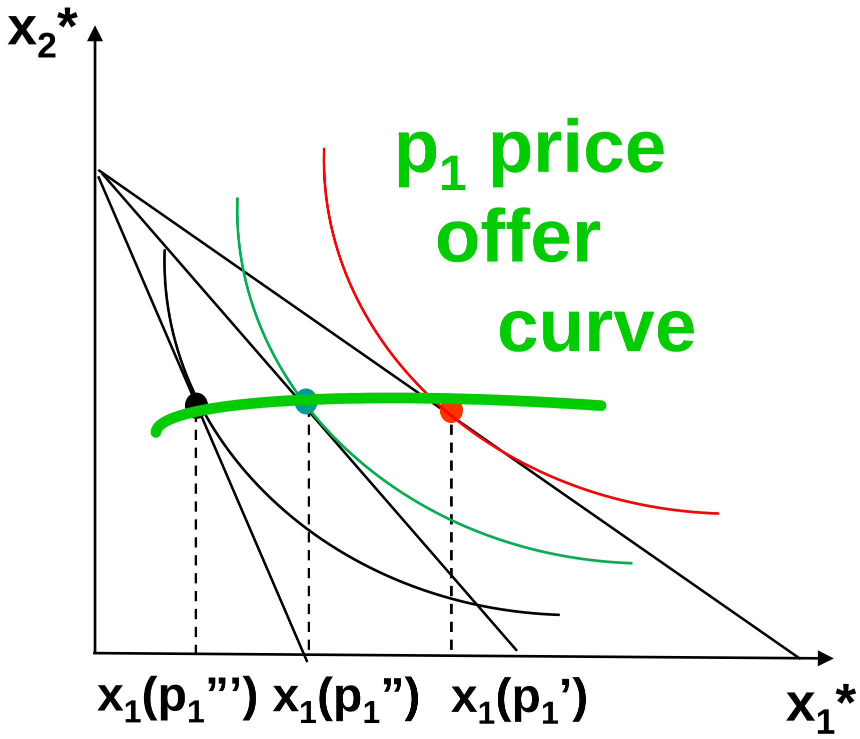
Giffen good

Own price changes

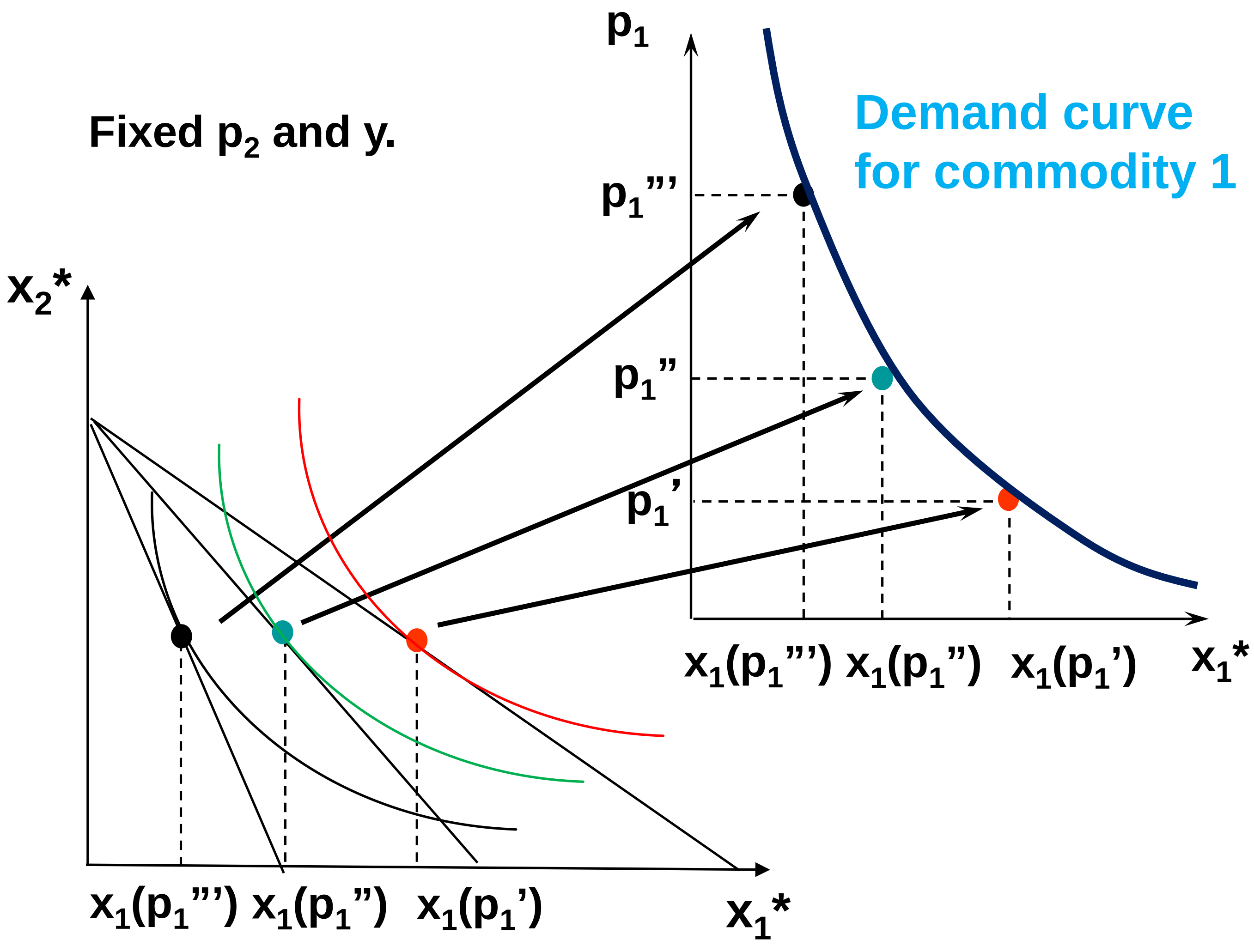
- Price offer curves
  - Represents the bundle that would be demanded at different own prices with the income and other prices being held fixed
- Demand curves
  - A graph of the demand function

