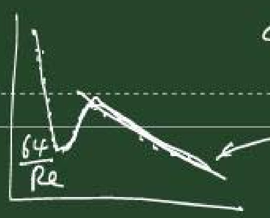
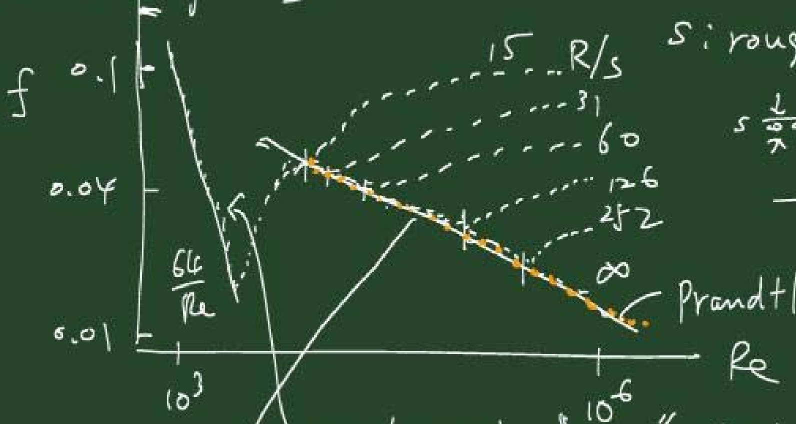
$$\frac{\Delta p\, D}{\frac{1}{2}\rho \bar{U}^2 L} = f$$

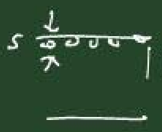


pipe flow

- wall roughness

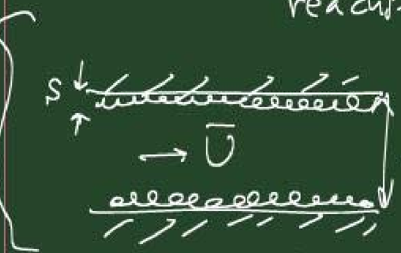


s : roughness length scale



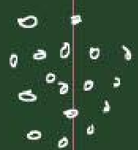
roughness has "little" effect in laminar regime

↳ the curves for each roughness follow the same line (Prandtl law) up to same Re #, before turning upward, and reaching an asymptote.



$D = 2R$ $\quad \bar{U} \uparrow$

$f \sim \Delta p \sim \tau_w$



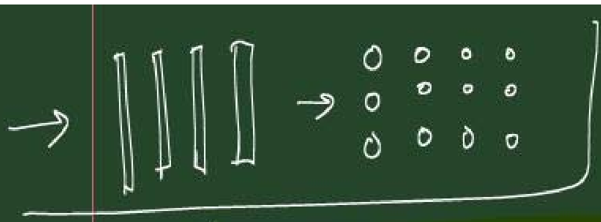
$\bar{U}_1 < \bar{U}_2$

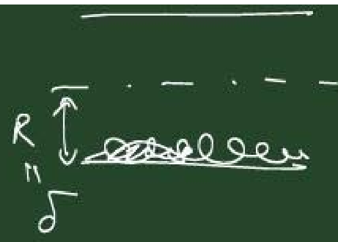
$Re_1 < Re_2$

turb. flows

$s_1^+ < s_2^+ = \frac{s}{\delta_v}$

$f_1 \approx f_{s_1}$ $\qquad f_2 > f_{s_2}$





- $\frac{d\langle U\rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}, \frac{s}{\delta_\nu}\right)$

inner layer $= \Phi\left(\frac{y}{\delta_\nu}, \frac{s}{\delta_\nu}\right)$ for $y/\delta \ll 1$

If $s/\delta_\nu \ll 1$, flow is unaffected by s.

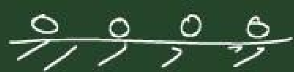
$s^+ = \frac{s u_\tau}{\nu}$ $\quad \therefore \frac{d\langle U\rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi_1\left(\frac{y}{\delta_\nu}\right)$

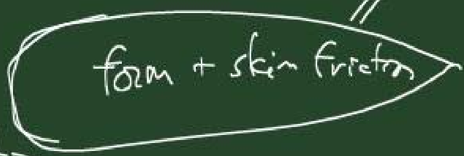
for large y/δ_ν, $\Phi_1 \approx \text{const} \sim 1/\kappa$

$\rightarrow \frac{\langle U\rangle}{u_\tau} = u^+ = \frac{1}{\kappa}\ln\left(\frac{y}{\delta_\nu}\right) + B$ for $s \ll \delta_\nu \ll y \ll \delta$

If $s/\delta_\nu \gg 1$, drag on roughness $\qquad$ o ← form drag

s^+ (wall unit)





form drag >> friction

$\rightarrow$ ν and δ_ν are not relevant parameters.

