Week 2, 8 & 10 March

Mechanics in Energy Resources Engineering
- Tension, Compression, and Shear

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Last week

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- Introduction to the course
- eTL

Outline
Chapter 1. Tension, Compression, and Shear

- Introduction to Mechanics of Materials
- Normal Stress and Normal Strain
- Mechanical Properties of Materials
- Elasticity, Plasticity, and Creep
- Linear Elasticity, Hooke’s Law, and Poisson’s Ratio
- Shear Stress and Shear Strain
- Allowable Stresses and Allowable Loads
- Design for Axial Loads and Direct Shear
- Review of statics

Outline

Chapter 1. 인장, 압축 및 전단

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- Introduction to Mechanics of Materials (재료역학)
- Normal Stress and Normal Strain (수직응력과 수직변형율)
- Mechanical Properties of Materials (역학적 성질)
- Elasticity, Plasticity, and Creep (탄성, 소성 및 크리프)
- Linear Elasticity, Hooke's Law, and Poisson's Ratio (선형탄성, Hooke의 법칙, 포아송비)
- Shear Stress and Shear Strain (전단응력과 전단변형율)
- Allowable Stresses and Allowable Loads (허용응력과 허용하중)
- Design for Axial Loads and Direct Shear (축하중과 직접전단의 설계)
- Review of Statics (정역학 정리)

Introduction to Mechanics of Materials
Three major Engineering Mechanics

- Solid mechanics:
  - Statics : rigid bodies (don't change in shape)
  - Strength of materials: deformable bodies (change in shape)
- Fluid mechanics:
  - fluid (water, oil, gas) flow in pipe or porous media
- Thermodynamics
  - deals with the internal energy of systems,
  - Temperature, heat, internal energy, entropy…

Introduction to Mechanics of Materials
Units

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- SI unit (The International System of Units; *le Système International d'unités*)
  - SI unit is the most widely used (except in the US)
  - SI base units important in mechanics:
    - m (meter, length), kg (mass), s (time), …
  - Derived units
    - Unit of Force: 1 N = 1kgm/s^2
    - Unit of Pressure: 1 Pa = 1 N/m^2
- USCS (US Customary System, or British Imperial System)
  - 1 feet = 0.3048 m,… , still widely used in Petroleum Engineering

Units (Prefixes)

