Naval Architecture & Ocean Engineering

- Ship Stability -

Ch.4 Concept of Righting Moment

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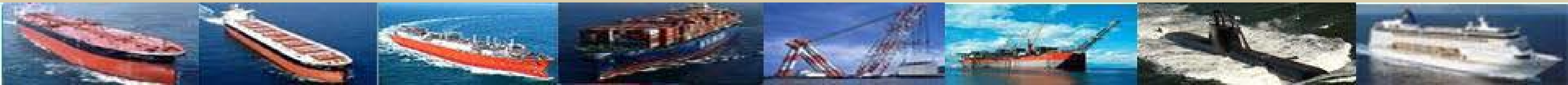
Prof. Kyu-Yeul Lee

Department of Naval Architecture and Ocean Engineering,
Seoul National University

Seoul National Univ.

@SDAL
Advanced Ship Design Automation Lab.
http://asdal.snu.ac.kr



Concept of Righting Force and Moment

Righting Moment

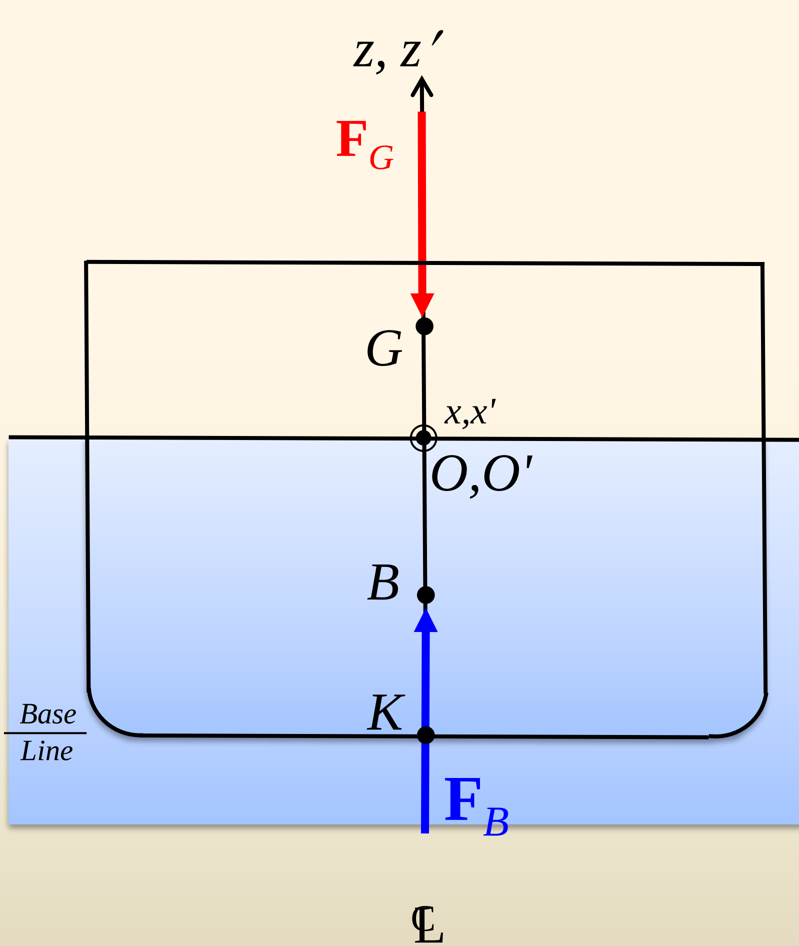
- When a ship is inclined, the moment tending to return to an upright position is called “righting moment”.
- There are transverse righting moment due to heel, and longitudinal righting moment due to trim.
- Because a ship is usually capsized due to heel, our main concern is whether transverse righting moment is enough or not, and that is a indication of ship stability.
- Transverse righting moment is expressed shortly as “righting moment”,
- Transverse stability is shortened to “stability”.

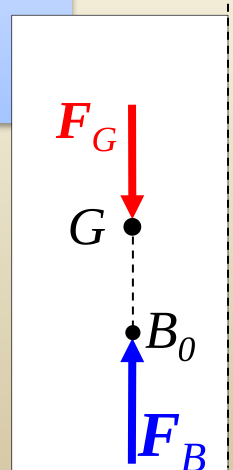
Transverse Righting Moment(1)

Heel case in ship hydrostatics

$$0 = \mathbf{M}_{Gravity}(\xi_4) + \mathbf{M}_{Buoyancy}(\xi_4) + \mathbf{M}_{T,Ext,static}(\xi_4)$$

$$0 = \mathbf{r}_G \times \mathbf{F}_{Gravity}(\xi_4) + \mathbf{r}_B \times \mathbf{F}_{Buoyancy}(\xi_4) + \mathbf{M}_{T,Ext,static}(\xi_4)$$





① $$\sum \mathbf{F_Z} = \mathbf{F_G} + \mathbf{F_B} = F_G\,\mathbf{k} + F_B\,\mathbf{k} = 0$$

(static equilibrium of force)

② Center of mass(G) and center of buoyancy (B) are in the same vertical line which is perpendicular to waterplane → y components of moment arms about origin O about z axis are same. (static equilibrium of moment)

