Stress path method (Lambe 1964, 1967)

- Settlement estimation based on realistic deformation characteristics measured from stress path tests which duplicate field stress paths and probable deformation modes of soil elements.

- A rational experimental approach to more exact estimation of field settlement.
- Typical stress path of saturated clay deposits under foundation loading

\[ \sigma'_w \]

\[ \sigma'_h \] (initial state)

\[ \sigma'_w + \Delta \sigma_v \] (immediately after loading)

\[ \sigma'_w + \Delta \sigma_v \] (after consolidation)

\[ \sigma'_h \] (lateral strain occurs)

\[ \Delta \sigma_h \neq K_o \Delta \sigma_v \]

\[ \epsilon_h \neq 0 \] (lateral strain occurs)

\( K_o \) stress condition

\( \neq 1D \) deformation mode

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* Stress state in p'-q diagram

I : $K_0$ initial state \( \left( \frac{\sigma'_{vi} + \sigma'_{hi}}{2}, \frac{\sigma'_{ui} - \sigma'_{hi}}{2} \right) \)

U : Immediately after loading \( \left( \frac{\sigma'_{vi} + \sigma'_{hi}}{2} + \Delta \sigma_v + \frac{\Delta \sigma}{2} - \Delta u_v, \frac{\sigma'_{ui} - \sigma'_{hi}}{2} + \Delta \sigma_v - \Delta \sigma_h \right) \)

C : After consolidation \( \left( \frac{\sigma'_{vi} + \sigma'_{hi}}{2} + \Delta \sigma_v + \frac{\Delta \sigma}{2}, \frac{\sigma'_{ui} - \sigma'_{hi}}{2} + \Delta \sigma_v - \Delta \sigma_h \right) \)

IU : Undrained path → immediate strain \( (\varepsilon'_{vi}) \) → immediate settlement \( (s_i) \)

UC : Consolidation path → consolidation strain \( (\varepsilon'_{vi}) \) → consolidation settlement \( (s_c) \)
1D oedometer test

\[ \sigma'_u \]

\[ \rightarrow \sigma'_v (= K_v \sigma'_e) \]

\[ K_v \text{ initial state (I)} \]

\[ \sigma'_u + \Delta \sigma_v \]

\[ \Delta \sigma_v = \Delta \sigma'_e \leftarrow \sigma'_u + \Delta \sigma_v \]

\[ \text{Immediately after loading (U')} \]

\[ (\text{total stress}) \]

\[ \sigma'_u + \Delta \sigma_v \]

\[ \sigma'_u + K_v \Delta \sigma_v \]

\[ \text{After consolidation (C')} \]

\[ q = \frac{\sigma'_v - \sigma'_h}{2} \]

\[ \Delta \sigma_v \]

\[ \Delta u_e \]

\[ \text{U'C': Consolidation path of oedometer test.} \]

\[ \rightarrow 1D \text{ consolidation strain (} \epsilon_{ve-1D} \text{) with no lateral strain} \]

\[ \rightarrow 1D \text{ consolidation settlement (} S_{c-1D} \text{) } \]
- Skempton and Bjerrum modification

  → 1D deformation mode of oedometer test

  → Governing stress increment : not $\Delta \sigma_{v}$ but $\Delta u_{e}$.

  → S&B consolidation strain : $\varepsilon_{vc=SB} = \varepsilon_{vc=1D} \times \frac{\Delta u_{e}}{\Delta \sigma_{v}}$

  → S&B consolidation settlement ($S_{c=SB}$)
- Methods that are commonly used to predict field settlement \( \left( S_t = S_i + S_c \right) \)

  1. 1D consolidation settlement \( \left( S_{c-1D} \right) \).

  2. Immediate settlement + 1D consolidation settlement \( \left( S_i + S_{c-1D} \right) \).

  3. Immediate settlement + S & B consolidation settlement \( \left( S_i + S_{c-SB} \right) \).

\[ \Rightarrow \] Unrealistic \( \kappa_s \) stress path and 1D deformation mode are assumed.

\[ \Rightarrow \] Can be expected to give an erratic approximation of field settlement
- Stress path method

• Lambe (1964, 1967)

• Settlement estimation based on realistic deformation characteristics measured from stress path tests which duplicate field stress paths and probable deformation modes of soil elements.

• A rational experimental approach to more exact estimation of field settlement.
i) Procedures: For a given structure,

① Divide subsoils into several layers and select average point of each layer.

