- Ship Stability -

Ch.7 Longitudinal Righting Moment

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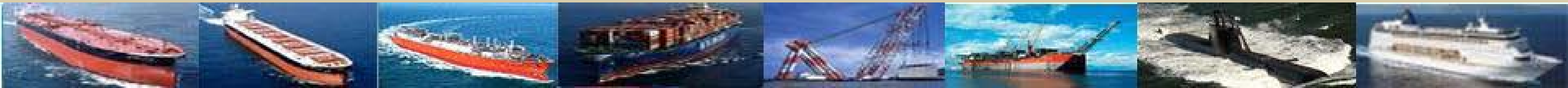


Sec.1 Longitudinal Righting Moment

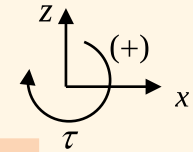
Sec.2 Calculation of BM_L,GZ_L in Wall Sided Ship

Sec.3 Transverse Righting Moment due to Movement of Cargo

Sec.4 Moment to Change 1cm Trim(MTC), Trim

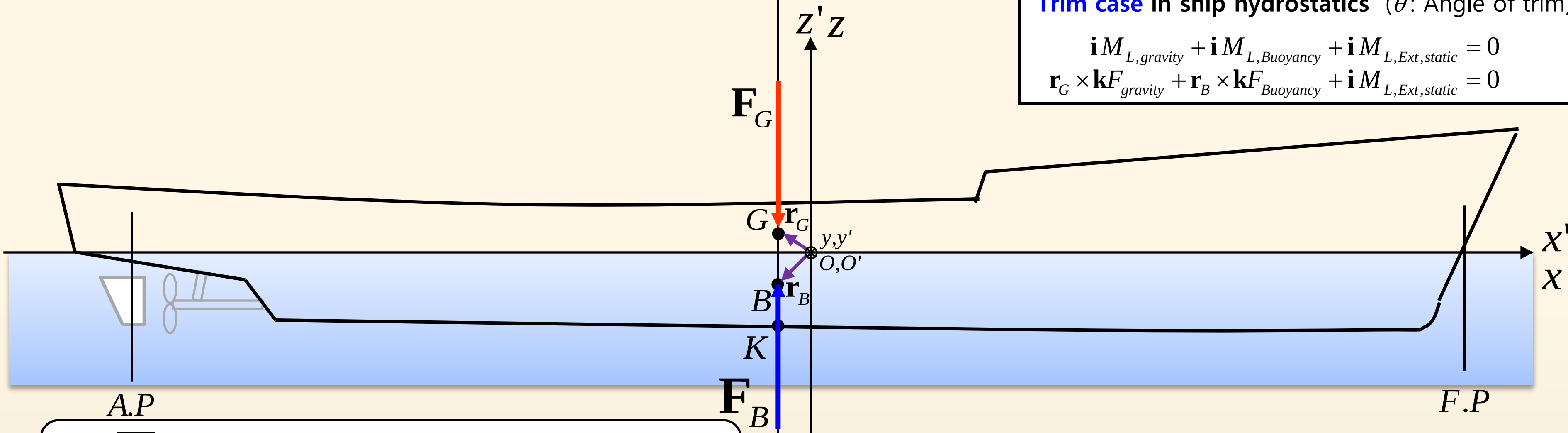
Longitudinal Righting Moment (1)

G: Center of mass
B: Center of buoyancy
$\mathbf{F}_G$: Weight of ship (=$\mathbf{W}$)
$\mathbf{F}_B$: Buoyancy (=$\rho g \nabla$)



Trim case in ship hydrostatics (θ: Angle of trim)

$$\mathbf{i}M_{L,gravity} + \mathbf{i}M_{L,Buoyancy} + \mathbf{i}M_{L,Ext,static} = 0$$
$$\mathbf{r}_G \times \mathbf{k}F_{gravity} + \mathbf{r}_B \times \mathbf{k}F_{Buoyancy} + \mathbf{i}M_{L,Ext,static} = 0$$



① $\sum \mathbf{F} = \mathbf{F}_G + \mathbf{F}_B = 0$, (static equilibrium of force)

⇨ $F_G\mathbf{k} + F_{B,z}\mathbf{k} = 0$ ⇨ $F_G = -F_{B,z}$

② Center of mass of ship(G) and center of buoyancy (B) are in the same vertical line which is perpendicular to waterplane which is perpendicular to waterline
→ Longitudinal moment about origin O about z axis are as follows

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_B \times \mathbf{F}_B$$

$$\boldsymbol{\tau}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & 0 & z_G \\ 0 & 0 & F_{G,z} \end{vmatrix} = \mathbf{i}x_G \cdot F_{G,z} \quad , \boldsymbol{\tau}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_B & 0 & z_B \\ 0 & 0 & F_{B,z} \end{vmatrix} = x_B \cdot F_{B,z}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B$$
$$= \mathbf{i}\left(x_G \cdot F_{G,z} + x_B \cdot F_{B,z}\right)$$

x_G, x_B are in same vertical line $\left(x_G = x_B\right)$

$$= \mathbf{i}\left(-x_B \cdot F_{G,z} + x_B \cdot F_{B,z}\right)$$

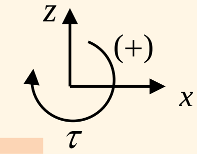
$\left(F_G = -F_{B,z}\right)$

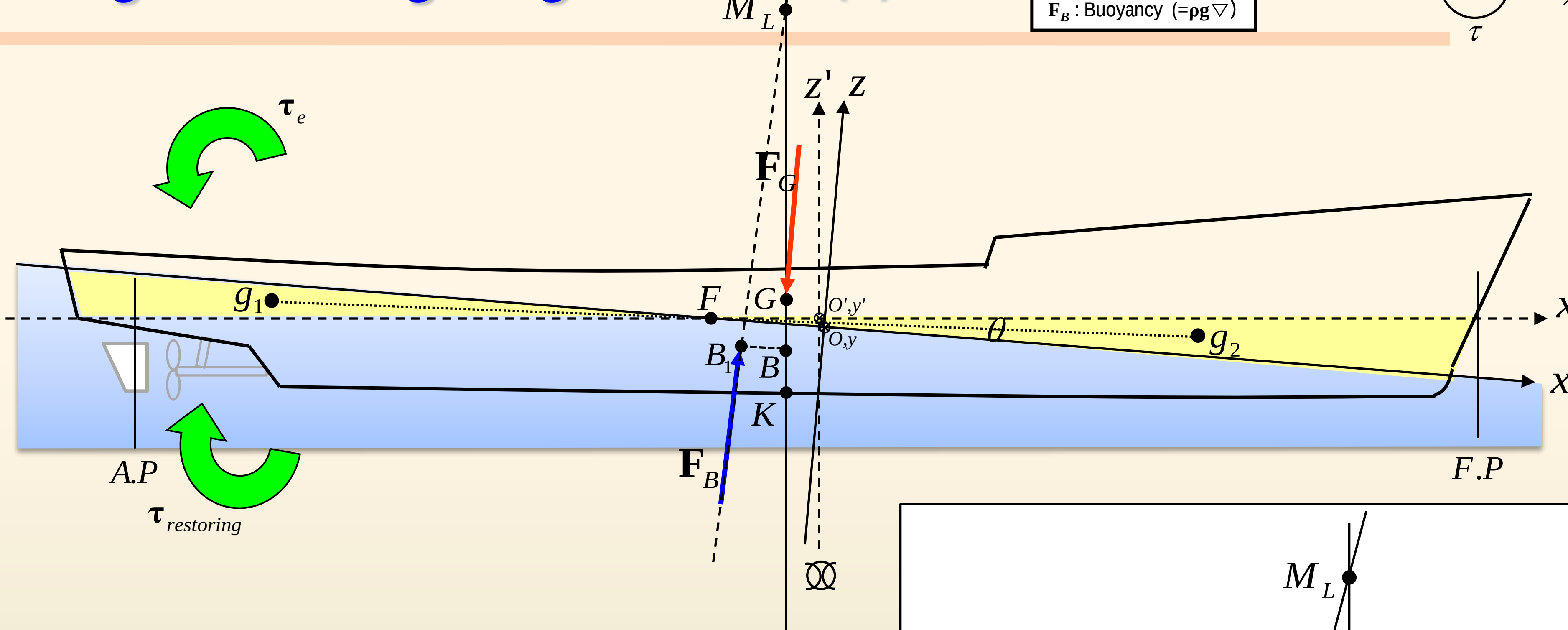
$$= \mathbf{i}\left(-x_B \cdot F_{B,z} + x_B \cdot F_{B,z}\right)$$
$$= 0$$

Ox′y′z′: Body fixed frame
Oxyz : Waterplane fixed frame

Longitudinal Righting Moment (2)

G: Center of mass
B: Center of buoyancy
$\mathbf{F}_G$: Weight of ship (=$\mathbf{W}$)
$\mathbf{F}_B$: Buoyancy (=$\rho g \nabla$)





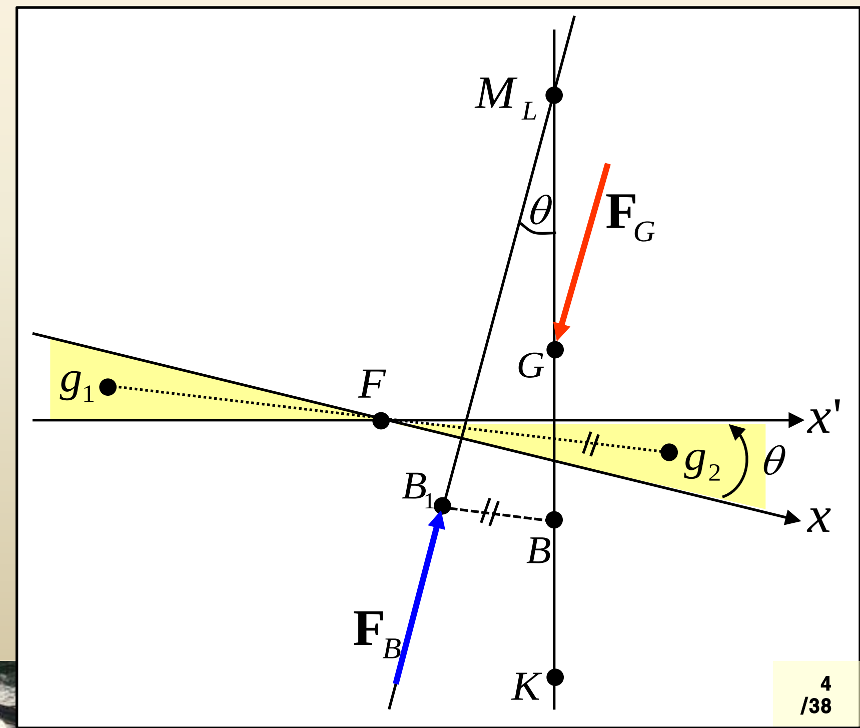
③ External moment (τ_e) is applied on the ship in counter-clockwise. (Negative moment is applied)

④ A ship is trimmed about origin F through an angle of θ.

⑤ Assuming that a ship is trimmed by an differential angle, a ship will be trimmed about a specific point F that immerged volume become equivalent to emerged volume.

F(LCF) : Longitudinal Center of Floatation

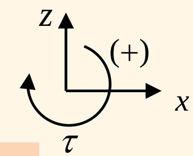
Ox'y'z': Body fixed frame
Oxyz : Waterplane fixed frame

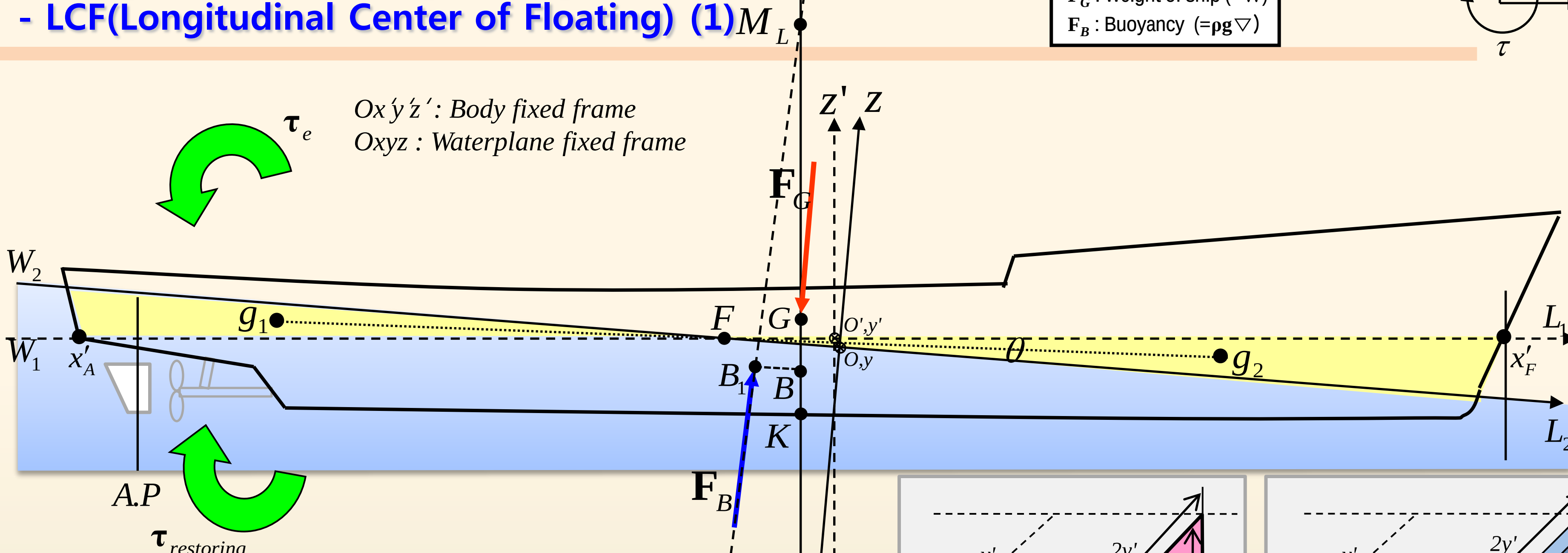


Longitudinal Righting Moment(3)

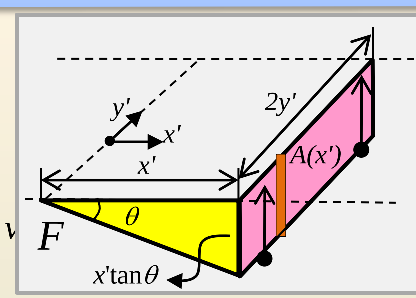
- LCF(Longitudinal Center of Floating) (1)

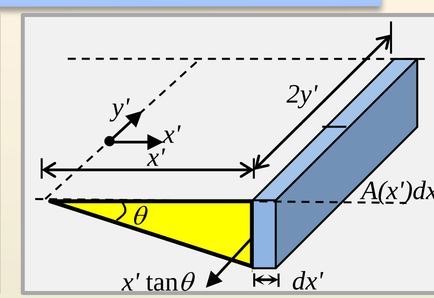
G: Center of mass
B: Center of buoyancy
$\mathbf{F}_G$: Weight of ship (=$\mathbf{W}$)
$\mathbf{F}_B$: Buoyancy (=$\rho g \nabla$)





Intersection point F between W_1L_1 (existing waterline) and W_2L_2 (new waterline)