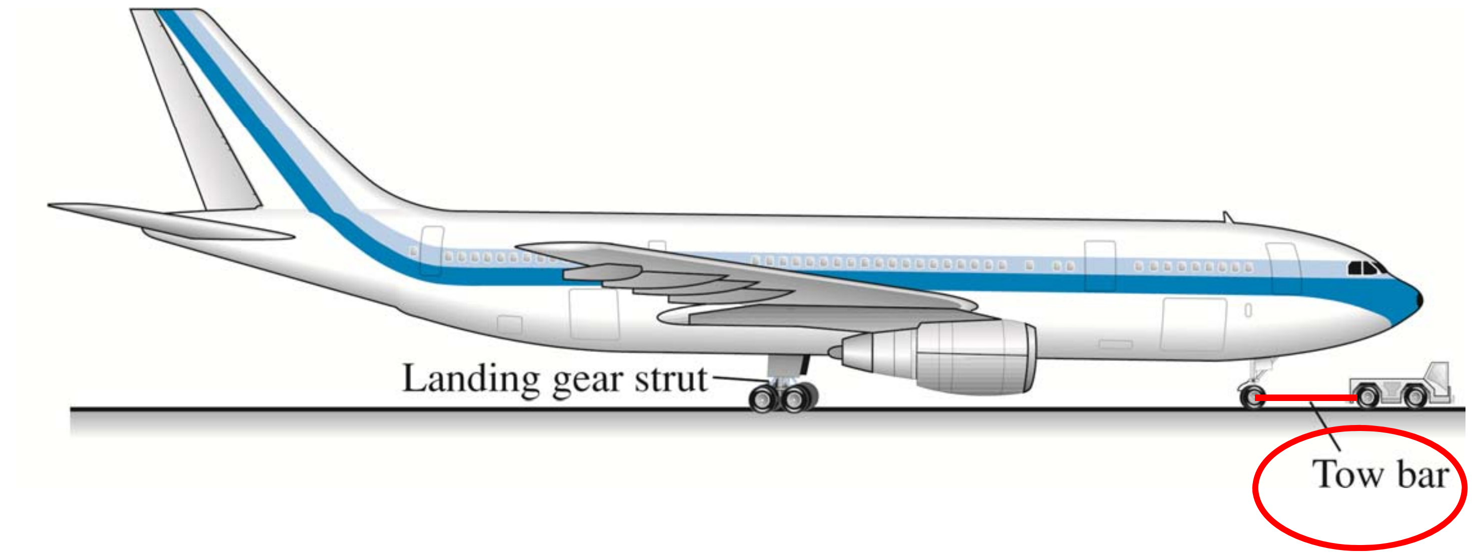
TABLE A-3 SI PREFIXES

Prefix	Symbol	Multiplication factor		
tera	T	10^{12}	=	1 000 000 000 000
giga	G	10^{9}	=	1 000 000 000
mega	M	10^{6}	=	1 000 000
kilo	k	10^{3}	=	1 000
hecto	h	10^{2}	=	100
deka	da	10^{1}	=	10
deci	d	10^{-1}	=	0.1
centi	c	10^{-2}	=	0.01
milli	m	10^{-3}	=	0.001
micro	μ	10^{-6}	=	0.000 001
nano	n	10^{-9}	=	0.000 000 001
pico	p	10^{-12}	=	0.000 000 000 001

Note: The use of the prefixes hecto, deka, deci, and centi is not recommended in SI.

Normal Stress and Strain

Normal Stress (수직응력)



- Tow bar is a prismatic bar (균일단면봉) in tension
- Isolate the segment as a free body
  → draw a Free Body Diagram

Normal Stress and Strain
Normal Stress

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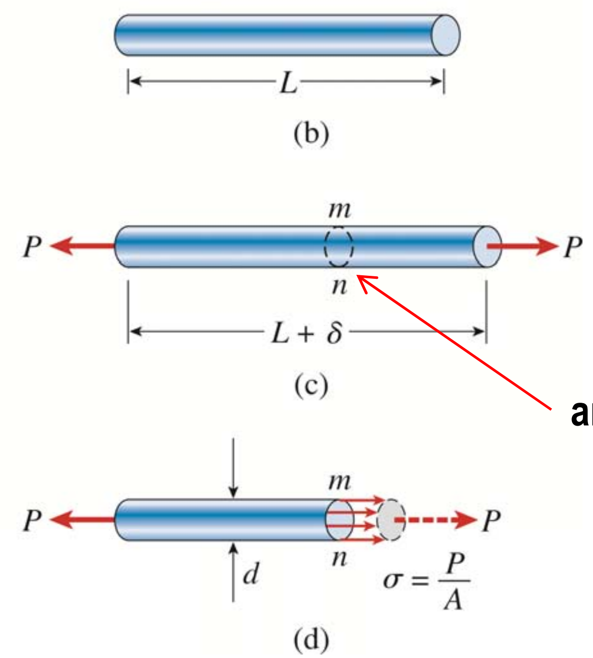


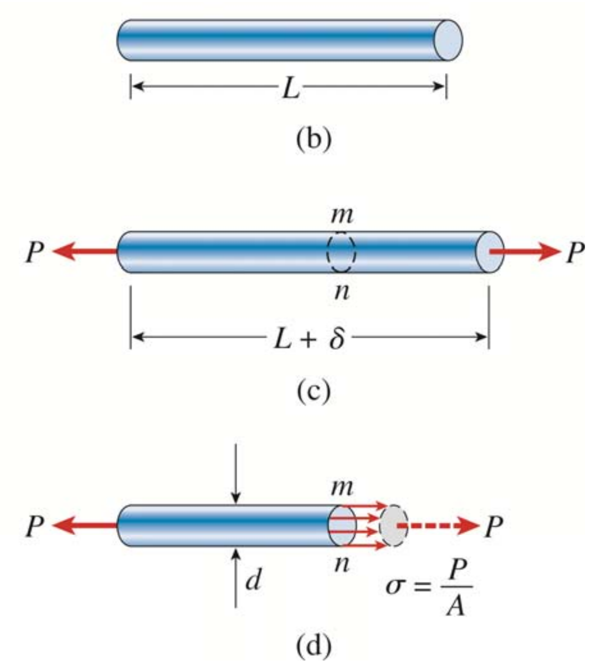
Before loading

After loading, increase of length by δ

an imaginary cut, at section mn

- Assumption:
  - Disregard the weight of the bar
  - The only active forces are the axial force P at the ends

