2.5 Consolidation Theory

Pore pressure

Parameter

Dissipation

(Consolidation Theory)

Loading

(Boundary

Condition Change)

Pore pressure

Volume

change

Dissipation rate associated

with soil deformation

(1) Consolidation Equation

● Governing Conditions

i) Equations of equilibrium

ii) Stress-strain relations for the mineral skeleton

iii) A continuity equation for the pore fluid

iii) → for 2-D flow

(Assumptions of validity of Darcy's law and small strains)

$$\underbrace{k_z \frac{\partial^2 h}{\partial z^2} + k_x \frac{\partial^2 h}{\partial x^2}}_{\text{Net flow of water into an element of soils.}} = \underbrace{\frac{1}{1+e}(e \frac{\partial S}{\partial t} + S \frac{\partial e}{\partial t})}_{\text{Rate of volume change if saturated } (\frac{\partial S}{\partial t} = 0, S = 1)}$$

Net flow of water into an element of soils.

Rate of volume change if saturated ($\frac{\partial S}{\partial t} = 0, S = 1$).

● One-dimensional consolidation with linear stress-strain relation

Horizontal dimension 》 thickness of the consolidating stratum

⇒ The same distribution of pore pressures and stress with depth for all vertical sections

⇒ Flow of water only in the vertical direction and no horizontal strain.

⇒ Governing Equations.

Equilibrium : (in vertical direction)

$$\sigma_v = \gamma z + surface\ \ stress$$

Stress-strain : $\dfrac{\partial e}{\partial \sigma_v} = -a_v$ (a_v is constant. ⇒ small strain assumption)

Continuity : $k\dfrac{\partial^2 h}{\partial z^2} = \dfrac{1}{(1+e)}\dfrac{\partial e}{\partial t}$ (full saturation, S=1, $\dfrac{\partial S}{\partial t} = 0$)

Combining the second and third equations,

$$\frac{k(1+e)}{a_v}\frac{\partial^2 h}{\partial z^2} = -\frac{\partial \overline{\sigma_v}}{\partial t}$$

Breaking the total head into its component parts,

$$h = h_e + \frac{u}{\gamma_w} = h_e + \frac{1}{\gamma_w}(u_{ss} + u_e)$$

$\partial^2 h_e \big/ \partial z^2 = 0$ and $\partial^2 u_{ss} \big/ \partial z^2 = 0$,

$$\frac{k(1+e)}{\gamma_w a_v}\frac{\partial^2 u_e}{\partial z^2} = -\frac{\partial \overline{\sigma_v}}{\partial t}$$

By defining the coefficient of consolidation c_v as

$$c_v = \frac{k(1+e)}{\gamma_w a_v} = \frac{k}{\gamma_w m_v}$$

↳ the coefficient of volume change.

and $\overline{\sigma_v} = \sigma_v - u = \sigma_v - (u_e + u_{ss})$ with $\dfrac{\partial u_{ss}}{\partial t} = 0$,

$$c_v \frac{\partial^2 u_e}{\partial z^2} = \frac{\partial u_e}{\partial t} - \frac{\partial \sigma_v}{\partial t}$$ $\rightarrow$ Terzaghi's consolidation equation.

● Two-dimensional consolidation of isotropic elastic material

Circular tank loads (axisymmetrical loading)

Long embankment (plain strain loading)

⇒ Consolidation with radial (or horizontal) flow and strain as well as vertical flow and strain.

∗ plane strain with isotropic elastic material

- For plane strain

$$\frac{\Delta V}{V_0} = \frac{(1-2\overline{\mu})}{\overline{E}}(\overline{\sigma_x} + \overline{\sigma_y} + \overline{\sigma_z})$$

$$\varepsilon_y = 0 \quad \rightarrow \quad \overline{\sigma_y} = \mu(\overline{\sigma_x} + \overline{\sigma_z})$$

$$= \mu(\overline{\sigma_v} + \overline{\sigma_h})$$

$$\frac{\Delta V}{V_0} = \frac{(1-2\overline{\mu})(1+\overline{\mu})}{\overline{E}}(\overline{\sigma_v} + \overline{\sigma_h})$$

$$\rightarrow \quad \Delta e = -\frac{\Delta V}{V_0}(1+e)$$

$$= -(1+e)\frac{(1-2\overline{\mu})(1+\overline{\mu})}{E}(\overline{\sigma_v} + \overline{\sigma_h}) \qquad \text{---------------------} \quad (1)$$

- For saturated soil

$$k\left(\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x^2}\right) = \frac{1}{1+e}\frac{\partial e}{\partial t} \quad \text{---} \quad (2)$$

Put (1) into (2)

$$k\left(\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x^2}\right) = -\frac{(1-2\overline{\mu})(1+\overline{\mu})}{E}\frac{\partial(\overline{\sigma_v} + \overline{\sigma_h})}{\partial t}$$

$$\frac{Ek}{(1-2\overline{\mu})(1+\overline{\mu})}\left(\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x^2}\right) = -\frac{\partial(\sigma_v - u_e + \sigma_h - u_e)}{\partial t}$$

$$\frac{Ek}{2\gamma_w(1-2\mu)(1+\mu)}\left(\frac{\partial^2 u_e}{\partial z^2} + \frac{\partial^2 u_e}{\partial x^2}\right) = \frac{\partial u_e}{\partial t} - \frac{1}{2}\frac{\partial(\sigma_v + \sigma_h)}{\partial t}$$

$\Rightarrow$ Consolidation equation for plane strain condition.

● Radial Consolidation

* Axisymmetric problems (transient radial flow but zero axial flow)

$\Rightarrow$ $\left\{ \begin{array}{l} \text{Triaxial test specimen with radial drainage} \\ \text{Vertical drains under preloading} \end{array} \right.$

$$\Rightarrow \ c_v\left(\frac{\partial^2 u_e}{\partial r^2} + \frac{1}{r}\frac{\partial u_e}{\partial r}\right) = \frac{\partial u_e}{\partial t} - \frac{\partial \sigma_v}{\partial t}$$