Assume that immersed volume is equals to emerged volume as w

Center of immerged volume : g_1, Center of emerged volume : g

Calculation of immerged volume

$$A_a(x') = \int dA = \int_{-y'}^{y'}\int_0^{z'} dz'\,dy' = \int_{-y'}^{y'}\int_0^{-x'\tan\theta} dz'\,dy'$$

$$= -\int_{-y'}^{y'} (x'\tan\theta)\,dy' = -2x'y'\tan\theta$$

$$v_a = -2\tan\theta \int_{x'_A}^{F} (x'y')\cdot dx'$$

Calculation of emerged volume

$$A_f(x') = \int dA = \int_{-y'}^{y'}\int_{z'}^{0} dz'\,dy' = \int_{-y'}^{y'}\int_{-x'\tan\theta}^{0} dz'\,dy'$$

$$= \int_{-y'}^{y'} (x'\tan\theta)\,dy' = 2x'y'\tan\theta$$

$$v_f = 2\tan\theta \int_{F}^{x'_F} (x'y')\cdot dx'$$

※ Calculation method for changed volume

$$v = \iiint dz'dy'dx'$$

$$= \int_{x'_A}^{x'_F}\int_0^{y'}\int_0^{z'} dz'dy'\,dx'$$

$$= \int_{x'_A}^{x'_F}\int_0^{y'} z'dy' \quad dx'$$

$$= \int_{x'_A}^{x'_F} A(x') \quad dx'$$

A_a : Sectional area of aft part

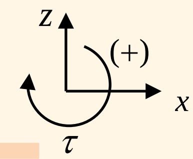
A_f: Sectional area of fore part

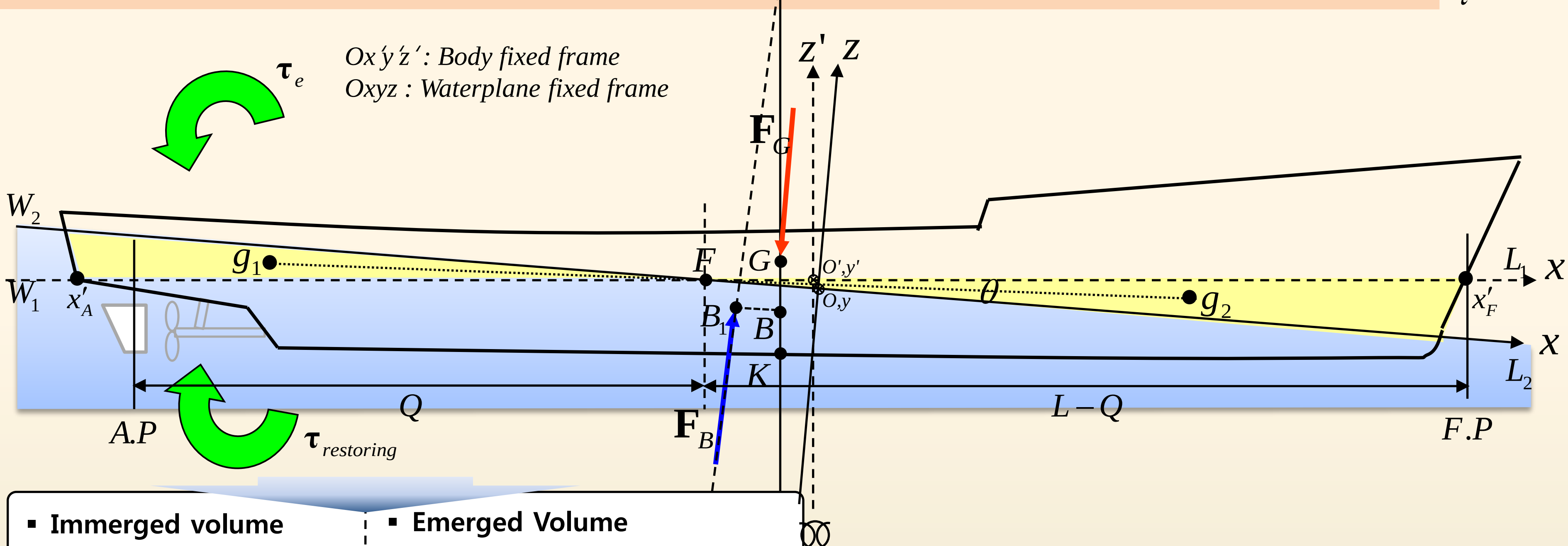
x_A' , x_F' : coordinate of x

Longitudinal Righting Moment(4)

- LCF(Longitudinal Center of Floating) (2)

G: Center of mass
B: Center of buoyancy
$\mathbf{F}_G$: Weight of ship (=$\mathbf{W}$)
$\mathbf{F}_B$: Buoyancy (=$\rho g \nabla$)





- **Immerged volume**

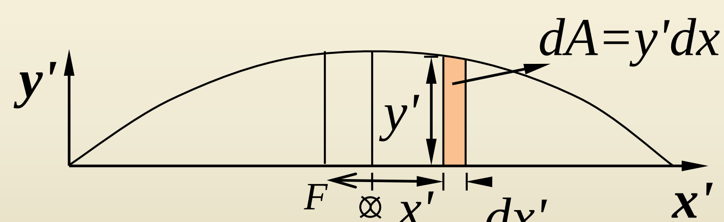
$$v_a = -2\tan\theta \int_{x'_A}^{F} (x'\,y') \cdot dx'$$

- **Emerged Volume**

$$v_f = 2\tan\theta \int_{F}^{x'_F} (x'\,y') \cdot dx'$$

Because we assumed immersed volume is equals to emerged volume

$$v_f - v_a = 0 \;\Rightarrow\; \underline{2\tan\theta \int_{x'_A}^{F} (x'\,y') \cdot dx'} + \underline{2\tan\theta \int_{F}^{x'_F} (x'\,y') \cdot dx'} = 0 \;\; \dots(a)$$



x_A', x_F': coordinate of x

$\underline{2\int_{x'_A}^{F} x'y'dx'}$ and $\underline{2\int_{F}^{x'_F} x'y'dx'}$ represent 1st moment of aft and forward waterplane area about origin F about y' axis

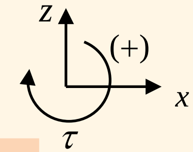
$$\sum M_F = \int_{x'_A}^{F} x'dA + \int_{F}^{x'_F} x'dA = \int_{x'_A}^{F}\int_{-y'}^{y'} x'\,dy'\,dx' + \int_{F}^{x'_F}\int_{-y'}^{y'} x'\,dy'\,dx' = \underline{2\int_{x'_A}^{F} x'y'dx'} + \underline{2\int_{F}^{x'_F} x'y'dx'} = 0$$

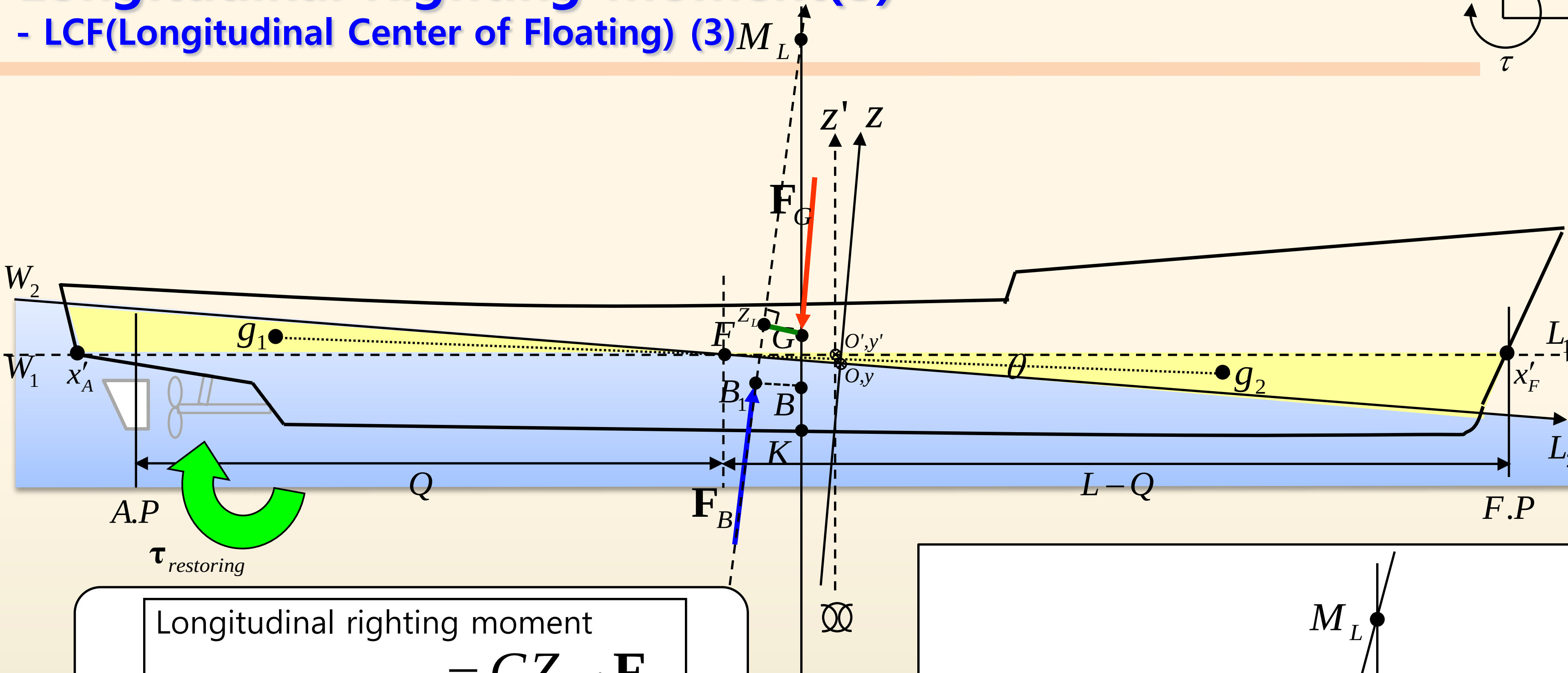
If we assume that F is center of waterplane area, Sum of those two terms is '0'. Because moment about centroid is '0'

Equation (a) and (b) are identically '0', if a ship is trimmed about cenroid of waterplane area. That means, for the ship is trimmed without change of displacement, ship have to be trimmed **about center of waterplane area**.

Longitudinal Righting Moment(5)

- LCF(Longitudinal Center of Floating) (3)



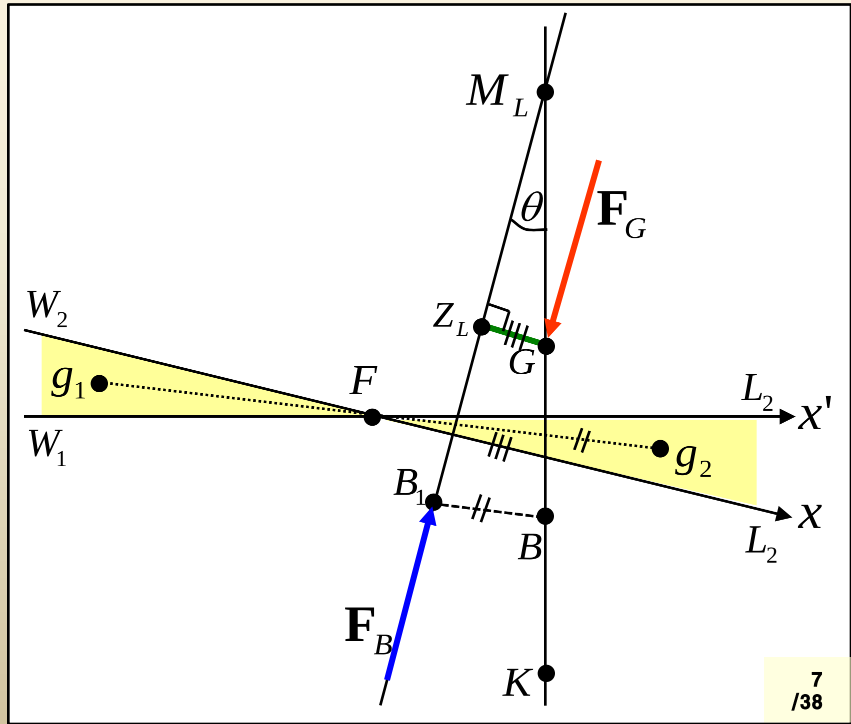


Longitudinal righting moment

$$= \underline{GZ_L} \cdot \mathbf{F}_B$$

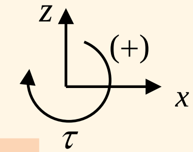
From geometrical figure with assumption that M_L does not change within small angle of trim (about 2°~5°)

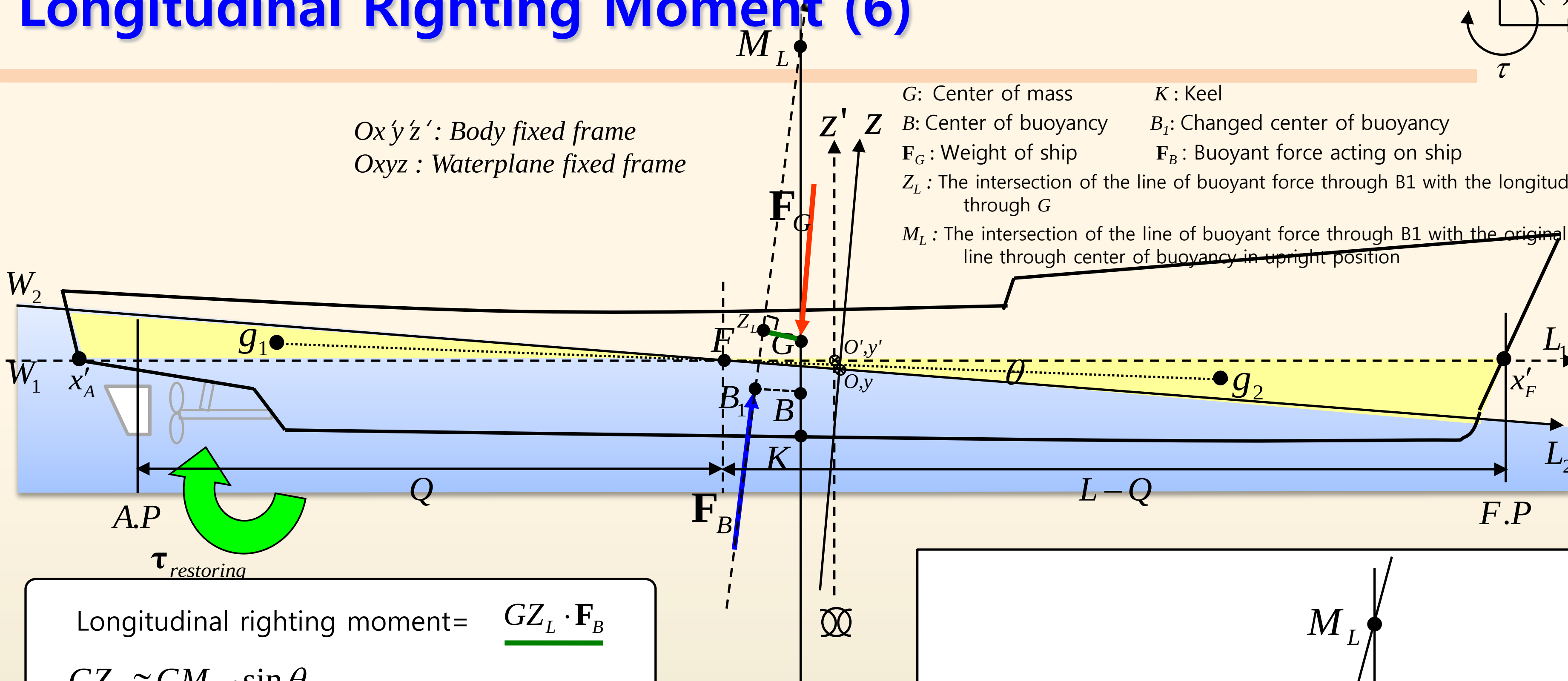
$$GZ_L \cong GM_L \cdot \sin\theta$$



G: Center of mass
K: Keel
B: Center of buoyancy
B_1: Changed center of buoyancy
$\mathbf{F}_G$: Weight of ship
$\mathbf{F}_B$: Buoyant force acting on ship
Z_L: The intersection of the line of buoyant force through B1 with the longitudinal line through G
M_L: The intersection of the line of buoyant force through B1 with the original vertical line through center of buoyancy in upright position

Longitudinal Righting Moment (6)





G: Center of mass — K : Keel

B: Center of buoyancy — B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship — $\mathbf{F}_B$: Buoyant force acting on ship

Z_L : The intersection of the line of buoyant force through B1 with the longitudinal line through G

M_L : The intersection of the line of buoyant force through B1 with the original vertical line through center of buoyancy in upright position

Longitudinal righting moment= $GZ_L \cdot \mathbf{F}_B$

$GZ_L \cong GM_L \cdot \sin\theta$

$$\boxed{GM_L = KB + BM_L - KG}$$

GM_L : **Longitudinal metacentric height**

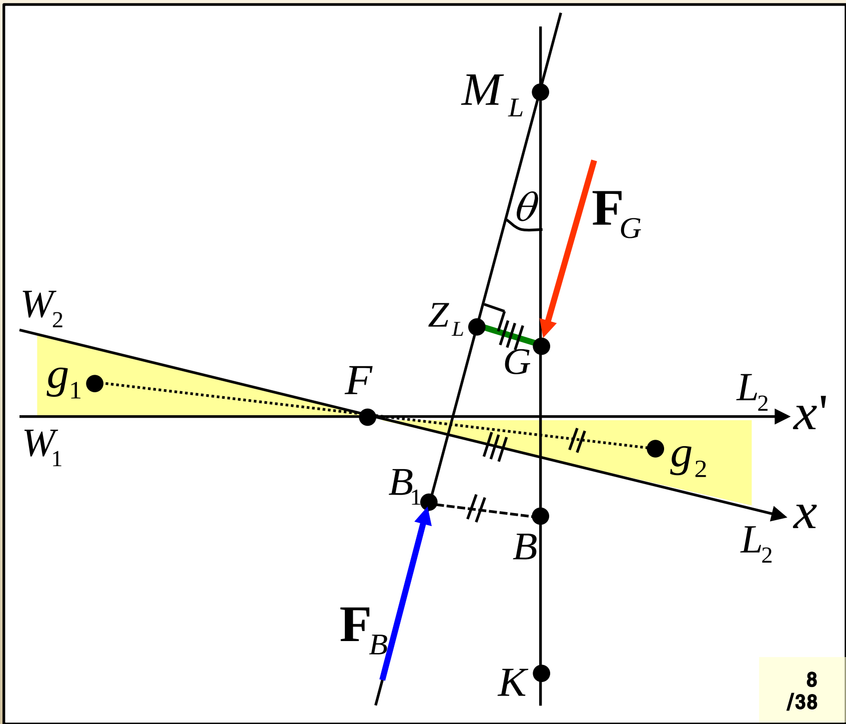
KB : **Vertical location of center of buoyancy**

BM_L : **Longitudinal Metacentric Radius**

KG : **Vertical location of Gravity**

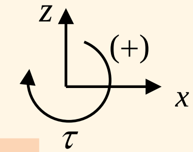
KB, BM is determined by geometric shape.