Normal Stress and Strain
Normal Stress



$$\sigma = \frac{P}{A}$$

P: Axial Force (N)
A: cross sectional area (m^2)

- Stress (σ)
  - Average force per unit area:
  - A measure of intensity of force
    - Stretched → tensile stress
    - Compressed → compressive stress
  - Stress act in perpendicular to cut surface: normal stress
  - … tangential…: shear stress
  - Sign convention: tensile (+), compressive (-)*

*** however, there can be different convention. e.g. rock**

Normal Stress and Strain
Normal Stress

- Unit of stress
  - Force per unit area = newtons per square meter
  - N/m^2 = Pa
  - Pa is a very small quantity (1 Pa = stress applied by 2~3 sheets of paper) → kPa or MPa is more often used.
  - If P has a magnitude of 27 kN and the diameter of the bar is 50 mm,

$$\sigma = \frac{P}{A} = \frac{P}{\pi d^2 / 4} = \frac{27kN}{\pi (50mm)^2 / 4} = 13.8\ MPa$$

Normal Stress and Strain
Normal Stress

- Uniformly distributed stress vs. concentrated stress

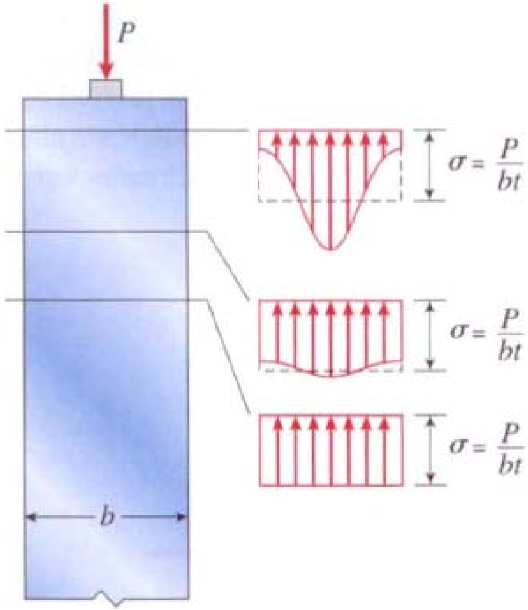


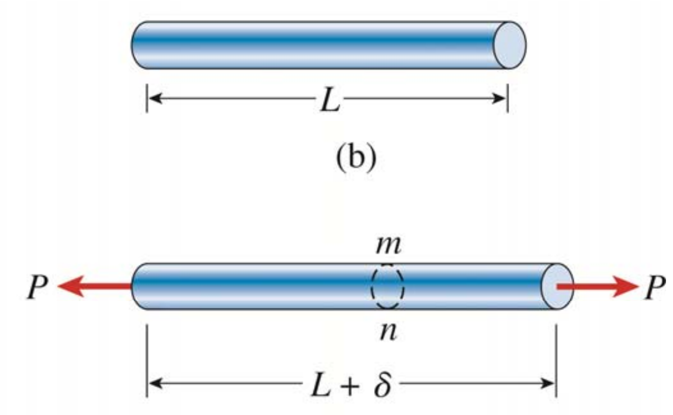
FIG. 2-60 Stress distributions near the end of a bar of rectangular cross section (width b, thickness t) subjected to a concentrated load P acting over a small area

  – Uniform stress exist except near the end

Normal Stress and Strain
Normal Strain (수직변형율)

