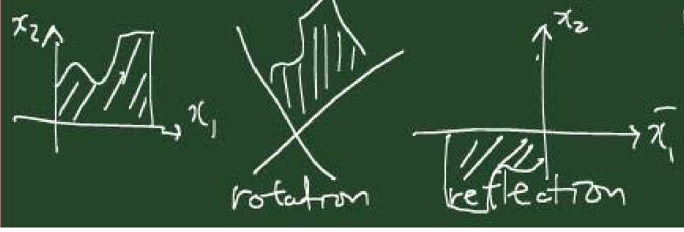
노트 제목 2010-03-09

- Transformation properties
- Reynolds number similarity

$$\frac{\partial U_i}{\partial x_i} = 0$$

$$\frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i} = -\frac{\partial p}{\partial x_j} + \frac{1}{Re} \nabla^2 U_j$$

- rotational and reflectional invariance



under reflection,

$\bar{x}_i = -x_i$

$\bar{U}_i = -U_i$

$$\underbrace{\frac{\partial U_i}{\partial t}}_{\Downarrow} + \underbrace{U_j \frac{\partial U_i}{\partial x_j}}_{} = \underbrace{-\frac{\partial p}{\partial x_i}}_{\parallel} + \frac{1}{Re} \underbrace{\nabla^2 U_i}_{\parallel}$$

$$-\frac{\partial \bar{U}_i}{\partial t} \quad (-\bar{U}_j)\frac{\partial(-\bar{U}_i)}{\partial(-\bar{x}_j)} \quad + \frac{\partial p}{\partial \bar{x}_i} \quad \bar{\nabla}^2(-\bar{U}_i)$$

$$\rightarrow \frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial \bar{x}_j} = -\frac{\partial p}{\partial \bar{x}_i} + \frac{1}{Re} \bar{\nabla}^2 \bar{U}_i \Rightarrow \text{invariant}$$

Transformed N-S eqs w/ rotation and reflection are the same. → rotational & reflectional invariance.

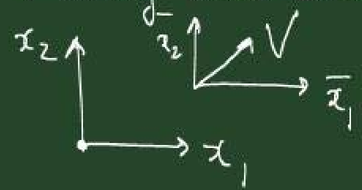
- time reversal → not invariant

U_i U_i due to viscous term

$t_1 \longrightarrow t_2$

· Galilean invariance

Moving frame with constant velocity V



A quantity that is the same in different inertial frames is said to be Galilean invariant.

$$\begin{cases} \bar{x} = x - Vt, \\ \bar{t} = t \\ \bar{U}(\bar{x}, t) = U(x,t) - V \end{cases}$$

$$\frac{\partial U_i}{\partial x_j} = \frac{\partial(\bar{U}_i + V)}{\partial \bar{x}_l}\frac{\partial \bar{x}_l}{\partial x_j} = \frac{\partial \bar{U}_i}{\partial \bar{x}_l}\delta_{lj} = \frac{\partial \bar{U}_i}{\partial \bar{x}_j}$$

$$\frac{\partial U_i}{\partial t} = \frac{\partial(\bar{U}_i + V)}{\partial \bar{t}}\frac{\partial \bar{t}}{\partial t} + \frac{\partial(\bar{U}_i + V)}{\partial \bar{x}_j}\frac{\partial \bar{x}_j}{\partial t}$$

$$= \frac{\partial \bar{U}_i}{\partial \bar{t}} + \frac{\partial \bar{U}_i}{\partial \bar{x}_j}(-V_j) = \frac{\partial \bar{U}_i}{\partial \bar{t}} - V_j\frac{\partial \bar{U}_i}{\partial \bar{x}_j}$$

$$\frac{D\bar{U}_i}{D\bar{t}} = \frac{\partial \bar{U}_i}{\partial \bar{t}} + \bar{U}_j\frac{\partial \bar{U}_i}{\partial \bar{x}_j} = \left(\frac{\partial U_i}{\partial t} + \cancel{V_j\frac{\partial U_i}{\partial x_j}}\right) + (U_j - \cancel{V_j})\frac{\partial U_i}{\partial x_j}$$

$$= \frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j} = \frac{DU_i}{Dt}$$

∴ Velocity gradient and fluid acceleration are Galilean invariant, whereas velocity and its partial time derivative are not.

N-S eq. $\longrightarrow$ $\overline{\text{N-S}}$ eq : same

∴ N-S eq. is Galilean invariant.

- N-S eq. is invariant under rectilinear acceleration of frame.

acceleration of frame

$$\rho \frac{\partial \bar{U}_i}{\partial \bar{t}} + \cdots = -\frac{\partial P}{\partial \bar{x}_i} + \cdots \boxed{-\rho A_i}$$

$$= -\frac{\partial}{\partial \bar{x}_i} \underbrace{\left(P + \rho \bar{x}_j A_j\right)}_{\bar{P}}$$

- N-S eq is <u>not invariant</u> under rotational acceleration
  - → generates centrifugal force $(-\bar{x}_i \bar{\Omega}_{ik} \bar{\Omega}_{kj})$, coriolis force $(-2\bar{U}_j \bar{\Omega}_{ij})$ and angular acceleration force $\left(-\bar{x}_i \frac{d\bar{\Omega}_{ij}}{dt}\right)$.