KG is determined by cargo loading condition



Calculation of BM_L, GZ_L (1)

- BM_L(Longitudinal Metacentric Radius)

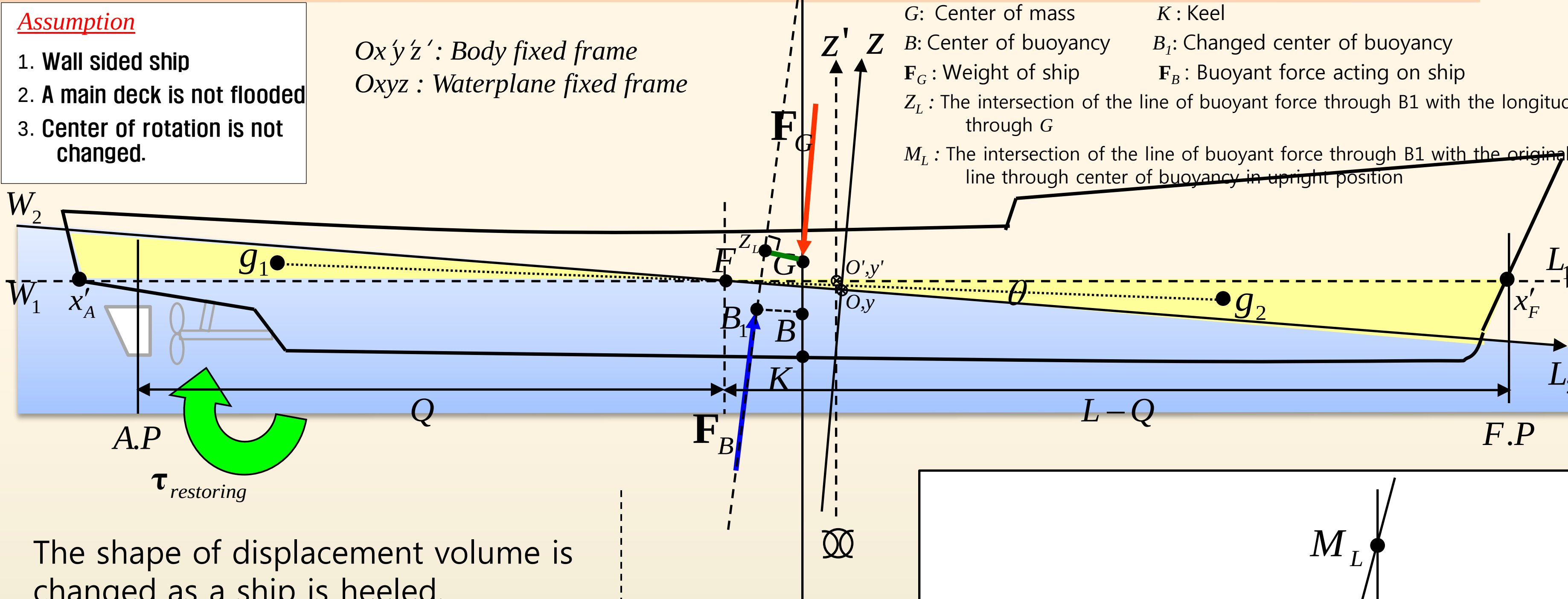


<u>Assumption</u>

1. Wall sided ship
2. A main deck is not flooded
3. Center of rotation is not changed.

Ox'y'z' : Body fixed frame
Oxyz : Waterplane fixed frame

G: Center of mass K : Keel
B: Center of buoyancy B_1: Changed center of buoyancy
$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship
Z_L : The intersection of the line of buoyant force through B1 with the longitudinal line through G
M_L : The intersection of the line of buoyant force through B1 with the original vertical line through center of buoyancy in upright position



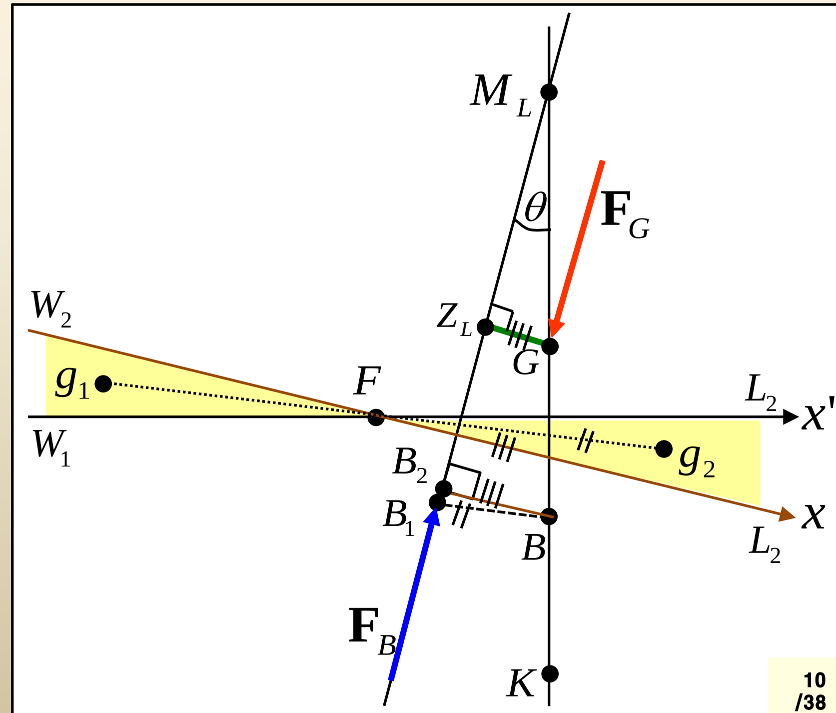
The shape of displacement volume is changed as a ship is heeled.

Relation between moving distance of center of changed displacement volume and moving distance of center of center of buoyancy is as follows.

$$BB_1 = \frac{\rho g\, v}{\rho g \nabla} gg_1$$

$$BB_1 = \frac{v}{\nabla} gg_1$$

∇ : Displacement volume
v : Changed displacement volume
BB_1: Moving distance of center of buoyancy
gg_1 : Moving distance center of changed displacement volume



Calculation of BM_L, GZ_L (2)

- BM_L(Longitudinal Metacentric Radius)

Assumption

1. Wall sided ship
2. A main deck is not flooded
3. Center of rotation is not changed.

$Ox'y'z'$: Body fixed frame

$Oxyz$: Waterplane fixed frame

G: Center of mass

K : Keel

B: Center of buoyancy

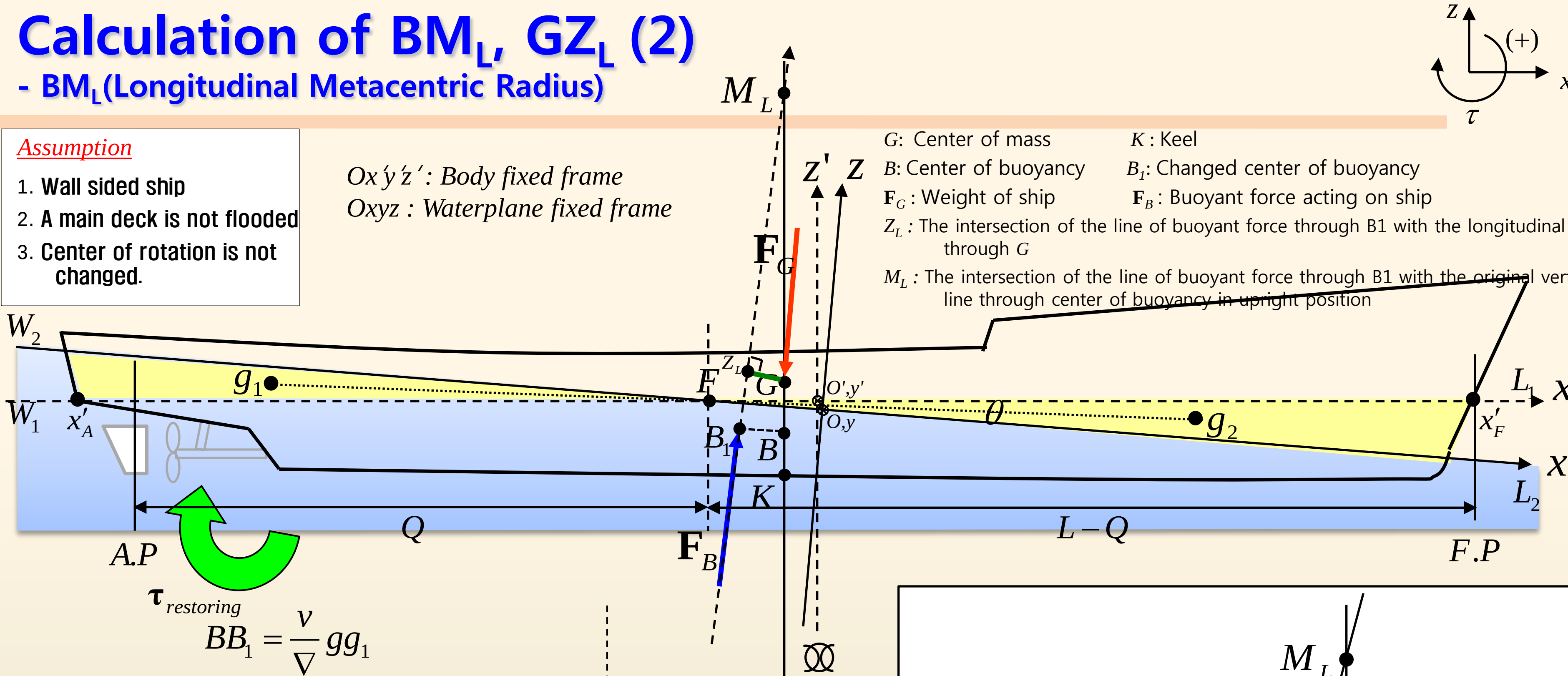
B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship

$\mathbf{F}_B$: Buoyant force acting on ship

Z_L : The intersection of the line of buoyant force through B1 with the longitudinal line through G

M_L : The intersection of the line of buoyant force through B1 with the original vertical line through center of buoyancy in upright position



$$BB_1 = \frac{v}{\nabla} gg_1$$

$$\angle B_1BB_2 = \angle gFL_2$$

$$\underbrace{BB_1 \cos(\angle B_1BB_2)}_{BB_2} = \frac{v}{\nabla} gg_1 \underbrace{\cos(gFL_2)}$$

$$BB_2 = \frac{v}{\nabla} \cdot g_{1'}g_{2'}$$

$$\boxed{BB_2 = \frac{1}{\nabla} v \cdot (-x_{v,a} + x_{v,f})}$$

L.H.S

$$\boxed{BB_2 = BM_L \cdot \sin\theta}$$

∇ : Displacement volume

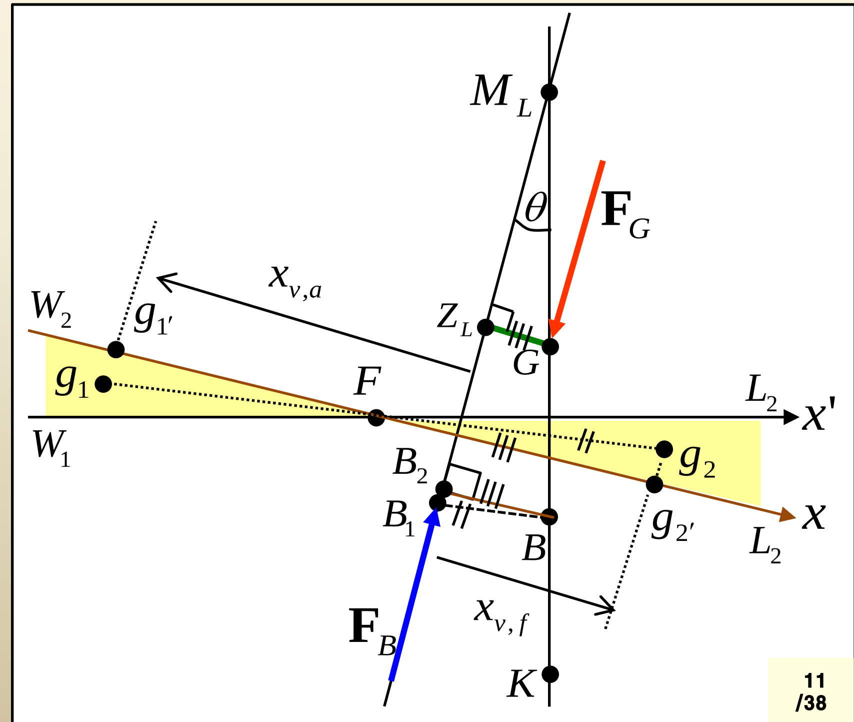
v : Changed displacement volume

BB_1: Moving distance of center of buoyancy

gg_1 : Moving distance center of changed displacement volume

$, x_{v,a}$: x coordinate center of changed displacement volume in aft

$, x_{v,f}$: x coordinate center of changed displacement volume in forward



Calculation of BM_L, GZ_L (3)
- BM_L(Longitudinal Metacentric Radius)

v : Changed displacement volume

∇: Displacement volume

(R.H.S)

$$\frac{1}{\nabla}\, v\cdot(-x_{v,a}+x_{v,f})$$

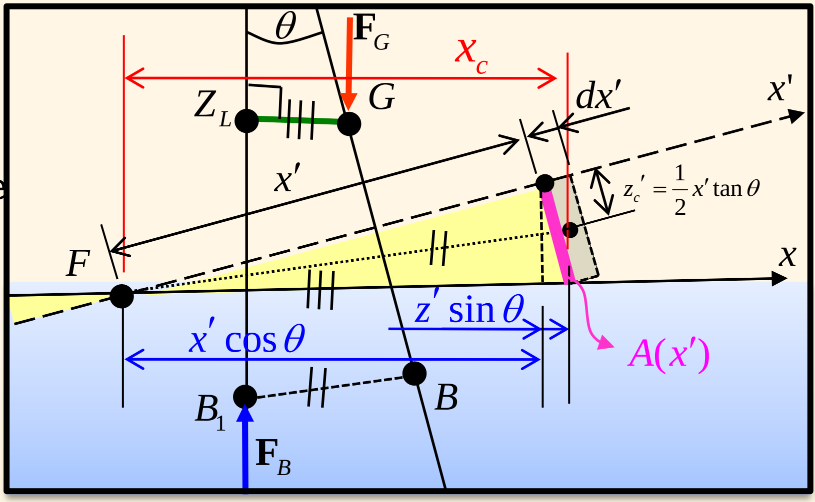
$, x_{v,a}$: x coordinate of center of changed displacement volume in aft

$, x_{v,f}$: x coordinate of center of changed displacement volume in forward

Represent center of buoyancy with respect to waterplane fixed frame $(x_{v,c}, z_{v,c})$ as one with respect to body fixed frame.

$$\begin{pmatrix} x_v \\ z_v \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x'_v \\ z'_v \end{pmatrix}$$

$$\Rightarrow \quad x_{v,c} = x'_{v,c}\cdot\cos\phi + z'_{v,c}\cdot\sin\phi$$



$$= \frac{1}{\nabla}\, v\cdot(-x'_{v,a}\cdot\cos\theta - z'_{v,a}\cdot\sin\theta + x'_{v,f}\cdot\cos\theta + z'_{v,f}\cdot\sin\theta)$$

$$= \frac{1}{\nabla}\left(-\boxed{v\cdot x'_{v,a}}\cos\theta + \boxed{v\cdot x'_{v,f}}\cos\theta \quad - \quad \boxed{v\cdot z'_{v,a}}\sin\theta \boxed{+v\cdot z'_{v,f}}\cdot\sin\theta\right)$$

Longitudinal moment of volume about origin F with respect to body fixed frame

Vertical moment of volume about origin F with respect to body fixed frame

Calculation of BM_L, GZ_L (4)

- BM_L(Longitudinal Metacentric Radius)

v : Changed displacement volume

∇: Displacement volume

(R.H.S)

$$\frac{1}{\nabla} v\cdot(-x_{v,a}+x_{v,f})$$

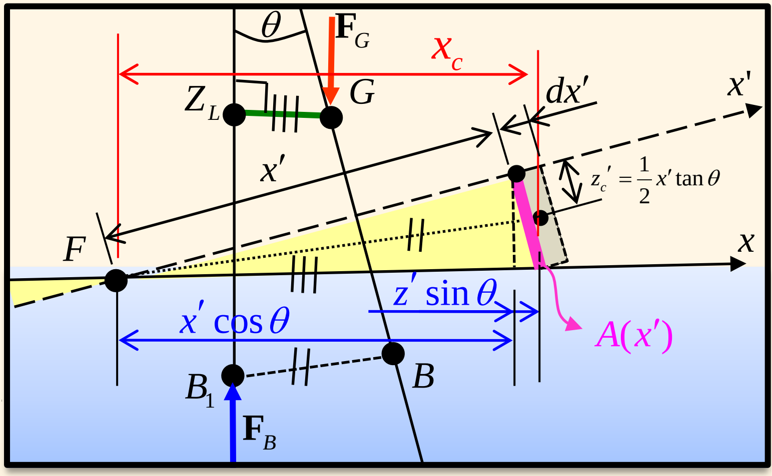
$, x_{v,a}$: x coordinate of center of changed displacement volume in aft

$, x_{v,f}$: x coordinate of center of changed displacement volume in forward

$$=\frac{1}{\nabla}\left(-\boxed{v\cdot x'_{v,a}}\cos\theta+\boxed{v\cdot x'_{v,f}}\cdot\cos\theta-\boxed{v\cdot z'_{v,a}}\sin\theta+\boxed{v\cdot z'_{v,f}}\sin\theta\right)$$

Longitudinal moment of volume about origin F with respect to body fixed frame

Vertical moment of volume about origin F with respect to body fixed frame



How to calculate 1st moment of volume?