② Determine field stress paths of the average points.

\[ \rightarrow \ k_0 \ \text{initial state} \left( \sigma_v^i, \ \sigma_h^i \right). \]

\[ \rightarrow \ \text{Stress increment} \left( \Delta \sigma_v, \ \Delta \sigma_h \right) \Leftrightarrow \text{the elastic theory.} \]
③ Duplicate the field stress paths in the laboratory.

→ Undisturbed samples.

→ TX tests for axisymmetric deformation mode (circular or square footing).

→ PS tests for plane strain deformation mode (strip footing, embankment).

→ Measure vertical strains ($\varepsilon_{vu}$, $\varepsilon_{vc}$).

Instantaneous loading $\rightarrow$ possibility to break soil structure.

(misleading deformation mode)

but providing $\varepsilon_{vu}$ and $\varepsilon_{vc}$, separately.

Stress rate loading $\rightarrow$ not breaking soil structure.

but not providing $\varepsilon_{vu}$ and $\varepsilon_{vc}$, separately.
Estimate settlements by integrating the vertical strains with depth.

\[ S_i = \int \varepsilon_{vu} \, dz = \sum \varepsilon_{vu} \Delta z \]

\[ S_c = \int \varepsilon_{vc} \, dz = \sum \varepsilon_{vc} \Delta z \]

\[ S_i = S_i + S_c \]
ii) Application example

Circular, flexible, smooth footing with radius $R$ and embedded depth $D_f$

- Sand Fill, $\gamma = 20\, \text{kN/m}^3$
- Saturated Clay, $\gamma = 17\, \text{kN/m}^3$

Kaolinite, NC state
- $w = 37.6\sim40.2$
- $C_c = 0.253\sim0.286$
- $C_r = 0.110\sim0.126$

$(Q_{net} = 135\, \text{t}, R = 5\, \text{m}, D_f = 2\, \text{m})$
① Divide subsoils into several layers and select average point of each layer.

② Determine field stress paths of the average points.

<table>
<thead>
<tr>
<th></th>
<th>Initial state</th>
<th>Stress increments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{vi}$ (kPa)</td>
<td>$\sigma_{ih}$ (kPa)</td>
</tr>
<tr>
<td>A</td>
<td>47.20</td>
<td>23.60</td>
</tr>
<tr>
<td>B</td>
<td>61.60</td>
<td>30.80</td>
</tr>
<tr>
<td>C</td>
<td>76.00</td>
<td>38.00</td>
</tr>
<tr>
<td>D</td>
<td>90.40</td>
<td>45.20</td>
</tr>
<tr>
<td>E</td>
<td>104.80</td>
<td>52.40</td>
</tr>
</tbody>
</table>
3. Duplicate the field stress paths in the laboratory (stress path test).

- 1st step: back pressure saturation $\rightarrow$ saturation of test specimen.

- 2nd step: $K_0$ consolidation $\rightarrow$ duplication of $K_0$ initial state (I).

$\rightarrow$ Slowly increase vertical stress up to $\sigma_{vi}'$ with $\varepsilon_h = 0$ condition by controlling cell pressure

or

$\rightarrow$ Slowly increase vertical and horizontal stresses up to $\sigma_{vi}'$ and $\sigma_h' = K_0 \sigma_{vi}'$. 

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• 3rd step: undrained loading → duplication of undrained path (IU).

→ Increase vertical and horizontal stresses by $\Delta \sigma_v$ and $\Delta \sigma_h$ under undrained condition.

→ Measure immediate strains $\varepsilon_{vu}$, $\varepsilon_{hu}$ and excess pore pressure $\Delta u_e$.

• 4th step: consolidation → duplication of consolidation path (UC).

→ Dissipate $\Delta u_e$ by opening drainage value.

→ Measure consolidation strains $\varepsilon_{vc}$ and $\varepsilon_{hc}$.
• Test results

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_{vu}$ (%)</th>
<th>$\varepsilon_{hu}$ (%)</th>
<th>$\Delta u_e$ (kPa)</th>
<th>$\varepsilon_{vc}$ (%)</th>
<th>$\varepsilon_{hc}$ (%)</th>
<th>$\varepsilon_{vt}$ (%)</th>
<th>$\varepsilon_{ht}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.118</td>
<td>-0.559</td>
<td>13.70</td>
<td>0.428</td>
<td>0.252</td>
<td>1.546</td>
<td>-0.307</td>
</tr>
<tr>
<td>B</td>
<td>3.465</td>
<td>-1.733</td>
<td>13.75</td>
<td>0.317</td>
<td>0.175</td>
<td>3.782</td>
<td>-1.558</td>
</tr>
<tr>
<td>C</td>
<td>0.771</td>
<td>-0.386</td>
<td>6.85</td>
<td>0.123</td>
<td>0.070</td>
<td>0.894</td>
<td>-0.316</td>
</tr>
<tr>
<td>D</td>
<td>0.286</td>
<td>-0.143</td>
<td>4.84</td>
<td>0.092</td>
<td>0.045</td>
<td>0.378</td>
<td>-0.098</td>
</tr>
<tr>
<td>E</td>
<td>0.088</td>
<td>-0.044</td>
<td>5.55</td>
<td>0.099</td>
<td>0.053</td>
<td>0.187</td>
<td>0.009</td>
</tr>
</tbody>
</table>