(2) Solution for uniform initial excess pore pressure

(a) $\dfrac{\partial \sigma_v}{\partial t} = 0$

(b) uniform distribution of initial excess pore pressure with depth

(c) drainage at both top and bottom of the consolidating stratum

● Solution

- By introducing nondimensional variables,

$Z = \frac{z}{H}$, $T = \frac{c_v t}{H^2}$ ⇒time factor

1-D consolidation equation becomes

$$\frac{\partial^2 u_e}{\partial Z^2} = \frac{\partial u_e}{\partial T}$$

- Initial and boundary conditions,

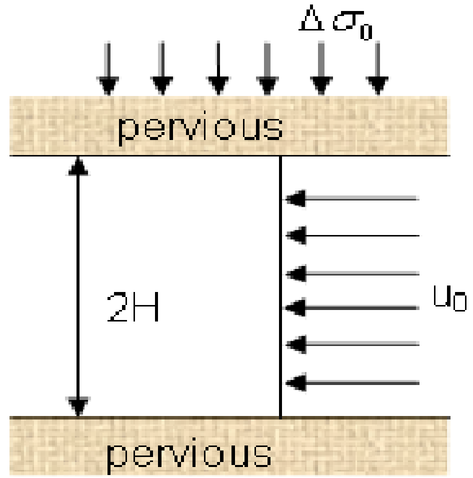
at $t = 0,\ \ u_e = u_0$ for $0 \le Z \le 2$

at all $t,\ \ u_e = 0$ for $Z = 0$ and $Z = 2$

- The solution is

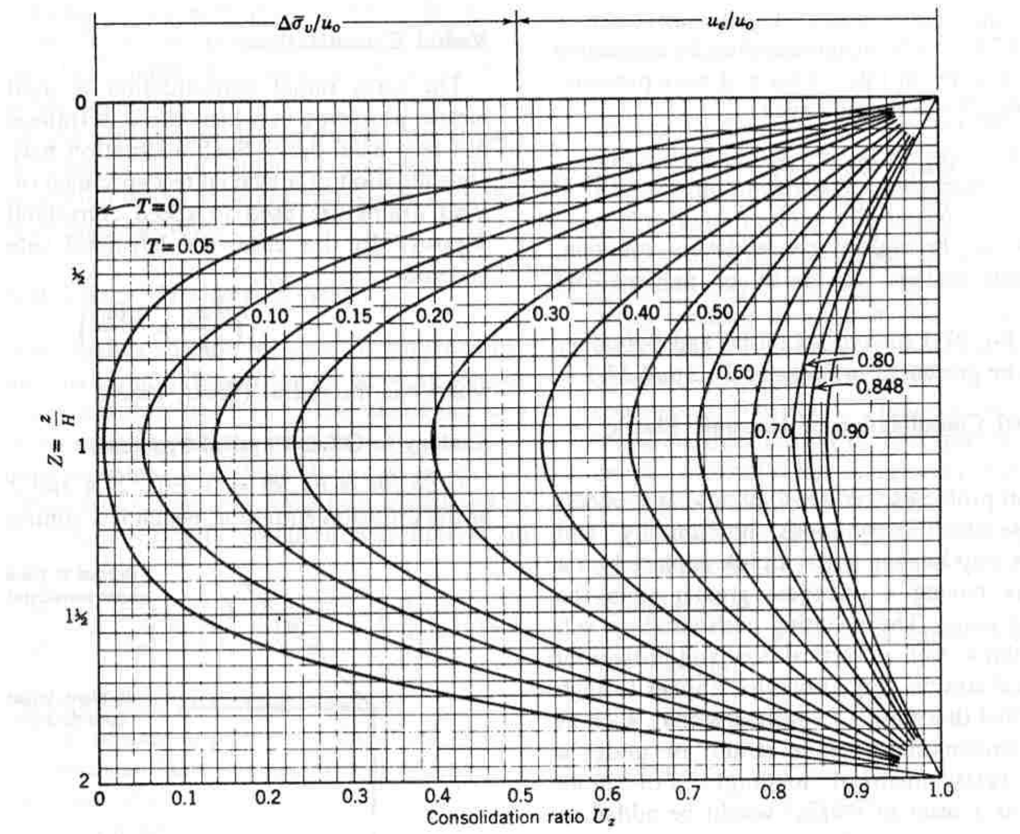
$$u_e = \sum_{m=0}^{\infty} \frac{2u_0}{M} (\sin MZ) e^{-M^2 T} \quad \text{--} \quad (1)$$

where $M = \frac{\pi}{2}(2m+1)$, m =1.2.3…



- Defining consolidation ratio, $U_Z = 1 - \dfrac{u_e}{u_0}$,

  Eq (1) can be portrayed in graphic form as below,



<Fig. 27.2> Consolidation ratio as function of depth and time factor: uniform initial excess pore pressure.

Notes

i) Immediately following application of the load, there are very large gradients near the top and bottom of the strata but zero gradient in the interior. Thus the top and bottom change in volume quickly, whereas there is no volume change at mid-depth until T exceeds about 0.05.

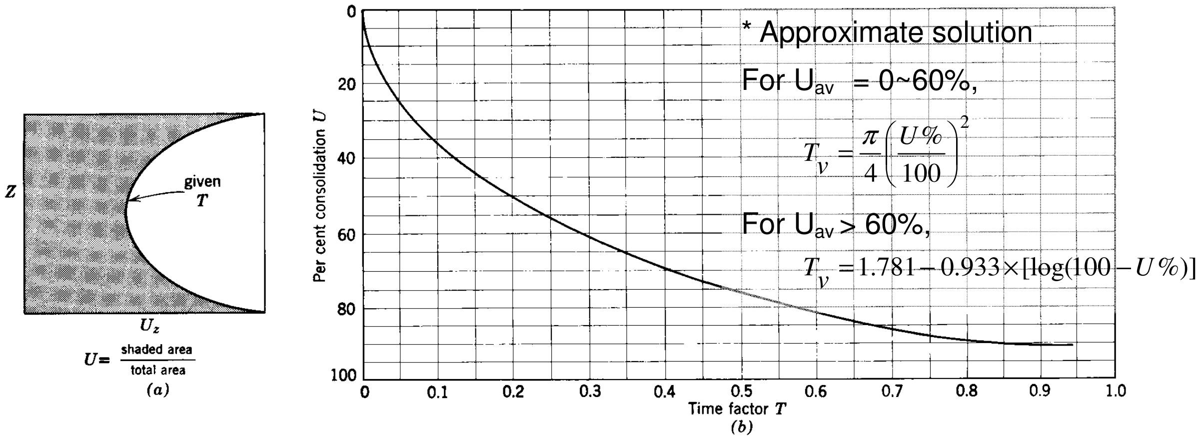
ii) For $T > 0.3$, the curves of U_z versus Z are almost exactly sine curves; i.e., only the first term in the series of eq (1) is now important.

iii) The gradient of excess pore pressure at mid-depth ($Z = 1$) is always zero so that no water flows across the plane at mid depth.