: It can be calculated by integral of 1st moment of area over the the length of ship.

$$M_{v,x'}=v\cdot x'_{v,c}=\iiint x'\,dy'\,dz'\,dx'=\int_{x'_a}^{x'_f}A(x')\,x'_c\,dx'$$

$$M_{v,z'}=v\cdot z'_{v,c}=\iiint z'\,dy'\,dz'\,dx'=\int_{x'_a}^{x'_f}A(x')\,z'_c\,dx'$$

$$=\frac{1}{\nabla}\left(-\cos\theta\int_{x'_a}^{F}A_a(x')\,x'_c\,dx'-\cos\theta\int_{F}^{x'_f}A_f(x')\,x'_c\,dx'+\sin\theta\int_{x'_a}^{F}A_a(x')\,z'_c\,dx'+\sin\theta\int_{F}^{x'_f}A_f(x')\,z'_c\,dx'\right)$$

$$=-\frac{1}{\nabla}\left(\int_{x'_a}^{F}A_a(x')\left(x'_c\cos\theta+z'_c\sin\theta\right)dx'+\int_{F}^{x'_f}A_f(x')\left(x'_c\cos\theta+z'_c\sin\theta_c\right)dx'\right)$$

Calculation of BM_L, GZ_L (5)

- BM_L(Longitudinal Metacentric Radius)

v : Changed displacement volume

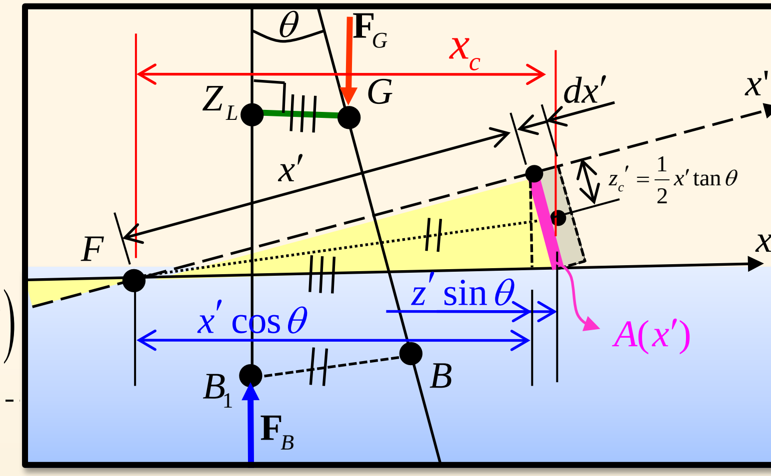
∇: Displacement volume

(R.H.S)

$$\frac{1}{\nabla}\, v\cdot\left(-x_{v,a}+x_{v,f}\right)$$

$, x_{v,a}$: x coordinate of center of changed displacement volume in aft

$, x_{v,f}$: x coordinate of center of changed displacement volume in forward



$$=\frac{1}{\nabla}\left(-v\cdot x'_{v,a}\cdot\cos\theta-v\cdot x'_{v,f}\cdot\cos\theta+v\cdot z'_{v,a}\cdot\sin\theta+v\cdot z'_{v,f}\cdot\sin\theta\right)$$

$$=-\frac{1}{\nabla}\left(\int_{x'_a}^{F}A_a(x')\left(x'_c\cos\theta+z'_c\sin\theta\right)dx'+\int_{F}^{x'_f}A_f(x')\left(x'_c\cos\theta+z'_c\sin\theta_c\right)dx'\right)$$

How to calculate sectional area?

In case of quadrilateral section

Sectional area of immerged region : A_a

$$A_a(x')=\int dA=\int_{-y'}^{y'}\int_{0}^{z'}dz'\,dy'=\int_{-y'}^{y'}\int_{0}^{-x'\tan\theta}dz'\,dy'$$

$$=\int_{-y'}^{y'}\left(-x'\tan\theta\right)dy'=-2x'\,y'\tan\theta$$

Sectional area of immerged region : A_f

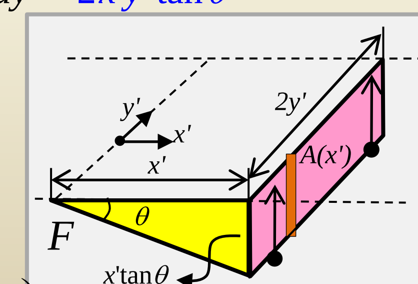
$$A_f(x')=\int dA=\int_{-y'}^{y'}\int_{z'}^{0}dz'\,dy'=\int_{-y'}^{y'}\int_{-x'\tan\theta}^{0}dz'\,dy'$$

$$=\int_{-y'}^{y'}\left(x'\tan\theta\right)dy'=2x'y'\tan\theta$$

How to calculate centroid of section?

$$x'_c=x' \qquad , z'_c=\frac{1}{2}x'\tan\theta$$

,(In case of quadrilateral section)



$$=\int_{x'_A}^{F}\left(2\tan\theta x'y'(x'\cos\theta+\frac{1}{2}x'\tan\theta\sin\theta)\right)dx'+\int_{F}^{x'_F}\left(2\tan\theta x'y'(x'\cos\theta+\frac{1}{2}x'\tan\theta\sin\theta)\right)dx'$$

Calculation of BM_L, GZ_L (6)

- BM_L(Longitudinal Metacentric Radius)

v : Changed displacement volume

∇: Displacement volume

(R.H.S)

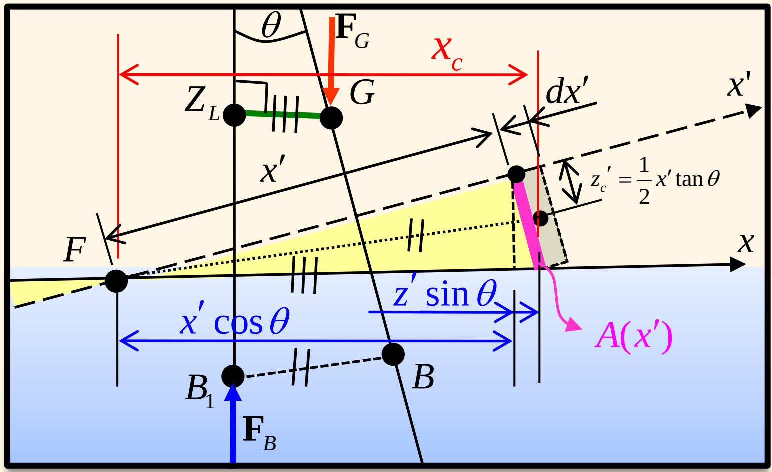
$$\frac{1}{\nabla} \ v\cdot\ (-x_{v,a}+x_{v,f})$$

$, x_{v,a}$: x coordinate of center of changed displacement volume in aft

$, x_{v,f}$: x coordinate of center of changed displacement volume in forward

$$=\frac{1}{\nabla}\left(-v\cdot x'_{v,a}\cdot\cos\theta - v\cdot x'_{v,f}\cdot\cos\theta + v\cdot z'_{v,a}\cdot\sin\theta + v\cdot z'_{v,f}\cdot\sin\theta\right)$$

$$=-\frac{1}{\nabla}\left(\int_{x'_a}^{F} A_a(x')(x'_c\cos\theta + z'_c\sin\theta)dx' + \int_{F}^{x'_f} A_f(x')(x'_c\cos\theta + z'_c\sin\theta_c)dx'\right)$$



$$=\frac{1}{\nabla}\left(\int_{x'_A}^{F}\left(2\tan\theta x' y'(x'\cos\theta + \frac{1}{2}x'\tan\theta\sin\theta)\right)dx' + \int_{F}^{x'_F}\left(2\tan\theta x' y'(x'\cos\theta + \frac{1}{2}x'\tan\theta\sin\theta)\right)dx'\right)$$

$$=\frac{1}{\nabla}\int_{x'_A}^{x'_F}\left(2\tan\theta x' y'(x'\cos\theta + \frac{1}{2}x'\tan\theta\sin\theta)\right)dx'$$

$$=\frac{1}{\nabla}\int_{x'_A}^{x'_F}\left(2\sin\theta x'^2 y' + x'^2 y'\tan^2\theta\sin\theta)\right)dx'$$

$$=\frac{1}{\nabla}\left(\sin\theta\underbrace{\boxed{\int_{x'_A}^{x'_F}(2x'^2y')dx'}}_{=I_L} + \frac{1}{2}\tan^2\theta\sin\theta\underbrace{\boxed{\int_{x'_A}^{x'_F}(2x'^2y')dx'}}_{=I_L}\right)$$

$$\because \left(I_L = \int_{x'_A}^{x'_F} 2x'^2y'\,dx'\right)$$

$$=\frac{1}{\nabla}\left(\sin\theta\cdot I_L + \frac{1}{2}\tan^2\theta\cdot\sin\theta\cdot I_L\right) \quad = \frac{1}{\nabla}\sin\theta\cdot I_L\left(1+\frac{1}{2}\tan^2\theta\right)$$

Calculation of BM_L, GZ_L (7)

- BM_L(Longitudinal Metacentric Radius)

v : Changed displacement volume

∇: Displacement volume

(R.H.S)

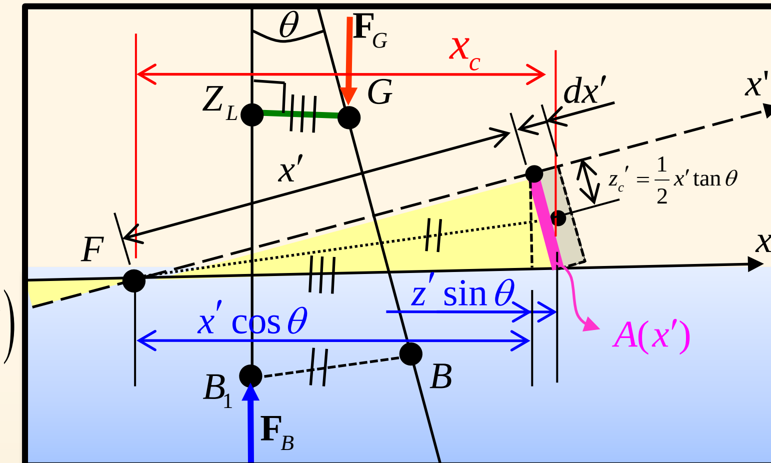
$$\frac{1}{\nabla} v \cdot (-x_{v,a} + x_{v,f})$$

$, x_{v,a}$: x coordinate of center of changed displacement volume in aft

$, x_{v,f}$: x coordinate of center of changed displacement volume in forward

$$= \frac{1}{\nabla}\left(-v \cdot x'_{v,a} \cdot \cos\theta - v \cdot x'_{v,f} \cdot \cos\theta + v \cdot z'_{v,a} \cdot \sin\theta + v \cdot z'_{v,f} \cdot \sin\theta\right)$$

$$= -\frac{1}{\nabla}\left(\int_{x'_a}^{F} A_a(x')(x'_c \cos\theta + z'_c \sin\theta)dx' + \int_{F}^{x'_f} A_f(x')(x'_c \cos\theta + z'_c \sin\theta_c)dx'\right)$$



$$= \frac{1}{\nabla}\left(\int_{x'_A}^{F}\left(2\tan\theta x' y'(x'\cos\theta + \frac{1}{2}x'\tan\theta\sin\theta)\right)dx' + \int_{F}^{x'_F}\left(2\tan\theta x' y'(x'\cos\theta + \frac{1}{2}x'\tan\theta\sin\theta)\right)dx'\right)$$

$$= \sin\theta\underbrace{\int_{x'_A}^{F}(2x'^2 y')dx'}_{I_L} + \frac{1}{2}\tan^2\theta\sin\theta\underbrace{\int_{x'_A}^{F}(2x'^2 y')dx'}_{I_L}$$

$$+\sin\theta\int_{F}^{x'_F}(2x'^2 y')dx' + \frac{1}{2}\tan^2\theta\sin\theta\int_{F}^{x'_F}(2x'^2 y')dx'$$

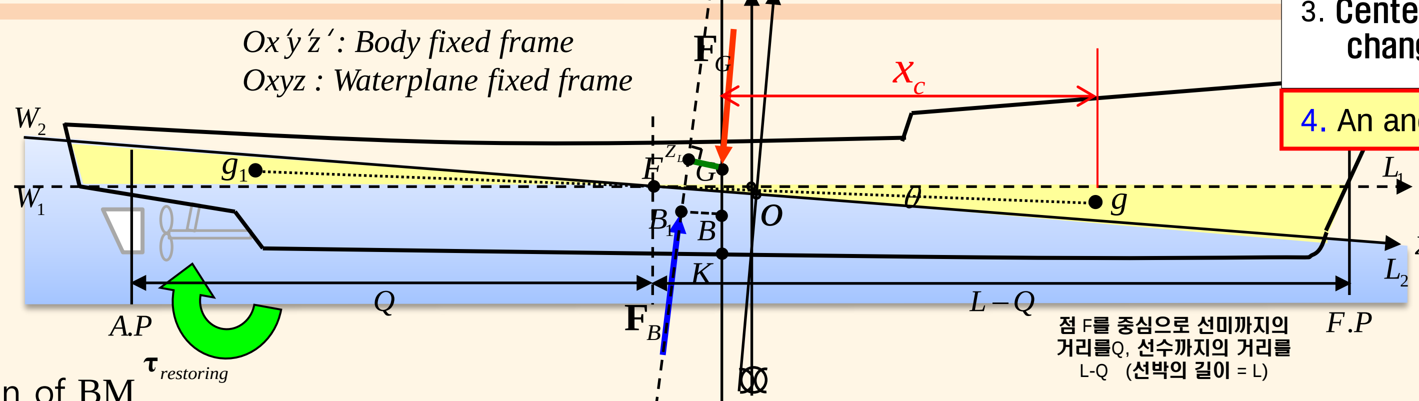
$$\because \left(I_L = \int_{x'_A}^{x'_F} 2x'^2 y'\, dx'\right)$$

$$= \sin\theta \cdot I_L + \frac{1}{2}\tan^2\theta \cdot \sin\theta \cdot I_L$$

$$= \sin\theta \cdot I_L\left(1 + \frac{1}{2}\tan^2\theta\right)$$

Calculation of BM_L, GZ_L (8)

- BM_L(Longitudinal Metacentric Radius)



Assumption

1. Wall sided ship.
2. A main deck is not flooded.
3. Center of rotation is not changed
4. An angle of trim θ is small.

G: Center of mass
B: Center of buoyancy
F_G : Weight of ship (=W)
F_B : Buoyancy (=$\rho g \nabla$)

- Derivation of BM

L.H.S R.H.S

$$\boxed{BB_2 = \frac{1}{\nabla} v \cdot (-x_{v,a} + x_{v,f})}$$

$$BB_2 = BM_L \cdot \sin\theta$$

$$\frac{1}{\nabla} v \cdot (-x_{v,a} + x_{v,f}) = \frac{1}{\nabla} \sin\theta\, I_L\left(1+\frac{1}{2}\tan^2\theta\right)$$

$$BM_L \cdot \sin\theta = \frac{1}{\nabla} \sin\theta\, I_L\left(1+\frac{1}{2}\tan^2\theta\right)$$

$$\boxed{BM_L = \frac{I_L}{\nabla}\left(1+\frac{1}{2}\tan^2\theta\right)}$$

Derivation of BM in case of small angle of trim

if θ is small,

$\tan\theta^2 \approx \theta^2 = 0$

$$BM_L = \frac{I_L}{\nabla}\left(1+\frac{1}{2}\tan^2\theta\right) \Rightarrow \boxed{BM_L = \frac{I_L}{\nabla}}$$

If we assume that θ is small,

$$\overline{BM_L} = \frac{I_L}{\nabla}$$

which is generally known as BM_L.

That BM_L does not consider change of center of buoyancy in vertical direction.