- Normal Strain (ε)
  - Elongation per unit length

$$\varepsilon = \frac{\delta}{L}$$

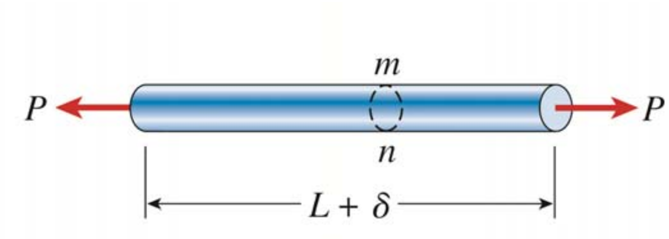


  - Tensile or compressive strain
  - Unit: dimensionless, (sometimes it is expressed as %)
  - If L equal to 2.0 m, elongation was 1.4 mm,

$$\varepsilon = \frac{\delta}{L} = \frac{1.4mm}{2.0m} = 0.0007 = 700 \times 10^{-6}$$

Normal Stress and Strain
Uniaxial (단축) stress and Strain

- Uniaxial stress and strain
  - Stress act only in one direction



  - More complicated cases with biaxial stress and strain will be covered later, e.g., in chapter 7.

Mechanical Properties of Materials

- We need to understand the mechanical behavior and mechanical properties of materials
- Materials subjected to loads in the laboratory
  - Measure force and elongation
  - Change of length → dial gauge or strain gauge

Mechanical Properties of Materials

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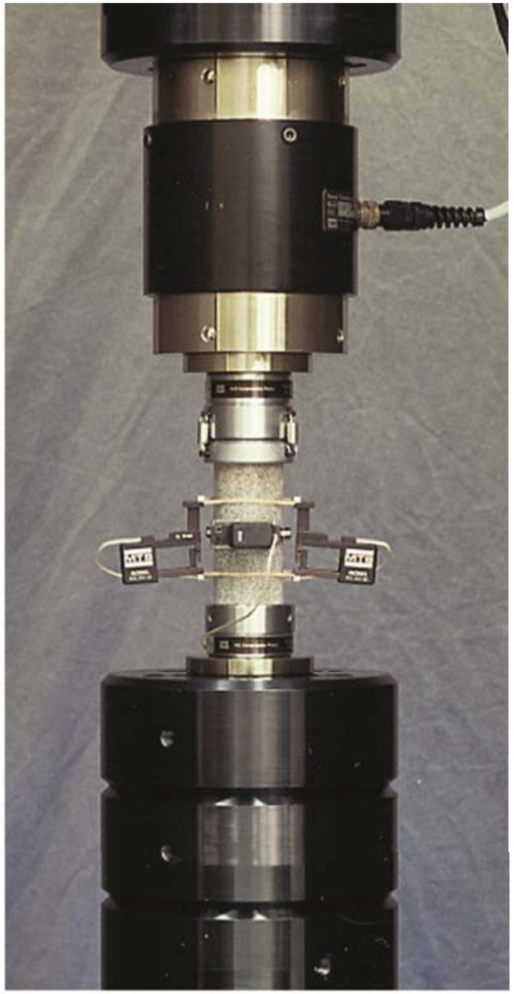
- Compression test of rock



Mechanical Properties of Materials
Stress-Strain diagrams

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- Expressing in terms of stress & strain is the most useful. Why?

