

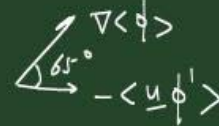
$$\langle u_j \phi' \rangle = -\Gamma_T \nabla \langle \phi \rangle$$

$$-\rho \langle u_i u_j \rangle + \frac{2}{3} \rho k \delta_{ij} = 2 \rho \nu_T \langle S_{ij} \rangle$$

- $\langle u_j \phi' \rangle$  is aligned with mean scalar gradient vector  $\nabla \langle \phi \rangle$ .

→ This is not true even in simple turb. flow.

e.g. angle bet  $\nabla \langle \phi \rangle$  and  $-\langle u_j \phi' \rangle$  is  $65^\circ$  in homo. turb. shear flow.



- $a_{ij} = -\rho \langle u_i u_j \rangle + \frac{2}{3} \rho k \delta_{ij} \sim \langle S_{ij} \rangle$   
proportional coeff.  $\rightarrow$  scalar for 6 different components

$$\underline{b} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \underline{c}$$

$$\underline{b} \sim \alpha \underline{c}$$

not true even in simple shear flow.

- In 2D turb. boundary layer flow, important flow variables are  $\langle u, u_z \rangle$  and  $\langle u_z \phi' \rangle$ .

$$\langle u_z \phi' \rangle = -\Gamma_T \frac{\partial \langle \phi \rangle}{\partial x_z}$$

$$\langle u, u_z \rangle = -\nu_T \frac{\partial \langle u_1 \rangle}{\partial x_z} \quad \therefore \text{scalar} \quad \therefore \text{not bad.}$$

scalar

- If  $\nu_T$  and  $\Gamma_T$  are properly modeled, the mean eqs can be solved.

- At high Re number and away from the wall,

$$\frac{\nu_T}{\nu} \text{ and } \frac{\Gamma_T}{\Gamma} \sim Re \gg 1$$

$\therefore$  molecular transport is negligible.

- $\sigma_T = \frac{\nu_T}{\Gamma_T}$  : turbulent Prandtl number

In most simple turb. flows,  $\sigma_T \sim \mathcal{O}(1)$ .

$$\checkmark \frac{\partial u}{\partial t} + \frac{\partial}{\partial y_j} (\rho_j u) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \quad \text{if } \frac{\partial p}{\partial x} \neq 0 \quad \text{Pr} = 1$$

$$\checkmark \frac{\partial \phi}{\partial t} + \frac{\partial}{\partial y_j} (\rho_j \phi) = \Gamma \nabla^2 \phi \quad u \doteq \phi \rightarrow \text{Similarity Let } u \& \phi$$

Question  $\frac{D\langle u \rangle}{Dt} = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x} + \nu \nabla^2 \langle u \rangle - \frac{\partial \langle u^2 \rangle}{\partial x}$

$\frac{D\langle \phi \rangle}{Dt} = -\frac{\partial \langle u \phi' \rangle}{\partial x}$

$\frac{\partial \langle u \phi' \rangle}{\partial x} \ll \frac{\partial \langle u \rangle}{\partial x} \ll \frac{\partial \langle v \rangle}{\partial y}$

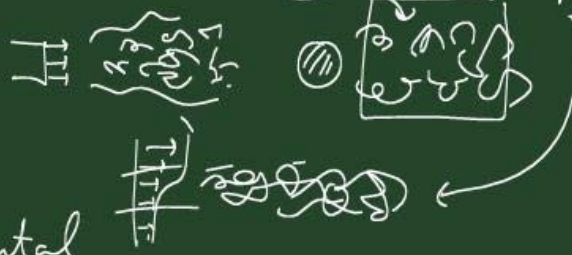
$\frac{\partial \langle u \phi' \rangle}{\partial x} \ll \frac{\partial \langle u \rangle}{\partial x} \ll \frac{\partial \langle v \rangle}{\partial y}$

$\therefore \langle uv \rangle \& \langle v \phi' \rangle$  are important.