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B = 0 + 0 = 0$$

O'x'y'z' : Body fixed frame

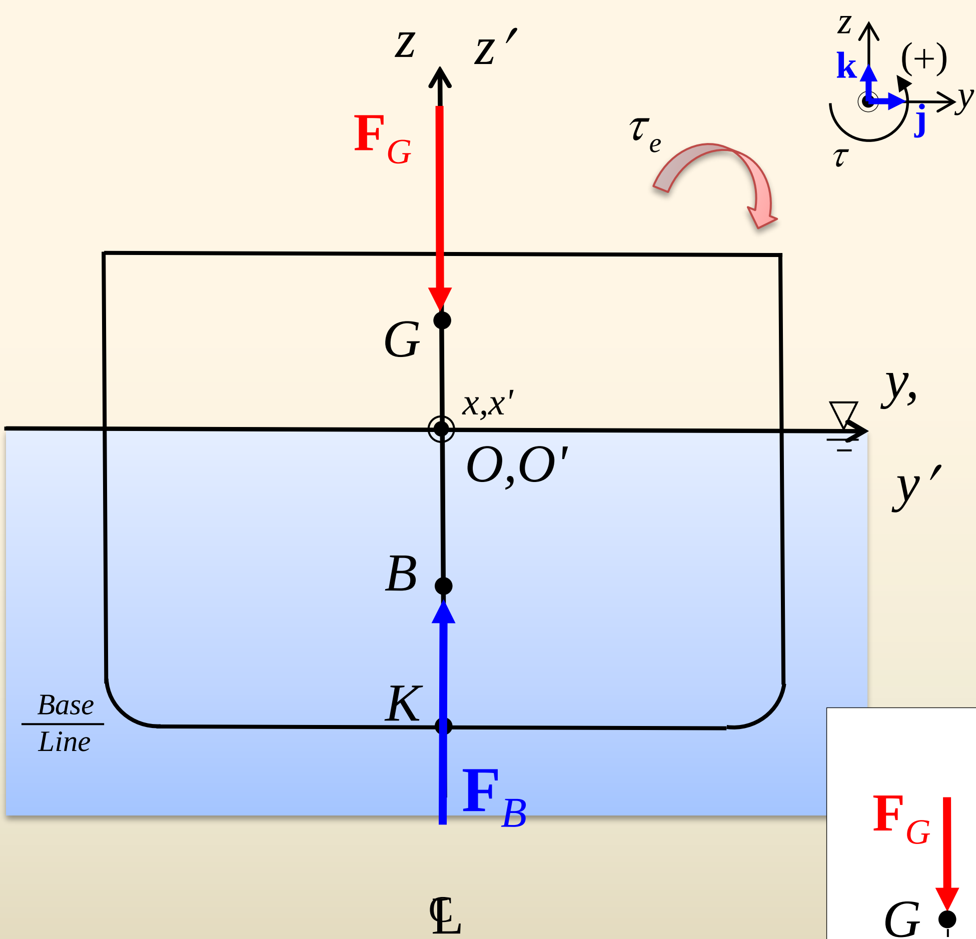
Oxyz : Waterplane fixed frame

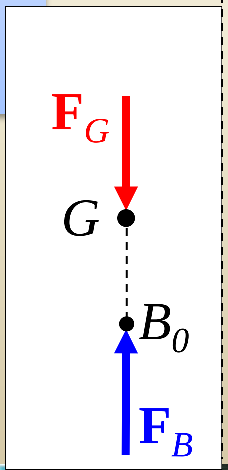
G: Center of mass K : Keel

B: Center of buoyancy

$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship

Transverse Righting Moment(2)





③ External moment ($\boldsymbol{\tau}_e$) is applied on the ship in clockwise.
(Negative moment is applied)

④ A ship is heeled about origin O through an angle of ϕ.

O'x'y'z' : Body fixed frame

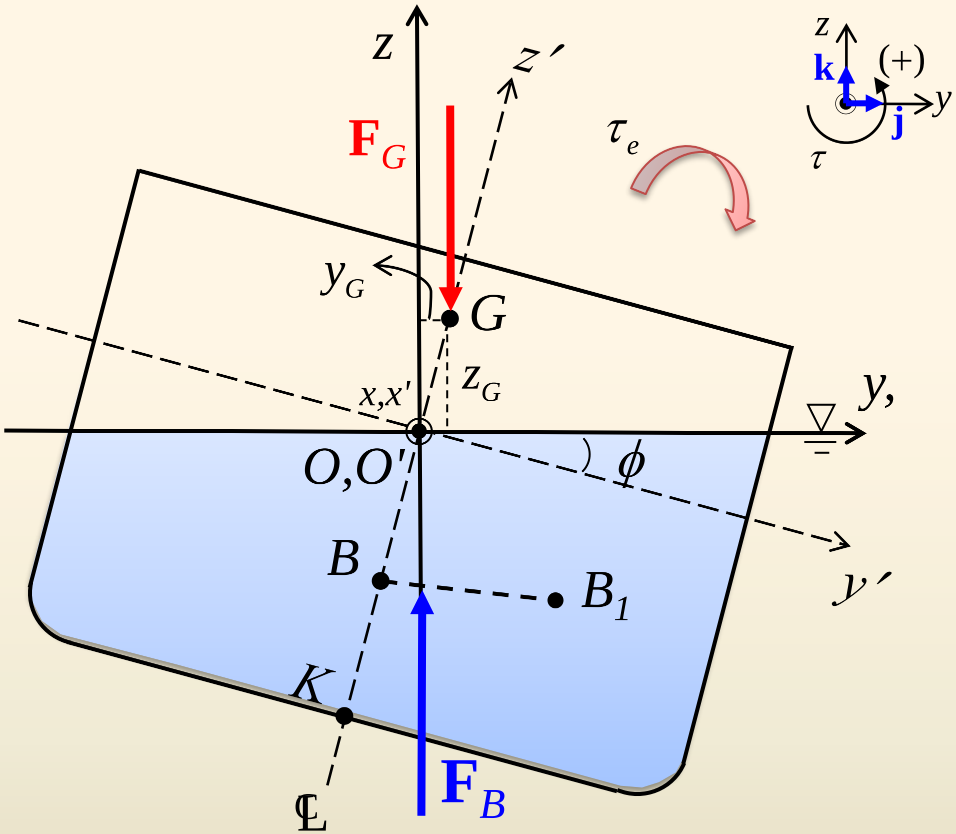
Oxyz : Waterplane fixed frame

G: Center of mass *K*:Keel

B: Center of buoyancy

$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship

Transverse Righting Moment(3)



③ External moment ($\boldsymbol{\tau}_e$) is applied on the ship in clockwise.
(Negative moment is applied)

④ A ship is heeled about origin O through an angle of ϕ.

⑤ Center of buoyancy is changed from B_0 to B_1.

Is G also changed?

: G also changed with respect to waterplane fixed frame.