＊ Average consolidation ratio

⇒ related to the total compression of the stratum at each stage of the consolidation process.

$$U = \frac{\text{Compression at time T}}{\text{Compression at end of consolidation}} = 1 - \frac{u_{e(ave)}(T)}{u_0} = 1 - \frac{1}{2u_0}\int_0^2 u_e(T)\,dZ$$



<Figure 27,3> Average consolidation ratio : **linear initial excess pore pressure**. (a) Graphical interpretation of average consolidation ratio. (b) U versus T.

Notes

i) U can be used for estimating (consolidation) settlement ratio($U = \frac{\rho_{ct}}{\rho_{cf}}$).

ii) U initially decreases rapidly.

iii) Theoretically, U never reaches 0. (Consolidation is never complete.)

$\Rightarrow$ For engineering purposes, T=1(92% consolidation) is often taken as the "end" of consolidation.

iv) Single drainage

Longest drainage path, H = total thickness of clay layer.

t_{consol} in single drainage = 4 t_{consol} in double drainage.

$U_{ave} - T$: same as double drainage.(Fig. 27.3)

$U_Z - T$: half of double drainage (upper or lower half of Fig. 27.2)

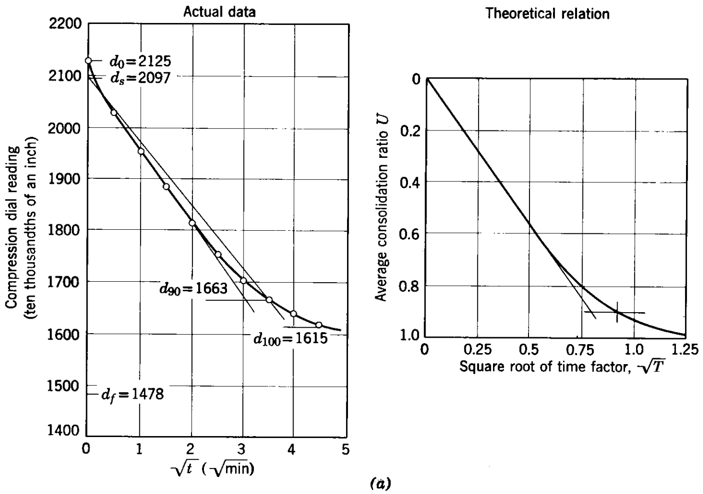
(3) Evaluation of c_v

c_v ⇐ Key parameter to estimate the rate of dissipation of pore pressure.

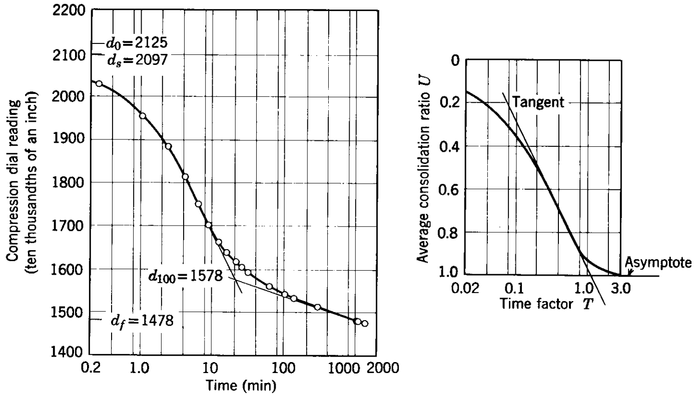
⇐ From oedometer test with undisturbed sample.

log t method → Casagrande

$\sqrt{t}$ method → Taylor



(a)



(b)

<Fig 27.4> Time-compression compression curves from laboratory tests. Analyzed for C_v by two methods. (a) Square root of time fitting method. (b) Log time fitting method. (From Taylor, 1948.)

* Arbitrary steps are required for the fitting methods

- a correction for the initial point.

  ⇒ apparatus errors or the presence of a small amount of air.

- an arbitrary determination of d_{90} or d_{100} ⇒ compression continues to occur after u_e is dissipated, due to secondary compression.

* $c_{v(\sqrt{t})} \approx (2 \pm 0.5)\ c_{v(\log\ t)}$

* c_v is dependent upon the level of applied stress and stress history. (Fig 24.5)

* It is very difficult to predict accurately the rate of settlement or heave.

(Rate of settlement)$_{\text{actually observed}}$ is (2~4) times faster than (Rate of settlement)$_{\text{predicted}}$

i) difficulties in measuring c_v.

ii) limitation of Terzaghi 1-D consolidation.

shortcomings in the linear theory of consolidation.

(assuming k or m_v are constant,…)

the two- and three- dimensional effects.

iii) size effect or heterogeneity of natural soil deposits ($c_{v(field)} > c_{v(lab)}$).

⇒ Predictions are useful for approximate estimation.

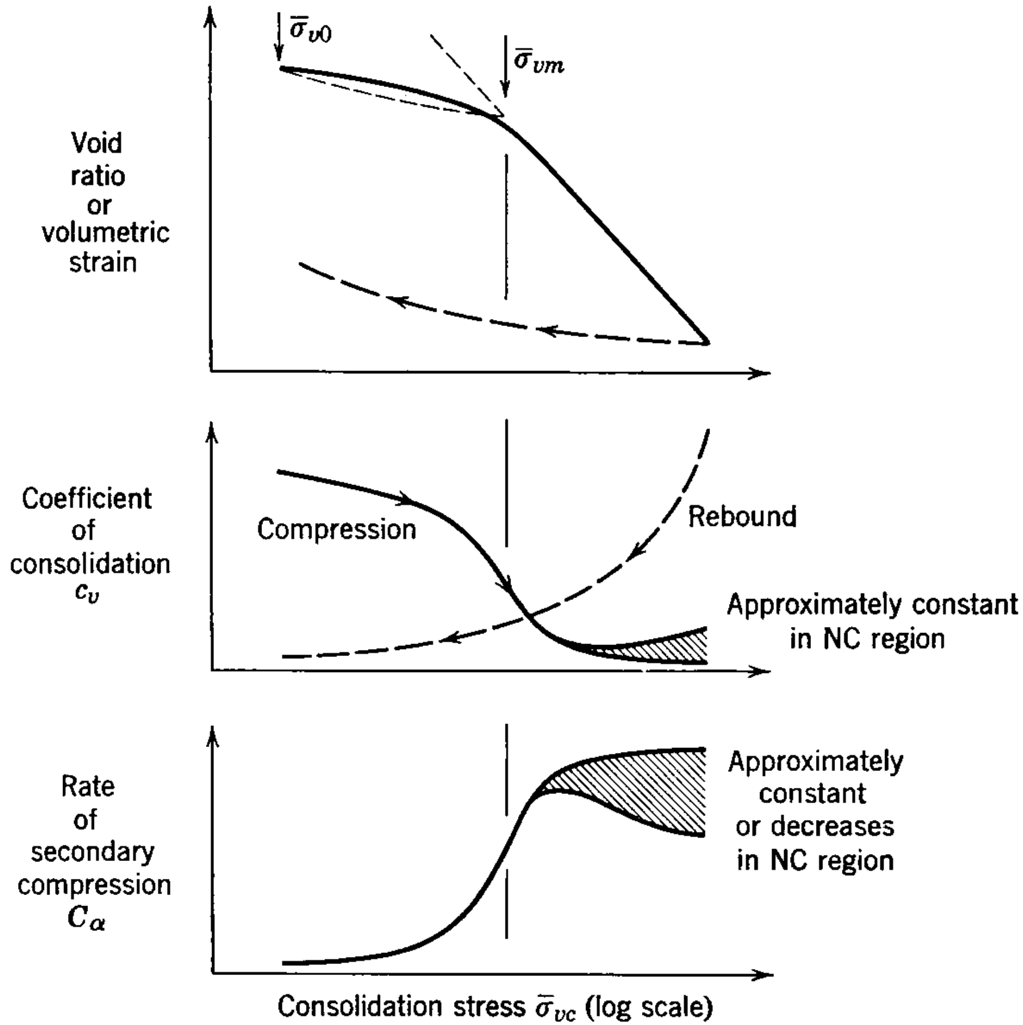
⇒ In critical case, field monitoring for rate of consolidation is indispensable.