In order to distinguish those, we will indicate two as follows

$$\boxed{BM_{L0} = \frac{I_L}{\nabla}\left(1+\frac{1}{2}\tan^2\theta\right)}$$,(Considering change of center of buoyancy in vertical direction)

$$\boxed{BM_L = \frac{I_L}{\nabla}}$$,(Not considering change of center of buoyancy in vertical direction)

Calculation of BM_L, GZ_L (9)

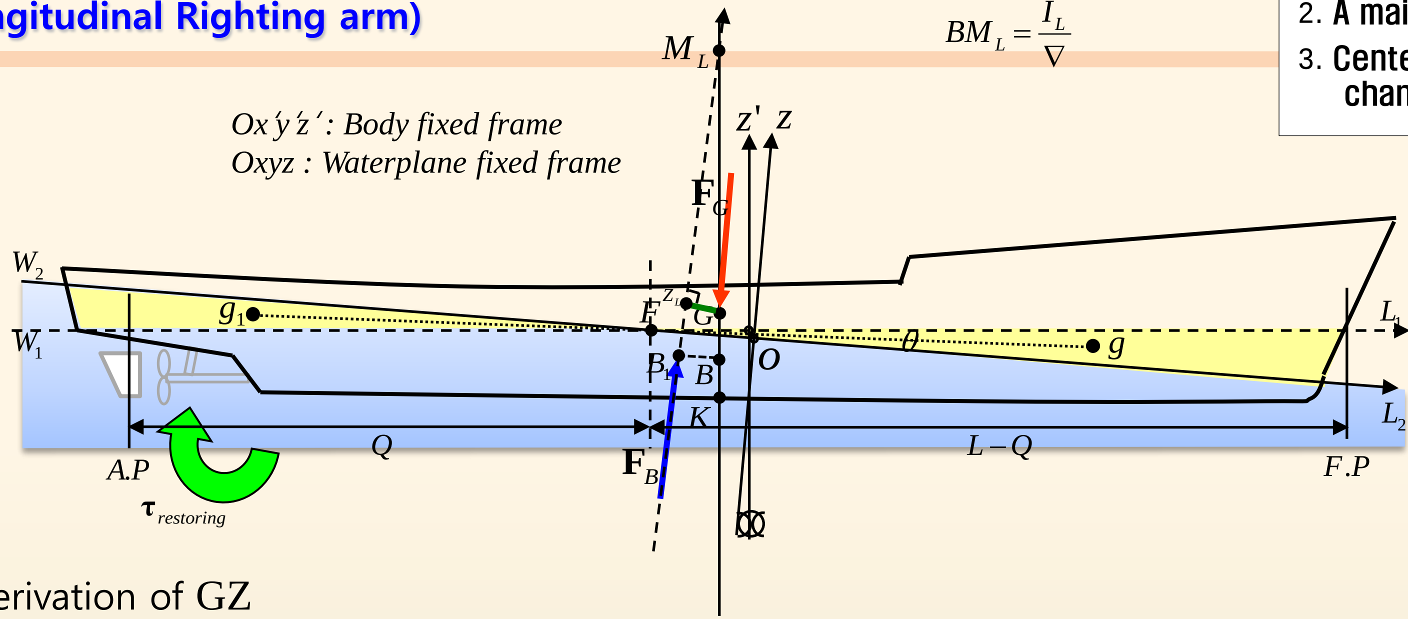
- GZ_L(Longitudinal Righting arm)

$$BM_{L0} = \frac{I_L}{\nabla}\left(1+\frac{1}{2}\tan^2\theta\right)$$

$$BM_L = \frac{I_L}{\nabla}$$

Assumption

1. Wall sided ship.
2. A main deck is not flooded.
3. Center of rotation is not changed



- G: Center of mass
- B: Center of buoyancy
- F_G : Weight of ship (=$\mathbf{W}$)
- F_B : Buoyancy (=$\rho g \nabla$)

- Derivation of GZ

$$GZ_L = KN_L - KG\sin\theta$$

$$= KM_L\sin\theta - KG\sin\theta$$

$$= (KB + BM_{L0})\sin\theta - KG\sin\theta$$

$$\downarrow \left(BM_{L0} = \frac{I_L}{\nabla}(1+\frac{1}{2}\tan^2\theta) \right)$$

$$= (KB + \frac{I_L}{\nabla}(1+\frac{1}{2}\tan^2\theta))\sin\theta - KG\sin\theta$$

$$= (KB + \frac{I_L}{\nabla}(1+\frac{1}{2}\tan^2\theta))\sin\theta - KG\sin\theta$$

$$= (KB + \frac{I_L}{\nabla} - KG)\sin\theta + \frac{1}{2}\frac{I_L}{\nabla}\tan^2\theta\sin\theta$$

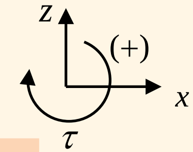
$$\downarrow \left(BM_L = \frac{I_L}{\nabla} \right)$$

$$\boxed{= (KB + BM_L - KG)\sin\theta + \frac{1}{2}BM_L\tan^2\theta\sin\theta}$$

$$\boxed{= GM_L\sin\theta + \frac{1}{2}BM_L\tan^2\theta\sin\theta}$$

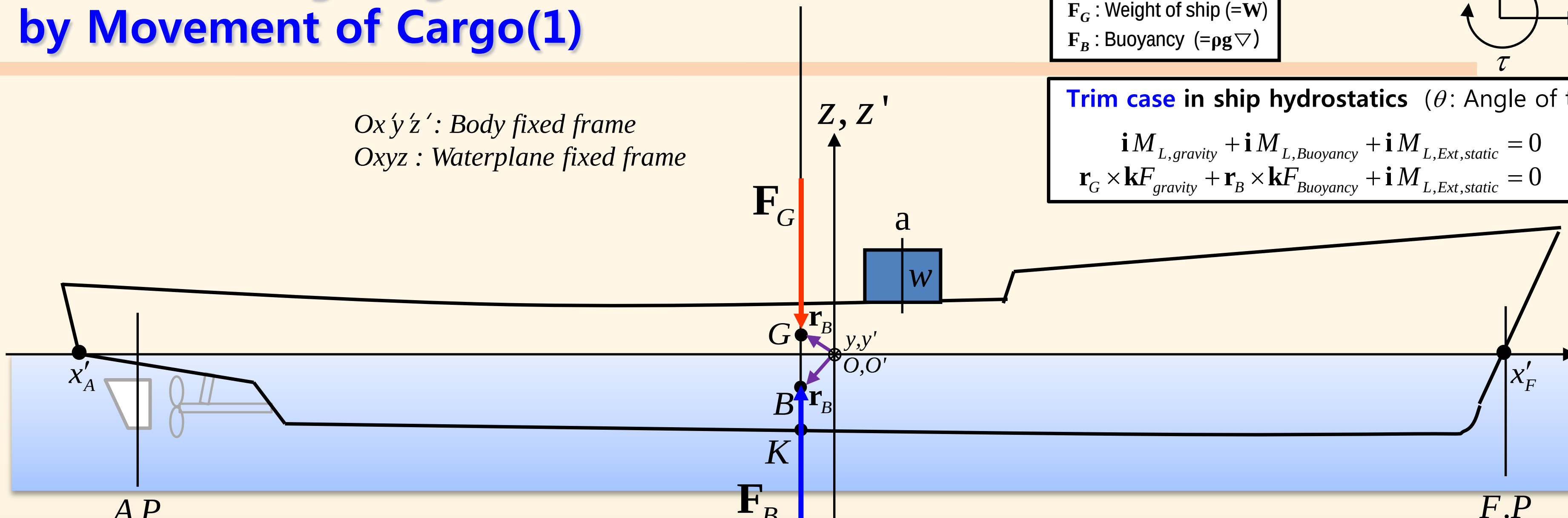
Transverse Righting Moment by Movement of Cargo(1)

G: Center of mass
B: Center of buoyancy
$\mathbf{F}_G$: Weight of ship (=$\mathbf{W}$)
$\mathbf{F}_B$: Buoyancy (=$\rho g \nabla$)



Trim case in ship hydrostatics (θ: Angle of trim)

$$\mathbf{i}M_{L,gravity} + \mathbf{i}M_{L,Buoyancy} + \mathbf{i}M_{L,Ext,static} = 0$$
$$\mathbf{r}_G \times \mathbf{k}F_{gravity} + \mathbf{r}_B \times \mathbf{k}F_{Buoyancy} + \mathbf{i}M_{L,Ext,static} = 0$$



① $\sum \mathbf{F} = \mathbf{F}_G + \mathbf{F}_B = 0$, (static equilibrium of force)

$\Rightarrow F_G\mathbf{k} + F_{B,z}\mathbf{k} = 0 \Rightarrow F_G = -F_{B,z}$

② Center of mass of ship(G) and center of buoyancy (B) and center of mass of cargo are(G_P) <u>in the same vertical line</u> which is perpendicular to waterplane

→ Longitudinal moment about origin O about z axis is as follows

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B$$
$$= \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_B \times \mathbf{F}_B$$

$$\boldsymbol{\tau}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & 0 & z_G \\ 0 & 0 & F_{G,z} \end{vmatrix} = \mathbf{i}x_G \cdot F_{G,z} \quad , \boldsymbol{\tau}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_B & 0 & z_B \\ 0 & 0 & F_{B,z} \end{vmatrix} = x_B \cdot F_{B,z}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B$$
$$= \mathbf{i}\left(x_G \cdot F_{G,z} + x_B \cdot F_{B,z}\right)$$

x_G, x_B are in same vertical line

$\downarrow$ $(x_G = x_B)$

$$= \mathbf{i}\left(-x_B \cdot F_{G,z} + x_B \cdot F_{B,z}\right)$$

$\downarrow$ $(F_G = -F_{B,z})$

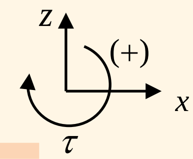
$$= \mathbf{i}\left(-x_B \cdot F_{B,z} + x_B \cdot F_{B,z}\right)$$
$$= 0$$

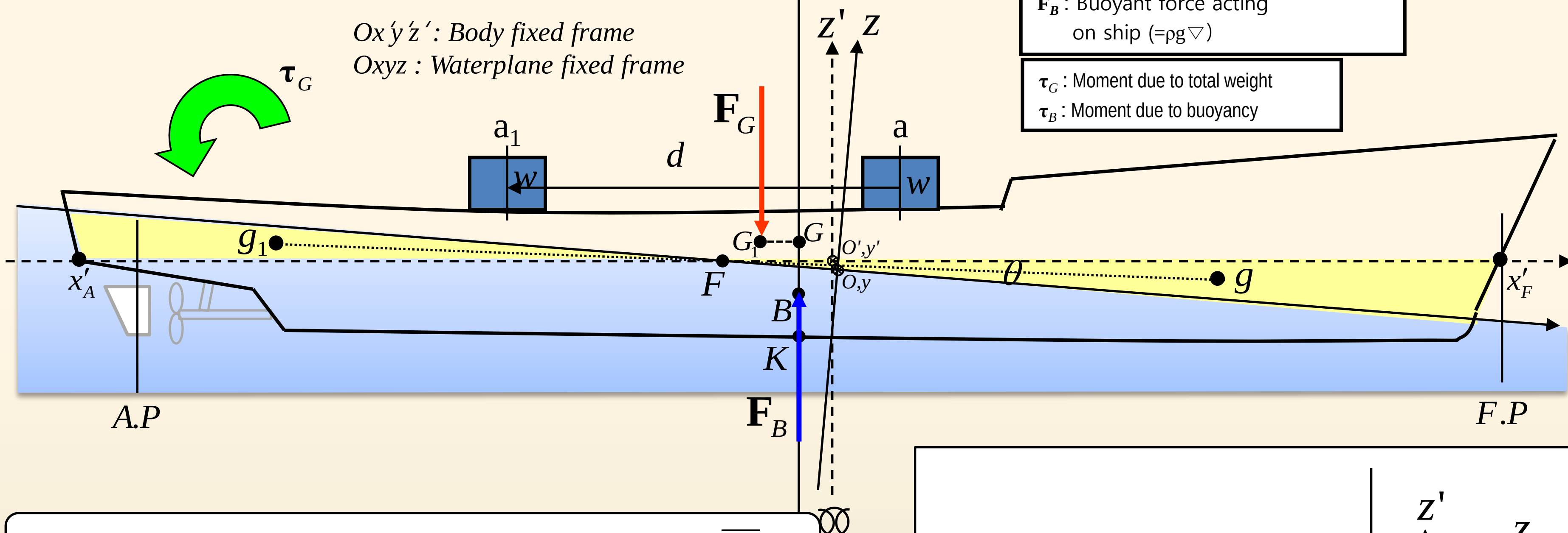
Ox'y'z': Body fixed frame
Oxyz : Waterplane fixed frame

Transverse Righting Moment by Movement of Cargo(2)

G : Center of total mass
G_1 : Changed center of total mass
B: Center of buoyancy
B_1: Changed center of buoyancy
$\mathbf{F}_G$: Total weight ($=\mathbf{W}=\mathbf{W}_{\mathbf{Ship}}+\mathbf{W}_{\mathbf{Weight}}$)
$\mathbf{F}_B$: Buoyant force acting on ship ($=\rho g \nabla$)

τ_G : Moment due to total weight
τ_B : Moment due to buoyancy





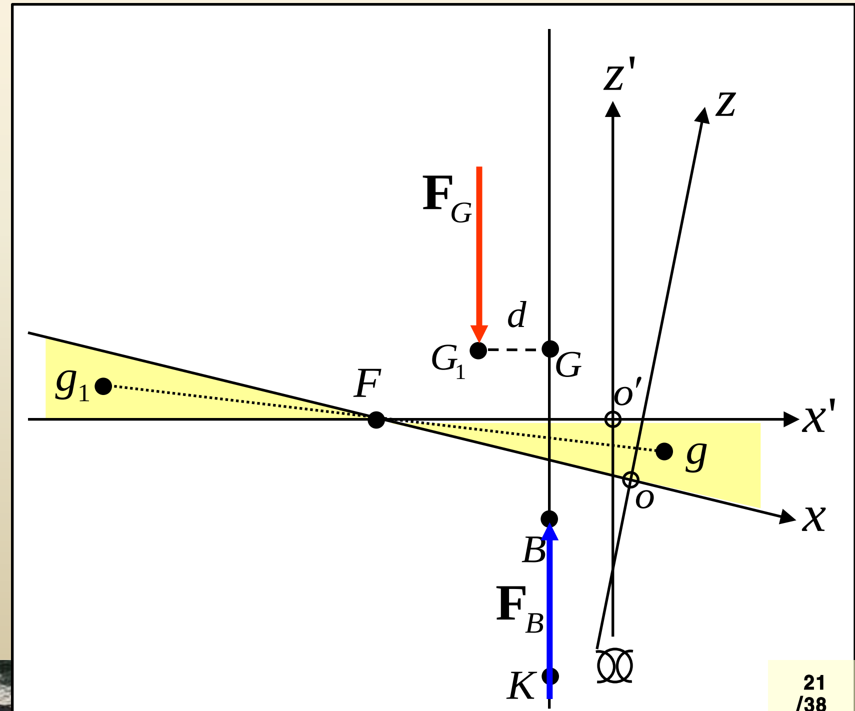
③ The cargo is moved to aft through a distance d, $(\overline{aa_1} = d)$

④ Center of mass of total weight is moved from G to G_1.

$$GG_1 = \frac{w}{F_G} d \quad ,(w\text{: Weight of cargo})$$

⑤ Assuming that a ship is trimmed by an differential angle, a ship will be trimmed about a specific point F that immerged volume become equivalent to emerged volume.