$(y_G, z_G) \rightarrow$ cause heeling moment

G: Center of mass | K: *Keel*

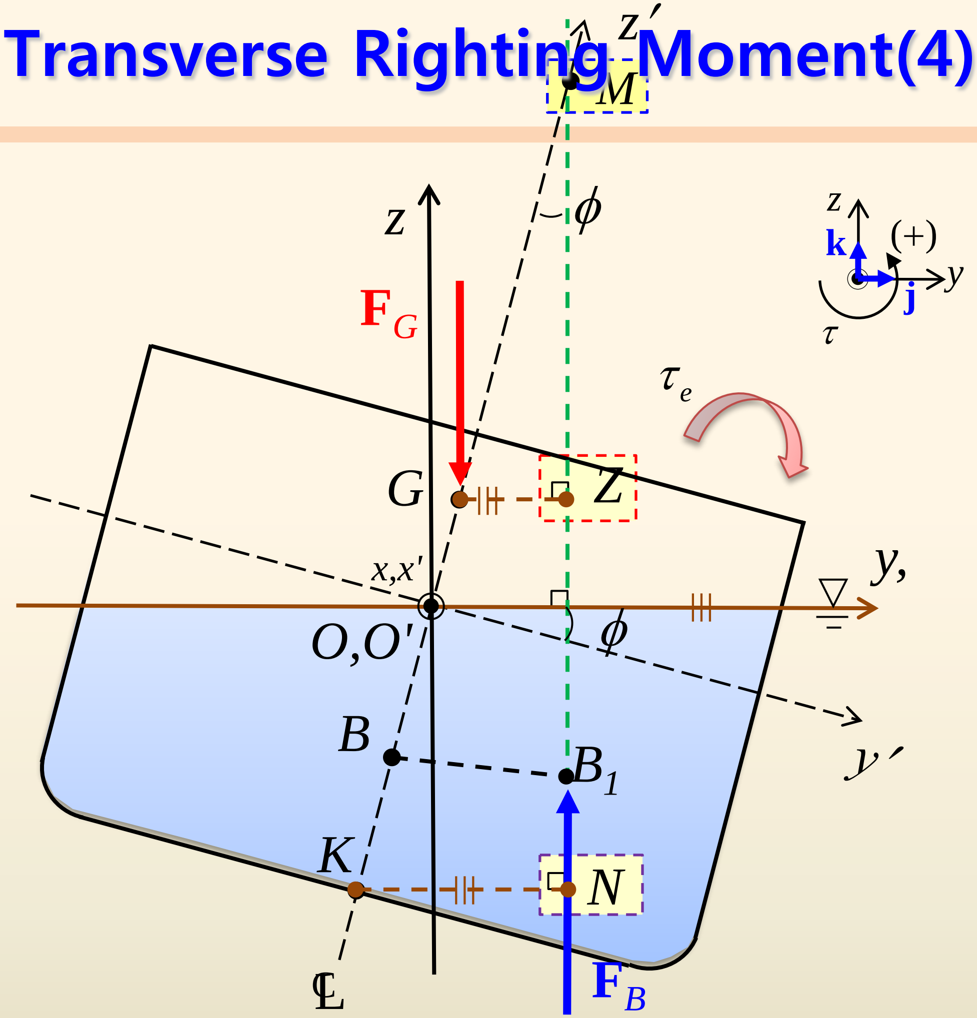
B: Center of buoyancy | B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship | $\mathbf{F}_B$: Buoyant force acting on ship

O'x'y'z' : Body fixed frame

Oxyz : Waterplane fixed frame

Transverse Righting Moment(4)



⑥ Define M, Z, N which is based on the vertical line through heeled center of buoyancy

- M: The intersection of the line of buoyant force through B_1 with the original vertical through the center of buoyancy in the upright position, which is in the ship's centerline plane
- Z: The intersection of the line of buoyant force through B_1 with the transverse line through G
- N: The intersection of the line of buoyant force through B1 with the transverse line through K

$O'x'y'z'$: Body fixed frame

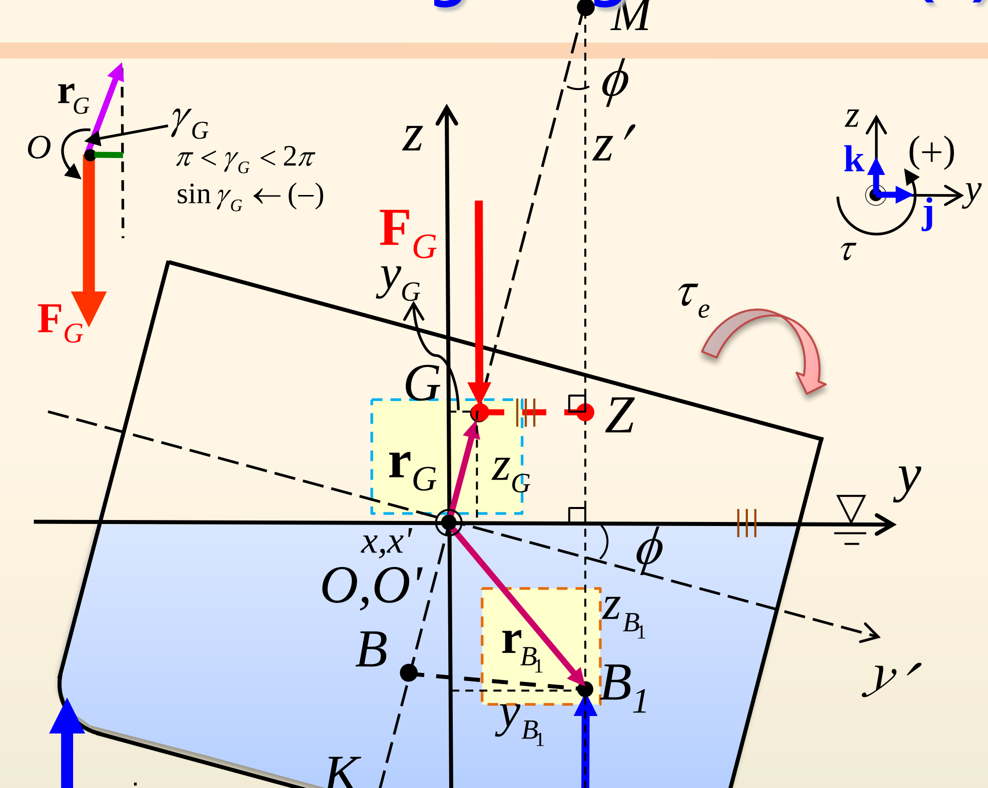
$Oxyz$: Waterplane fixed frame

G: Center of mass K:*Keel*

B: Center of buoyancy B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship

Transverse Righting Moment(5)



G: Center of mass *K*: Keel

B: Center of buoyancy B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship

Z: The intersection of the line of buoyant force through B_1 with the transverse line through *G*

M: The intersection of the line of buoyant force through B_1 with the centerline of the ship

⑦ Moments due to weight of ship and buoyant force are calculated as follows

$$\sum \boldsymbol{\tau}_r = \boldsymbol{\tau}_G + \boldsymbol{\tau}_B$$

$$= \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_B \times \mathbf{F}_B$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y_G & z_G \\ 0 & 0 & F_G \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y_{B_1} & z_{B_1} \\ 0 & 0 & F_B \end{vmatrix}$$

$$= y_G \cdot F_G\,\mathbf{i} + y_{B_1} \cdot F_B\,\mathbf{i}$$

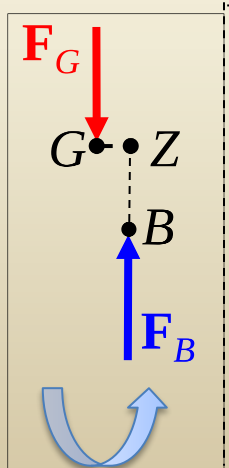
$$\sum \mathbf{F} = \mathbf{F}_G + \mathbf{F}_B = 0$$

$$\Rightarrow F_G\mathbf{k} + F_B\mathbf{k} = 0 \quad \Rightarrow \quad F_G = -F_B$$