Table 27.1 Typical values of c_v

Liquid Limit	Lower Limit (Recompression)	Undisturbed Virgin Compression	Upper Limit (Remolded)
30	3.5×10^{-2}	5×10^{-3}	1.2×10^{-3}
60	3.5×10^{-3}	1×10^{-3}	3×10^{-4}
100	4×10^{-4}	2×10^{-4}	1×10^{-4}

Table 27.2 Typical values of c_α

	c_α
Normally consolidated clays	0.005 to 0.02
Very plastic soils ; organic soils	0.03 or higher
Precompressed clays with OCR > 2	Less than 0.001



<Fig 27.5> Typical variation in coefficient of consolidation and rate of secondary compression with consolidation stress.

(4) Other one-dimensional solutions

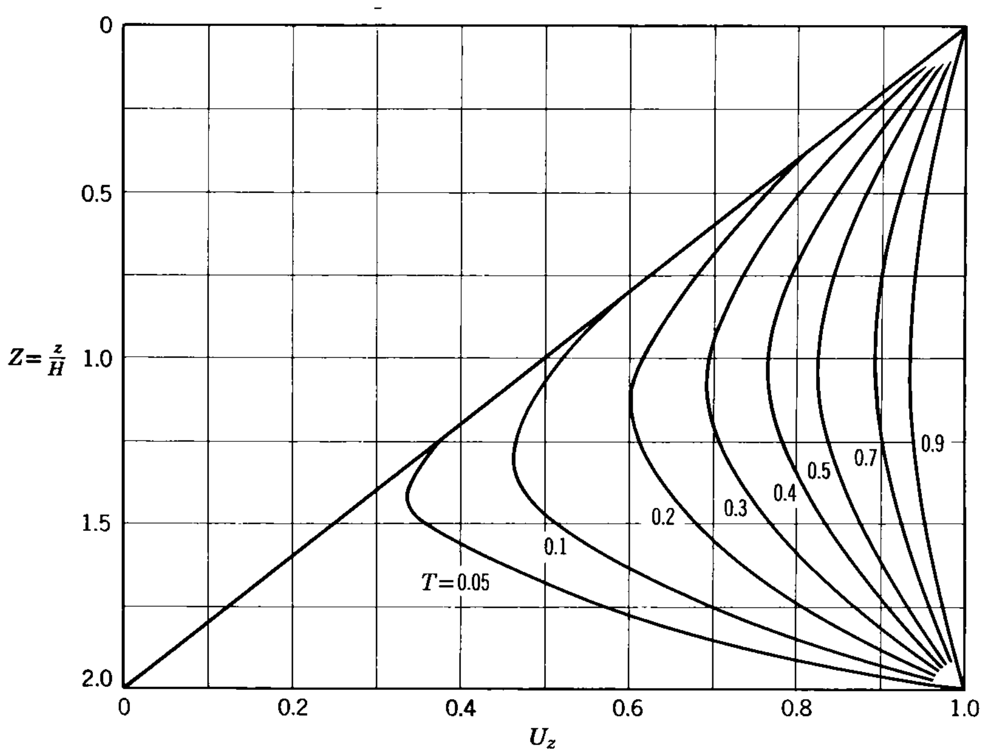
i) Triangular Initial Excess Pore Pressure

⇒ Arises when only bc's change at only one boundary of a stratum.

<One example ; (Fig. 26.4)>

⇒ U_Z in terms of Z and T (Fig. 27.6)

⇒ Double drainage condition : even though one way drainage took place (H is one half of the total thickness of stratum.).



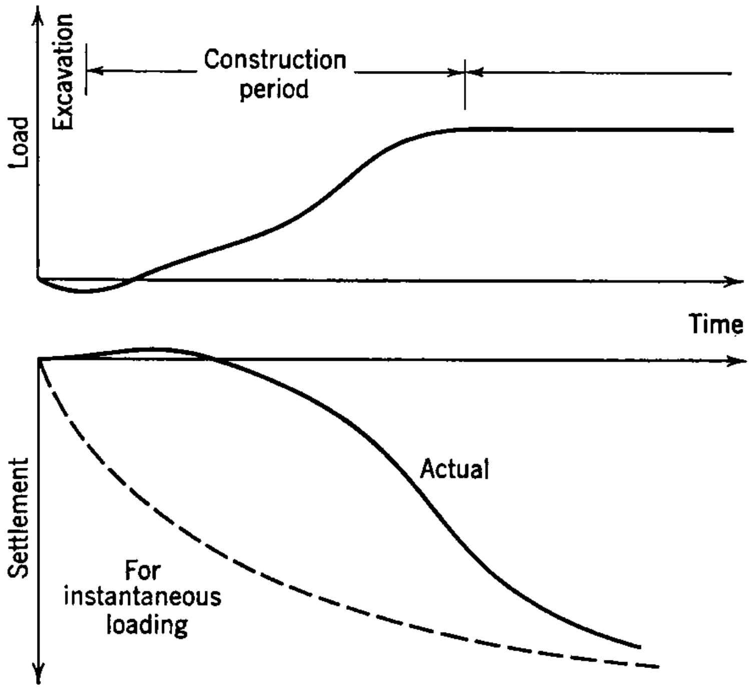
<Fig. 27.6> U_Z versus Z for triangular initial excess pore pressure distribution.
(Upside down of Fig 26.4)

⇒ U_{ave} vs. T ; same as for uniform initial excess pore pressure.