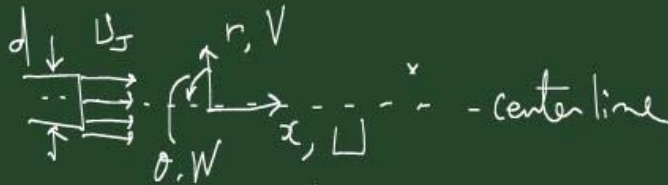
ch 5. Free shear flows. → jet, wake, mixing layer

'free' ?

→ remote from wall



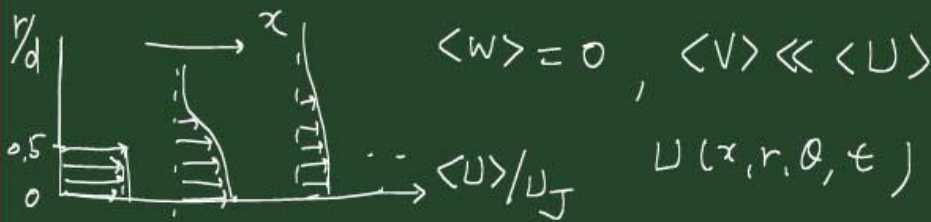
5.1 Round jet : experimental observations



statistically stationary, axisymmetric

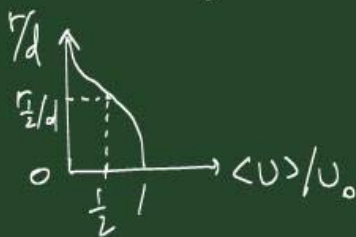
$U_j$ : jet-exit velocity.  $Re = U_j d / \nu$

• nozzle (jet) details.



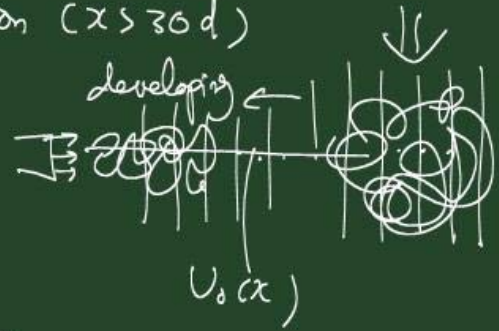
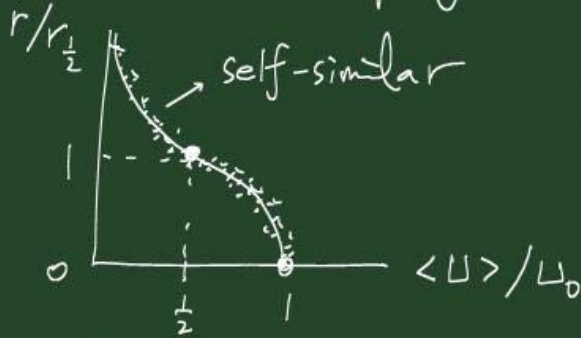
• Centerline velocity:  $U_0(x) = \langle U(x, r=0, \theta=0, t) \rangle$

Jet half-width  $r_{1/2}(x)$ :  $\langle U(x, r=r_{1/2}(x), \theta=0, t) \rangle = \frac{1}{2} U_0(x)$



As  $x$  increases,  
jet decays (i.e.  $U_0(x)$  decreases)  
& jet spreads (i.e.  $r_{1/2}(x)$  increases)

• Beyond the developing region ( $x \geq 30d$ )



self-similarity : important concept in turbulent flow  
consider a quantity  $Q(x, y)$

characteristic scales  $Q_0(x)$  and  $\delta(x)$

Define  $\xi = y/\delta(x)$

$$\tilde{Q}(\xi, x) = Q(x, y) / Q_0(x)$$

If  $\tilde{Q}(\xi, x) = \hat{Q}(\xi)$ ,  $Q(x, y)$  is self-similar.

• Axial variation of scales



$$\rightarrow \frac{U_0(x)}{U_j} = \frac{B}{(x-x_0)/d}$$

↙ empirical constant

virtual origin  $x_0$