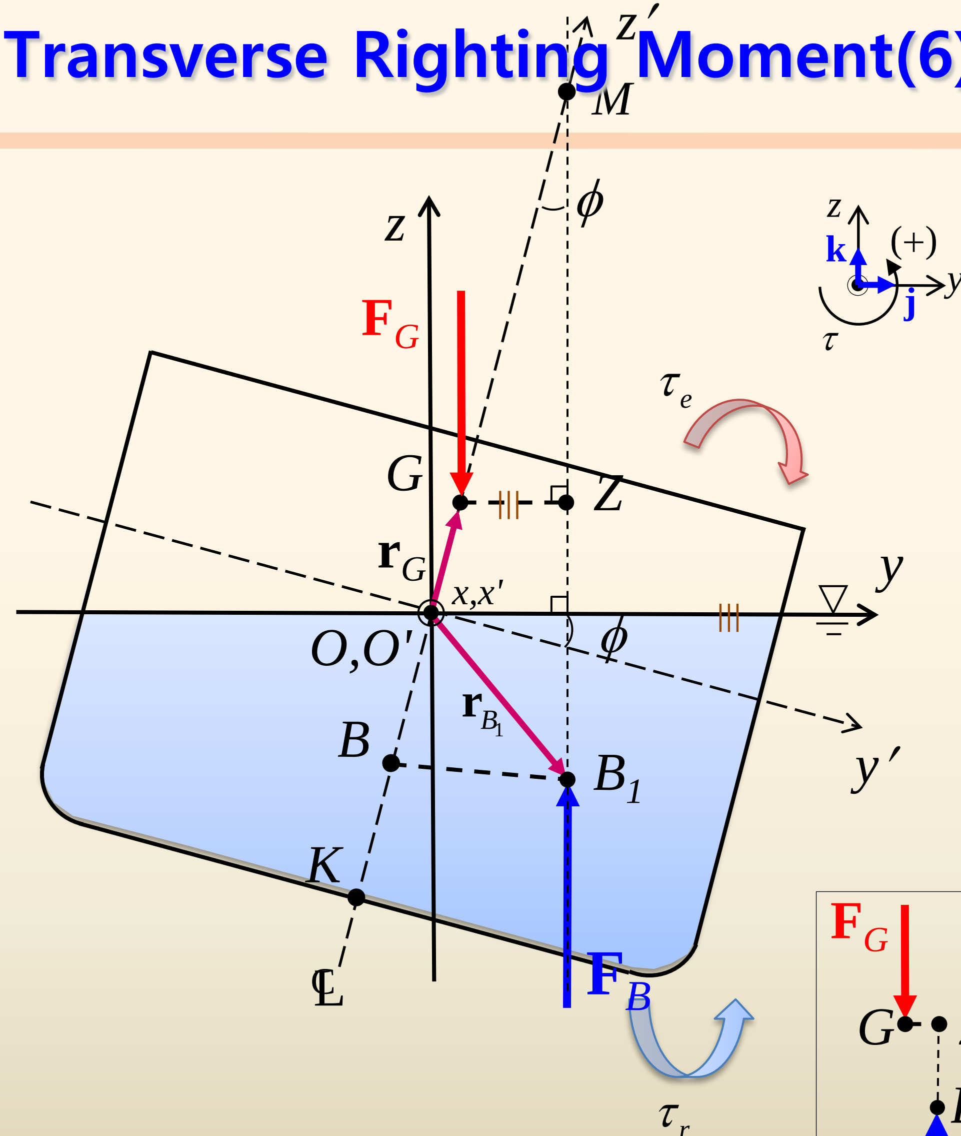
$$\sum \boldsymbol{\tau}_r \mathbf{i} = (-y_G + y_{B_1}) \cdot F_B$$

Key Point!

$$\boxed{= \mathbf{i}\, GZ \cdot F_B}$$



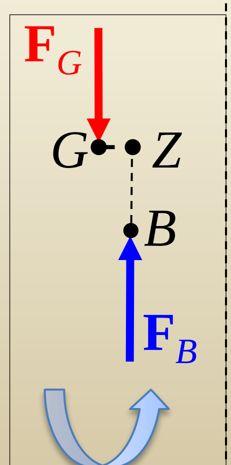
Transverse Righting Moment(6)



⑧ B_1 moves to the right side until transverse righting moment and external moment are balanced.

Then a ship is in the static equilibrium at position of angle ϕ of heel

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_e + \boldsymbol{\tau}_r = 0$$

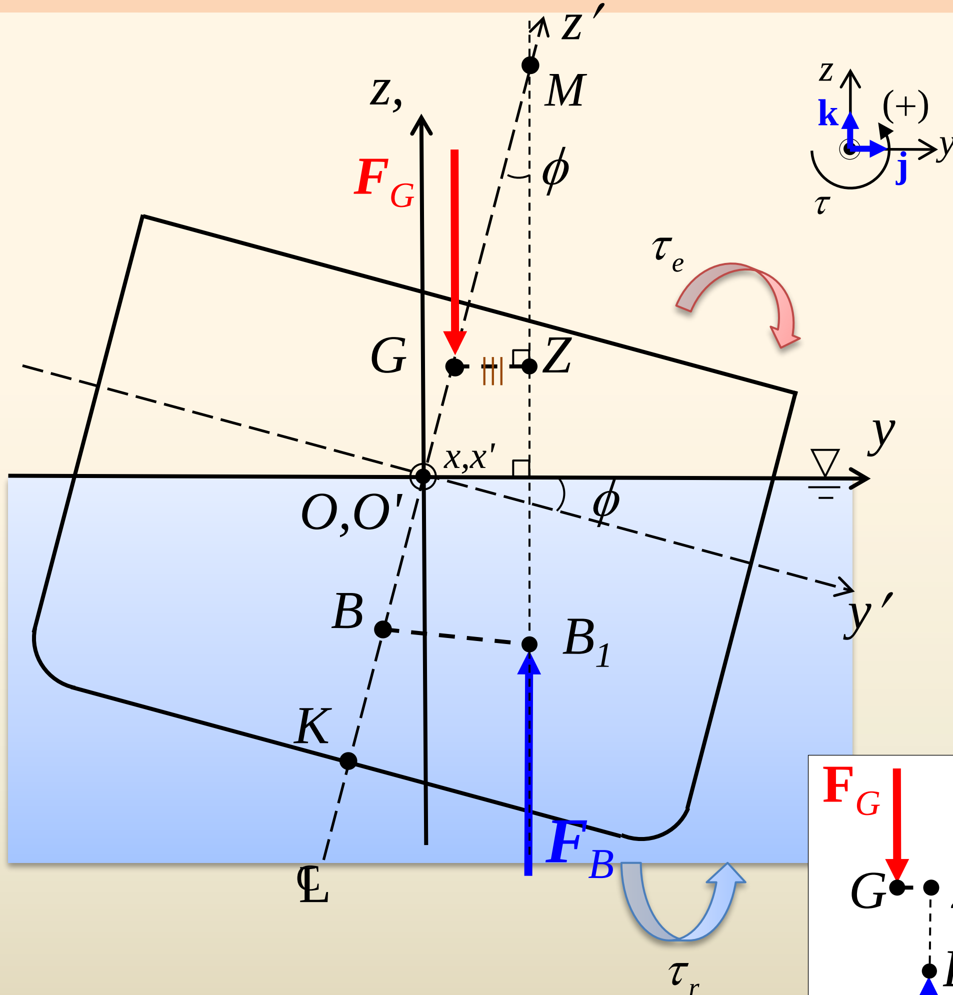


G: Center of mass
K: Keel
B: Center of buoyancy
B_1: Changed center of buoyancy
$\mathbf{F}_G$: Weight of ship
$\mathbf{F}_B$: Buoyant force acting on ship
Z: The intersection of the line of buoyant force through B_1 with the transverse line through G
M: The intersection of the line of buoyant force through B_1 with the centerline of the ship