⇒ Fig. 27.3

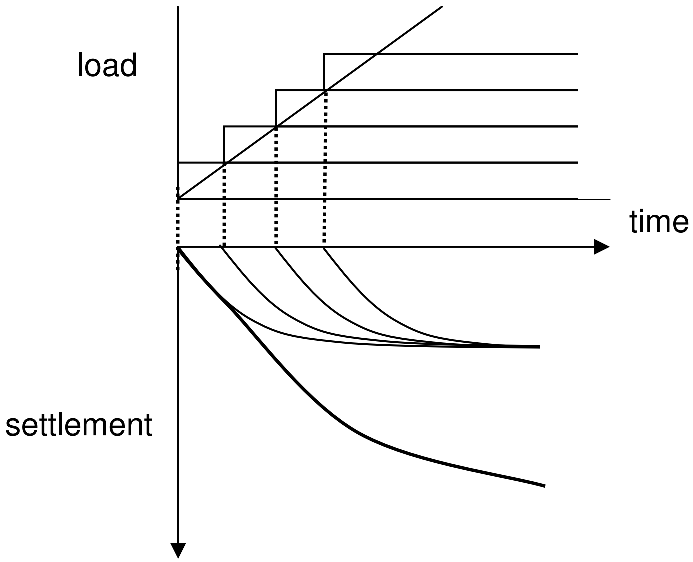
ii) Time varying load

$$\partial \sigma_v / \partial t \neq 0$$



<Fig. 27.7> Settlement from time-varying load.

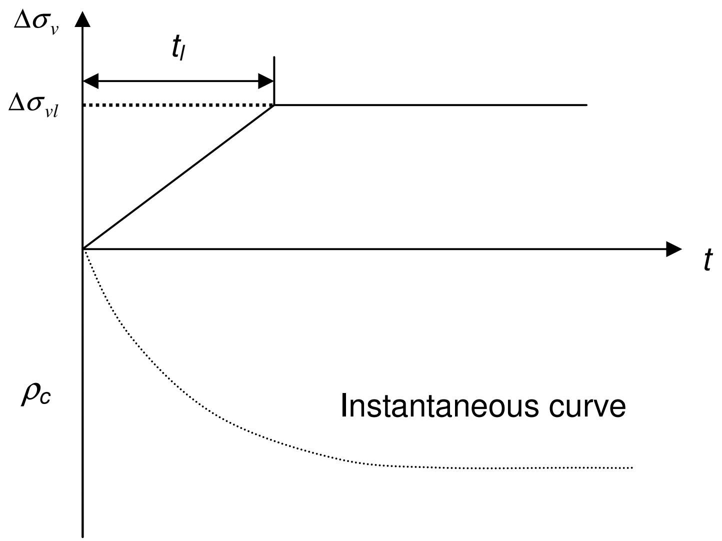
① Superposition



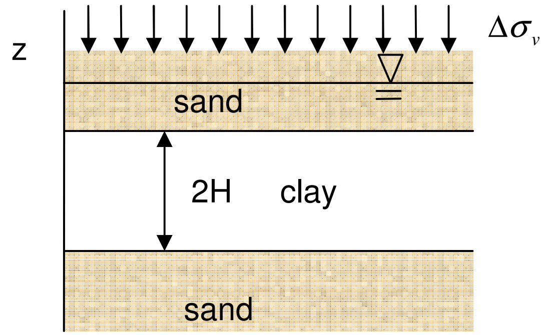
② Approximate method (linear increase in load followed by a constant load) ⇒ Taylor(1948)

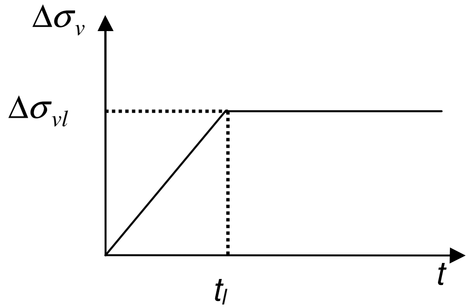
$\rho(t < t_l) =$ (Load fraction) $\times \rho$ at 1/2t from instantaneous curve

$\rho(t > t_l) = \rho$ at $t - 0.5t_l$ from instantaneous curve



③ Olson(1977) presented a mathematical solution for time-varying load.





For $T \le T_l (= \frac{c_v t_l}{H^2})$

Excess pore pressure, $u = \sum_{m=0}^{\infty} \frac{2\Delta\sigma_{vl}}{M^3 T_l} \sin\frac{MZ}{H}\left[1-\exp(-M^2T)\right]$

Average consolidation ratio,

$$U_{ave} = \frac{T}{T_l}\left\{1-\frac{2}{T}\sum_{m=0}^{\infty}\frac{1}{M^4}\left[1-\exp(-M^2T)\right]\right\}$$

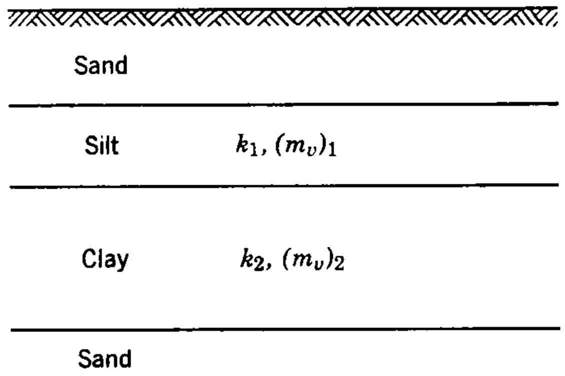
where $M = \frac{\pi}{2}(2m+1),\ m = 1, 2, 3, \cdots$

For $T \ge T_l$

$$u = \sum_{m=0}^{\infty} \frac{2\Delta\sigma_{vl}}{M^3 T_l}\left[\exp(-M^2T)-1\right]\sin\frac{MZ}{H}\exp(-M^2T)$$

$$U_{ave} = \left\{1-\frac{2}{T}\sum_{m=0}^{\infty}\frac{1}{M^4}\left[\exp(-M^2T)-1\right]\right\}\exp(-M^2T)$$

iii) More than One Consolidation Layer



<Fig. 27.8> Consolidation problem with two compressible strata.

