Lecture 13

Risk and Reliability: Failure Mode

Statistics for
Civil & Environmental Engineers

Three Basic Configurations of a System

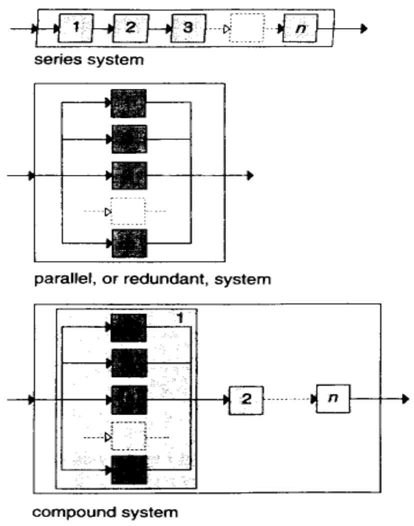

FIGURE 9.2.1
Block diagrams of series, parallel, and compound system layout.

A Series System

❖ **The Reliability**

$$r = \Pr[A_1^c A_2^c \dots A_n^c] = \Pr[A_1^c]\Pr[A_2^c]\dots\Pr[A_n^c] = \prod_{i=1}^{n}(1-p_i)$$

where A_i= failure of the ith component,
p_i= probability of failure of the ith component

❖ **The Overall Probability of Failure**

➢ **for p_i= p for all i,**

A Parallel (or Redundant) System

❖ **The Reliability**

$$r = \Pr[(A_1 A_2 \dots A_n)^c] = 1 - \Pr[A_1 A_2 \dots A_n] = 1 - \prod_{i=1}^{n} p_i$$

where A_i= failure of the ith component,
p_i= probability of failure of the ith component

❖ **The Overall Probability of Failure**

➢ **for p_i= p for all i,**

A Combined System

- **The Reliability of a System with *n* Redundant Components each *m* Series Components**

Where r_{ij}= the reliability of the *j*th series subcomponent of the *i*th redundant component

Example (1-1)

Example 9.18. Pipe network. Consider the part of a pipeline network for an urban water supply shown in Fig. 9.2.2. Knowing the individual probabilities of rupture for each pipe, p_i, and the corresponding reliabilities $r_i = 1 - p_i$, $i = 1, \ldots, 5$, consider the reliability of the system with respect to node *d*. This is the probability that node *d* does not remain isolated because no water passes through it. From Fig. 9.2.2 it is seen that for this condition to hold at least one of the following routes must work: (1, 3), (2, 4), (1, 5, 4), or (2, 5, 3). Let us call these routes *A*, *B*, *C*, and *D*, respectively. For each route (that is series component) the probability of failure is obtained as shown in the rightmost term of Eq. (9.2.8). That is,

$$\Pr[A] = 1 - r_1 r_3 \text{ (i.e., route } A \text{ does not work)},$$
$$\Pr[B] = 1 - r_2 r_4,$$
$$\Pr[C] = 1 - r_1 r_4 r_5,$$
$$\Pr[D] = 1 - r_2 r_3 r_5.$$

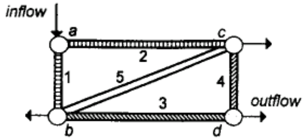

FIGURE 9.2.2
Part of pipeline network for urban water supply showing series, without pipe *bc*, and redundant components, with pipe *bc*.

Example (1-2)

These routes form four parallel-series (i.e., redundant) systems. However, the events described here are *not* independent. For example, routes $A = (1, 3)$ and $C = (2, 5, 3)$ have pipe 3 in common. Therefore, their joint effects must be considered. Routes *A* and *B* are independent. Thus, from Eq. (9.2.8),

$$\Pr[AB] = (1 - r_1 r_3)(1 - r_2 r_4) = (1 - r_1 r_3 - r_2 r_4 + r_1 r_2 r_3 r_4).$$

On the other hand, routes *C* and *D* are *not* independent. From the addition rule of probability,

$$\begin{aligned}\Pr[CD] &= \Pr[C] + \Pr[D] - \Pr[C + D] \\ &= (1 - r_1 r_4 r_5) + (1 - r_2 r_3 r_5) - (1 - r_1 r_2 r_3 r_4 r_5) \\ &= 1 - r_1 r_4 r_5 - r_2 r_3 r_5 + r_1 r_2 r_3 r_4 r_5.\end{aligned}$$

Proceeding in this way, the probability of failure of the system is found as follows:

$$\begin{aligned}\Pr[ABCD] = 1 - r_s = 1 &- r_1 r_4 r_5 - r_2 r_3 r_5 + r_1 r_2 r_3 r_4 r_5 \\ &- r_1 r_3 + r_1 r_3 r_4 r_5 + r_1 r_2 r_3 r_5 - r_1 r_2 r_3 r_4 r_5 \\ &- r_2 r_4 + r_1 r_2 r_4 r_5 + r_2 r_3 r_4 r_5 - r_1 r_2 r_3 r_4 r_5 \\ &+ r_1 r_2 r_3 r_4 - r_1 r_2 r_3 r_4 r_5 - r_1 r_2 r_3 r_4 r_5 + r_1 r_2 r_3 r_4 r_5.\end{aligned}$$

Example (1-3)

