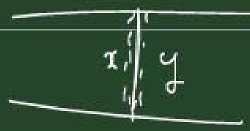
Final exam on June 8 (Tuesday) 6:30pm

노트 제목 2010-05-20

Normal shock



- Normal shock in a perfect gas $(h = c_p T)$

Energy eq: $c_p T_x + \frac{V_x^2}{2} = c_p T_y + \frac{V_y^2}{2} = c_p T_0$

$$T_{0x} = T_{0y}$$

$$\frac{T_y}{T_x} = \frac{T_y}{T_{0y}} \cdot \frac{T_{0x}}{T_x} = \frac{1 + \frac{k-1}{2} M_x^2}{1 + \frac{k-1}{2} M_y^2} \quad \text{— ①}$$

$$p = \rho R T$$

$$\frac{T_y}{T_x} = \frac{p_y}{p_x} \cdot \frac{\rho_x}{\rho_y} = \frac{p_y}{p_x} \frac{V_y}{V_x} = \frac{p_y}{p_x} \cdot \frac{M_y c_y}{M_x c_x} = \frac{p_y}{p_x} \frac{M_y}{M_x} \sqrt{\frac{T_y}{T_x}}$$

$$\rightarrow \frac{T_y}{T_x} = \left(\frac{p_y}{p_x}\right)^2 \left(\frac{M_y}{M_x}\right)^2$$

$$\xrightarrow{①} \frac{p_y}{p_x} = \frac{M_x}{M_y} \cdot \frac{\sqrt{1 + \frac{k-1}{2} M_x^2}}{\sqrt{1 + \frac{k-1}{2} M_y^2}} \quad : \text{Fanno line}$$

Likewise, use mtm. cont. state eqs.

$$\rightarrow \frac{p_y}{p_x} = \frac{1 + k M_x^2}{1 + k M_y^2} \quad : \text{Rayleigh line}$$

Across the normal shock, $\left.\frac{p_y}{p_x}\right|_{\text{Fanno}} = \left.\frac{p_y}{p_x}\right|_{\text{Rayleigh}}$

$\rightarrow \quad M_y^2 = \dfrac{M_x^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_x^2 - 1} < 1 \quad$ — (!!)

given

M_x	M_y
P_x	P_y
ρ_x	T_y
T_x	

$\rightarrow \quad \dfrac{P_y}{P_x} = \dfrac{2k}{k+1} M_x^2 - \dfrac{k-1}{k+1} \quad$ — (!!!)

$\rightarrow \quad \dfrac{T_y}{T_x} = \dfrac{\left(1 + \frac{k-1}{2} M_x^2\right)\left(\frac{2k}{k-1} M_x^2 - 1\right)}{\frac{(k+1)^2}{2(k-1)} M_x^2}$

$\rightarrow \quad \dfrac{\rho_y}{\rho_x} = \dfrac{P_y}{P_x} \Big/ \dfrac{T_y}{T_x} = \cdots \quad ft(M_x)$

$\rightarrow \quad \dfrac{V_y}{V_x} = \dfrac{\rho_x}{\rho_y} = \cdots$

$\dfrac{P_{0y}}{P_{0x}} = \dfrac{P_{0y}}{P_y} \dfrac{P_y}{P_x} \dfrac{P_x}{P_{0x}} = \cdots \quad ft(M_x)$

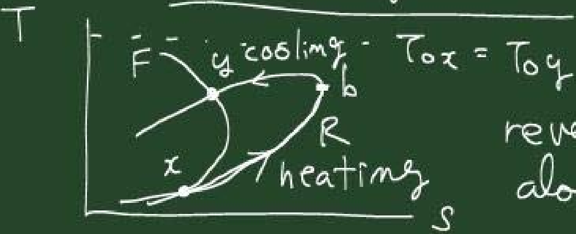
normal shock

isentropic process

entropy change

$s_y - s_x = c_p \ln \dfrac{T_y}{T_x} - R \ln \dfrac{P_y}{P_x} = \cdots \quad ft(M_x)$

$s_y > s_x$ if $M_x > 1$.



reversible heating & cooling along Rayleigh line ($x \rightarrow y$)

Then $Q_{rev} = \int T ds = 0$ $\because T_{0x} = T_{0y}$ $\quad s_b - s_x$

Heating @ low temp $Q_H = \int_x^b T ds \to |\Delta s_x^b|$

cooling @ high 〃 $Q_C = \int_b^y T ds \to |\Delta s_b^y|$ $\quad s_b - s_y$

$s_x \to s_y$

$\boxed{s_y > s_x}$

• Rankine - Hugoniot relation

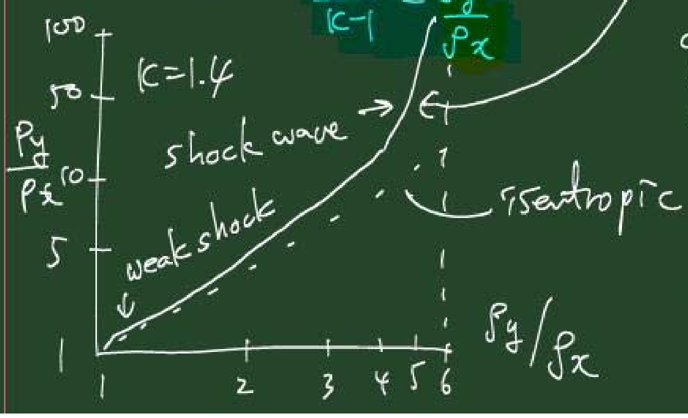
Relation between p_y/p_x and ρ_y/ρ_x

Use (i) - (iii) $\to$ $$\frac{T_y}{T_x} = \frac{\frac{p_y}{p_x}\left[1 + \frac{k-1}{k+1}\frac{p_y}{p_x}\right]}{\frac{p_y}{p_x} + \frac{k-1}{k+1}}$$

$$\frac{\rho_y}{\rho_x} = \frac{p_y}{p_x}\Big/\frac{T_y}{T_x} = \frac{\left(\frac{k+1}{k-1}\right)\frac{p_y}{p_x} + 1}{\frac{p_y}{p_x} + \frac{k-1}{k+1}}$$

$$\to \frac{p_y}{p_x} = \frac{\frac{k+1}{k-1}\frac{\rho_y}{\rho_x} - 1}{\frac{k+1}{k-1} - \frac{\rho_y}{\rho_x}}$$

