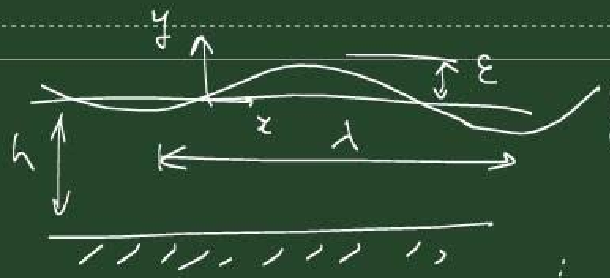
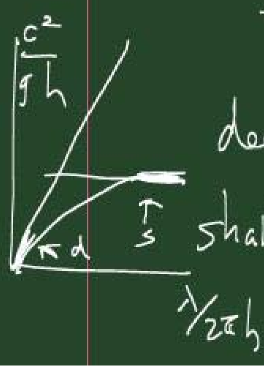
노트 제목 2010-05-06



$y = \eta(x,t)$

$= \varepsilon \sin \frac{2\pi}{\lambda}(x - ct)$

$\rightarrow \frac{c^2}{gh} = \frac{\lambda}{2\pi h} \tanh \frac{2\pi h}{\lambda}$



deep liquids ($h \gg \lambda$), $\frac{c^2}{gh} = \frac{\lambda}{2\pi h} \rightarrow c = fn(\lambda)$

shallow liquids ($h \ll \lambda$), $\frac{c^2}{gh} = 1 \rightarrow c = fn(h)$

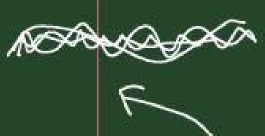
Arbitrary shaped wave → superposition of sinusoidal waves

↳ use Fourier transf.

→ decomposition of different wavelengths

In the case of shallow liq, $c = fn(h)$

→ waveforms will not change because



they have same speed of propagation.

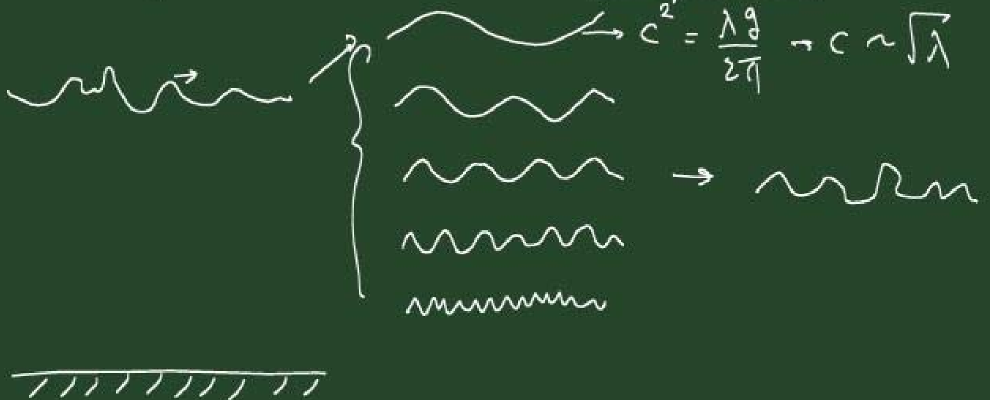
$c \sim f(h)$

In the case of deep liq, $c = fn(\lambda)$

→ each wave has its own speed of propagation

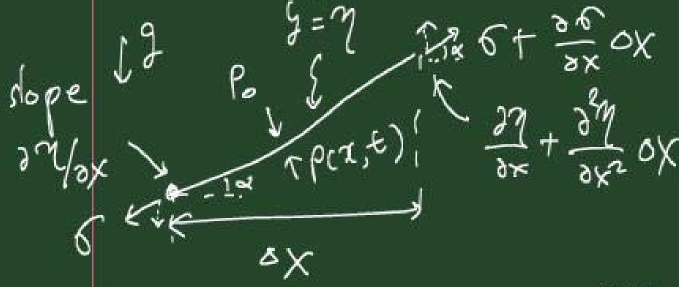
→ waveforms will change continuously.