Own price changes

Fixed p_2 and m.



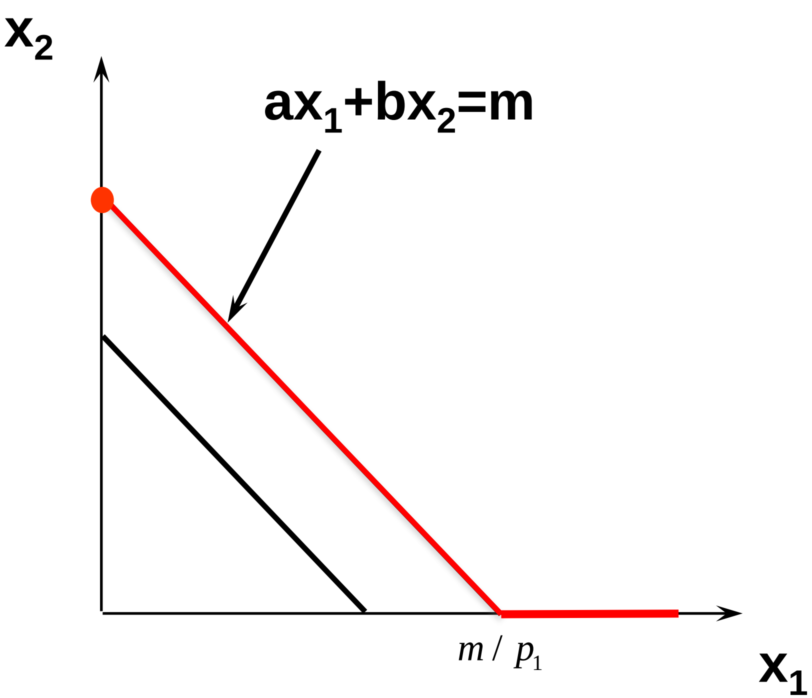
Own price changes



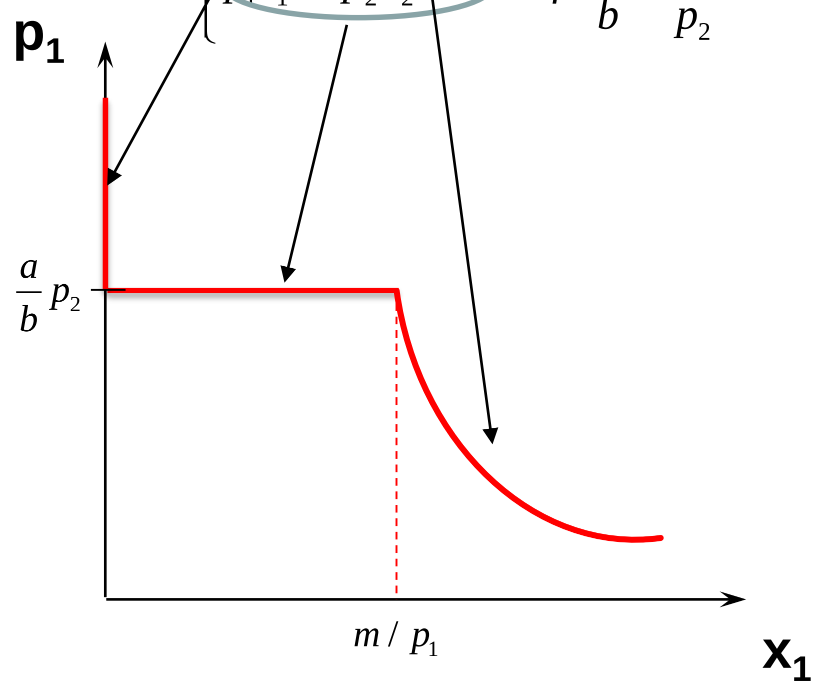
Examples: Perfect substitutes

$$u(x_1, x_2) = ax_1 + bx_2$$

Demand function =
$$\begin{cases} x_1^* = \dfrac{m}{p_1},\ x_2^* = 0 & \text{if } \dfrac{a}{b} > \dfrac{p_1}{p_2} \\ x_1^* = 0,\ x_2^* = \dfrac{m}{p_2} & \text{if } \dfrac{a}{b} < \dfrac{p_1}{p_2} \\ p_1x_1^* + p_2x_2^* = m & \text{if } \dfrac{a}{b} = \dfrac{p_1}{p_2} \end{cases}$$



Price offer curve for good 1



Demand curve for good 1

Other good's price changes

- How a consumer's demand for a good changes as other good's price changes
- The good i is a substitute for good j if

$$\frac{\partial x_i(\tilde{p}, m)}{\partial p_j} > 0$$

- The good i is a complement for good j if

$$\frac{\partial x_i(\tilde{p}, m)}{\partial p_j} < 0$$

Comparative Statics: Methodology

- In mathematical methods, comparative statics can be done by <u>determining the sign of partial differentials</u>
- Two-good case with equality constraint

$$\max_{x_1, x_2} U(x_1, x_2)$$

$$s.t.\ p_1 x_1 + p_2 x_2 = m$$

  - First-order conditions (F.O.C.)

$$-p_1 x_1(p_1, p_2, m) - p_2 x_2(p_1, p_2, m) + m = 0$$

$$\frac{\partial u\left(x_1(p_1, p_2, m), x_2(p_1, p_2, m)\right)}{\partial x_1} - \lambda p_1 = 0$$

$$\frac{\partial u\left(x_1(p_1, p_2, m), x_2(p_1, p_2, m)\right)}{\partial x_2} - \lambda p_2 = 0$$

Comparative Statics: Methodology

- Differentiating w.r.t. p_i, and arranging in matrix form