$O'x'y'z'$: Body fixed frame

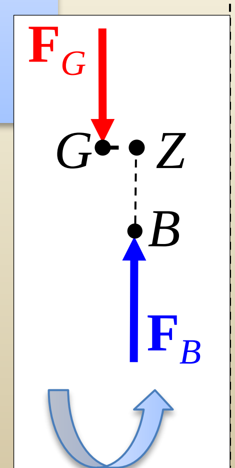
$Oxyz$: Waterplane fixed frame

Transverse Righting Moment(7)



⑨ Remove external moment on ship.

⑩ A ship **returns to upright floating position** due to transverse righting moment ($\boldsymbol{\tau}_e$)



G: Center of mass K: Keel

B: Center of buoyancy B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship

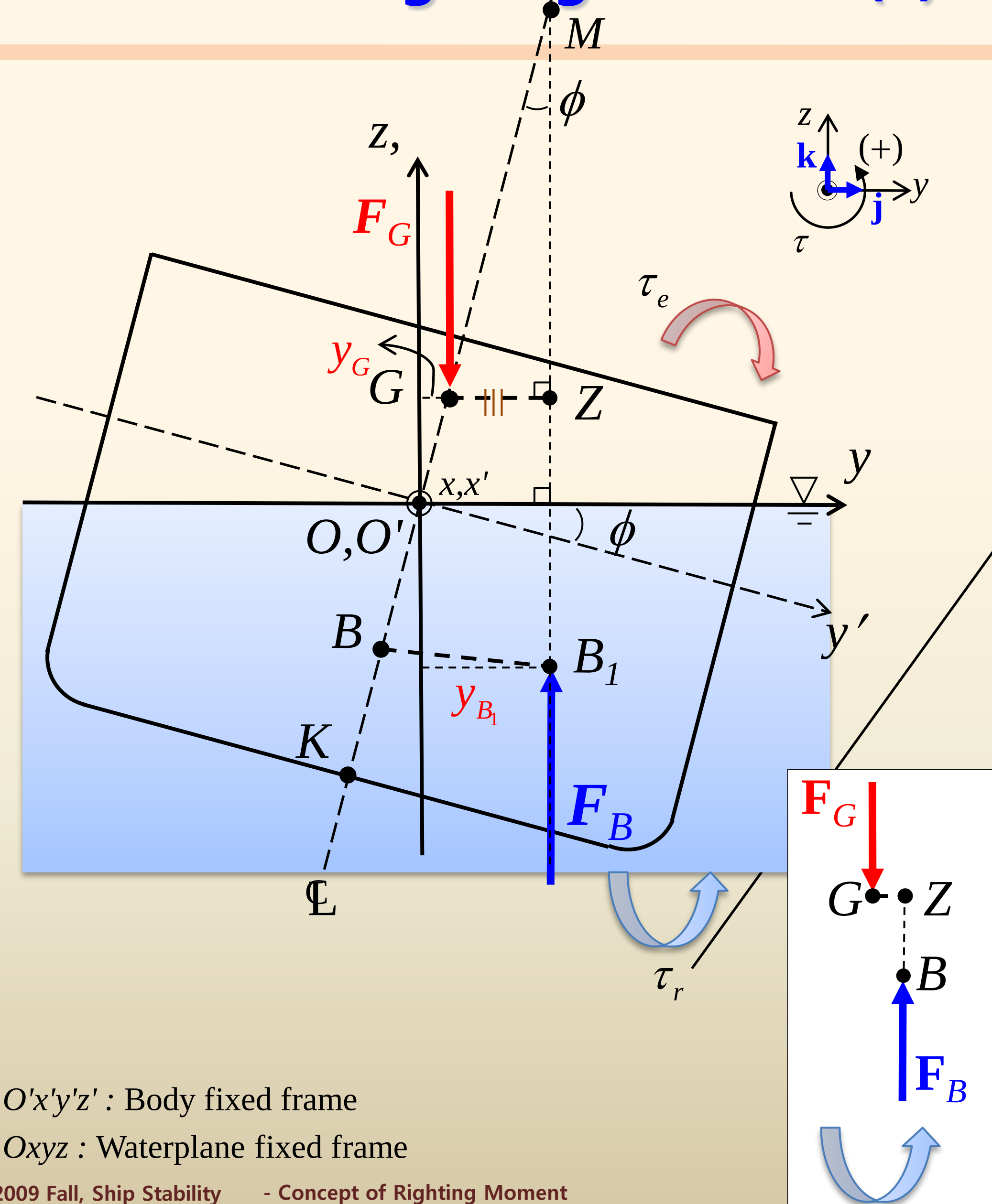
Z : The intersection of the line of buoyant force through B_1 with the transverse line through G

M : The intersection of the line of buoyant force through B_1 with the centerline of the ship

$O'x'y'z'$: Body fixed frame

$Oxyz$: Waterplane fixed frame

Transverse Righting Moment(8)



⑨ Remove external moment on ship.

⑩ A ship **returns to upright floating position** due to transverse righting moment ($\boldsymbol{\tau}_r$)

Moment to return to upright position

→ Righting moment

$$\sum \boldsymbol{\tau}_r \mathbf{i} = \underbrace{(-y_G + y_{B_1})}_{y_G,\ y_{B1}\text{ in waterplane fixed frame}} \cdot F_B$$

