Chap. 10. Making and Using Vortices

· vorticity

$$\bar{\xi} \equiv \nabla \times \bar{V}$$

· angular velocity $\quad \bar{\omega} = \hat{i}\,\omega_x + \hat{j}\,\omega_y + \hat{k}\,\omega_z$

$$= \frac{1}{2}\,\mathrm{curl}\,\bar{V}$$

$$= \frac{1}{2}\,\nabla \times \bar{V}$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

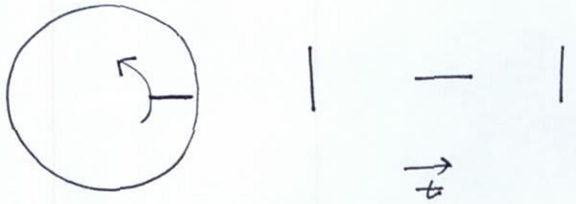
$$\bar{\xi} = 2\bar{\omega}.$$

* Vorticity for rigid body rotation (forced vortex). e.g. dish on turntable

$$v_r = 0, \quad v_\theta = \omega r$$

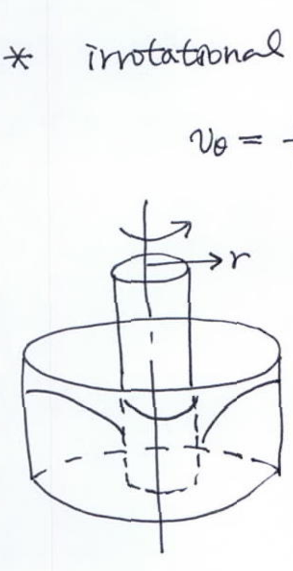
$$\bar{\xi} = \nabla \times \bar{V} = \frac{1}{r}\begin{vmatrix} \hat{e}_r & r\hat{e}_\theta & \hat{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ v_r & v_\theta & v_z \end{vmatrix} = 2\omega\,\hat{e}_z$$

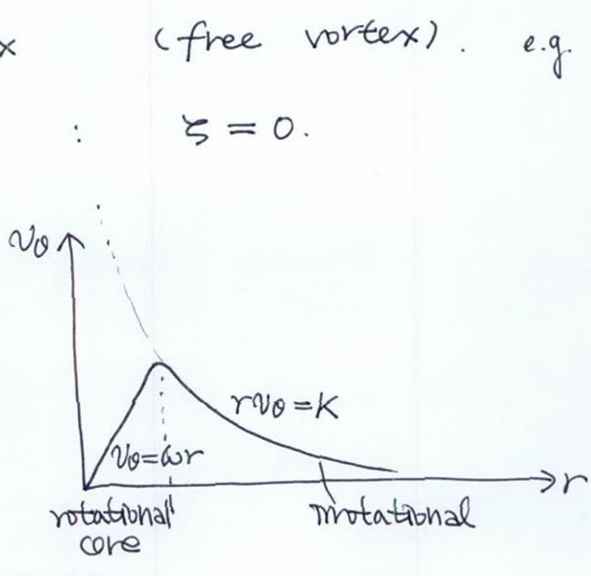
∴ rotational flow



* irrotational vortex (free vortex). e.g. fluid drawn down a plug-hole

$$v_\theta = \frac{K}{r} \quad : \quad \xi = 0.$$





§ Vortex (pl. vortices)

- motion of fluid swirling rapidly around a center
- spiral motion with closed streamlines

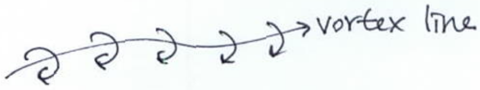
* properties of vortices

- fluid pressure lowest in the center

  e.g. tornado, dust devil

- vortex lines can start and end at the boundary of the fluid or form closed loops.

  cannot start or end in the fluid.



- two or more vortices that are approximately parallel and circulating in the same direction will merge to form a single vortex

  $$\text{circulation } \Gamma_{\text{merged vortex}} = \sum_i \Gamma_i$$

- In an ideal fluid, vortex energy can never be dissipated and the vortex would persist forever. It is only through dissipation of a vortex due to viscosity that a vortex line can end in the fluid, rather than at the boundary of the fluid.

- A pair of vortices with opposite circulations repel each other

e.g. Vorticella : small torroidal vortex to draw edible particles within reach

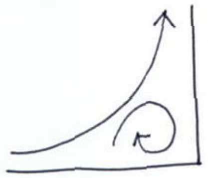
Fig. 10.4

§ Making and using vortices near interfaces : Fig. 10.6

(a) flow across a furrow

: particle resuspension
enhanced material exchange
induced flow in and out of porous substrata