F(LCF) : Longitudinal Center of Floatation

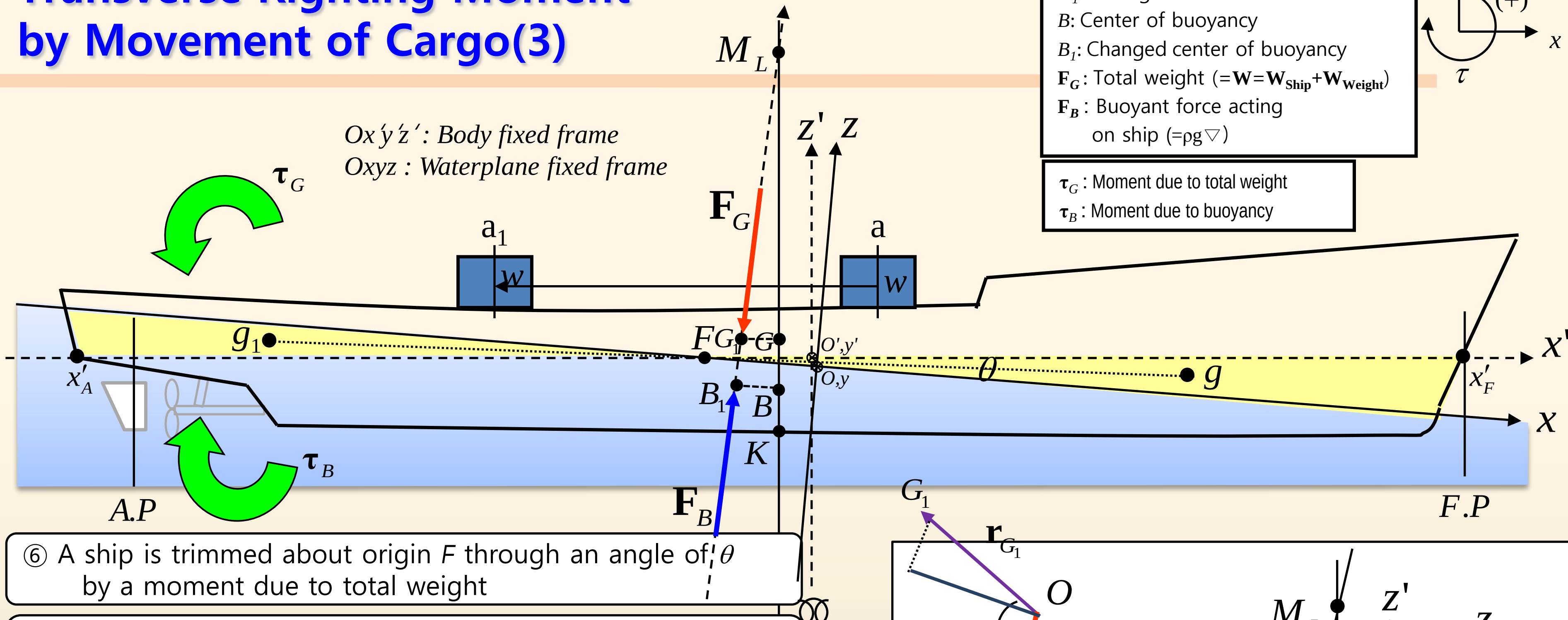


Transverse Righting Moment by Movement of Cargo(3)

F(LCF) : Longitudinal Center of Floatation

G : Center of total mass
G_1: Changed center of total mass
B: Center of buoyancy
B_1: Changed center of buoyancy
$\mathbf{F}_G$: Total weight ($=\mathbf{W}=\mathbf{W}_{\mathbf{Ship}}+\mathbf{W}_{\mathbf{Weight}}$)
$\mathbf{F}_B$: Buoyant force acting on ship ($=\rho g \nabla$)

$\boldsymbol{\tau}_G$: Moment due to total weight
$\boldsymbol{\tau}_B$: Moment due to buoyancy



⑥ A ship is trimmed about origin F through an angle of θ by a moment due to total weight

⑦ Center of buoyancy is changed from B to B_1

⑧ Changed center of mass(G_1) and changed center of buoyancy (B_1) are in the same vertical line which is perpendicular to waterplane. Then it become in static equilibrium

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B = \mathbf{r}_{G_1} \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

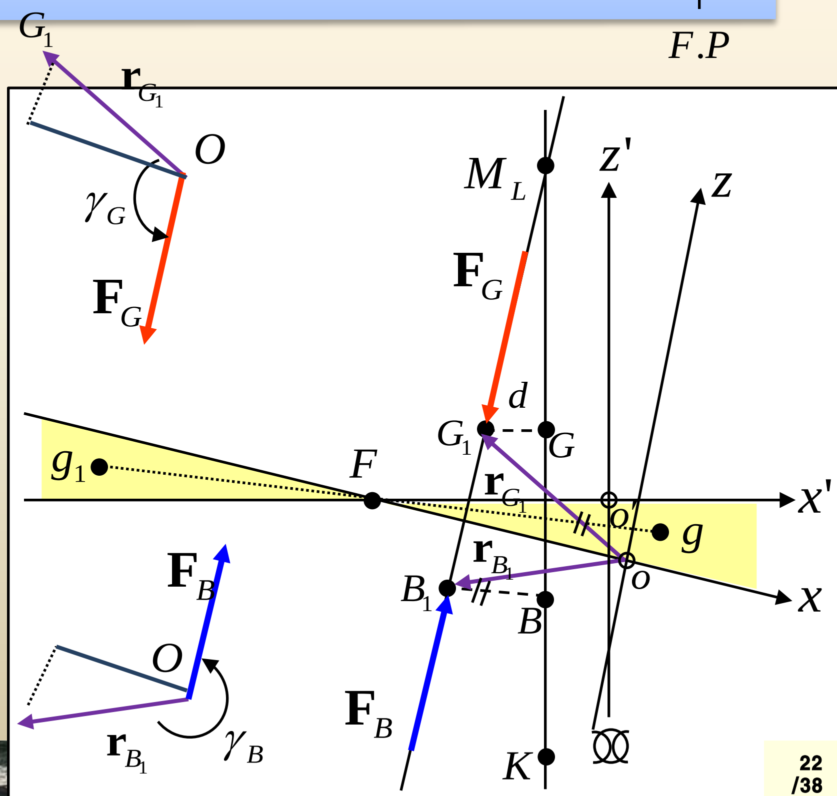
$$\boldsymbol{\tau}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{G_1} & 0 & z_{G_1} \\ 0 & 0 & F_{G,z} \end{vmatrix} = x_{G_1} \cdot F_{G,z} \quad , \boldsymbol{\tau}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{B_1} & 0 & z_{B_1} \\ 0 & 0 & F_{B,z} \end{vmatrix} = x_{B_1} \cdot F_{B,z}$$

$$= \mathbf{i}\left(x_{G_1} \cdot F_{G,z} + x_{B_1} \cdot F_{B,z}\right)$$

$$\sum \mathbf{F} = \mathbf{F}_G + \mathbf{F}_B = 0 \;\Rightarrow\; F_G = -F_{B,z}$$

$$= \mathbf{i}\left(-x_{B_1} \cdot F_{G,z} + x_{B_1} \cdot F_{B,z}\right)$$,because they lay in the same vertical line $x_{G_1} = x_{B_1}$

$$= 0$$



Sec.1 Longitudinal Righting Moment

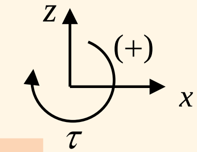
Sec.2 Calculation of BM_L,GZ_L in Wall Sided Ship

Sec.3 Transverse Righting Moment due to Movement of Cargo

Sec.4 Moment to Change 1cm Trim(MTC), Trim

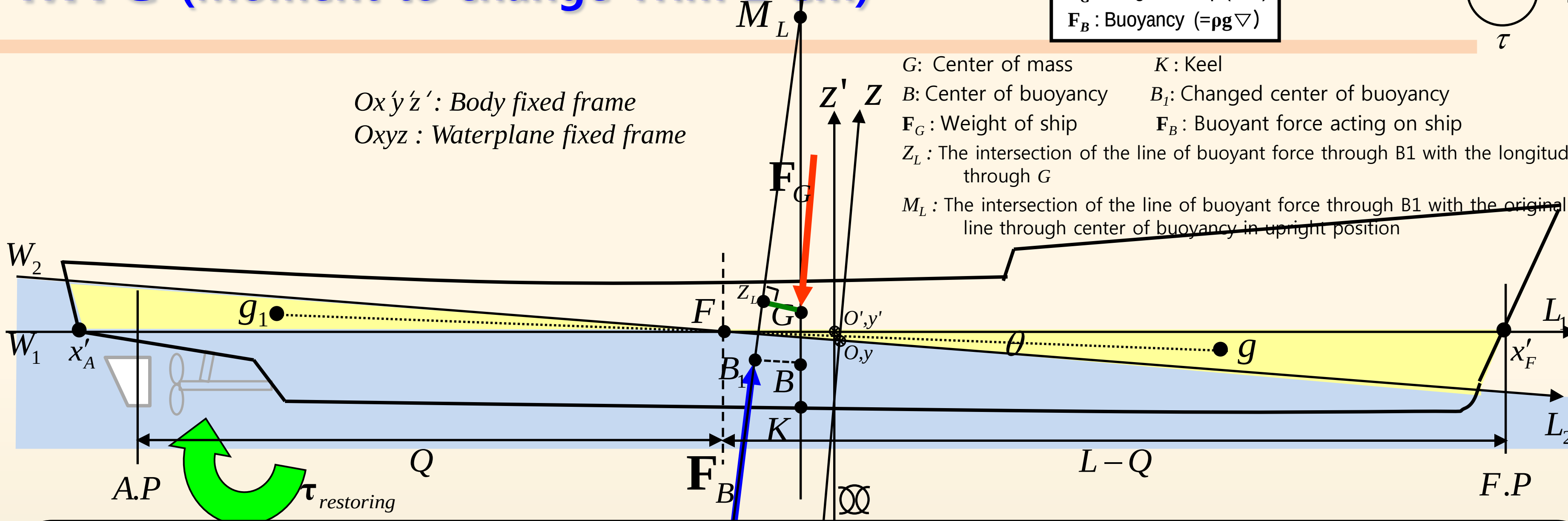
MTC (Moment to change Trim 1 Cm)

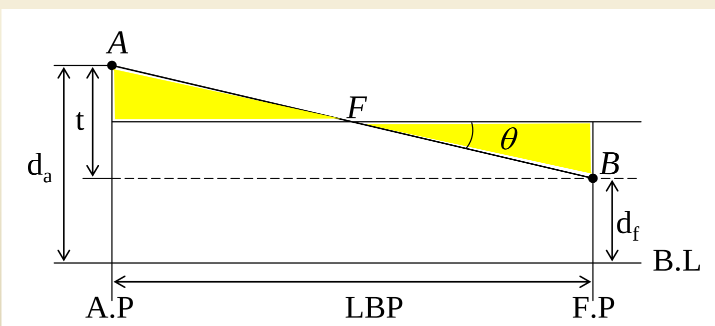
G: Center of mass
B: Center of buoyancy
$\mathbf{F}_G$: Weight of ship (=$\mathbf{W}$)
$\mathbf{F}_B$: Buoyancy (=$\rho g \nabla$)



G: Center of mass $\quad$ K : Keel
B: Center of buoyancy $\quad$ B_1: Changed center of buoyancy
F_G : Weight of ship $\quad$ F_B : Buoyant force acting on ship
Z_L : The intersection of the line of buoyant force through B1 with the longitudinal line through G
M_L : The intersection of the line of buoyant force through B1 with the original vertical line through center of buoyancy in upright position

$Ox'y'z'$: Body fixed frame
$Oxyz$: Waterplane fixed frame





$$\text{Trim(t)} : \mathrm{d_a} - \mathrm{d_f}\,, \qquad \sin\theta = \frac{t}{AB}$$

If a angle of trim θ is small,

$$\sin\theta \approx \tan\theta = \frac{t_{\text{trim}}}{LBP}$$

If the ship is in static equilibrium at angle of trim θ,

Longitudinal heeling moment = Longitudinal righting moment

$$= \Delta \cdot GZ_L \cong \Delta \cdot GM_L \cdot \sin\theta$$

The moment is considered as the moment to make 1cm trim.
So, the moment to change trim 1 cm is

$$\boxed{MTC = \Delta \cdot GM_L \cdot \frac{1}{LBP \cdot 100}}$$

From the equation $\quad GM_L = KB + BM_L - KG$ $\quad$,(unit conversion for cm)

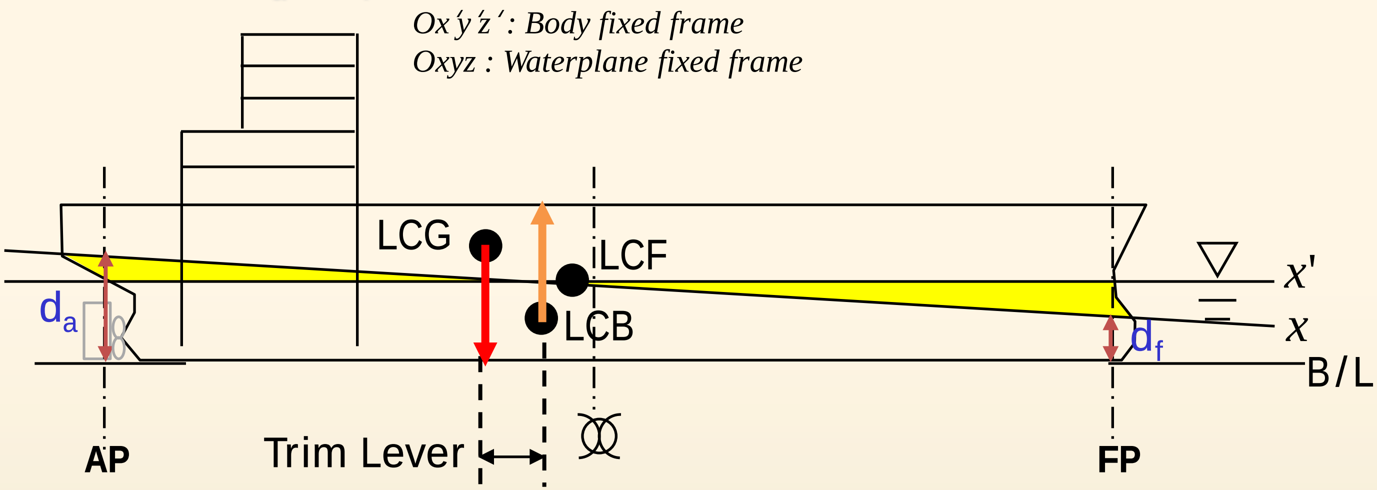
If we assume that KB, KG is much smaller than BM_L, they can be omitted

$$GM_L \approx BM_L$$

$$\boxed{\therefore\ MTC = \Delta \cdot BM_L \cdot \frac{1}{LBP \cdot 100}}$$

Trim

☑ **Trim : d_a - d_f (Positive : Trim by stern, Negative : Trim by bow)**



- Calculation of Trim using MTC

  · MTC : moment to change trim one centimeter over the LBP

$$\boxed{Trim \times MTC \times \underbrace{100}_{\text{(m 단위 환산)}}} = \boxed{\Delta \times Trim\ Lever}$$

Longitudinal heeling moment to cause trim

Moment caused by difference of location between center of mass and center of buoyancy

$$Trim\ Lever = LCB - LCG$$

$$\boxed{Trim[m] = \frac{\Delta \times Trim\ Lever}{MTC \times 100}}$$

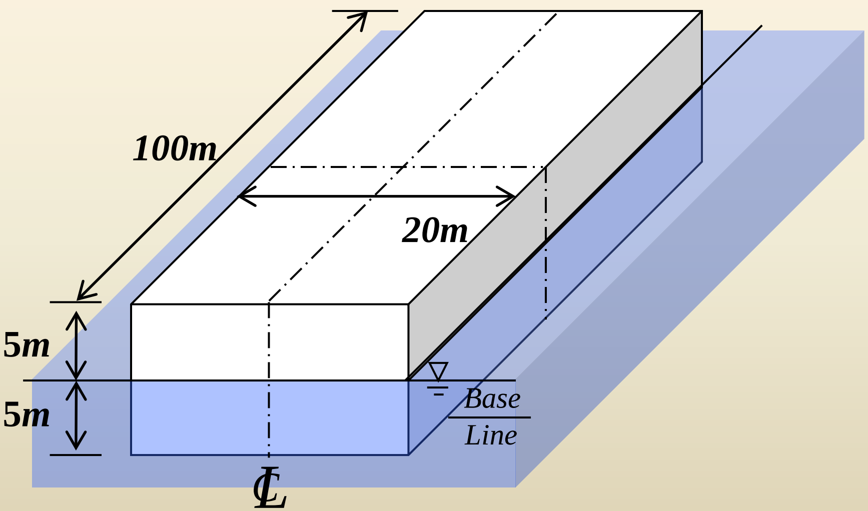
$$MTC = \frac{\Delta \times GM_L}{100 \times LBP}$$

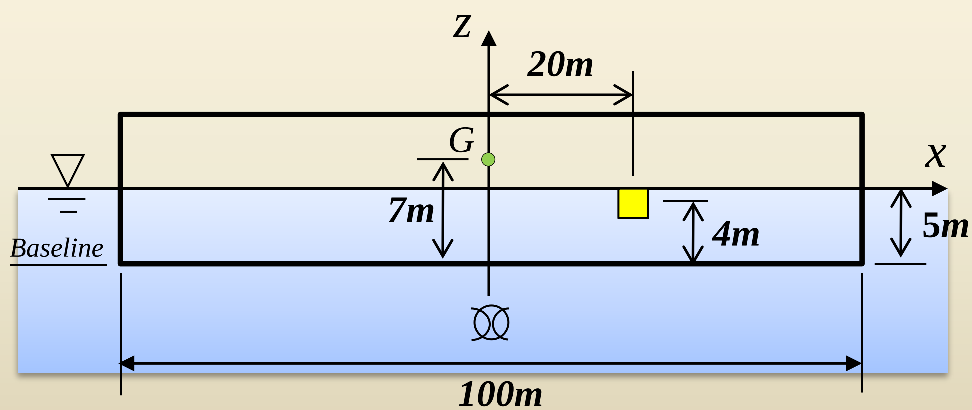
Examples

Problem> Calculation of Trim of Barge

Question)

A barge 100 m long, 12 m beam and 10 m deep is floating on an even keel at 6 m draft. Vertical center of mass of the ship is 7 m from the baseline.
A cargo of 1,000 ton is loaded in the location 20 m forward from centerline, 4 m upward from the baseline.
Find the draft forward and aft.