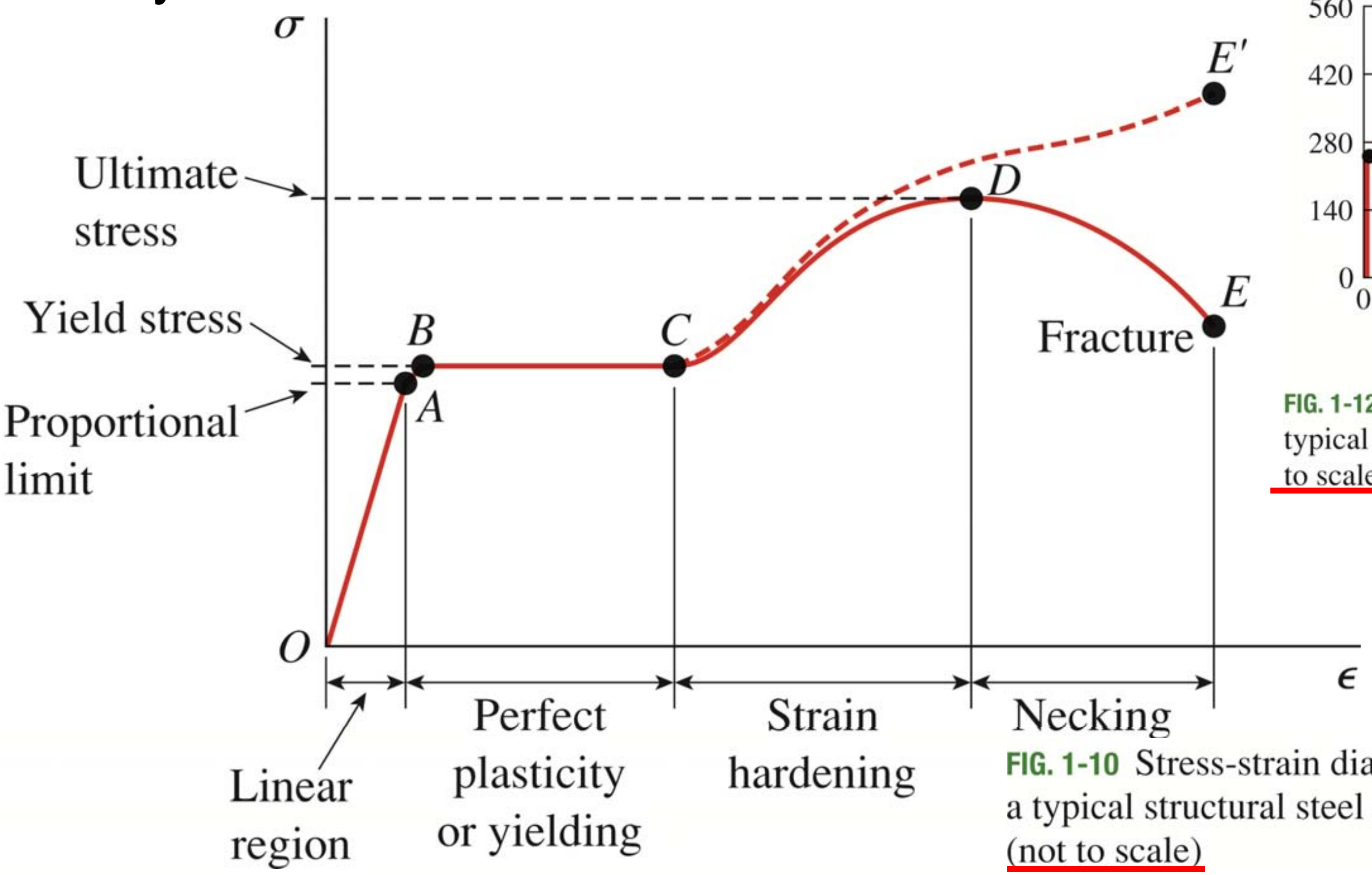

FIG. 1-10 Stress-strain diagram for a typical structural steel in tension (not to scale)

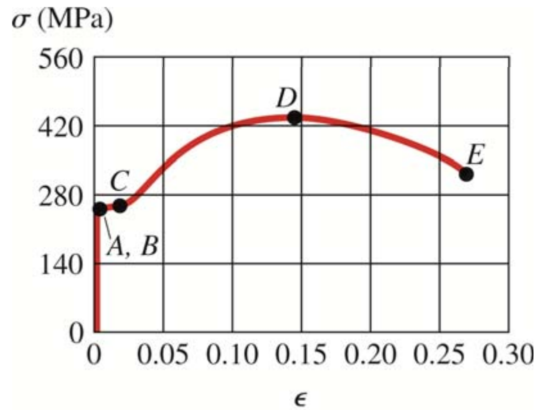

FIG. 1-12 Stress-strain diagram for a typical structural steel in tension (drawn to scale)

