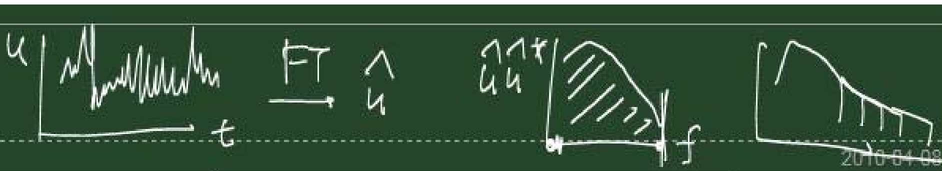


노트 제목 2010-04-08

- Kolmogorov scales : characteristic scales of the smallest turbulent motions

  ε and ν

  $\varepsilon = 2\nu s_{ij} s_{ij}$

  length scale : $\eta = (\nu^3/\varepsilon)^{\frac{1}{4}}$

  time scale : $\tau_\eta = (\nu/\varepsilon)^{\frac{1}{2}}$

  velocity scale : $u_\eta = (\nu\varepsilon)^{\frac{1}{4}}$

  $\eta = \nu^a \varepsilon^b \quad [L]$

Comparison w/ mean-flow scales $r_{\frac{1}{2}}$ and U_0

$(Re_0 = U_0 r_{\frac{1}{2}} / \nu)$

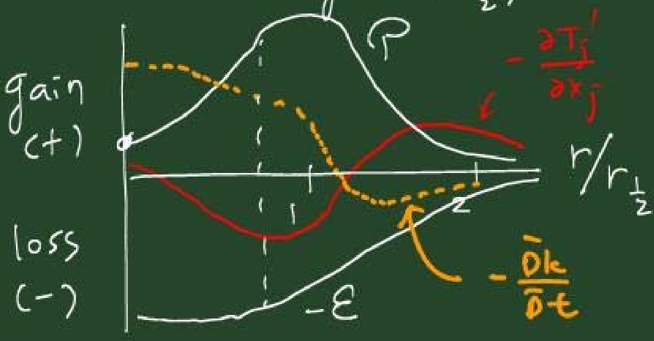
$(\hat{\varepsilon} = \varepsilon / (U_0^3 / r_{\frac{1}{2}}))$ ← non-dimensional and indep. of Re.

$$\left.\begin{aligned} \frac{\eta}{r_{\frac{1}{2}}} &= Re_0^{-\frac{3}{4}} \hat{\varepsilon}^{-\frac{1}{4}} \\ \frac{\tau_\eta}{r_{\frac{1}{2}}/U_0} &= Re_0^{-\frac{1}{2}} \hat{\varepsilon}^{-\frac{1}{2}} \\ \frac{u_\eta}{U_0} &= Re_0^{-\frac{1}{4}} \hat{\varepsilon}^{\frac{1}{4}} \end{aligned}\right\} \text{decrease as } Re_0 \text{ increases}$$

• $\frac{\eta u_\eta}{\nu} = 1$: Re # based on the Kolmogorov scale is unity → motions on these scales are strongly affected by viscosity.

• Budget of k $\quad \frac{\bar{D}k}{\bar{D}t} + \frac{\partial}{\partial x_j} T_j' = P - \varepsilon$.

(normalized by $U_0^3/r_{\frac{1}{2}}$)



$P \simeq \varepsilon$ @ $r/r_{\frac{1}{2}} \simeq 0.6$

($\simeq 0.8\varepsilon$)

$\tau = \frac{k}{\varepsilon}$ (turbulence decaying time scale) or time to dissipate an amount of energy k at constant rate ε.

$\tau_p = \frac{k}{P}$ (turbulence producing time scale) or time to produce k at the rate P

$\tau \simeq \tau_p \simeq 3S^{-1}$: three times the timescale of the imposed shear S^{-1}.

∴ turbulence is long-lived.

- Pseudo-dissipation $\tilde{\varepsilon} = \nu \langle \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \rangle$

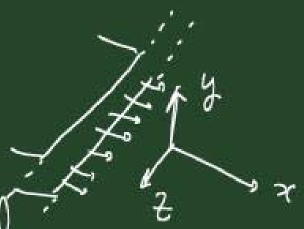
$$\tilde{\varepsilon} = \underset{\uparrow}{\varepsilon} - \nu \frac{\partial^2 \langle u_i u_j \rangle}{\partial x_i \partial x_j} \leftarrow \text{very small.}$$

true dissipation ($= \nu s_{ij} s_{ij}$)

5.4 Other self-similar flows

- Plane jet

same as for circular jet



$U_0(x) = \langle U(x, y=0) \rangle$: centerline vel.

$y_{\frac{1}{2}}(x)$: $\langle U(x, y=y_{\frac{1}{2}}) \rangle = \frac{1}{2} U_0$ jet half-width

self-similarity & mtm flux conservation

$$\rightarrow \frac{dy_{\frac{1}{2}}}{dx} = S \doteqdot 0.10 \Rightarrow y_{\frac{1}{2}} \sim x$$

$$U_0(x) \sim x^{-\frac{1}{2}}$$

$$\hat{\nu}_T(\xi) = \frac{\nu_T}{U_0 y_{\frac{1}{2}}} \rightarrow \nu_T \sim x^{\frac{1}{2}}, \quad \xi = y/y_{\frac{1}{2}}$$

const.