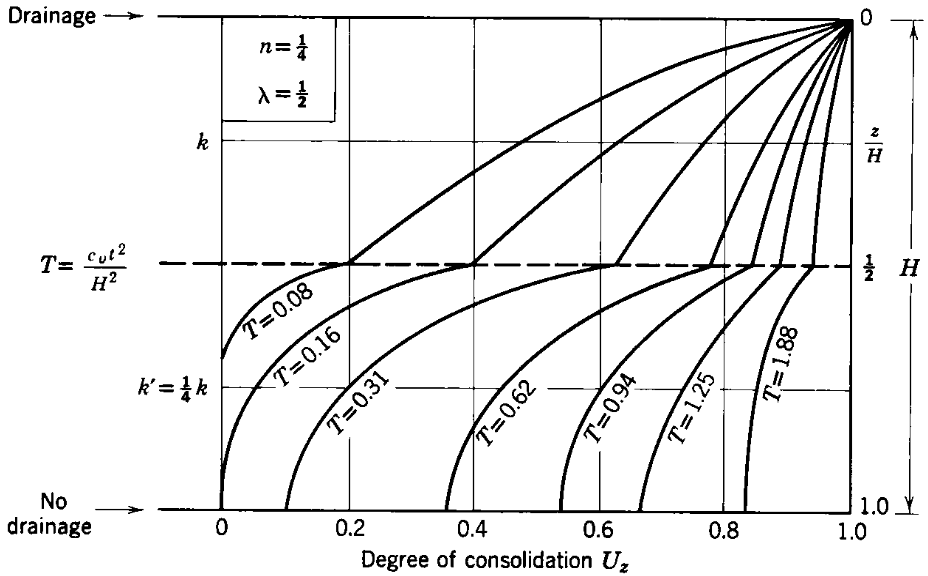
⇒ Consolidation behavior of multilayered system depends on the relative values of k and m_v for the two strata.

⇒ In Fig 27.9, for the conditions of

c_v (for upper layer) =4× c_v (for lower layer) with single drainage.

The upper layer consolidates much more quickly.

Pore pressure in the upper layer must persist until the end of consolidation.



<Fig. 27.9> Consolidation of two layers. C_v and k for bottom layer are 1/4 of values for upper layer. T is based upon C_v of upper layer(from Luscher,1965).

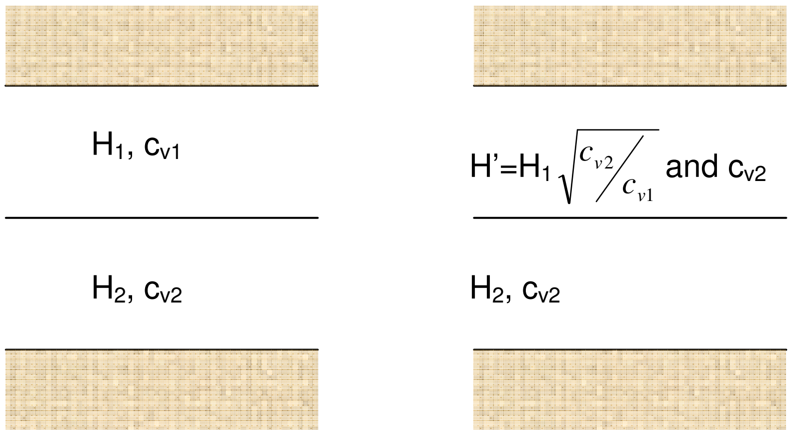
⇒ No general chart solutions are possible. ⇒ Numerical solution

⇒ Soils of $c_{v1} > 20c_{v2}$ with double drainage can be reasonably evaluated in two separate stages ;

a) first the more permeable stratum consolidation with single drainage.

b) then the less permeable stratum consolidating with double drainage.

⇒ Approximate solution for the layered system



(5) Two- and Three-Dimensional Consolidation

- True three-dimension → couples the equilibrium of total stresses and the continuity of the soil mass (flow of water)

Consolidation rate is high near the free drainage surface. → Stiffness is not constant along the bottom of the structure.

↓

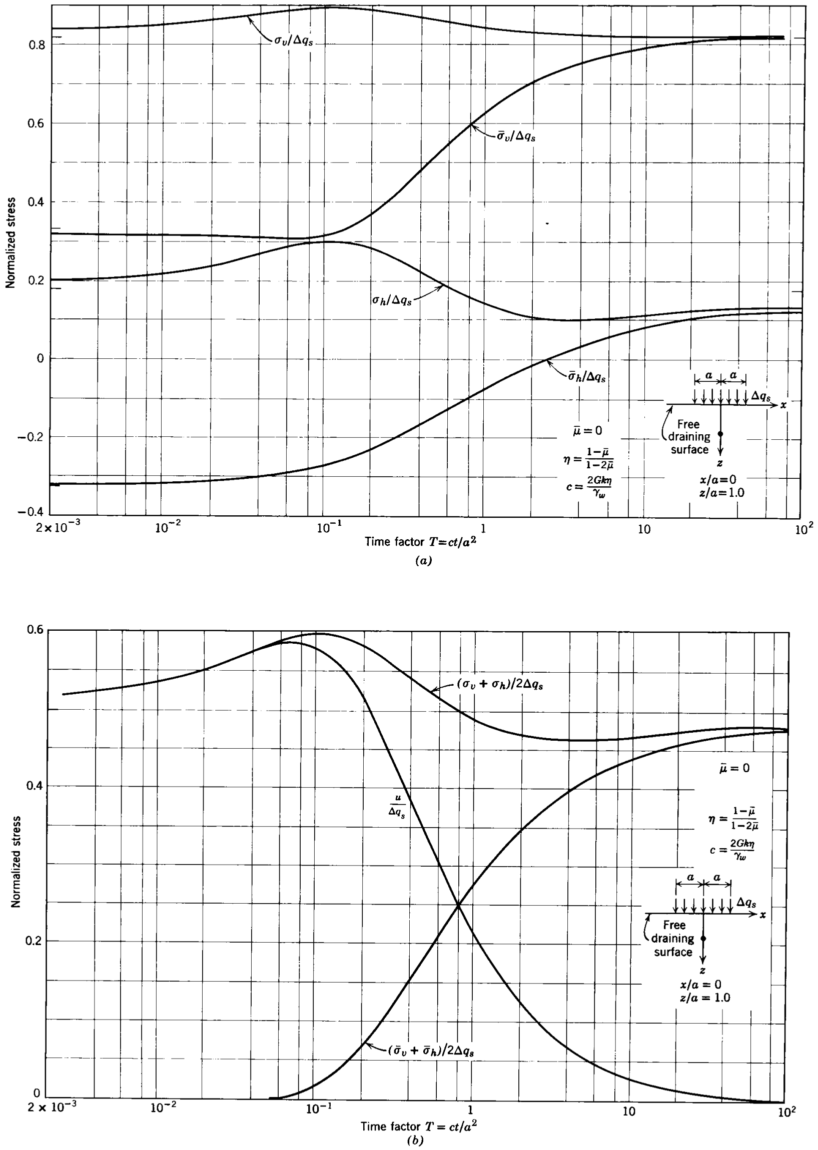
Stress is concentrated at less consolidated zone where stiffness is high. → Increase of u_e and total stress at zone far from the drainage

(but both u_e and total stress are the same at the beginning and at the end of consolidation.)