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4 Estimate settlements by integrating the vertical strains with depth.

\[ S_i = \int \varepsilon_{vu} \, dz = \frac{(1.118 + 3.465 + 0.771 + 0.286 + 0.088)}{100} \times 2000 = 114.56 \text{mm} \]

\[ S_c = \int \varepsilon_{vc} \, dz = \frac{(0.428 + 0.317 + 0.123 + 0.092 + 0.099)}{100} \times 2000 = 21.18 \text{mm} \]

\[ S_i = S_i + S_c = 135.74 \text{mm} \]
### iii) Comparison with conventional methods

<table>
<thead>
<tr>
<th></th>
<th>Stress path method</th>
<th>Oedometer</th>
<th>S&amp;B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{yu}$ (%)</td>
<td>$\varepsilon_{yc}$ (%)</td>
<td>$\varepsilon_{yt}$ (%)</td>
</tr>
<tr>
<td>A</td>
<td>1.118</td>
<td>0.428</td>
<td>1.546</td>
</tr>
<tr>
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<td>3.465</td>
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<td>0.099</td>
<td>0.187</td>
</tr>
</tbody>
</table>

$S_y = 114.56 \text{mm}$  \quad $S_c = 21.18 \text{mm}$  \quad $S_t = 135.4 \text{mm}$  \quad $S_{c-ID} = 66.86 \text{mm}$  \quad $S_{c-SB} = 50.16 \text{mm}$
If field conditions are far from being 1D,

- \( S_{c-1D} < S_i \) ← Based on total settlement

\[ \therefore S_{c-1D} \rightarrow \text{Underestimation of field total settlement.} \]

- \( S_{c-1D} > S_{c-SB} > S_c \) ← Based on consolidation settlement

\[ \therefore S_i + S_{c-1D} \rightarrow \text{Overestimation of field total settlement.} \]

\[ \therefore S_i + S_{c-SB} \rightarrow \text{Overestimation of field total settlement.} \]

\[ \rightarrow \text{But closer to field total settlement than } S_i + S_{c-1D}. \]
iv) Limitations of stress path method

① Applicability of the elastic theory.

- Soils do not behave as linear elastic materials.
- $\Delta \sigma_v$ and $\Delta \sigma_h$ estimated based on the elastic theory may be erratic.
  → Overestimation of $\Delta \sigma_v$ and high underestimation of $\Delta \sigma_h$
  → Harr (1977) proposed an alternative approach using probabilistic theory.
  → However, no other way
② Change of stress increments during consolidation.
• Decrease of Poisson's ratio \((ν_u = 0.5 \rightarrow ν_d = 0.1 \sim 0.4)\) \(\rightarrow\) Decrease of \(Δσ_h\)

• Realistic inclined consolidation path UC* can not be duplicated using the conventional stress path testing scheme. (Why?)

• An efficient stress path testing scheme was newly devised by Kim (2004).
  \(\rightarrow\) Back pressure equalization followed by actively-controlled consolidation.
  \(\rightarrow\) Any arbitrary consolidation path can be duplicated.
  \(\rightarrow\) Exact Deformations of a tested consolidation path can be continuously measured.
  (One path by One test)
③ Too excessive experimental work.

- A number of laborious tests are required for every structure.
- Different types of structures require mostly different series of tests.
- Various design alternatives can not be easily examined.
- Modification of design factors during construction can not be readily reflected.
* Conventional Stress Path Method

A given structure

Field stress paths

Experimental duplication

Settlement estimation
A more practical approach of stress path method was proposed by Kim (2004).
→ Characteristic behaviors of deformation which can cover all probable field stress paths are evaluated in advance by an economically-designed experimental program.

→ Settlements of various structures or design alternatives can be routinely estimated without additional tests by simply substituting their corresponding field stress paths into the characteristic behaviors.

→ Practicality of the proposed approach was maximized in the manner of minimizing experimental effort required to establish the characteristic behaviors of deformation.