Mechanical Properties of Materials
Stress-Strain diagrams

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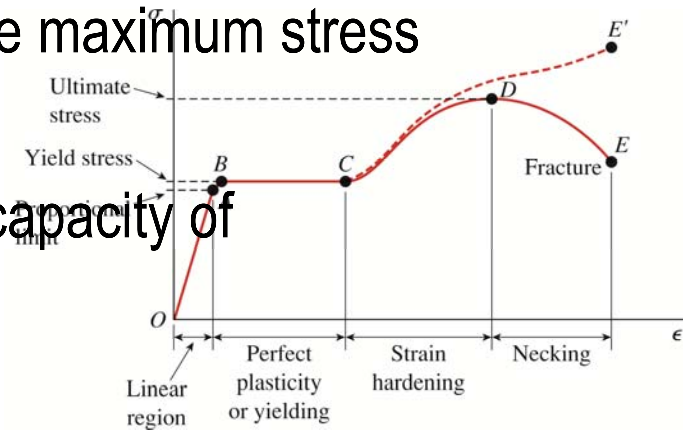
- **Yield stress (항복응력)**: stress when considerable elongation of the specimen occur with no noticeable increase in the stress
  - Very clear in steel, not clear for rock
- **Perfectly plastic (완전소성)**: deforms without an increase in the applied load
- **Strain hardening (변형 경화)**: increased resistance of the material to further deformation
- **Ultimate stress** (or ultimate strength): the maximum stress material can resist
- **Strength (강도)**: a general term for the capacity of a structure to resist loads



Mechanical Properties of Materials
Stress-Strain diagrams

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– **<u>Ductile (연성)</u>** : large permanent deformation before failure
  - steel
– **<u>Brittle (취성)</u>** :relative small permanent deformation before failure
  - Rock, concrete

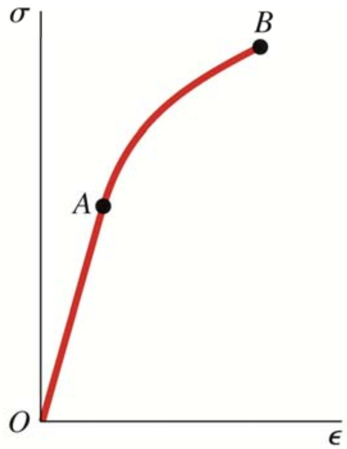

FIG. 1-16 Typical stress-strain diagram for a brittle material showing the proportional limit (point A) and fracture stress (point B)

Elasticity, Plasticity, and Creep
Elastic behavior

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- **Elastic**
  - When a material returns to its original dimension upon unloading
  - Linear elastic or nonlinear elastic
- Elastic limit (E in the graph)
  - The limit of elastic region
- Residual strain
  - Strains after complete removal of the load
  - Residual elongation of the bar is called the 'permanent set'.

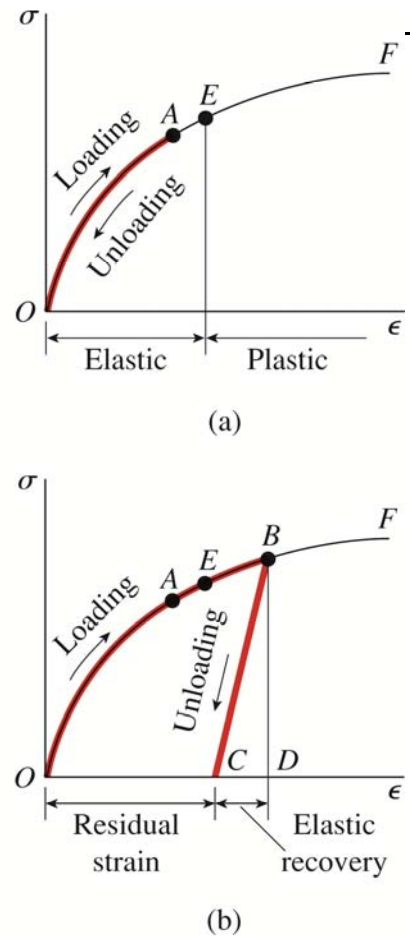

FIG. 1-18 Stress-strain diagrams illustrating (a) elastic behavior, and (b) partially elastic behavior