$$\boxed{= \underbrace{GZ}_{\text{Righting arm}} \cdot F_B\, \mathbf{i}}$$

$\tau_r \rightarrow \tau_{righting}$

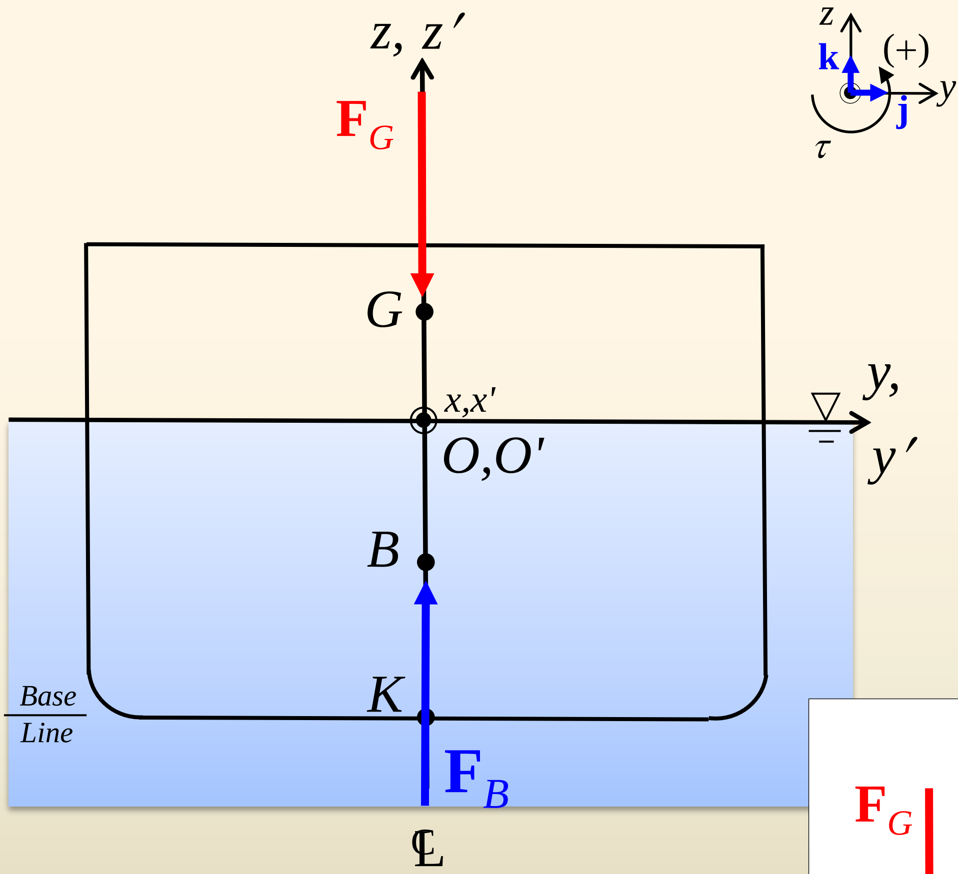
Righting Moment

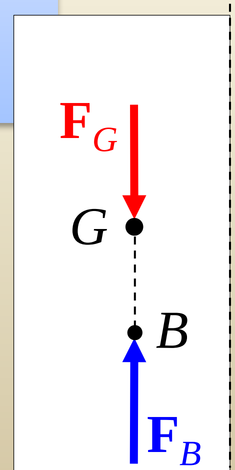
G: Center of mass
K : Keel
B: Center of buoyancy
B_1: Changed center of buoyancy
$\mathbf{F}_G$: Weight of ship
$\mathbf{F}_B$: Buoyant force acting on ship
Z : The intersection of the line of buoyant force through B_1 with the transverse line through G
M : The intersection of the line of buoyant force through B_1 with the centerline of the ship

$O'x'y'z'$: Body fixed frame

$Oxyz$: Waterplane fixed frame

Transverse Righting Moment(9)





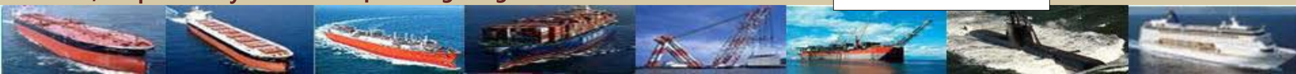
⑪ Center of mass(G) and center of buoyancy (B) are in the same vertical line which is perpendicular to waterplane. Then it becomes in static equilibrium of moment.

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_e + \boldsymbol{\tau}_{righting}$$
$$= 0 + 0$$
$$= 0$$

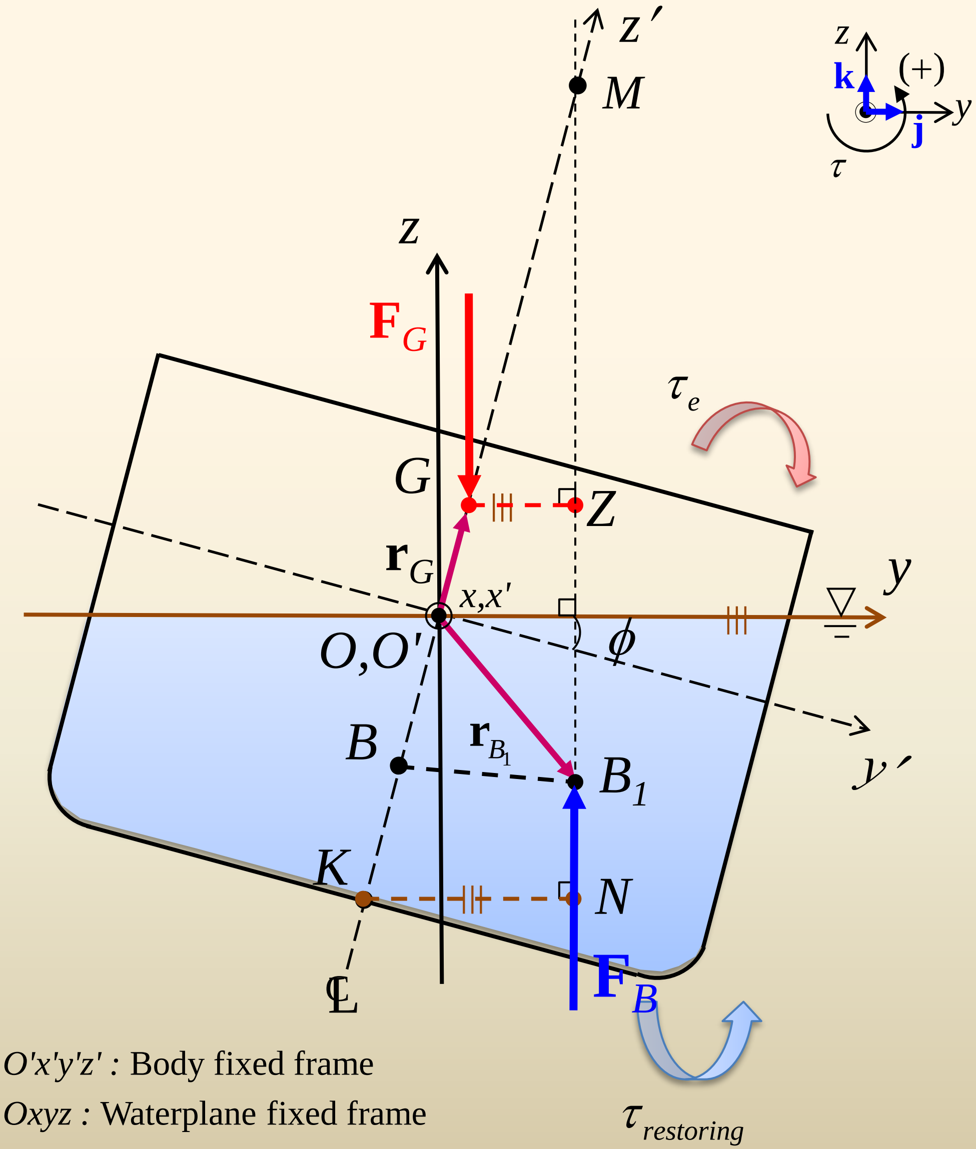
O'x'y'z' : Body fixed frame

Oxyz : Waterplane fixed frame

G: Center of mass
K : Keel
B: Center of buoyancy
B_1: Changed center of buoyancy
$\mathbf{F}_G$: Weight of ship
$\mathbf{F}_B$: Buoyant force acting on ship



