노트 제목 2010-03-11

- Standardization: zero mean and unit variance

$$\hat{u} \equiv (u - \langle u \rangle)/\sigma_u$$

$$\mathrm{var}(\hat{u}) = 1 = \int_{-\infty}^{\infty} \hat{u}^2 \underline{\hat{f}(\hat{u})}\, d\hat{u}$$

$$\mathrm{var}(u) = \int_{-\infty}^{\infty} (\underbrace{u - \langle u \rangle}_{= \sigma_u \hat{u}})^2 f(u)\, \underbrace{du}_{= \sigma_u d\hat{u}} = \sigma_u^2$$

$$= \int_{-\infty}^{\infty} \underline{\sigma_u^{\cancel{2}}}\, \hat{u}^2 \underline{f(u)}\, d\hat{u} = \cancel{\sigma_u^2}$$

$$\therefore \hat{f}(\hat{u}) = \sigma_u f(u) \rightarrow \hat{f}(\hat{V}) = \sigma_u f(V) = \sigma_u f(\langle u \rangle + \sigma_u \hat{V})$$

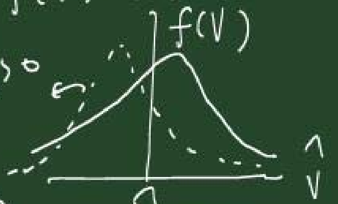
standardized PDF of V

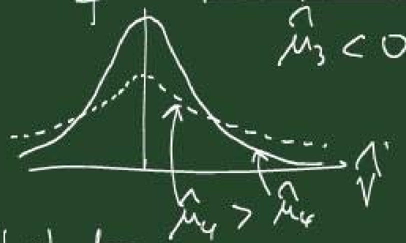
$$\hat{\mu}_n = \frac{\langle u^n \rangle}{\sigma_u^n} = \frac{\mu_n}{\sigma_u^n} = \int_{-\infty}^{\infty} \hat{V}^n \hat{f}(\hat{V})\, d\hat{V}$$

$\underline{\hat{\mu}_0 = 1}$, $\underline{\hat{\mu}_1 = 0}$, $\underline{\hat{\mu}_2 = 1}$

$\hat{\mu}_3$: skewness

$\hat{\mu}_4$: flatness (or kurtosis)

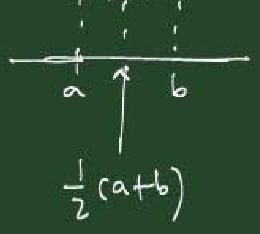




⊙ Examples of Probability distributions

- U is uniform in $a \le V < b$

$$\rightarrow f(V) = \begin{cases} \frac{1}{b-a} & \text{for } a \le V < b \\ 0 & \text{elsewhere} \end{cases}$$



$$\langle U \rangle = \int_{-\infty}^{\infty} V f(V)\, dV = \int_a^b V \cdot \frac{1}{b-a}\, dV = \frac{1}{2}(a+b).$$

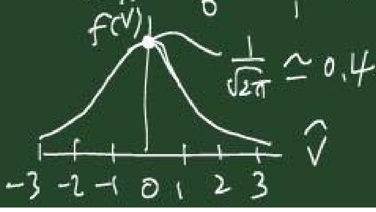
$$\sigma_u^2 = \int_{-\infty}^{\infty} (V - \langle U \rangle)^2 f(V)\, dV = \frac{1}{12}(b-a)^2$$

$$\hat{\mu}_3 = 0, \quad \hat{\mu}_4 = \frac{9}{5}$$

- U is normally (or Gaussian) distributed with mean μ and standard deviation σ.

$$\rightarrow f(V) = N(V; \mu, \sigma^2) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}(V-\mu)^2/\sigma^2\right]$$

$$\hat{U} = \frac{U-\mu}{\sigma}, \quad \hat{f}(\hat{V}) = \sigma f(V) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}V^2}$$



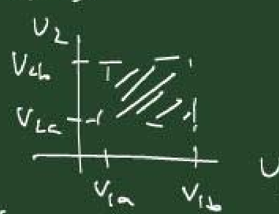
$$\frac{1}{\sqrt{2\pi}} \simeq 0.4 \qquad \hat{\mu}_3 = 0, \quad \hat{\mu}_4 = 3 \quad \Big| \quad \hat{\mu}_5 = 0, \quad \hat{\mu}_6 = 15.$$