→ This process is called as '<u>dispersion</u>'



$$c^2 = \frac{\lambda g}{2\pi} \rightarrow c \sim \sqrt{\lambda}$$

4. Effect of surface tension



small amplitude ε

$$\tan\alpha = \frac{\partial \eta}{\partial x} \simeq \sin\alpha$$

$$\Sigma F_y = (p - p_0)\,\delta x + \left(\sigma + \frac{\partial \sigma}{\partial x}\delta x\right)\left(\frac{\partial \eta}{\partial x} + \frac{\partial^2 \eta}{\partial x^2}\delta x\right) - \sigma \frac{\partial \eta}{\partial x} = 0$$

$$\rightarrow p - p_0 + \sigma \frac{\partial^2 \eta}{\partial x^2} + \underbrace{\frac{\partial \sigma}{\partial x}}_{\hookrightarrow\, 0 \text{ if } \sigma \simeq \text{const}} \frac{\partial \eta}{\partial x} = 0$$

$$\rightarrow p(x,t) = p_0 - \sigma \frac{\partial^2 \eta}{\partial x^2}$$

Dynamic bdry cond.

$$\frac{\partial^2\phi}{\partial t^2}(x,0,t) + \frac{1}{\rho}\frac{\partial p^{(x,t)}}{\partial t} + g\frac{\partial\phi}{\partial y}(x,0,t) = 0$$

$$\frac{\partial p}{\partial t} = \cancel{\frac{\partial p_0}{\partial t}}^{\,0} - \sigma\frac{\partial}{\partial t}\left(\frac{\partial^2\eta}{\partial x^2}\right)$$

$$= -\sigma\frac{\partial^2}{\partial x^2}\left(\frac{\partial\eta}{\partial t}\right)$$ kinematic b.c.

$$= -\sigma\frac{\partial^2}{\partial x^2}\frac{\partial\phi}{\partial y}(x,0,t)$$

$$\therefore\ \frac{\partial^2\phi}{\partial t^2}(x,0,t) - \frac{\sigma}{\rho}\frac{\partial^3\phi}{\partial x^2\partial y}(x,0,t) + g\frac{\partial\phi}{\partial y}(x,0,t) = 0$$

Surface tension does not change governing eq., kinematic b.c, and bed b.c. except dynamic b.c.

$$\rightarrow \phi(x,y,t) = C_2\cos\frac{2\pi}{\lambda}(x-ct)\left\{\tanh\frac{2\pi h}{\lambda}\sinh\frac{2\pi y}{\lambda} + \cosh\frac{2\pi y}{\lambda}\right\}$$

Substitute into dynamic b.c.

$$\rightarrow \frac{c^2}{gh} = \frac{\lambda}{2\pi h}\left[1+\frac{\sigma}{\rho g}\left(\frac{2\pi}{\lambda}\right)^2\right]\tanh\frac{2\pi h}{\lambda}.$$

if $\sigma \ll 1$, $\frac{c^2}{gh} = \frac{\lambda}{2\pi h}\tanh\frac{2\pi h}{\lambda}$

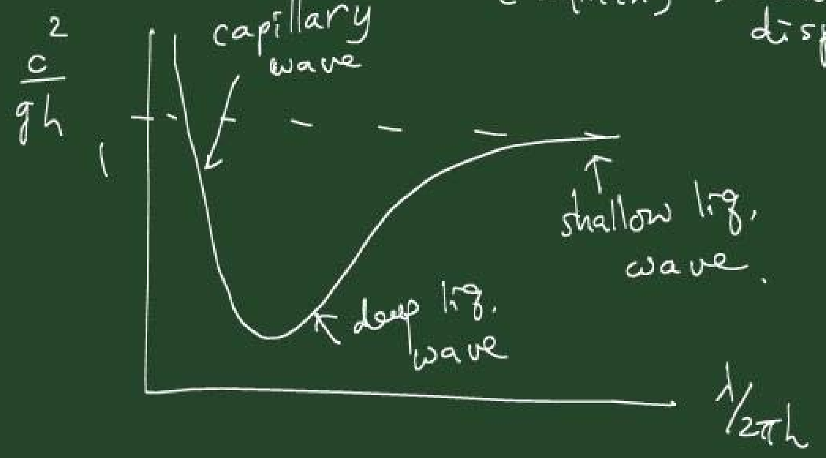
$\therefore$ the effect of surface tension is to increase the propagation speed of the wave.