$-\Omega \times \Omega \times \bar{x}$

$-2\Omega \times \bar{u}$

$-\frac{d\Omega}{dt} \times \bar{x}$

- For 2D flow, consider steady rotation
  - → these three terms go to zero.
  - → N-S eq. is invariant w/ steady rotation of the frame for 2D flow.

Ch.3 Statistical description of turbulent flows.

⊙ random nature of turbulence

→ not deterministic → rely on statistical description.

⊙ Characterization of random variables

· U is random ⇐ its value is inherently unpredictable

So, theory aims at determining the probability of events.

↓

probability density function (PDF)

• event $B \equiv \{ U < V_b \}$

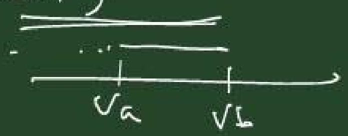
$C \equiv \{ V_a \le U < V_b \}$ for $V_b > V_a$

probability of the event B is

$$p = P(B) = P\{ U < V_b \}$$

• Cumulative distribution function (CDF)

$$F(V) \equiv P\{ U < V \}$$



Then, $P(B) = P\{ U < V_b \} = F(V_b)$

$$P(C) = P\{ V_a \le U < V_b \} = P\{ U < V_b \} - P\{ U < V_a \}$$
$$= F(V_b) - F(V_a)$$

$F(-\infty) = 0$, $F(\infty) = 1$, $F(V_b) \ge F(V_a)$ for $V_b > V_a$

F is a non-decreasing ft.

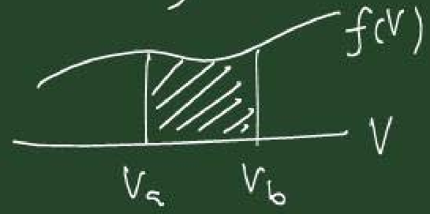
• Probability density function (PDF)

$$f(V) \equiv \frac{dF(V)}{dV} = \lim_{\Delta V \to 0} \frac{F(V+\Delta V) - F(V)}{\Delta V} \ge 0$$

$$\int_{-\infty}^{\infty} f(V)\, dV = 1 \quad \text{(normalization condition)}$$

$$f(-\infty) = f(\infty) = 0$$

$$P\{V_a \le U < V_b\} = F(V_b) - F(V_a) = \int_{V_a}^{V_b} f(V)\,dV$$



$$\downarrow$$

$$\int_{-\infty}^{\infty} f(V)\,dV = 1$$

$$\because P\{-\infty < U < \infty\} = 1$$

$$P\{V \le U < V + dV\} = F(V+dV) - F(V) = f(V)\,dV$$

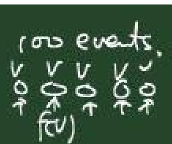
$\Rightarrow$ PDF $f(V)$ is the probability per unit distance

$\rightarrow$ called probability <u>density</u> ft.

- Two or more random variables have the same PDF

  $\rightarrow$ they are said to be identically distributed
  or statistically identical

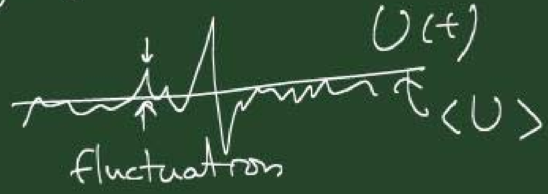
- mean (or expectation) of U



$$\langle U \rangle = \int_{-\infty}^{\infty} V f(V)\,dV \qquad \int_{-\infty}^{\infty} U f(U)\,dU$$

more generally $\underbrace{\langle Q(U) \rangle}_{\text{mean of } Q} = \int_{-\infty}^{\infty} Q(V) f(V)\,dV$

$$\langle\langle U \rangle\rangle = \langle U \rangle$$



- fluctuation of U

$$u \equiv U - \langle U \rangle$$

- variance (mean-square fluctuation)

$$\mathrm{var}(U) \equiv \langle u^2 \rangle = \int_{-\infty}^{\infty} (V - \langle V \rangle)^2 f(V)\,dV$$

- standard deviation (root-mean-square fluctuation)
  rms

$$\mathrm{sdev}(U) = \sqrt{\mathrm{var}(U)} = \langle u^2 \rangle^{\frac{1}{2}} \equiv u' \quad (\text{or } \sigma_u)$$

- n^{th} central moment

$$\mu_n \equiv \langle u^n \rangle = \int_{-\infty}^{\infty} (V - \langle V \rangle)^n f(V)\, dV$$

$$\mu_0 = 1, \quad \mu_1 = 0, \quad \mu_2 = \sigma_\mu^2$$