: Rankine-Hugoniot relation for perfect gas across a normal shock wave



$k=1.4$

$\frac{k+1}{k-1} = \frac{2.4}{0.4} = 6$

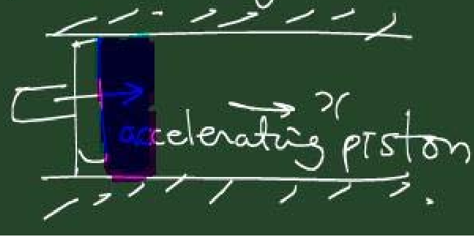
As $\frac{\rho_y}{\rho_x} \to 6$, $\frac{p_y}{p_x} \to \infty$

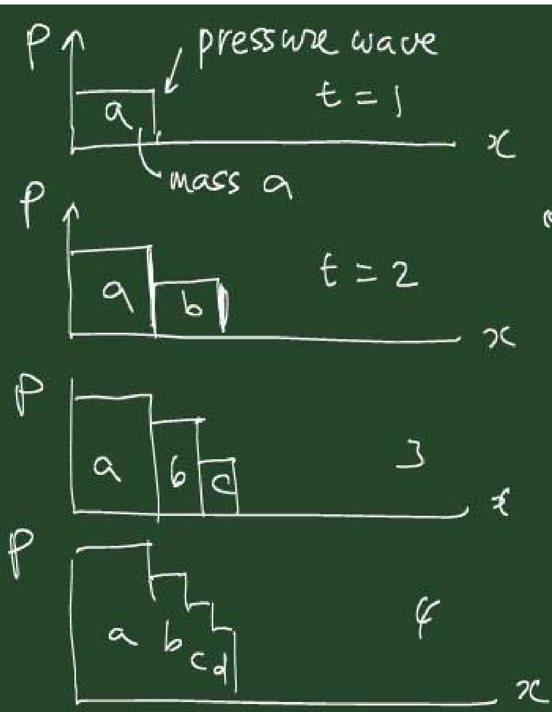
For diatomic gas, density can at most increase by a factor of six, whereas the press. ratio may reach infinity.

Weak shocks are nearly isentropic $\quad p = c\rho^k$

↳ a shock in which the percentage rise in press. is small.

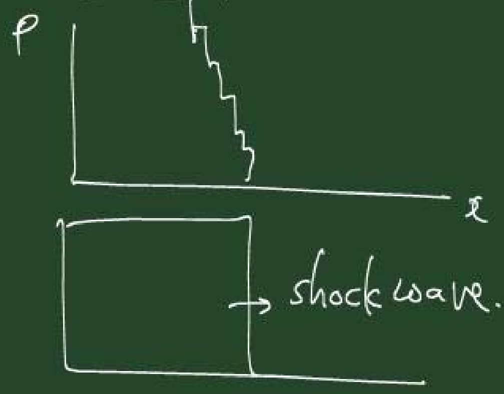
- Formation of shock waves





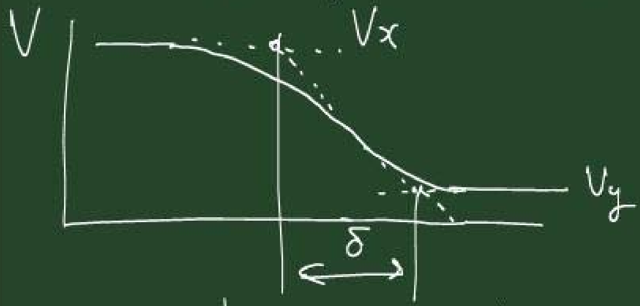
P of mass a ↑

"a" receives further increase in P → T ↑ → c ↑



- Thickness of shock waves

  order of magnitude analysis



$$\delta \equiv \frac{V_x - V_y}{(dV/dx)_{max}}$$

in a shock, press term, inertia ", viscous " } of same order of magnitude

$$\rightarrow \cancel{\tfrac{4}{3}}\,\mu^* \frac{d}{dx}\left(\frac{dV}{dx}\right) \simeq \rho^* V^* \frac{dV}{dx} \qquad (^*: @\, M=1)$$

$\cancel{\tfrac{4}{3}} \approx 1$

$$\rightarrow \mu^* \cdot \frac{\cancel{(V_x-V_y)}/\delta}{\delta} \simeq \rho^* V^* \frac{\cancel{V_x-V_y}}{\cancel{\delta}}$$

$$\rightarrow \frac{\rho^* V^* \delta}{\mu^*} \simeq 1$$

$$\frac{\rho^* V^* \delta}{\mu^*} = Re_{shock}$$

From kinetic theory, $\frac{\delta}{l^*} \simeq \frac{5}{8} Re \simeq 1$

l^* ↑ mean free path

$\rightarrow \delta \sim l^*$ (mean free path)

In reality, $\delta \approx 2 \sim 4\, l^*$