For deep wgs, $\frac{2\pi h}{\lambda} \gg 1$

$$\rightarrow \frac{c^2}{gh} = \frac{\lambda}{2\pi h}\left(1+\frac{\sigma}{\rho g}\left(\frac{2\pi}{\lambda}\right)^2\right)$$

if $\frac{\sigma}{\rho g}\left(\frac{2\pi}{\lambda}\right)^2 >> 1$, $\frac{c^2}{gh} = \frac{2\pi\sigma}{\rho g \lambda h}$: Capillary wave

$c = fn(\lambda) \rightarrow$ wave disperses.



5. shallow-liquid waves of arbitrary form

$$u(x,t) = f_1(x-\sqrt{gh}\,t) + g_1(x+\sqrt{gh}\,t) \qquad \frac{c^2}{gh} = 1$$

$$\eta(x,t) = f_2(\;\;\text{"}\;\;) + g_2(\;\;\text{"}\;\;)$$

shape of the wave doesn't change as it moves along the surface.

6. Complex potential for traveling waves

For $\eta(x,t) = \varepsilon \sin\frac{2\pi}{\lambda}(x-ct)$

$$\phi = c_2 \cos\frac{2\pi}{\lambda}(x-ct)\left[\tanh\frac{2\pi h}{\lambda}\sinh\frac{2\pi y}{\lambda} + \cosh\frac{2\pi y}{\lambda}\right]$$

kinematic b.c. $\frac{\partial\phi}{\partial y} = \frac{\partial\eta}{\partial t}$

$$\rightarrow c_2 = -\frac{c\varepsilon}{\tanh\frac{2\pi h}{\lambda}}.$$

$$\therefore \phi(x,y,t) = -c\varepsilon\cos\frac{2\pi}{\lambda}(x-ct)\left[\sinh\frac{2\pi y}{\lambda} + \coth\frac{2\pi h}{\lambda}\cosh\frac{2\pi y}{\lambda}\right]$$

$$\left(\frac{c^2}{gh} = \frac{\lambda}{2\pi h}\tanh\frac{2\pi h}{\lambda}\right)$$

To obtain $F = \phi + i\psi$

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

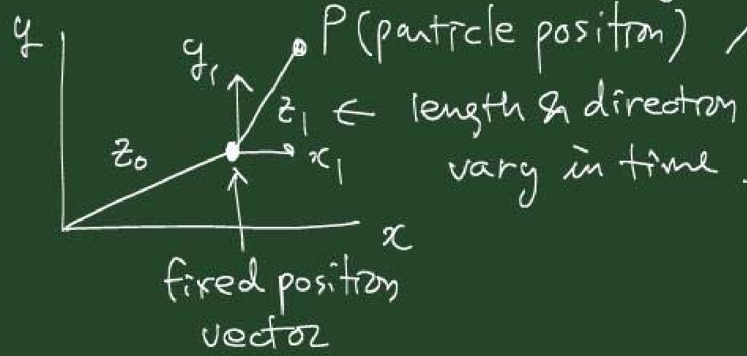
$$\Rightarrow \psi(x,y,t) = c\varepsilon\sin\frac{2\pi}{\lambda}(x-ct)\left[\cosh\frac{2\pi y}{\lambda} + \coth\frac{2\pi h}{\lambda}\sinh\frac{2\pi y}{\lambda}\right]$$