- Pseudo three-dimensional theory uncouples these two phenomena.

$\Rightarrow$ the assumption that the total stresses are constant, so that the rate of change of u_e is equal to the rate of volume change.

$\rightarrow$ strictly true in one-dim. case.



<Fig.27.10> Two-dimensional consolidation beneath a strip load. (From Schiffman et al., 1967).

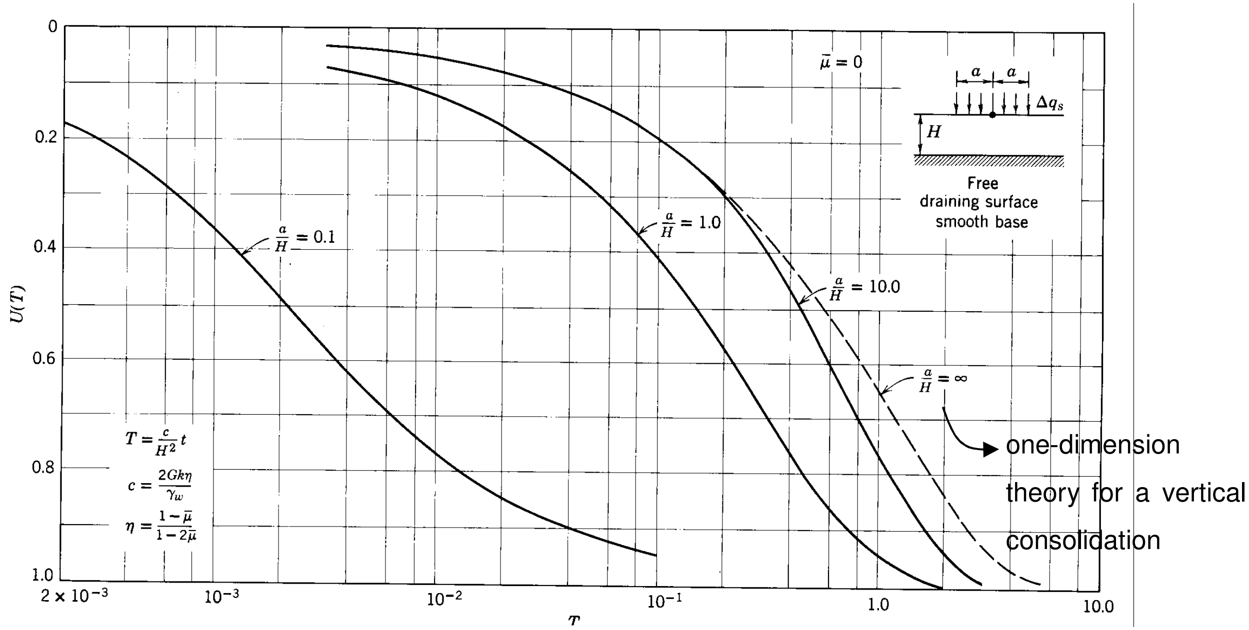
- From Kim and Chung(1999)

Difference of total stress between at the beginning and at the end of consol, due to the change of poisson's ratio ν during consolidation.

	$\Delta\sigma_v$	$\Delta\sigma_h$
elastic theory	No difference	beginning > end
plastic theory	beginning $\geq$ end	beginning > end

`

- Circular load on a stratum



< Fig 27.11> Consolidation of stratum under circular load
(From Gibson, Schiffman, and Pu, 1967).

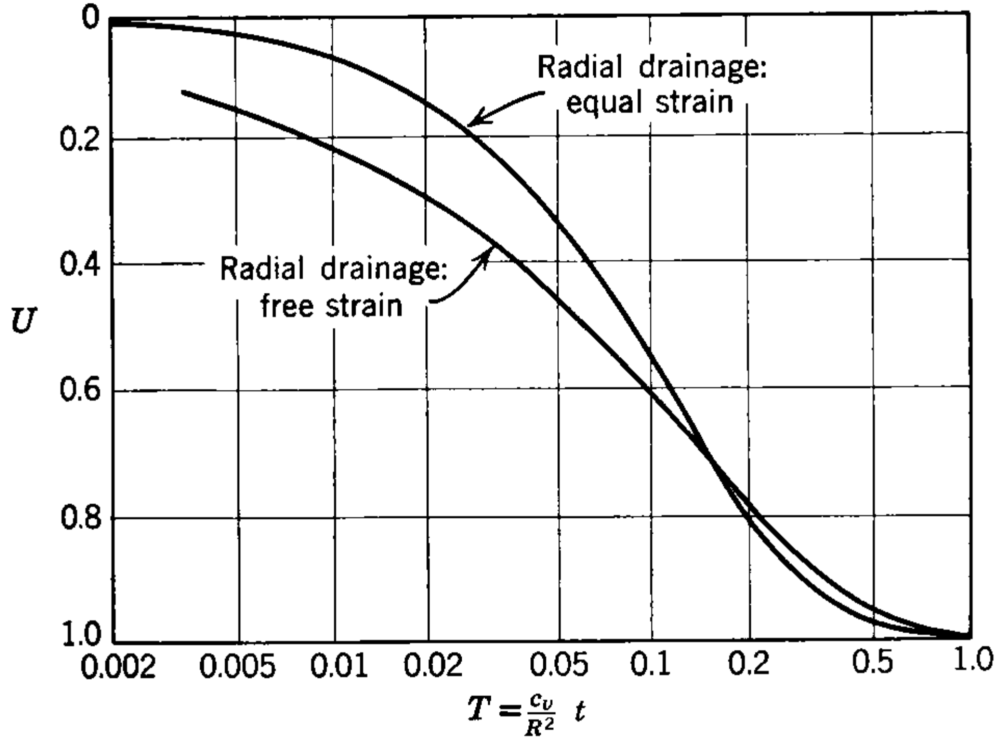
→ The effect of <u>radial drainage</u> (Significantly speeds up the consolidation process)

→ Use of the ordinary one dimensional theory can be quite conservative.

(6) Pseudo Two-and Three-Dimensional Consolidation

- Assumption : The total stresses remain constant.
- Still useful in practice

i) Radial Consolidation in Triaxial Test



<Fig 27.12> Average consolidation ratio for radial drainage in triaxial test (After Scott, 1963).