$\rightarrow \frac{d\langle U\rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi_R\left(\frac{y}{s}\right)$ for $s/\delta_\nu \gg 1$ and $y/\delta \ll 1$

for $y/s \gg 1$, turb. is determined by local processes, indep of s $\rightarrow$ so, same as for smooth wall

$\rightarrow \Phi_R = \frac{1}{\kappa}$

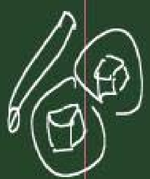
$\rightarrow \; u^+ = \frac{1}{\kappa}\ln\frac{y}{s} + B_2$ for $\delta_v \ll s \ll y \ll \delta$.

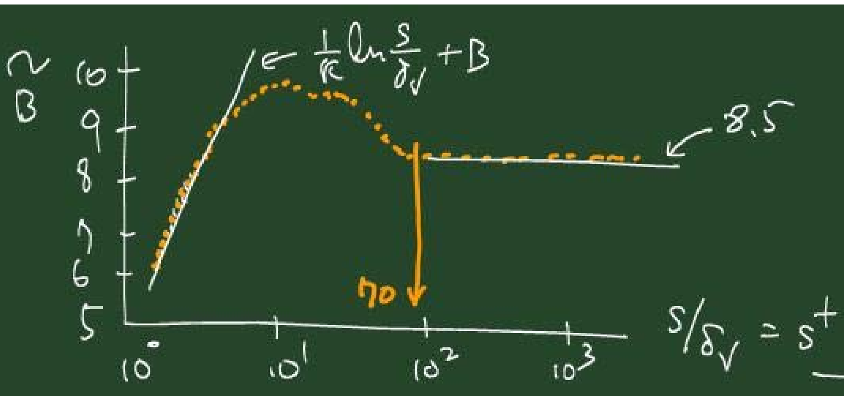
If $s/\delta_v \simeq 1$, for $y/\delta_v \gg 1$ and $y/s \gg 1$
$s^+ = s/\delta_v$

$$\boxed{u^+ = \frac{1}{\kappa}\ln\frac{y}{s} + \tilde{B}\left(\frac{s}{\delta_v}\right)} \quad \left(\frac{1}{\kappa}\ln\frac{y}{\delta_v} + \tilde{B}^*\left(\frac{s}{\delta_v}\right)\right)$$

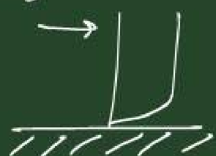
for $s/\delta_v \ll 1$, $u^+ = \frac{1}{\kappa}\ln\frac{y}{\delta_v} + B$
smooth wall $\quad = \frac{1}{\kappa}\ln\frac{y}{s} + \underbrace{\frac{1}{\kappa}\ln\frac{s}{\delta_v} + B}_{\tilde{B}}$

for $s/\delta_v \gg 1$, $u^+ = \frac{1}{\kappa}\ln\frac{y}{s} + B_2$
fully rough wall





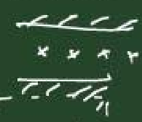
$\bar{U} \uparrow$
$u_\tau \uparrow$
$s^+ \uparrow$