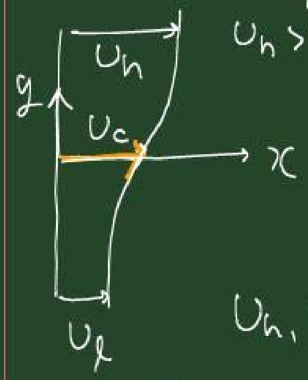
$$Re_0 = U_0 y_{\frac{1}{2}}/\nu \sim x^{\frac{1}{2}}$$

$$Re_T = U_0 y_{\frac{1}{2}}/\nu_T \text{ indep of } x \quad Re_T \simeq 31$$

boundary layer eq + self-similarity

$$\rightarrow \bar{f}(\xi) = \frac{\langle U \rangle}{U_0} = \operatorname{sech}^2(\alpha \xi)$$

- Plane mixing layer



$U_h > U_l > 0$

U_h / U_l

$U_c = \frac{1}{2}(U_h + U_l)$: char. conv. vel.

$U_s = U_h - U_l$: char. vel. diff.

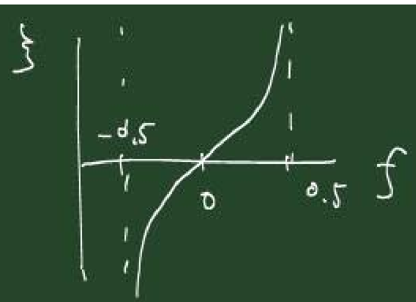
U_h, U_l, U_c, U_s are indep. of x.

y_α s.t. $\langle U(x, y_\alpha)\rangle = U_l + \alpha(U_h - U_l)$

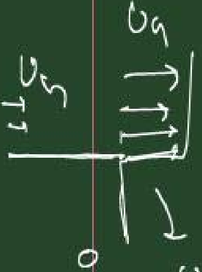
char. length $\delta(x) = y_{0.9}(x) - y_{0.1}(x)$

$\bar{y}(x) = \frac{1}{2}[y_{0.9}(x) + y_{0.1}(x)]$

$$\xi = \frac{y - \bar{y}(x)}{\delta(x)}$$



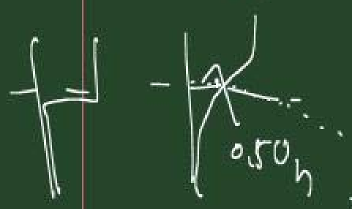
$$f(\xi) = \frac{\langle U\rangle - U_c}{U_s}$$



$f(\pm\infty) = \pm 0.5$

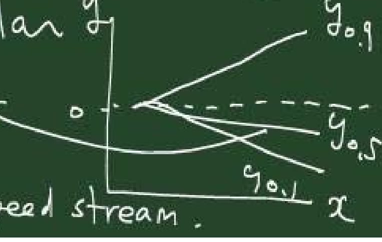
$f(\pm\frac{1}{2}) = \pm 0.4$

- $U_l/U_h = 0$: exp. results confirm that mixing layer is self similar



flow is not symmetric about $y = 0$!

It spreads into the low-speed stream.

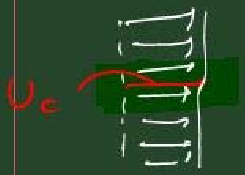


- boundary layer eq. + self-similarity

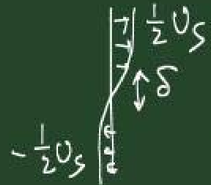
  $\rightarrow \delta \sim x$

$$S \equiv \frac{U_c}{U_s}\frac{d\delta}{dx} = \text{const.} \quad (0.06 \sim 0.11)$$

- $U_\ell / U_h \rightarrow 1$ (limiting case)

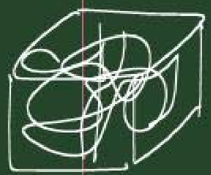


To an observer travelling in x direction at U_c



$$\frac{d\delta}{dt} = U_c \frac{d\delta}{dx} = U_c S = \text{const.}$$

$\rightarrow \delta \sim t$

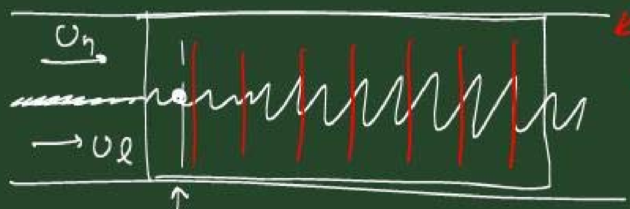


flow becomes statistically 1-D and time-dependent.

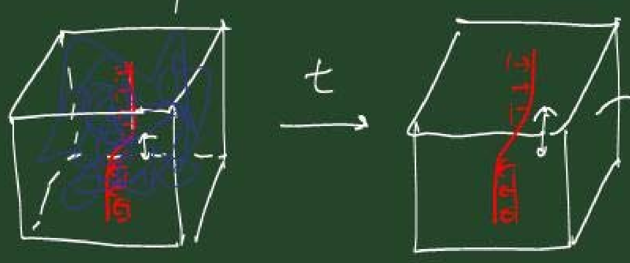
$\rightarrow$ It is called 'temporal mixing layer' (as opposed to spatial mixing layer)

Rogers & Moser (1994).

Exp.



Num.



$\langle \cdot \rangle = \langle \cdot \rangle (y)$

- (x, y, z, t)

Mixing layer

$U_s = \text{const.}$

$\delta \sim x$

$Re_0 = \frac{U_s \delta}{\nu} \sim x$

$\nu_T = U_s \delta \hat{\nu}_T \sim x$

Flow rate of k: $\dot{k}(x) = \int_{-\infty}^{\infty} \langle U \rangle k \, dy$

$\sim U_c U_s^2 \delta \sim x$

(jet & wake : $\dot{k} \sim x^{-1}$)

$P > \varepsilon$