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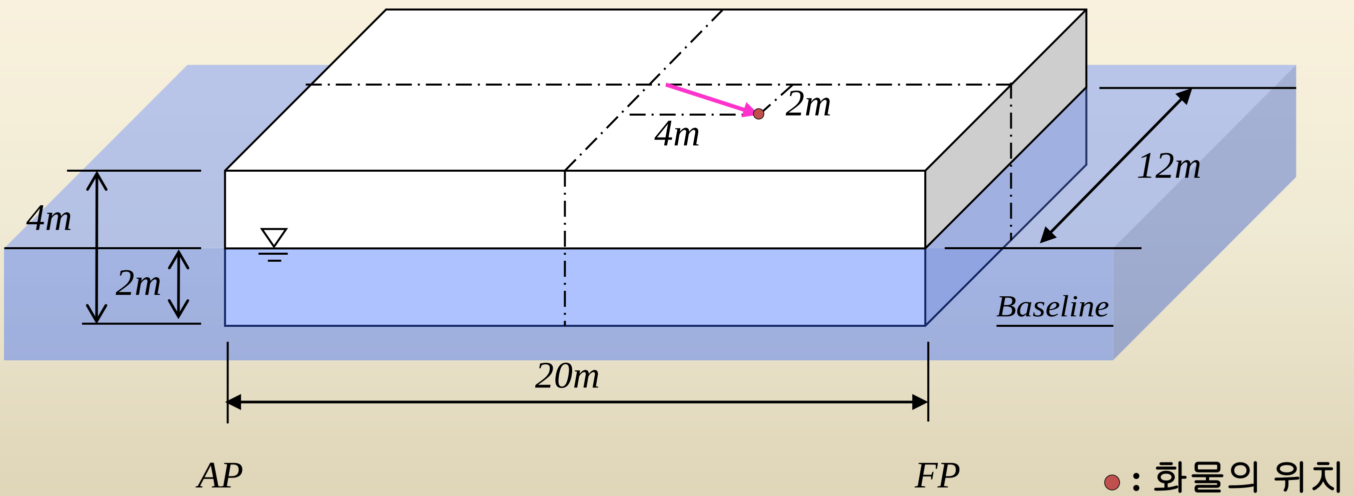
Problem> Calculation of Draft at Bow and Stern caused by Movement of Cargo

Question)

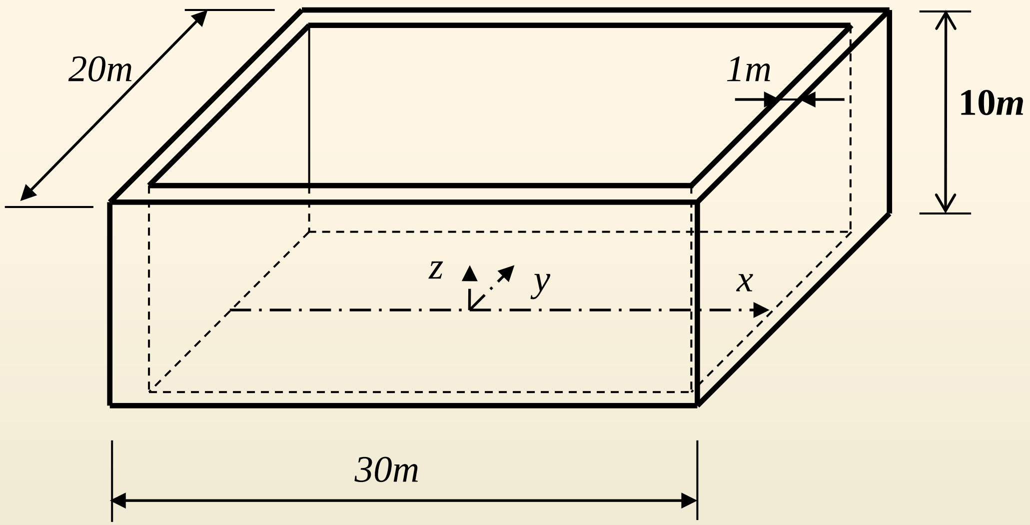
A barge is floating in fresh water at draft 2m. The length, breadth, depth of the barge are 20m, 12m, 4m. A cargo on the deck is shifted through a distance of 4 m to forward and a distance 2 m to starboard.
Calculate port and starboard draft in FP and AP. KG of barge is 2 m.



Problem> Calculation of Trim and Trim Angle of Barge(1)

Question)

There is a ship of density ρ_m=1.0 ton/m^3 and length, breadth, depth of the ship are 28m, 18m, 9m. Answer questions about the barge.



① Find the lightweight in light ship condition, and the draft in fresh water.
And if the ship is in sea water, what will the draft is changed?

② Following cargos are supposed to be loaded in the barge.

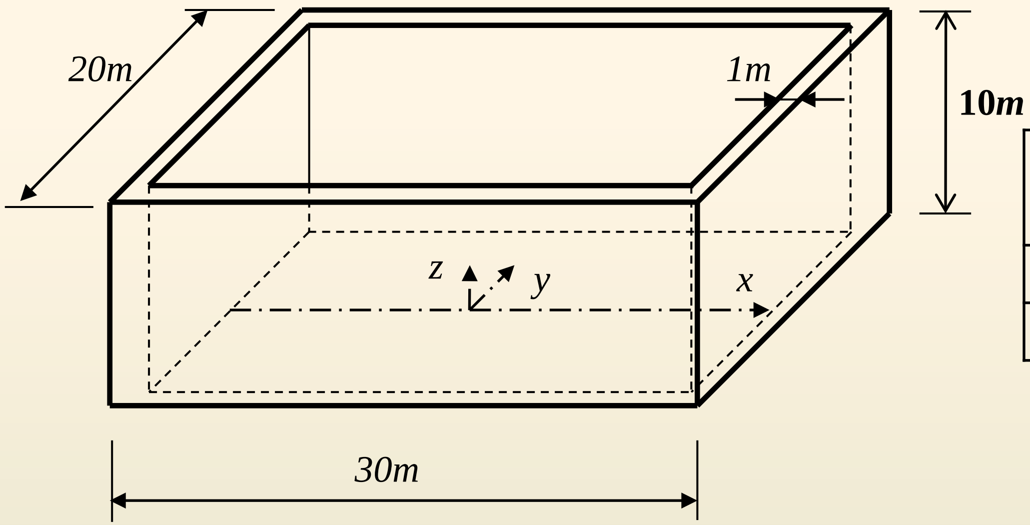
Cargo	Unit Weight	No. of Cargo	Loading location(cm)		
			x	y	z
Cargo 1	100 ton	3	0	0	1
Cargo 2	150 ton	2	-5	0	1

Find ⓐdeadweight(DWT) ⓑTPC ⓒMTC ⓓTrim ⓔdraft forward and aft ⓕLCB ⓖLCG

Problem> Calculation of Trim and Trim Angle of Barge(2)

Question)

There is a ship of density ρ_m=1.0 ton/m^3 and length, breadth, depth of the ship are 28m, 18m, 9m. Answer questions about the barge.



② Following cargos are supposed to be loaded in the barge.

Cargo	Unit Weight	No. of Cargo	Loading location(cm)		
			x	y	z
Cargo 1	100 ton	3	0	0	1
Cargo 2	150 ton	2	-5	0	1

Find ⓐdeadweight(DWT) ⓑTPC ⓒMTC ⓓTrim ⓔdraft forward and aft ⓕLCB ⓖLCG

③ When the cargo 2 is removed from the result of problem ②, calculate LCB, LCG after removing.

④ When the cargo 1 are shifted to the right (along the y axis) through a distance of 5 m from the result of problem ③, calculate an angle of heel of the barge.

Problem> Calculation of Trim Forward and Aft

Question)

A bulk carrier of 150,000 ton deadweight, 264 m LBP is floating on an even keel at 16.9 m draft, and cargo holds are full of cargo.
When cargo in No.1 cargo hold is discharged on port, calculate drafts forward and aft. The transverse center of mass of cargo is in centerline, and longitudinal center of mass of cargo is 107.827 m from midship.
Find draft forward and aft by using hydrostatic table of this ship.

(첨부 자료) *DWT* 150,000 *ton* Bulk Carrier, *Lbp* = 264 *m*

HYDROSTATIC TABLE

DRAFT (EXT.) (M)	DISPL EXT. (MT)	TPC (MT/CM)	MTC (MT*M/CM)	L.C.B (M)	L.C.F (M)	KMT (M)	Cb	WETSUR (M^2)
15.200	150450	105.4	1906.1	9.464	2.107	18.717	0.8128	17013
15.220	150667	105.5	1907.1	9.454	2.081	18.714	0.8129	17023
15.240	150883	105.5	1908.1	9.443	2.055	18.712	0.8130	17019
15.260	151100	105.5	1909.0	9.432	2.029	18.709	0.8131	17032
15.280	151316	105.5	1910.0	9.422	2.004	18.706	0.8132	17062
15.300	151532	105.5	1911.0	9.411	1.978	18.704	0.8133	17078
15.320	151749	105.6	1911.9	9.400	1.953	18.701	0.8134	17065
15.340	151965	105.6	1912.9	9.389	1.928	18.699	0.8135	17074
15.360	152182	105.6	1913.9	9.379	1.903	18.696	0.8136	17093
15.380	152399	105.6	1914.8	9.368	1.878	18.694	0.8137	17128
15.400	152615	105.6	1915.8	9.357	1.854	18.691	0.8138	17123
15.420	152832	105.6	1916.7	9.347	1.829	18.689	0.8139	17121
15.440	153049	105.7	1917.7	9.336	1.805	18.686	0.8140	17132
15.460	153265	105.7	1918.6	9.325	1.781	18.684	0.8141	17153
15.480	153482	105.7	1919.6	9.314	1.757	18.682	0.8142	17180
15.500	153699	105.7	1920.5	9.304	1.733	18.679	0.8143	17190
15.520	153916	105.7	1921.5	9.293	1.709	18.677	0.8144	17217
15.540	154133	105.8	1922.4	9.282	1.685	18.675	0.8145	17234
15.560	154350	105.8	1923.3	9.271	1.662	18.673	0.8146	17210
15.580	154567	105.8	1924.3	9.261	1.638	18.671	0.8147	17192

DRAFT (EXT.) (M)	DISPL EXT. (MT)	TPC (MT/CM)	MTC (MT*M/CM)	L.C.B (M)	L.C.F (M)	KMT (M)	Cb	WETSUR (M^2)
16.600	165679	106.7	1968.5	8.709	0.589	18.598	0.8196	17711
16.620	165898	106.7	1969.3	8.698	0.570	18.597	0.8197	17722
16.640	166116	106.7	1970.1	8.687	0.552	18.597	0.8198	17733
16.660	166335	106.7	1970.9	8.676	0.534	18.596	0.8199	17744
16.680	166554	106.7	1971.6	8.665	0.516	18.595	0.8200	17756
16.700	166773	106.7	1972.4	8.655	0.498	18.595	0.8201	17767
16.720	166991	106.7	1973.2	8.644	0.480	18.594	0.8202	17777
16.740	167210	106.8	1974.0	8.633	0.462	18.594	0.8203	17788
16.760	167429	106.8	1974.7	8.622	0.444	18.593	0.8204	17799
16.780	167648	106.8	1975.5	8.611	0.426	18.592	0.8205	17810
16.800	167867	106.8	1976.2	8.601	0.408	18.592	0.8206	17821
16.820	168086	106.8	1977.0	8.590	0.390	18.591	0.8207	17831
16.840	168305	106.8	1977.7	8.579	0.372	18.591	0.8208	17841
16.860	168524	106.8	1978.4	8.568	0.354	18.590	0.8209	17851
16.880	168743	106.9	1979.1	8.558	0.336	18.590	0.8209	17861
16.900	**168962**	**106.9**	**1980.1**	**8.547**	**0.324**	**18.590**	**0.8210**	**17871**
16.920	169181	106.9	1980.9	8.536	0.308	18.590	0.8211	17881
16.940	169400	106.9	1981.7	8.525	0.292	18.589	0.8212	17891
16.960	169619	106.9	1982.6	8.514	0.276	18.589	0.8213	17901
16.980	169839	106.9	1983.4	8.504	0.261	18.589	0.8214	17911

NOTE : POSITIVE SIGN(+) OF L.C.B & L.C.F MEANS FORWARD DIRECTION OF MIDSHIP.
DISPLACEMENT(EXT) IS BASED ON SEA WATER S.G OF 1.025.

Problem> Calculation of Trim of Barge

Question)

There is a ship operating in fresh water as shown in the picture. Answer following questions.

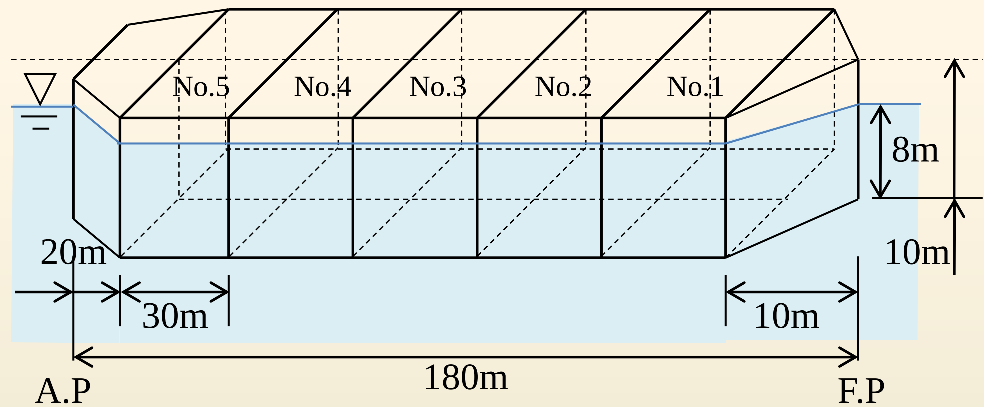
Densities of aft-body, cargo hold, fore-body are $\rho_m = 1.0$ ton/m3.

① Calculate the displacement of this ship.

② Find LCF, LCB, LCG, KG of the ship.

③ When the cargo of 0.6 ton/m^3 density is loaded homogenously in each cargo hold (Total 10 holds : No.1 ~ No.5 Hold x (port & starboard)). Calculate the deadweight(DWT) and lightweight(LWT) of the ship.

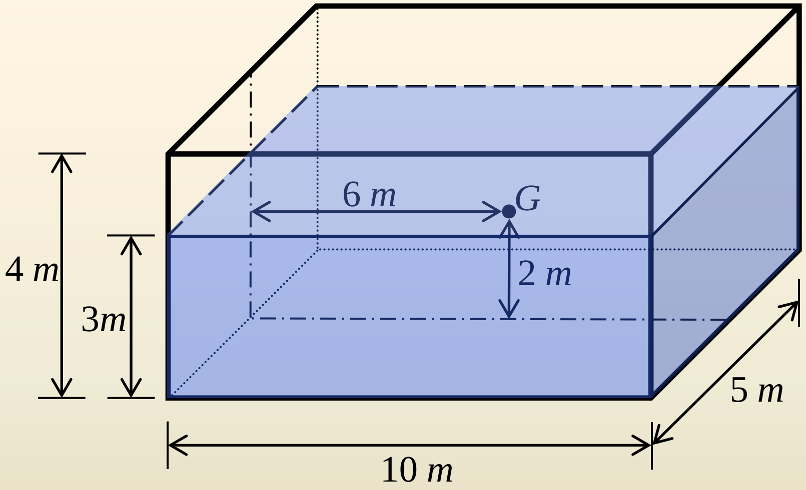
④ In real ship, Loading/unloading cargo cause shape of waterplane to be changed. In that case, explain the procedure of calculating change of trim.



Problem> Calculation of Position of Barge

Question)

A rectangular barge is floating on an even keel at a draft of 3 m. The center of mass is changed to 2 m from the baseline, 6 m from aft, then calculate the position of the barge.