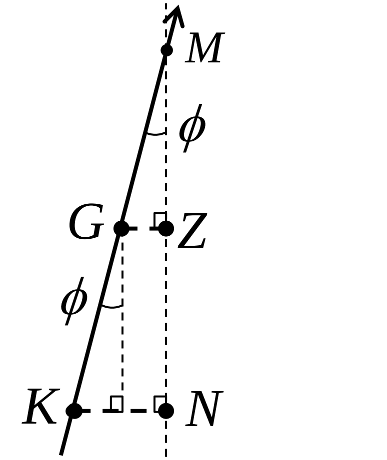
Transverse Righting Moment(10)



$O'x'y'z'$: Body fixed frame

$Oxyz$: Waterplane fixed frame

※ GZ can be calculated in geometrical shape.



$$GZ = KN - KG\sin\phi$$

G: Center of mass

K : Keel

B: Center of buoyancy

B_1: Changed center of buoyancy

$\mathbf{F}_G$: Weight of ship

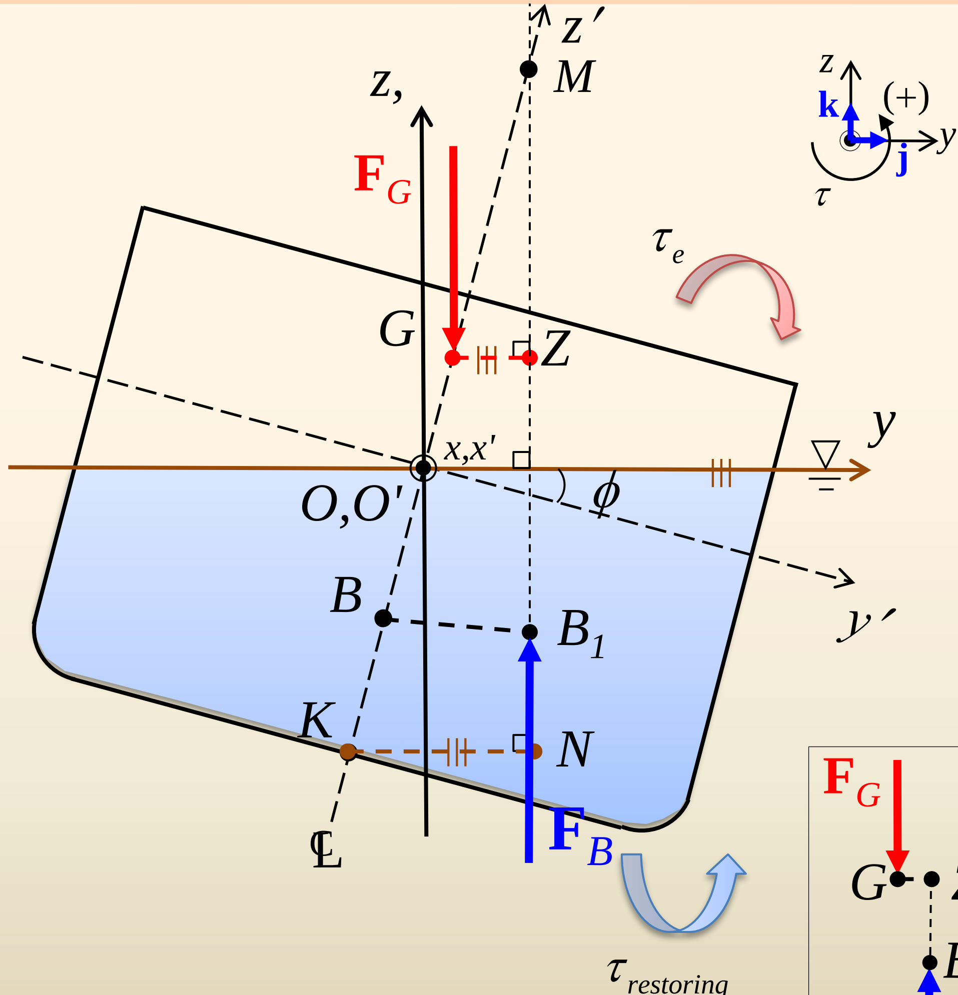
$\mathbf{F}_B$: Buoyant force acting on ship

Z : The intersection of the line of buoyant force through B1 with the transverse line through G

M : The intersection of the line of buoyant force through B1 with the centerline of the ship

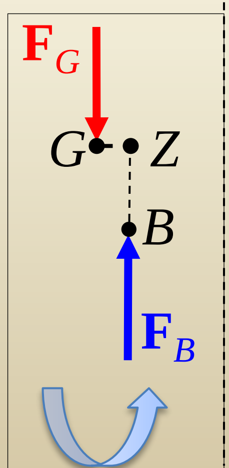
K : The intersection of the line of buoyant force through B1 with the transverse line through K

Transverse Righting Moment(11)



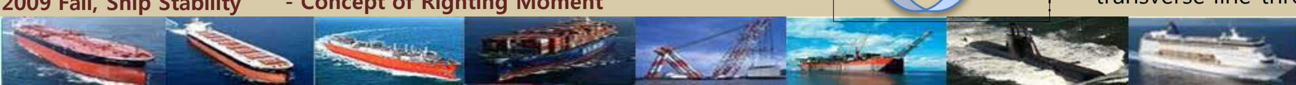
- **Righting Moment :** Moment to return the ship to the upright floating position (Moment of statical stability))

- **Stability :** Capability to return the ship from heeled position to the upright floating position when the cause of heeling is removed