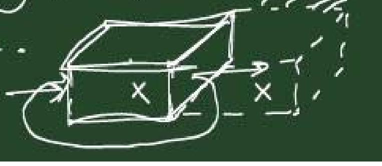


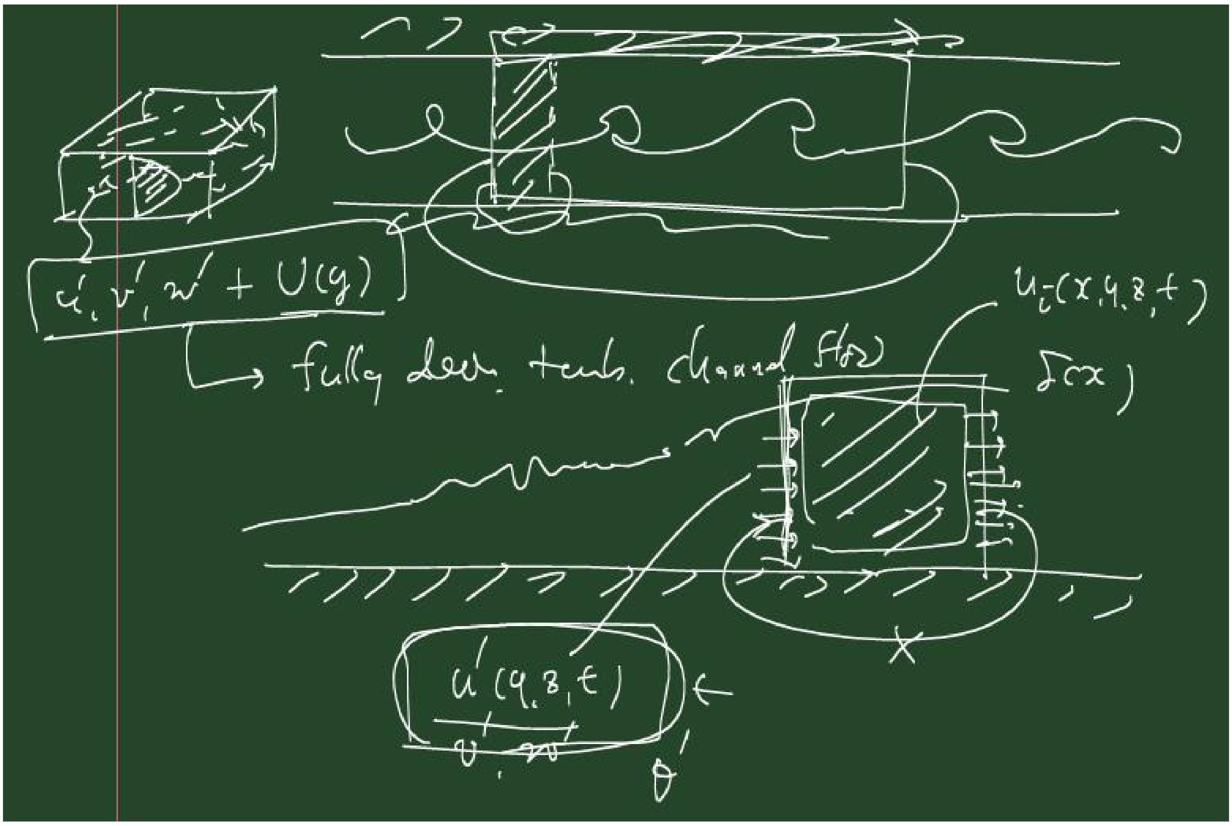
7.3 Boundary layers – external flow

Primary differences from fully developed channel flow:

① bdry layer develops continuously in x
$\delta(x)$ increases in x.

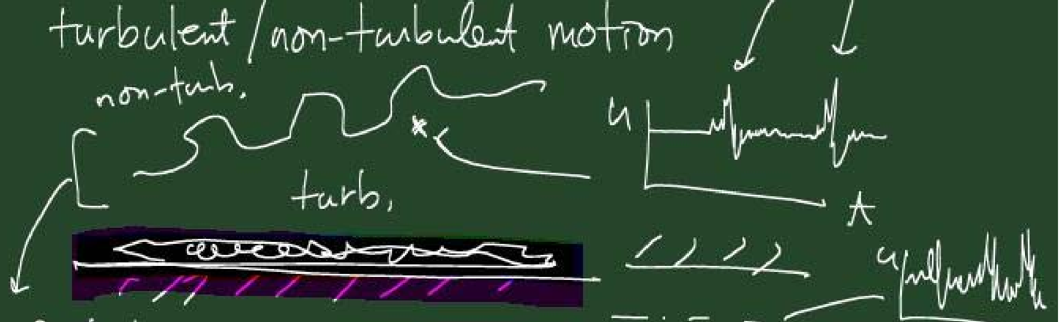






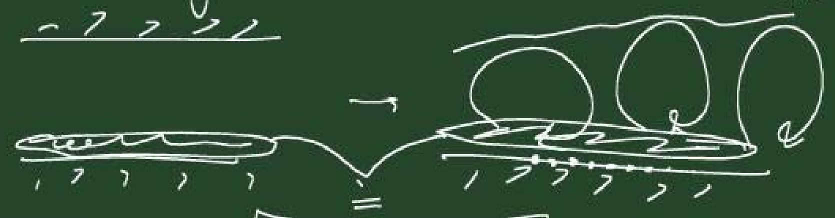
② $\tau_w(x)$ is not known a priori.

③ Outer part of flow consists of intermittent turbulent/non-turbulent motion



defect layer:
departure from log law is more significant.

But the behavior in the inner layer ($y/\delta(x) < 0.1$) is essentially the same as that in channel flow.



$\boxed{p_w(x,z,t)} \xrightarrow{FT} \boxed{\hat{p}_w(k_x,k_z,\omega)} \to \hat{p}_w \hat{p}_w^{*}$

↳ power spectrum

$\tau_w(x,z,t)$