Lecture 5

Discrete Probability Distribution (2)

Statistics for
Civil & Environmental Engineers

Poisson Distribution(1)

- **Description**
  - a Bernoulli process defined over an interval of time (or space) so that p is the probability that an event may occur during the time interval where $\triangle t \to 0$, $p \to 0$, np= constant
- **Assumptions**
  - The probability of more than one arrival in the subinterval then becomes zero
  - If one partitions the interval t into sufficiently small subintervals of equal length Δt, the probability of an arrival over such a subinterval tends to $\lambda \Delta t$
  - The occurrence of an arrival within a subinterval independent of occurrences in other disjoint subintervals

Poisson Distribution(2)

- **Poisson pmf**
  - $p_X(x) = \Pr[X = x; \nu] = \nu^x e^{-\nu} / x!$ for $x = 1, 2, \ldots$ and $\nu > 0$
    $= 0$ otherwise
- **Poisson cdf**
  - $F_X(k) = \sum_{x=0}^{k} \nu^x e^{-\nu} / x!$ for $k = 0, 1, 2, \ldots$

  where $x =$ number of occurrence of a specific event
  $\nu =$ population parameter

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Poisson Distribution(3)

- **Population parameter**
  - **Mean=**
  - **Variance=**
  - **Skewness coef.=**
- **Parameter estimates**
  - $\hat{\nu} = \overline{X}$
- **Comments**
  - **events must be independent**
  - **Poisson is a convenient approximation for the binomial distribution when *n* is "large" and *p* is "small"**

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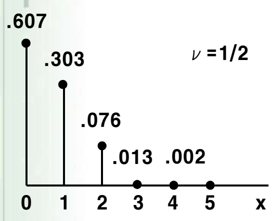
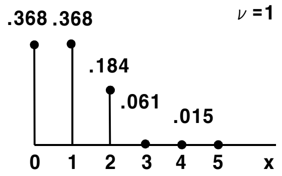
Poisson Distribution(4)

- **Property**
  - **the sum of two random variables with parameters v_1 and v_2 is a Poisson random variable with parameters $v = v_1 + v_2$**
- **Distribution shapes**





Poisson Distribution Example(1)

Example 4.4. Atmospheric pollution. Atmospheric dust particles at a particular location cause an environmental problem. The number of particles within a unit volume are observed by focusing a powerful microscope on the particles and making counts. The results of tests on 100 such volumes are shown in Table 4.1.1.

TABLE 4.1.1
Poisson distribution of dust particles in the atmosphere

	Particles in unit volume					
	0	**1**	**2**	**3**	**4**	**>4**
Observed frequency	13	24	30	18	7	8
Poisson frequency	12	26	27	19	10	6

Poisson Distribution Example(2)

Example 4.5. Reliability of machinery at a water treatment plant. All of the pumps at a water treatment plant have been made to the same specifications by a single manufacturer. From tests made over four-week periods, it has been ascertained that there are on average two breakdowns during each period. A new plant manager assumes that the problem is not serious if there are no more than four breakdowns over a period of four weeks. What is the probability of such an occurrence?

Poisson Distribution Example(3)

Example 4.7. Strength of timber. From past data, an engineer has estimated a probability of $p = .01$ that timber delivered at a construction site from a particular source is below specification. If 150 joists of timber are necessary for a particular construction job, determine the minimum number that should be ordered so that the chance of not having the required number of suitable joists is less than 10 percent.