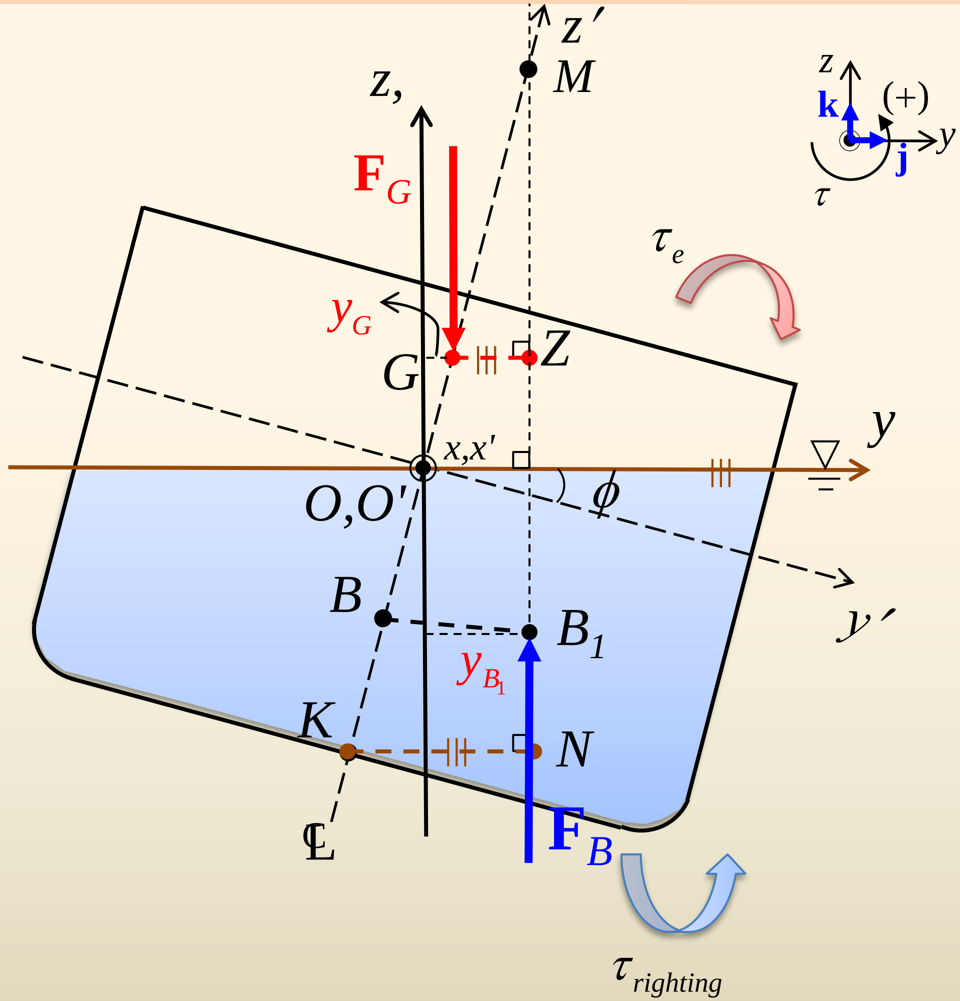


G: Center of mass K : Keel
B: Center of buoyancy B_1: Changed center of buoyancy
$\mathbf{F}_G$: Weight of ship $\mathbf{F}_B$: Buoyant force acting on ship
Z : The intersection of the line of buoyant force through B1 with the transverse line through G
M : The intersection of the line of buoyant force through B1 with the centerline of the ship
K : The intersection of the line of buoyant force through B1 with the transverse line through K

$O'x'y'z'$: Body fixed frame
$Oxyz$: Waterplane fixed frame



Transverse Righting Moment(12)



O'x'y'z' : Body fixed frame

Oxyz : Waterplane fixed frame

- **Righting Moment :** Moment to return the ship to the upright floating position (Moment of statical stability))

- **Transverse Righting Moment**

$$\tau_{righting} = F_B \cdot \underline{GZ}\,\mathbf{i}$$

$\mathbf{F}_B$ is given as force equilibrium,

$\mathbf{F}_B = -\,\mathbf{F}_G$

We should know $\underline{GZ}$ in order to know transverse righting moment

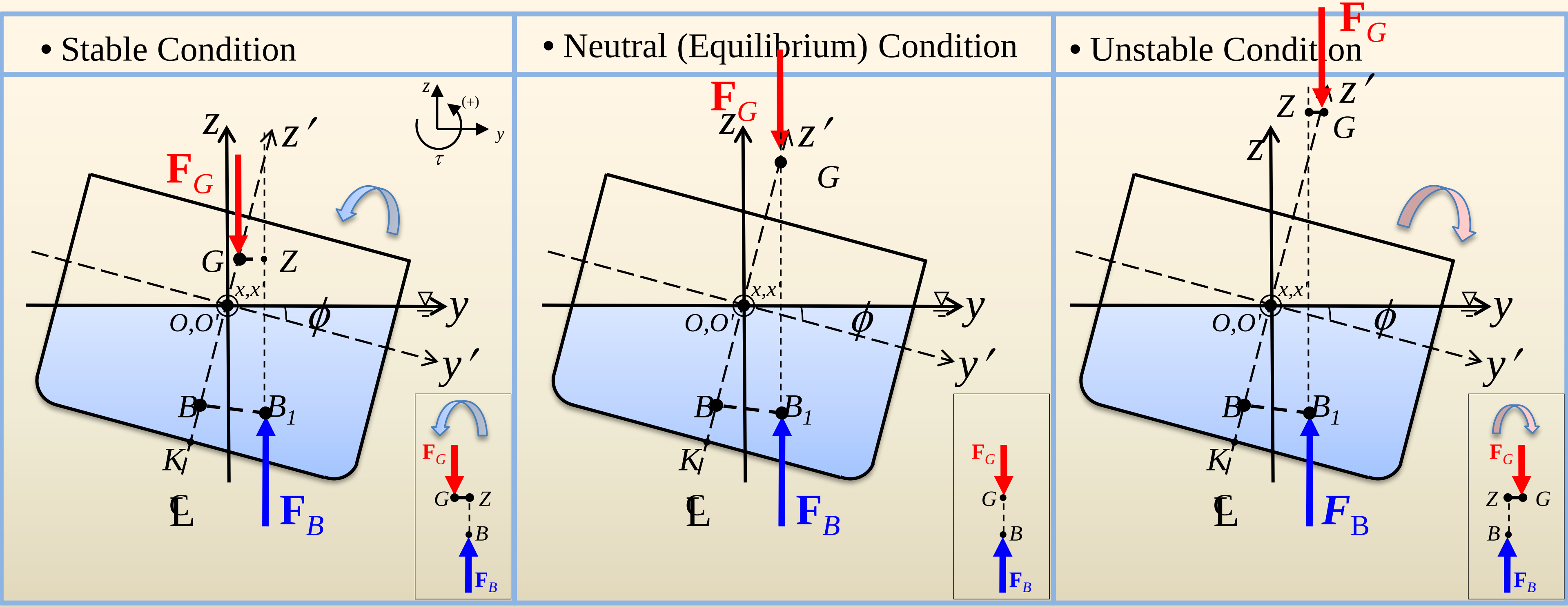
- **Transverse Righting Moment**

$$\tau_{righting} = F_B \cdot \underline{GZ}\,\mathbf{i}$$

$$= \underbrace{(-y_G + y_{B_1})}_{\text{With respect to waterplane fixed frame}} \cdot F_B\,\mathbf{i}$$

Stability of Ship
- Stable / Neutral / Unstable Condition

- **Righting Moment :** Moment to return the ship to the upright floating position (Moment of statical stability))



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