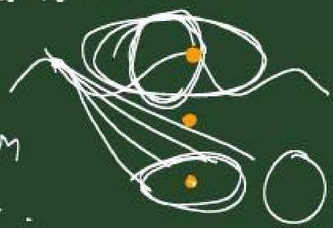
$$\Rightarrow F(z,t) = \phi + i\psi$$

$$= \cdots = \cdots$$

$$\rightarrow \boxed{F(z,t) = -\frac{c\varepsilon}{\sinh\frac{2\pi h}{\lambda}}\cos\frac{2\pi}{\lambda}(z-ct+ih) \quad \text{for } \eta(x,t) = \varepsilon\sin\frac{2\pi}{\lambda}(x-ct) \qquad z = x+iy}$$

7. Particle paths for traveling waves





particle position : $z_1 = x_1 + i y_1$

$z_1^* = x_1 - i y_1$

$$\frac{dz_1^*}{dt} = \frac{dx_1}{dt} - i\frac{dy_1}{dt} = u - iv = \frac{dF}{dz}$$

$$= \frac{\frac{2\pi}{\lambda}c\varepsilon}{\sinh\frac{2\pi h}{\lambda}} \sin\frac{2\pi}{\lambda}(z - ct + ih)$$

$$\rightarrow z_1^* = \frac{\varepsilon}{\sinh\frac{2\pi h}{\lambda}} \cos\frac{2\pi}{\lambda}(z - ct + ih) + \cancel{f(x,y)}\Big|_{t=0}$$

$$= -\frac{1}{c}F(z,t)$$

$$x_1 - iy_1 = -\frac{1}{c}\phi - \frac{1}{c}i\psi$$

$$\Rightarrow x_1 = -\phi/c = \cdot\ \cdot\ \cdot$$

$$y_1 = \psi/c = \cdot\ \cdot\ \cdot$$

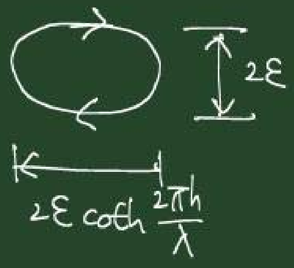
$$\Rightarrow \left(\sin^2\frac{2\pi}{\lambda}(x-ct) + \cos^2\frac{2\pi}{\lambda}(x-ct) = 1\right)$$

$$\Rightarrow \frac{x_1^2}{\varepsilon^2\left[\sinh\frac{2\pi y}{\lambda} + \coth\frac{2\pi h}{\lambda}\cosh\frac{2\pi y}{\lambda}\right]^2} + \frac{y_1^2}{\varepsilon^2\left[\cosh\frac{2\pi y}{\lambda} + \coth\frac{2\pi h}{\lambda}\sinh\frac{2\pi y}{\lambda}\right]^2} = 1$$

$\Rightarrow$ trajectory of a fluid particle (fn. of y)

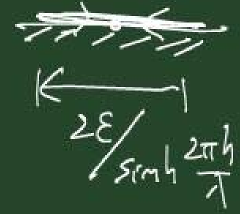
For particles which lie on the free surface, $y=0$

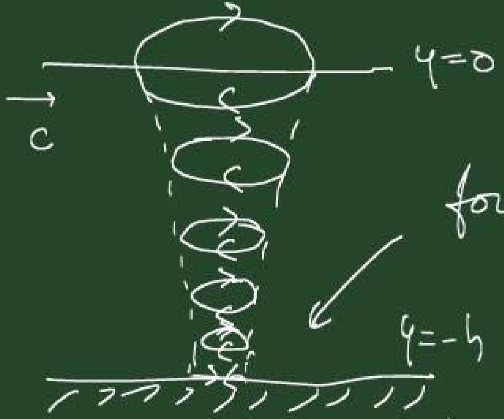
$$\frac{x_1^2}{\left(\varepsilon\coth\frac{2\pi h}{\lambda}\right)^2}+\frac{y_1^2}{\varepsilon^2}=1$$