- Ship Stability -

Ch.8 Heeling Moment caused by Fluid in Tanks
(Free Surface Moment)

2009

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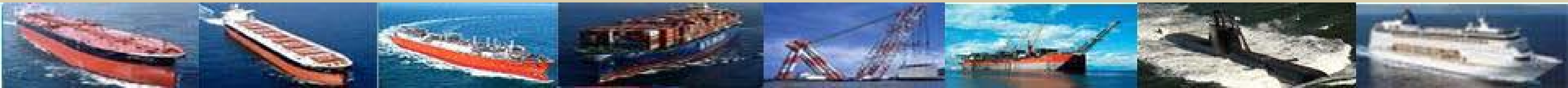
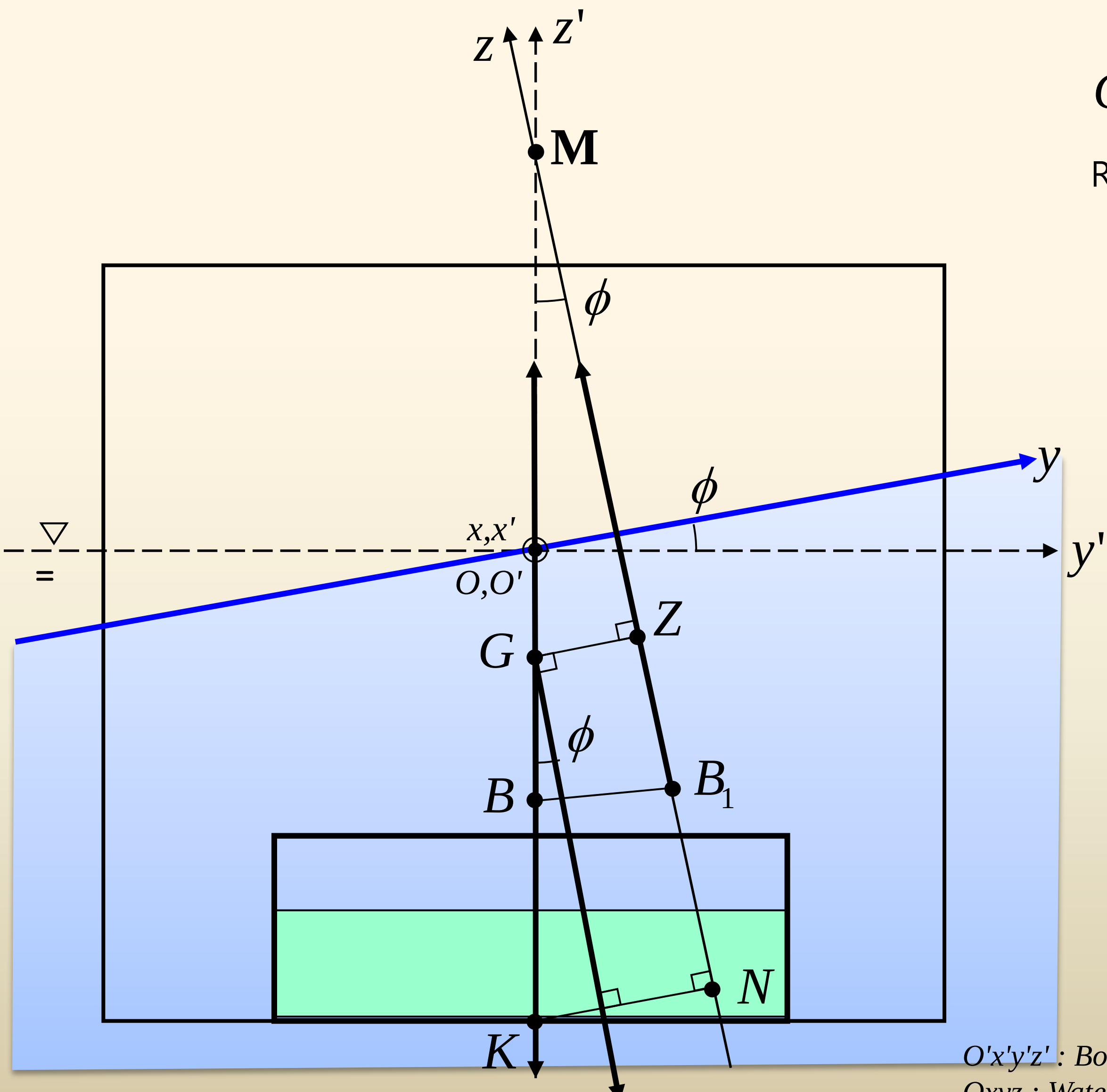


Heeling Moment caused by Fluid in Tanks (1)
- Case1 : Solid Cargo fixed to Cargo Hold

$$\tau_{restoring} = GZ \cdot F_B$$
$$GZ = GM \cdot \sin\phi$$
$$GM = KB + BM - \boxed{KG}$$



$GZ // KN$

Righting arm GZ

$$GZ = KN - KG\sin\phi$$
$$= GM\sin\phi$$

O'x'y'z' : Body fixed frame

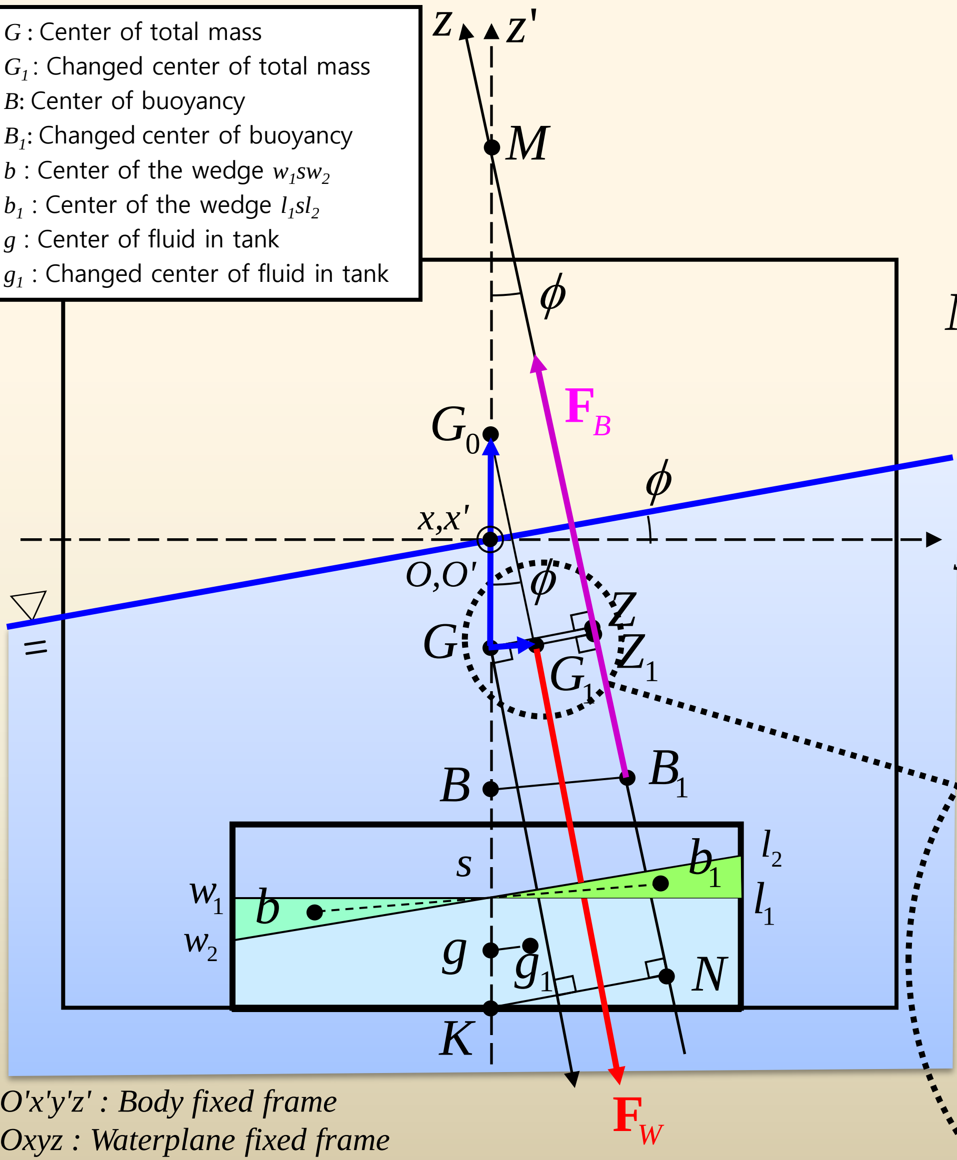
Oxyz : Waterplane fixed frame

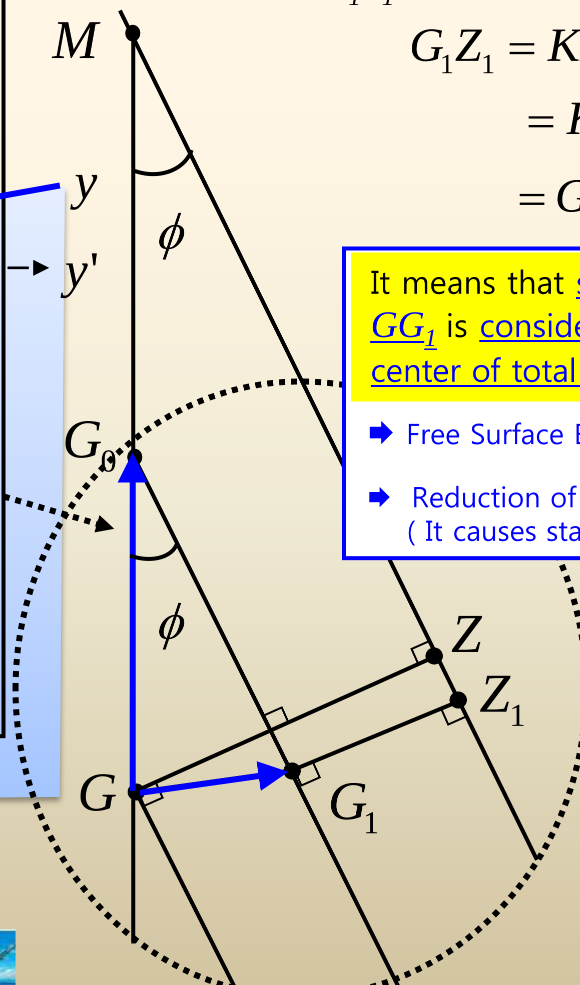
Heeling Moment caused by Fluid in Tanks (2)
- Case2 : Fluid Cargo in Cargo Hold (1)

cf) In case that fixed cargo of solid is loaded

Righting arm	$\mathbf{GZ} = \mathbf{KN} - \mathbf{KG}\sin\phi$ $= \mathbf{GM}\sin\phi$

G : Center of total mass
G_1: Changed center of total mass
B: Center of buoyancy
B_1: Changed center of buoyancy
b : Center of the wedge w_1sw_2
b_1 : Center of the wedge l_1sl_2
g : Center of fluid in tank
g_1 : Changed center of fluid in tank





O'x'y'z' : Body fixed frame
Oxyz : Waterplane fixed frame

If a ship is heeled, plane of fluid in tank is changed to be parallel to waterplane.

$$bb_1 // gg_1 // GG_1$$

From the geometric shape, righting arm G_1Z_1 is as follows

$$G_1Z_1 = KN - (KG + GG_0)\sin\phi$$
$$= KN - (KG_0)\sin\phi$$
$$= G_0M\sin\phi$$

It means that shift of center of total mass GG_1 is considered as the elevation of center of total mass from G to G_0.

➡ Free Surface Effect (FSE)

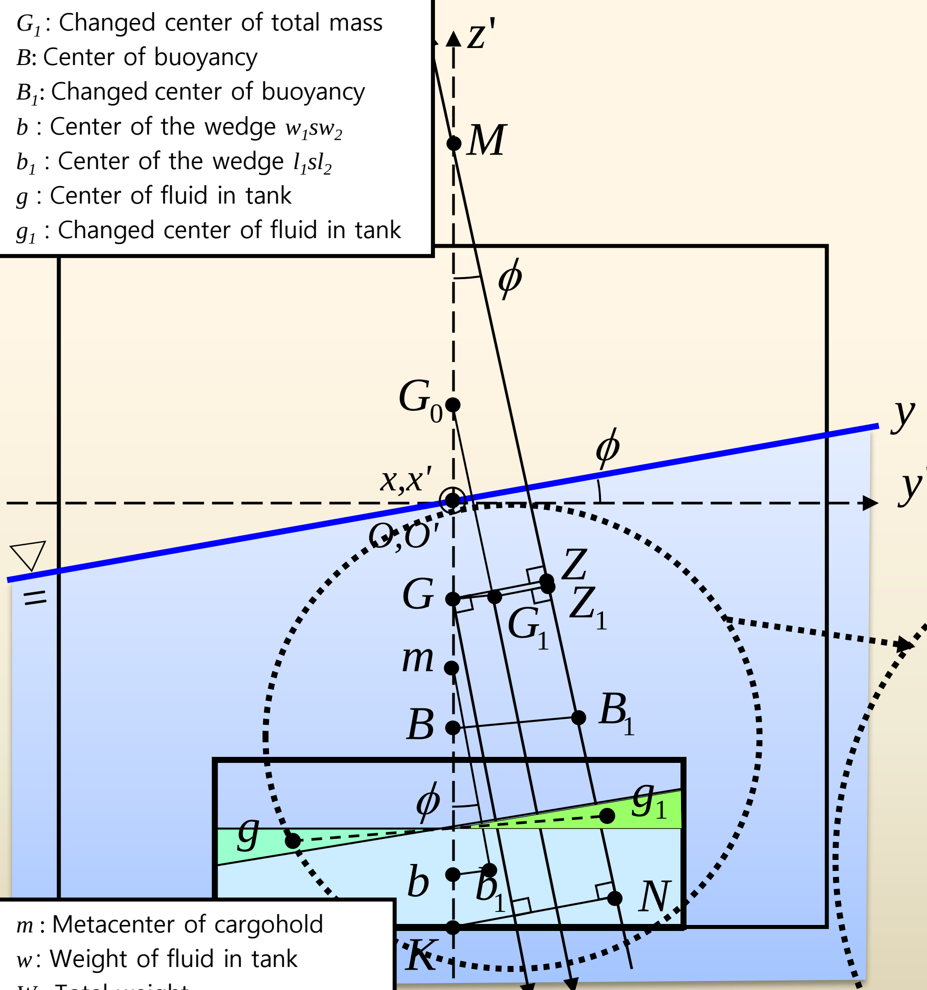
➡ Reduction of righting arm
(It causes stability to be worse.)

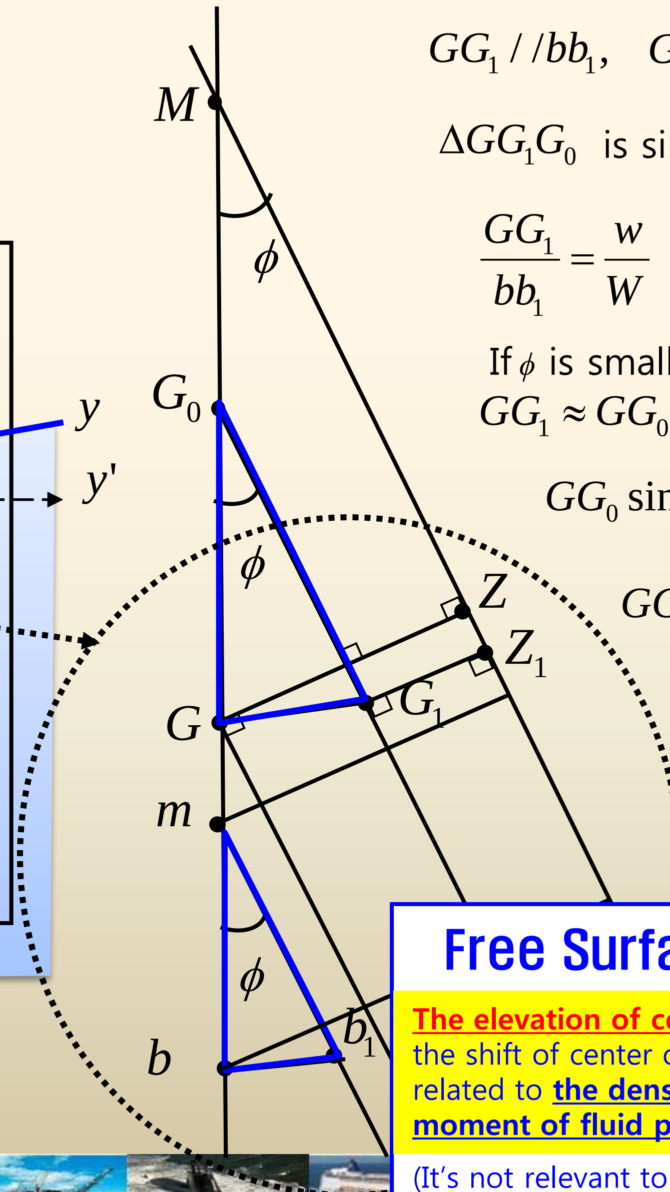
$$\tau_{restoring} = GZ \cdot F_B$$
$$GZ = GM \cdot \sin\phi$$
$$GM = KB + BM - \boxed{KG}$$
$$\Downarrow$$
$$\boxed{KG_0}$$

Heeling Moment caused by Fluid in Tanks (3)
- Case2 : Fluid Cargo in Cargo Hold (2)

$$\tau_{restoring} = GZ \cdot F_B$$
$$GZ = GM \cdot \sin\phi$$
$$GM = KB + BM - \boxed{KG}$$
$$\Downarrow$$
$$\boxed{KG_0}$$

G : Center of total mass
G_1 : Changed center of total mass
B: Center of buoyancy
B_1: Changed center of buoyancy
b : Center of the wedge $w_1 s w_2$
b_1 : Center of the wedge $l_1 s l_2$
g : Center of fluid in tank
g_1 : Changed center of fluid in tank





$$GG_1 // bb_1, \quad GG_0 // bm, \quad G_1G_0 // b_1m$$

ΔGG_1G_0 is similar to Δbb_1m

$$\frac{GG_1}{bb_1} = \frac{w}{W} \Rightarrow \boxed{GG_1 = \frac{w}{W} bb_1}$$

If ϕ is small,

$$GG_1 \approx GG_0 \sin\phi, \quad b_1m \approx bm \sin\phi$$

$$GG_0 \sin\phi = \frac{w}{W} bm \sin\phi$$

$$GG_0 = \frac{w}{W} bm$$

$$= \frac{w}{W}\frac{i_T}{v} = \frac{\rho_F g v}{\rho_{SW} g \nabla}\frac{i_T}{v}$$

$$= \frac{\rho_F}{\rho_{SW}}\frac{i_T}{\nabla}$$

m : Metacenter of cargohold
w: Weight of fluid in tank
W: Total weight
i_T: 2nd moment of fluid plane area in tank about x' axis
ρ_F : Density of fluid in tank
ρ_{SW} : Density of sea water
∇ : Displacement volume
v : Volume of fluid in tank

Free Surface Moment $\rho_F i_T$

The elevation of center of total mass due to the shift of center of mass of fluid in tank is related to **the density of fluid in tank** and **2nd moment of fluid plane area in tank**.

(It's not relevant to weight of fluid in tank)

Problem> Free Surface Effect

Question)

Sea water is filled partially in tank with waterplane of rectangular shape.

If longitudinal bulkhead is installed in center of tank, how much GM will be changed?