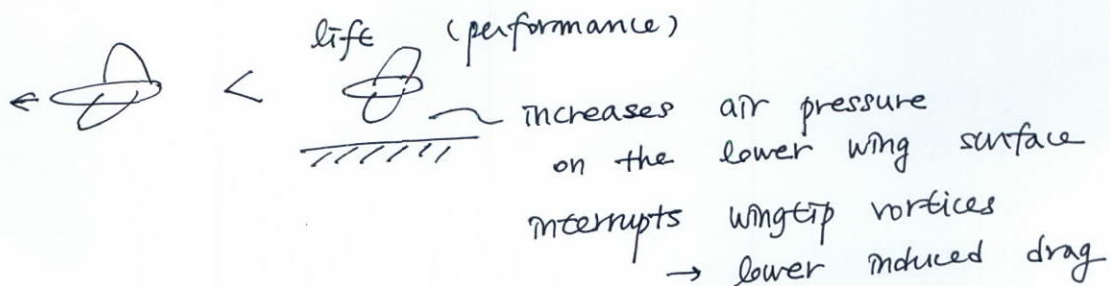
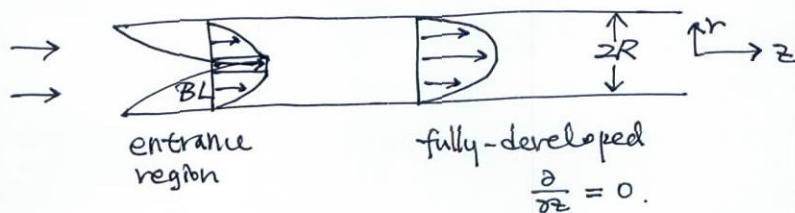


~ fish schooling

(2) Ground effect



Chap. 13. Flows within pipes and other structures



Navier-Stokes eq (cyl. coords)

$$\rho \frac{DV_z}{Dt} = - \frac{\partial p}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right] v_z$$

$$\rightarrow 0 = - \frac{dp}{dz} + \mu \frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) u$$

B.C.  $u(r=R) = 0$

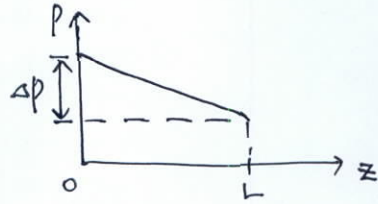
$$\frac{du}{dr}(r=0) = 0.$$

$$u = \left( - \frac{dp}{dz} \right) \frac{1}{4\mu} (R^2 - r^2)$$

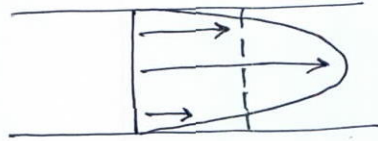
$$- \frac{dp}{dz} = \frac{\Delta p}{L}$$

$$Q = \int_0^R u(2\pi r) dr$$

$$= \frac{\pi R^4}{8\mu} \left( \frac{\Delta p}{L} \right)$$

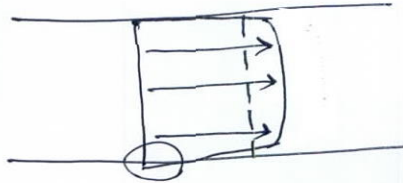


\* Turbulent pipe flow



laminar .  $f = \frac{64}{Re_d}$

(same  $U_{av}$ )

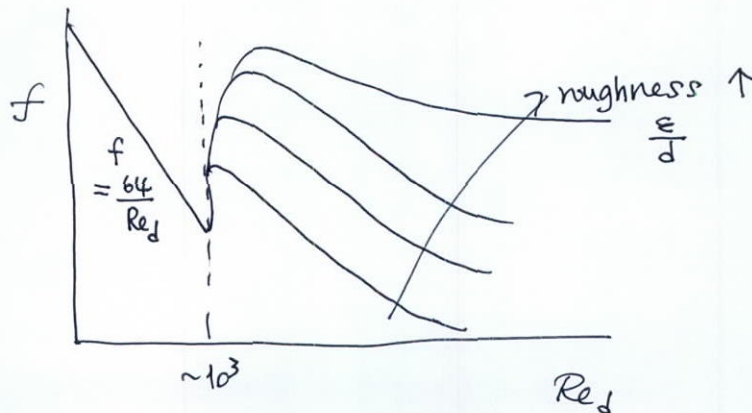


turbulent

higher vel. gradient  $\rightarrow \tau_w \uparrow$

friction factor =  $\frac{8\tau_w}{\rho U_{av}^2}$

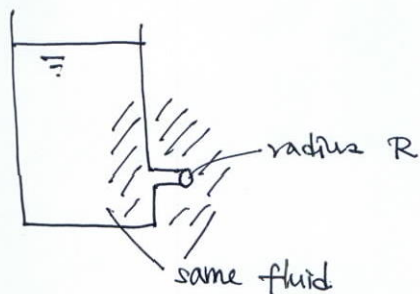
The Moody chart



\* Resistance

$$R = \frac{\Delta p}{Q} = \frac{8\mu L}{\pi R^4} \quad (\text{laminar})$$

### § Flow through circular apertures (submerged)



$$Re < 3 : Q = \frac{R^3 \Delta P}{3\mu}$$

$$\text{higher } Re : Q = C_0 \pi R^2 \left( \frac{2\Delta P}{\rho} \right)^{1/2}$$

$$C_0 : \text{orifice coefficient} = f_n(Re_d)$$

### Chap. 14. Internal flows in organisms

- most circulatory systems of animals

~ pulsatile flow of non-Newtonian fluids in pipes of time varying cross-sectional areas and shapes

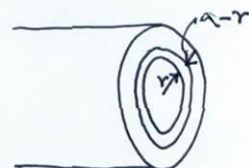
### § Circumventing the parabolic profile



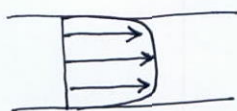
(1) How far is flow from a wall?

- distance index

$$D_i = \frac{1}{\pi a^2 \bar{u}} \int_0^a u(a-r) \cdot 2\pi r dr / a$$



① plug flow (slug flow)



$$\bar{u} = u = \text{const.}$$

$$D_i = \frac{2\pi}{\pi a^2 \bar{u}} u \int_0^a (a-r) r dr$$

$$= \frac{2}{a^2} \cdot \left( \frac{a^3}{2} - \frac{a^3}{3} \right) = \frac{1}{3}$$