$$\begin{bmatrix} 0 & -p_1 & -p_2 \\ -p_1 & u_{11} & u_{12} \\ -p_2 & u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \lambda}{\partial p_1} \\ \dfrac{\partial x_1}{\partial p_1} \\ \dfrac{\partial x_2}{\partial p_1} \end{bmatrix} \equiv \begin{bmatrix} x_1 \\ \lambda \\ 0 \end{bmatrix}$$

- Solving for $\partial x_1 / \partial p_1$ via Cramer's rule gives,

$$\frac{\partial x_1}{\partial p_1} = \frac{\begin{vmatrix} 0 & x_1 & -p_2 \\ -p_1 & \lambda & u_{12} \\ -p_2 & 0 & u_{22} \end{vmatrix}}{|\bar{\mathbf{H}}|} = \lambda \frac{\begin{vmatrix} 0 & -p_2 \\ -p_2 & u_{22} \end{vmatrix}}{|\bar{\mathbf{H}}|} - x_1 \frac{\begin{vmatrix} -p_1 & u_{12} \\ -p_2 & u_{22} \end{vmatrix}}{|\bar{\mathbf{H}}|} \begin{matrix} \leq \\ \geq \end{matrix} 0 \ (?)$$

Comparative Statics: Methodology

- Differentiating w.r.t. m, and arranging in matrix form

$$\begin{bmatrix} 0 & -p_1 & -p_2 \\ -p_1 & u_{11} & u_{12} \\ -p_2 & u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \lambda}{\partial m} \\ \dfrac{\partial x_1}{\partial m} \\ \dfrac{\partial x_2}{\partial m} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

- Solving for $\partial x_1 / \partial m$ via Cramer's rule gives,

$$\frac{\partial x_1}{\partial m} = \frac{\begin{vmatrix} 0 & -1 & -p_2 \\ -p_1 & 0 & u_{12} \\ -p_2 & 0 & u_{22} \end{vmatrix}}{|\bar{\mathbf{H}}|} = \frac{\begin{vmatrix} -p_1 & u_{12} \\ -p_2 & u_{22} \end{vmatrix}}{|\bar{\mathbf{H}}|} \begin{matrix} \le \\ \ge \end{matrix} 0 \ (?)$$