⓪ <u>Joint</u> random variables

velocity (U_1, U_2, U_3)

- CDF of the joint random variables (U_1, U_2)

$$F_{12}(V_1, V_2) \equiv P\{U_1 < V_1,\ U_2 < V_2\}$$



- Joint PDF (JPDF) of U_1 and U_2

$$f_{12}(V_1, V_2) \equiv \frac{\partial^2}{\partial V_1 \partial V_2} F_{12}(V_1, V_2)$$

$$P\{V_{1a} \le U_1 < V_{1b},\ V_{2a} \le U_2 < V_{2b}\} = \int_{V_{1a}}^{V_{1b}} \int_{V_{2a}}^{V_{2b}} f_{12}(V_1, V_2)\, dV_2\, dV_1$$

$$\int_{-\infty}^{\infty} f_{12}(V_1, V_2)\, dV_1 = f_2(V_2), \quad \int_{-\infty}^{\infty} f_{12}(V_1, V_2)\, dV_2 = f_1(V_1)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{12}(V_1, V_2)\, dV_1 dV_2 = 1$$

- Mean of $Q(U_1, U_2)$

$$\langle Q(U_1, U_2)\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(V_1, V_2)\, f_{12}(V_1, V_2)\, dV_1 dV_2$$

- Covariance of U_1 and U_2

$$\mathrm{cov}(U_1, U_2) \equiv \langle u_1 u_2 \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (V_1 - \langle U_1\rangle)(V_2 - \langle U_2\rangle)\, f_{12}(V_1, V_2)\, dV_1 dV_2$$

correlation coefficient $\rho_{12} \equiv \underline{\langle u_1 u_2 \rangle} \big/ \left(\langle u_1^2\rangle\langle u_2^2\rangle\right)^{\frac{1}{2}}$

$\left[\langle u_1 u_1\rangle\langle u_2 u_2\rangle\right]^{\frac{1}{2}}$

$\rho_{12} = 0$: U_1 and U_2 are uncorrelated (?)

$\rho_{12} = 1$: " perfectly correlated

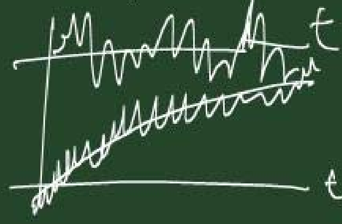
$\rho_{12} = -1$: " " negatively correlated

⊙ Random processes

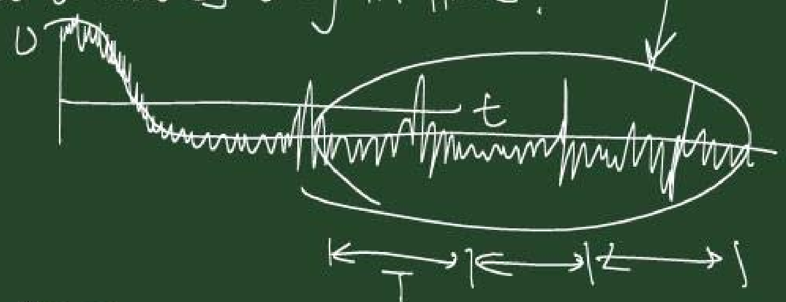
. A process is statistically stationary

→ all multi-time statistics are invariant under a shift in time, i.e., for all T,

$$f(V_1, t_1+T; V_2, t_2+T, \cdots; V_N, t_N+T) = f(V_1, t_1; V_2, t_2; \cdots; V_N, t_N)$$



• A turbulent flow can reach a statistically steady state in which the statistics are indep. of time although the flow variables vary in time.



• Autocovariance $R(s) \equiv \langle u(t)\, u(t+s) \rangle$

Autocorrelation ft. $\rho(s) = \dfrac{\langle u(t)\, u(t+s) \rangle}{\langle u(t) \rangle^2}$

$\rho(0) = 1$, $|\rho(s)| \le 1$, $\rho(s) = \rho(-s)$: ρ is an even ft.

If $U(t)$ is periodic w/ period T, $\rho(s) = \rho(s+T)$

In most turbulent flow, $\rho(s) \to 0$ as $s \to \infty$.

• Integral timescale

$$\bar{\tau} = \int_0^\infty \rho(s)\, ds$$