- Boundary conditions on axial strains

(a) equal strain $\quad >$ little difference in the results.

(b) free strain

- Radial drainage + axial drainage

$$U_{vh} = 1 - (1 - U_v)(1 - U_h)$$

↑ ↖

Fig 27.3 Fig 27.12

ii) Drain wells

- Used to speed consolidation of a clay stratum.

  <reducing drainage path and inducing radial flow>.

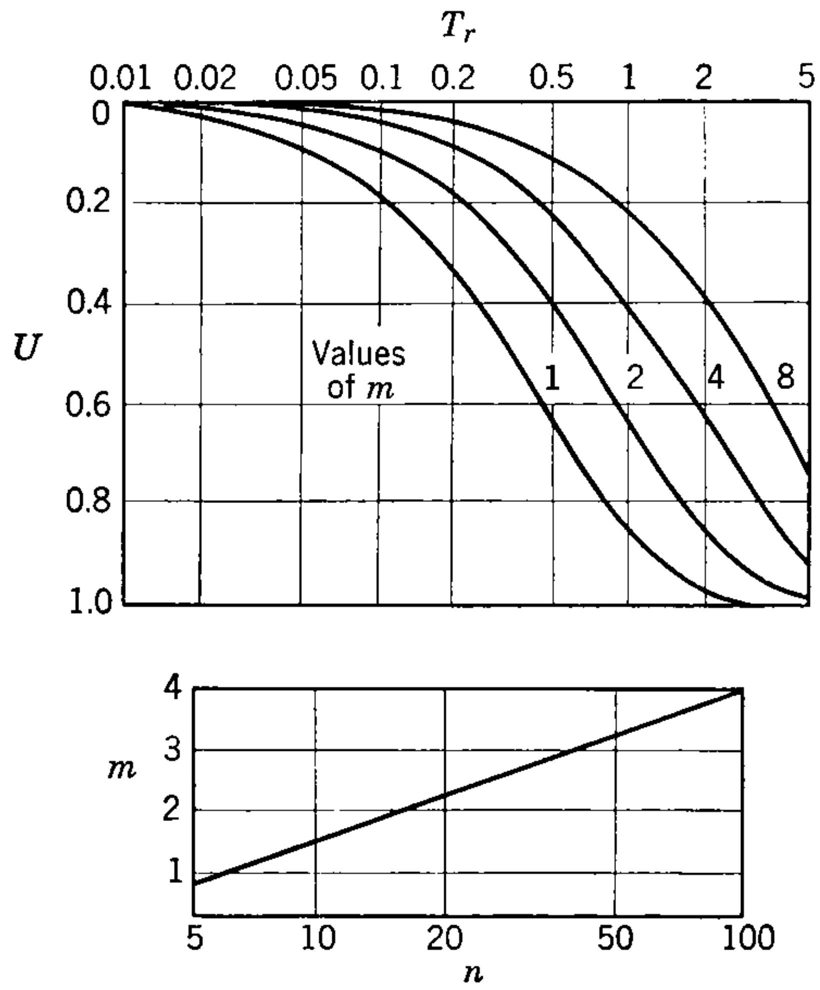
- Based on one-dimensional (vertical) strain along with three- dimensional water flow.

- For radial drainage

  $U - T_r (= \frac{c_v t}{r_e^{\,2}})$ relations for ideal cases(no smearing effect and no well resistance).

  where r_e = one-half of the well spacing,

  $r_w (= {r_e}/{n})$= the radius of the drain well.



<Fig. 27.13> Average consolidation ratio for radial drainage by vertical drains (After Scott, 1963)

(7) Secondary Compression

- Two Phases of the compression

i) primary consolidation $\rightarrow$ related to dissipation of excess pore pressure.

ii) secondary compression $\rightarrow$ important for highly plastic soils and especially for organic soils.

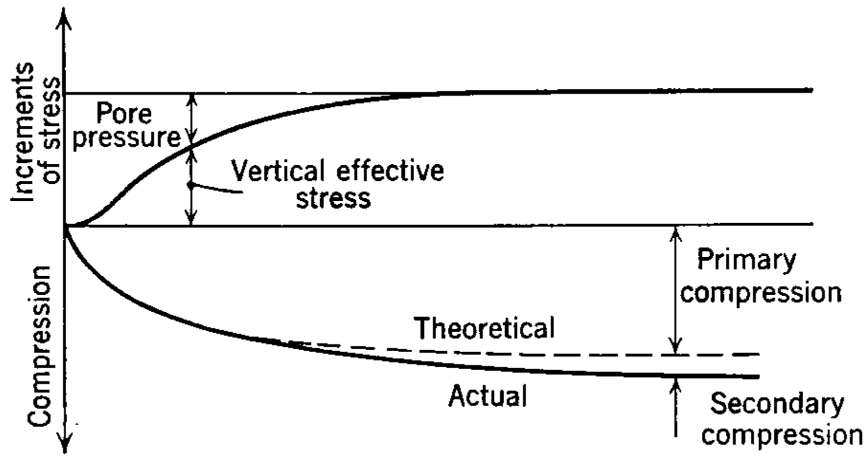


Fig. 27.14 Primary and secondary compression.

- Typical curves of stress versus void ratio for a normally consolidated clay.

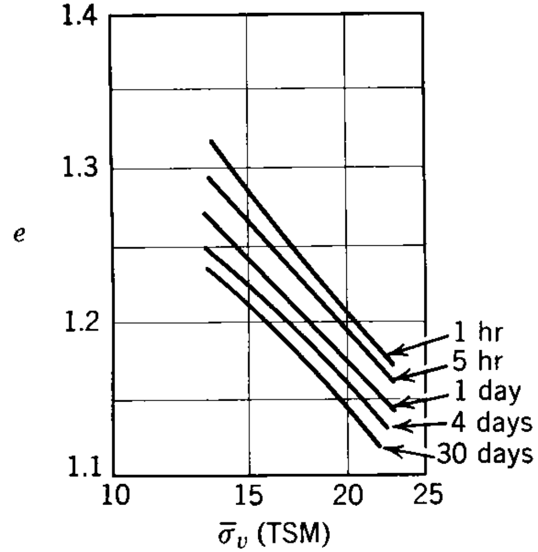


Fig. 27.15 e versus log $\bar{\sigma}_v$ as function of duration of secondary compression (After Bjerrum, 1967).

● c_{α} is expressed for the magnitude of secondary compression

$$c_{\alpha} = (1 + e_0) c_{\alpha\varepsilon}$$

- c_{α} for recompression < c_{α} for virgin compression
- The ratio of secondary to primary compression increases with decreasing the ratio of stress increment to initial stress