노트 제목 2010-04-29

$E(k) = C\varepsilon^{\frac{2}{3}}k^{-\frac{5}{3}}$

$E_{11}(k_1) = C_1\varepsilon^{\frac{2}{3}}k_1^{-\frac{5}{3}}$

$E_{11}^{(k_1)} \sim \langle u_1(\underline{x})\, u_1(\underline{x} + x_1 \underline{e}_1)\rangle$

Saddoughi & Veeravalli (1995)

NASA-Ames

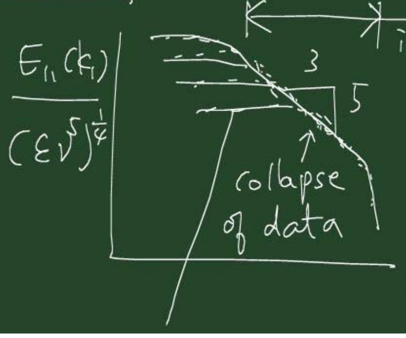
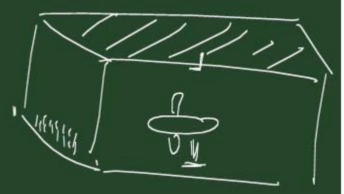


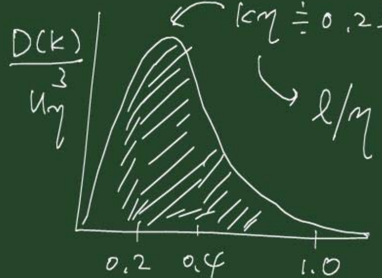
Fig. 6.14



model spectrum represents the data accurately

$f_\eta(k\eta) = \exp\left(-\frac{3}{2}C(k\eta)^{\frac{4}{3}}\right)$: Pao spectrum

not that good as compared to the model spectrum.



$k\eta \doteqdot 0.26$

$l/\eta \doteqdot 24$

$\left(lk = 2\pi \quad l = \frac{2\pi}{k} = \frac{2\pi}{0.26}\eta \simeq 24\eta \right)$

motions responsible for bulk of dissipation $(0.1 < k\eta < 0.75;\ 8 < l/\eta < 60)$ are

$D(k)$: dissipation spectrum

$= 2\nu k^2 E(k)$

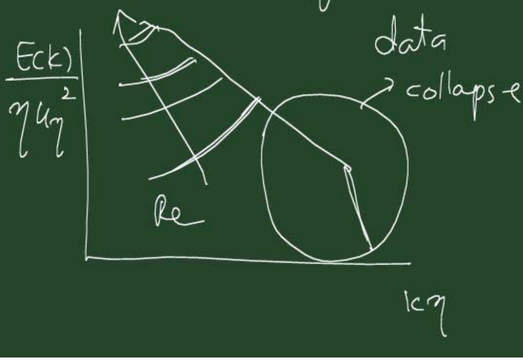
$\ell \simeq \eta \rightarrow k\ell = 2\pi$

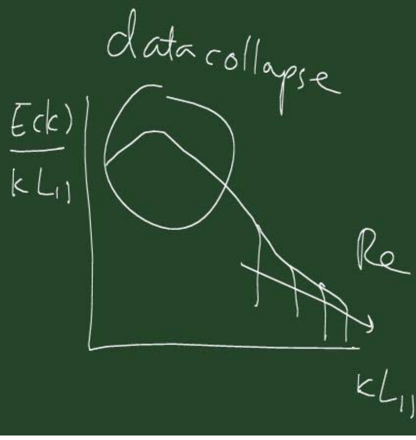
$\rightarrow k = \frac{2\pi}{\ell} = \frac{2\pi}{\eta}$

$\rightarrow k\eta = 2\pi$

Considerably larger than the Kolmogorov scale.

• Reynolds number effect



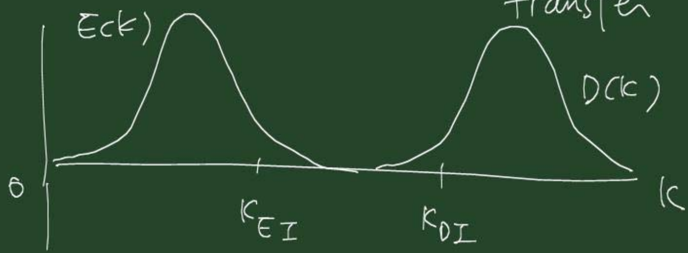


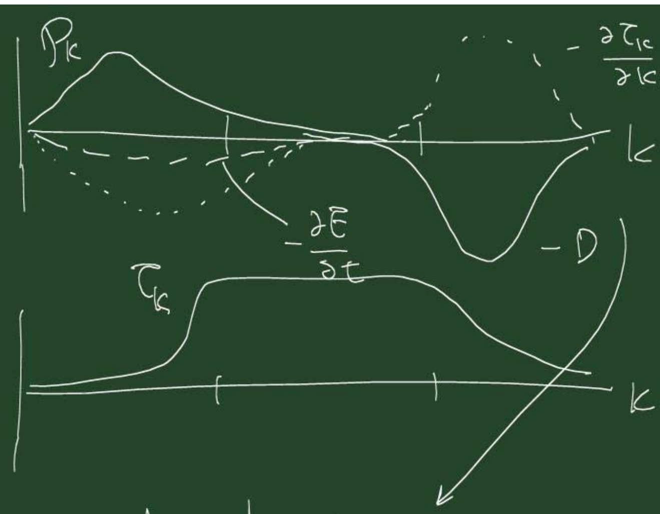
6.6 Spectral view of energy cascade

$$\frac{\partial}{\partial t} E(k,t) = P_k(k,t) - \frac{\partial}{\partial k} \mathcal{T}_k(k,t) - 2\nu k^2 E(k,t)$$