Note, incidentally, that there are no squared terms in such multiplications; the effect of a pipe is considered only once in the same product. Hence,

$$\begin{aligned}1 - r_s = 1 &- r_1 r_3 - r_2 r_4 - r_1 r_4 r_5 - r_2 r_3 r_5 + r_1 r_3 r_4 r_5 + r_1 r_2 r_3 r_5 \\ &+ r_1 r_2 r_4 r_5 + r_2 r_3 r_4 r_5 + r_1 r_2 r_3 r_4 - 2 r_1 r_2 r_3 r_4 r_5.\end{aligned}$$

If $r_1 = r_2 = r_3 = r_4 = r_5 = r$,

$$r_s = 2r^2 + 2r^3 - 5r^4 + 2r^5.$$

If we remove pipe 5, we can apply Eq. (9.2.8) directly (for two independent routes):

$$r_s' = 1 - (1 - r^2)(1 - r^2).$$

Thus

$$r_s' = 2r^2 - r^4,$$

and

$$r_s - r_s' = 2r^3 - 4r^4 + 2r^5 = 2r^3(1 - 2r + r^2).$$

This expression shows that $0 < r_s - r_s' < 1$ for $0 < r < 1$. For example, if $p = .01$, $r = .99$, and

$$r_s = 2 \times .99^2 + 2 \times .99^3 - 5 \times .99^4 + 2 \times .99^5 = .9998,$$
$$r_s' = 2 \times .99^2 - .99^4 = .9996.$$

Thus, removal of one of the pipes causes only a negligible decrease in the reliability if p is small; if, however, $p = .5$, the decrease is 1 16.

The *k*-out-of-*m* Model

❖ **The Reliability**

❖ **The Probability of Failure**

The *k*-out-of-*m* Model

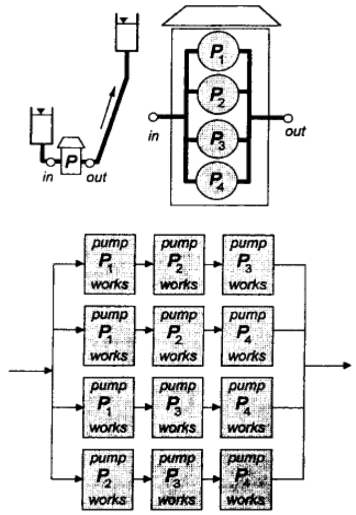

FIGURE 9.2.3
Pumping station and pipeline connecting two tanks. The station has four parallel pumps, three of which must operate to ensure that the target flow reaches the upper tank. The associated block diagram shows four reliability paths, each of them having three components.

Example (2-1)

Example 9.19. Flight planning. An airline company is planning the types and numbers of carriers to be used between two cities at night. Current estimates are that the minimum number of passengers is 100, the maximum is 250, and the average is 175.

Three different options are considered for the aircraft with the following number of seats: 260, 180, and 110, as shown in Figure 9.2.4.

One needs to find the reliability of each demand for each of the three options, assuming a constant risk p that an airplane is out of service. The block diagrams displaying the failure paths and the corresponding reliabilities are also given (see Eq. (9.2.11)). The curves in the lower half of Fig. 9.2.4 show how the different reliabilities vary with the individual probability of failure p. As expected, the system with two parallel elements is seen to have the highest reliability. The redundant configuration displays the smallest risk if $0 < p < \frac{1}{2}$. The risk is always higher for the series configurations. In a practical situation, different individual reliabilities are likely for each aircraft. Costs also need to be accounted for. This method can be applied to water resources management, road traffic, structural foundation, and other problems.

Example (2-2)

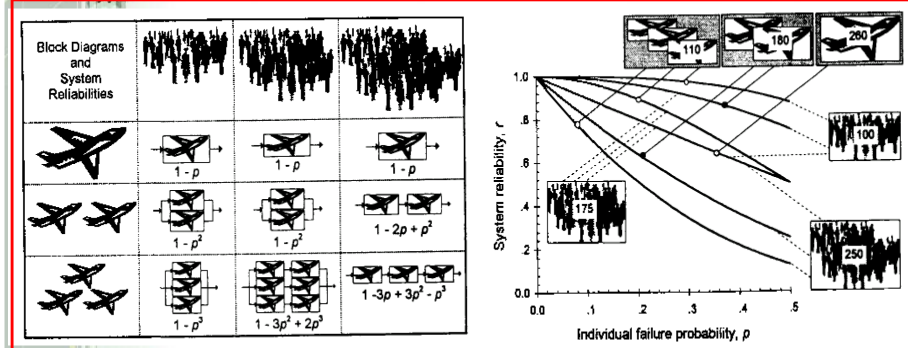

FIGURE 9.2.4
Block diagrams and system reliabilities for different configurations of flight transportation, with a plot of reliability r against the individual probability of failure p.

Example (2-3)

If m = k in Eq. (9.2.11), $p_f = 1 - (1 - p)^m$; that is, Eq. (9.2.3) is obtained. For $k = 1$, $p_f = p^m$; that is, Eq. (9.2.7) is obtained. When all component reliabilities are equal, an m-out-of-m system is equivalent to a series configuration with m independent components; conversely, a 1-out-of-m system is equivalent to a redundant configuration with m independent components. Accordingly, these two configurations give the bounds for the probability of failure, since

$$\prod_{i=1}^{m} p_i \le p_f \le \sum_{i=1}^{m} p_i, \tag{9.2.12}$$

where the p_i are the individual probabilities of failure of the m independent system components. For example, if $m = 10$ and $p_i = p = 10^{-2}$, the lower bound is 10^{-20} and the upper one is 10^{-1}. This range is rather wide, so some better procedure is necessary to tighten the bounds in order to approach complex systems with a large number of components.[8]