At $y=-h$, $-\dfrac{\varepsilon}{\sinh\frac{2\pi h}{\lambda}} \le x_1 \le \dfrac{\varepsilon}{\sinh\frac{2\pi h}{\lambda}}$

$y_1=0$



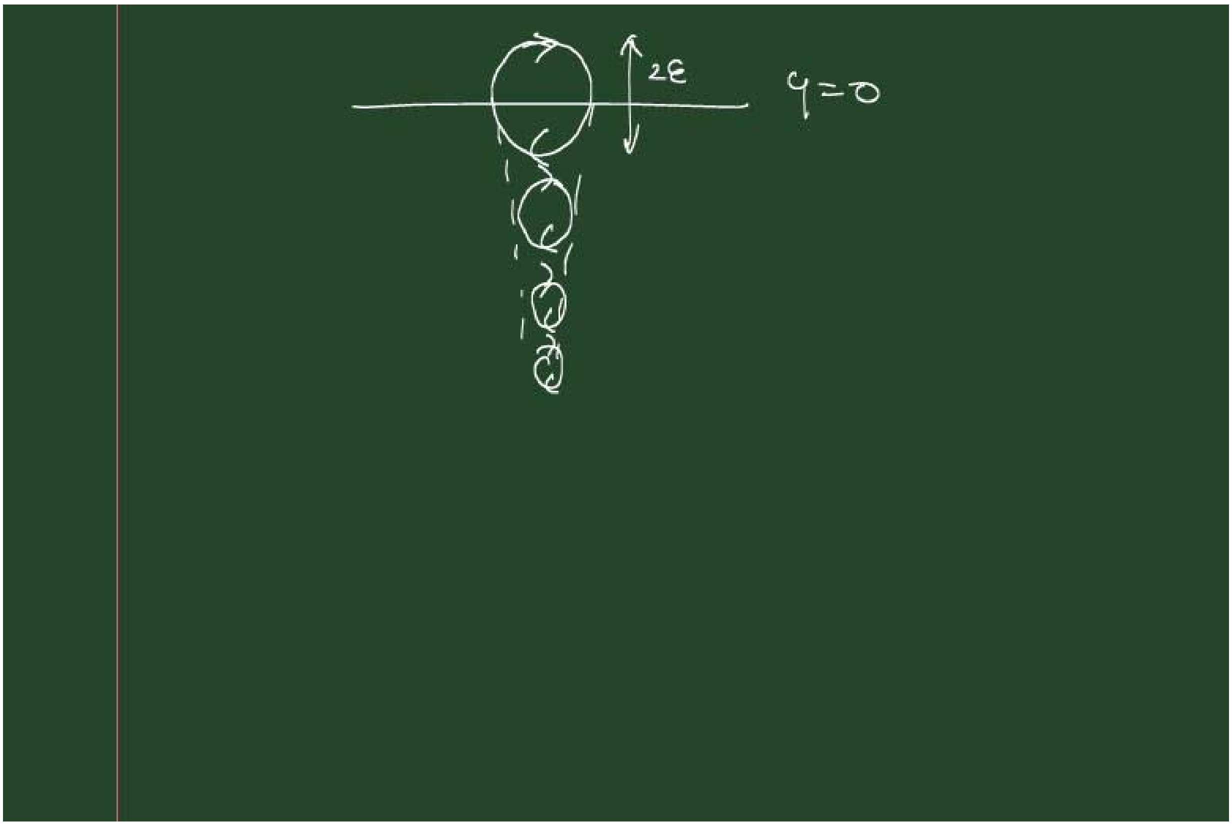


for shallow lqs, the ellipses become elongated in the x direction.

For deep lqs., $\frac{2\pi h}{\lambda} \gg 1 \rightarrow \coth\frac{2\pi h}{\lambda}=1$

$$\rightarrow x_1^2+y_1^2=\varepsilon^2\left[\sinh\frac{2\pi y}{\lambda}+\cosh\frac{2\pi y}{\lambda}\right]^2$$

circles


2ε
y=0