Production — Spectral transfer — Dissipation $D(k)$

$\frac{\partial u_i}{\partial t} \xrightarrow{FT} \frac{\partial \hat{u}_i}{\partial t}$





For energy containing range, $\frac{dk}{dt} \doteq P - T_{EI}$

In the inertial subrange, spectral transfer only, $T_{EI} \sim T_{DI}$

in the dissipation range, $T_{DI} \sim \varepsilon$

6.7 Limitations

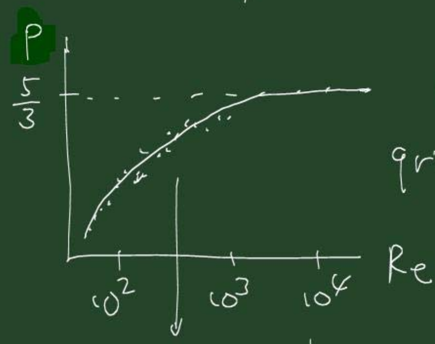
Kolmogorov hypothesis → oversimplification

energy cascade consists of one-way transfer of energy from eddies of size l to those of somewhat smaller size (e.g. $\frac{1}{2} l$)

and this energy transfer depends solely on motions of size l.

→ is good for 'high' Re #.

Q: How high is really high?



$E(k) \sim k^{-p}$

grid turbulence

Kolmogorov

$p = \frac{5}{3}$

In reality, Kolmogorov $-\frac{5}{3}$ spectrum is approached slowly as Re increases.

... People have tried to overcome the shortcomings.

- .. the issue wasn't so important
  because small scales contain little energy
  and so have little direct effect on the flow.

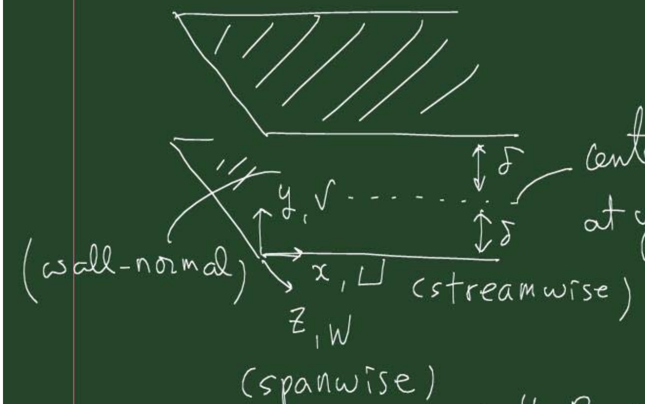
Ch.7 Wall flows

most turbulent flows are bounded by one or more solid surfaces.

- internal flow : flow through pipes & ducts.
- external flow : flow around aircraft, ...

★ Fully developed channel flow
  " " pipe flow
flat-plate boundary layer flow
→ mean velocity vector is parallel (or nearly parallel) to the wall.

7.1 Channel flow



$w \neq 0$

$\langle w \rangle = 0$

'fully developed' flow

$\frac{\partial}{\partial x}\langle \cdot \rangle = 0$; statistically stationary,

$Re \equiv (2\delta)\bar{U}/\nu$: bulk Reynolds #. ← ft. of y ← 1-D

$\bar{U} = \frac{1}{\delta}\int_0^\delta \langle U \rangle dy$: bulk velocity

$Re_0 \equiv U_0 \delta / \nu$ U_0 : centerline vel.

laminar for $Re < 1350$

turbulent for $Re > 1800$

continuity : $\cancel{\frac{\partial \langle U \rangle}{\partial x}}_{0} + \frac{\partial \langle V \rangle}{\partial y} = 0 \rightarrow$ $\langle V \rangle_w = 0$

$\rightarrow \langle V \rangle = 0$

y-mtm eq. : $0 = -\frac{d}{dy}\langle v^2 \rangle - \frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial y}$ mean p. on bottom wall

integrate $\int_0^y$: $\langle v^2 \rangle + \frac{1}{\rho}\langle p \rangle = \frac{1}{\rho} P_w + \cancel{\langle v^2 \rangle_w}_{0}$

$$\rightarrow \cancel{\frac{\partial \langle v^2 \rangle}{\partial x}}_{0} + \frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial x} = \frac{1}{\rho}\frac{dP_w}{dx}$$

x-mtm. eq : $0 = \nu \frac{d^2 \langle U \rangle}{dy^2} - \frac{d}{dy}\langle uv \rangle - \frac{1}{\rho}\boxed{\frac{\partial \langle p \rangle}{\partial x}}$

$$\rightarrow \frac{d}{dy}\left(\rho\nu\frac{d\langle U \rangle}{dy} - \rho\langle uv \rangle\right) = \frac{dP_w}{dx} = \text{const}$$

$\frac{d}{dy}\tau$ → total shear stress

ft. of y

ft. of x

$\rightarrow \boxed{\tau(y) = \tau_w \left(1 - \frac{y}{\delta}\right)}$; linear shear stress profile

$\tau_w = \tau(y=0)$: wall shear stress

$$\rightarrow -\frac{dP_w}{dx} = \frac{\tau_w}{\delta} > 0$$

$$\left.\begin{aligned} C_f &\equiv \frac{\tau_w}{\frac{1}{2}\rho U_\delta^2} \\ C_f &\equiv \frac{\tau_w}{\frac{1}{2}\rho \bar{U}^2} \end{aligned}\right\} \text{skin-friction coeff.}$$