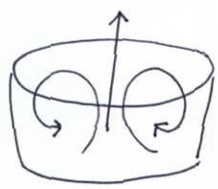
(b) sharp corner



(c) sharp cross-flow edge

separation

(d) flow across and within a pit or cup

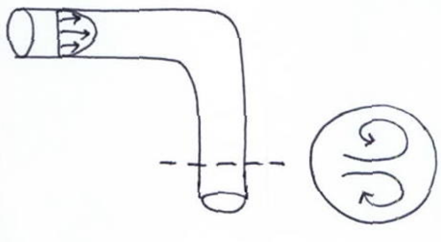


: release of propagules (soredia)
by lichen

(e) inside of droplets

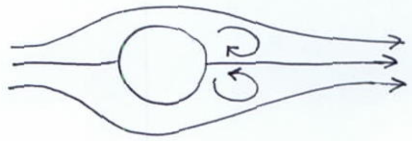
(f) wakes of jets

(g) bent pipes

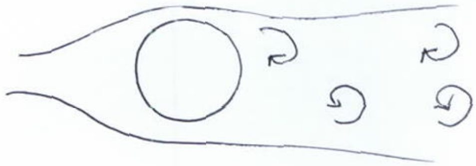


(h) rears of cylinders

$10 < Re < 40$ attached vortices



$40 < Re < 2 \times 10^5$ Karman vortex street



* ascending - paired vortices

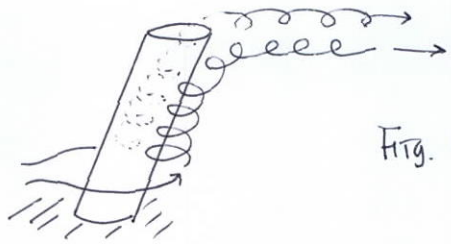


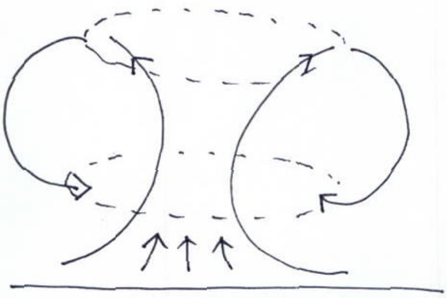
Fig. 10.7. detritus feeding

* Vortex digging

Fig. 10.9

§ Thermal vortices

· When winds are light and the ground heats the lowest part of the atmosphere



ascending vortex ring

e.g. tree (heating of leaves)
plowed field (more absorptive of solar radiation than surrounding vegetation)

highways through vegetated areas

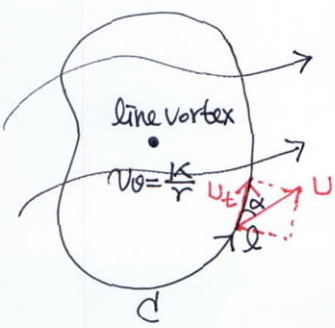
· users : large terrestrial birds that soar (hawks, vultures, eagles)
~ keep turning with a sufficiently narrow radius to stay in the locally ascending part of the torus.

ballooning spiders

silk thread

locust swarms ?

§ Circulation



$$\Gamma = \oint_l U_t \, dl$$

$$U_t \, dl = \bar{U} \cdot d\bar{s} = U \cos\alpha \, dl$$

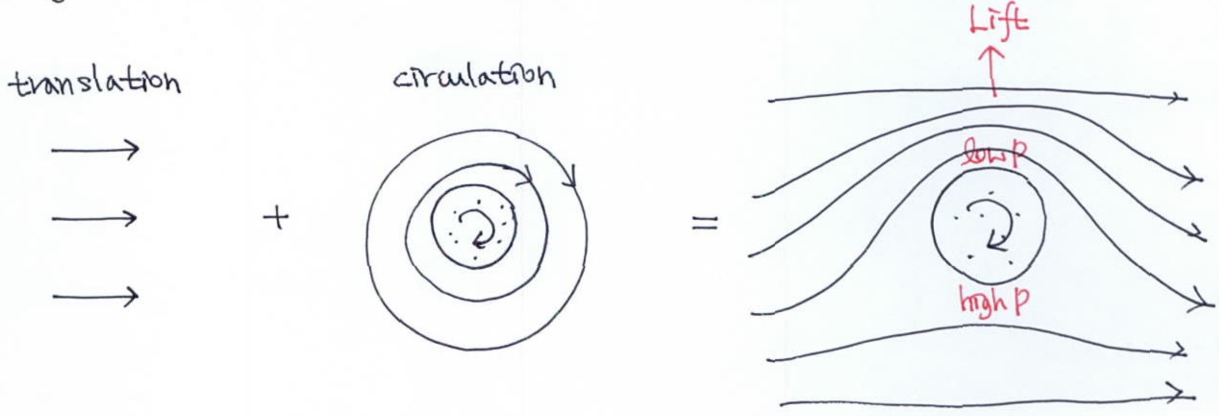
$$\Gamma = \oint_{r=R} \frac{K}{r} \, dr = \frac{K}{R} (2\pi R) = 2\pi K$$

$$\xi \text{ (vorticity)} = \lim_{r \to 0} \frac{\Gamma}{\pi r^2}$$

— circulation around an infinitesimal circuit

§ The origin of lift (for cylinder and sphere)

· The Magnus effect



Kutta-Joukowski theorem :

If an irrotational air stream surrounds a closed curve with circulation, a force is set up perpendicular to that air stream.

$$\frac{F}{l} = \rho U \Gamma \qquad \text{for cylinder}$$

* autorotation

e.g. tumbling card
winged seeds (samaras)

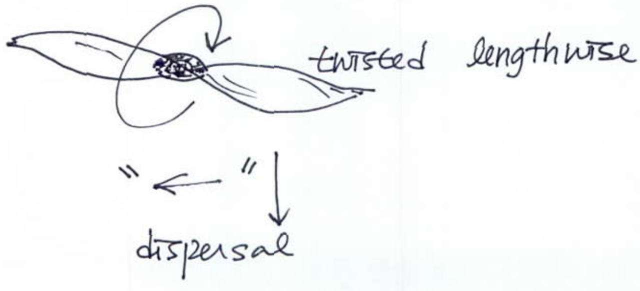


Fig. 10. 13