Elasticity, Plasticity, and Creep
Creep and Relaxation

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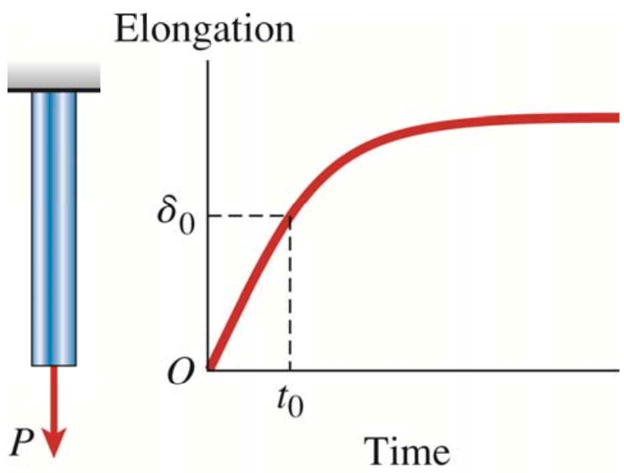
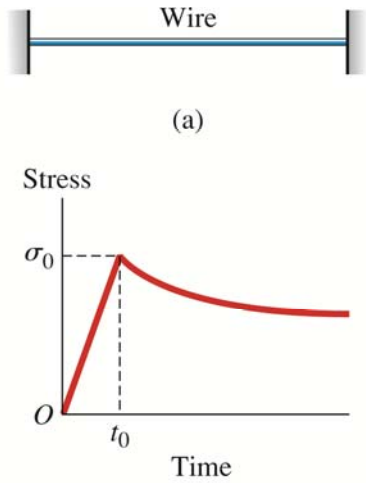
- Creep
  - time dependent behavior of materials
  - Deformation when the stress is constant
- Relaxation
  - Change of stress when the deformation is fixed



Load was fixed as P when $t = t_0$

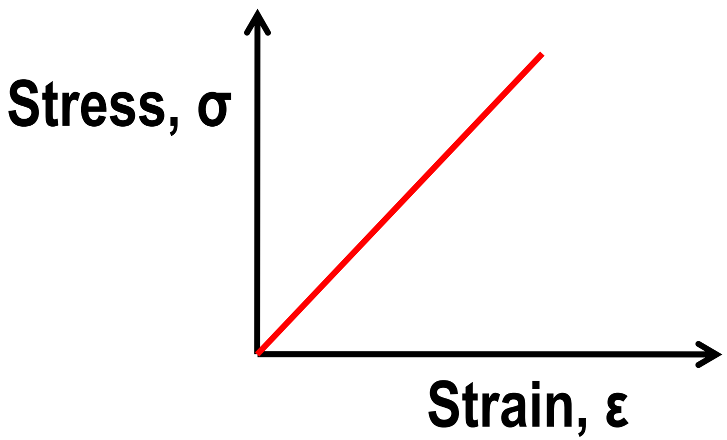


Deformation was fixed when $t = t_0$

I don't think she can stay with the weight for such a long time....There must be creep...

Linear Elasticity, Hooke's Law, and Poisson's Ratio

- Linearly elastic
  - When a material behaves elastically and also exhibits a liner relationship between stress and strain
  - Extremely important ← we want structures and machines works in this region



Linear Elasticity, Hooke's Law, and Poisson's Ratio

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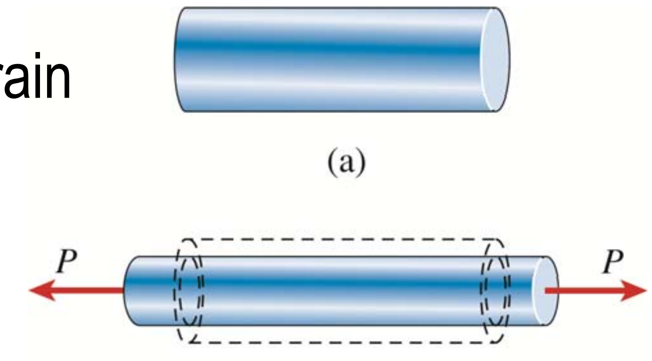
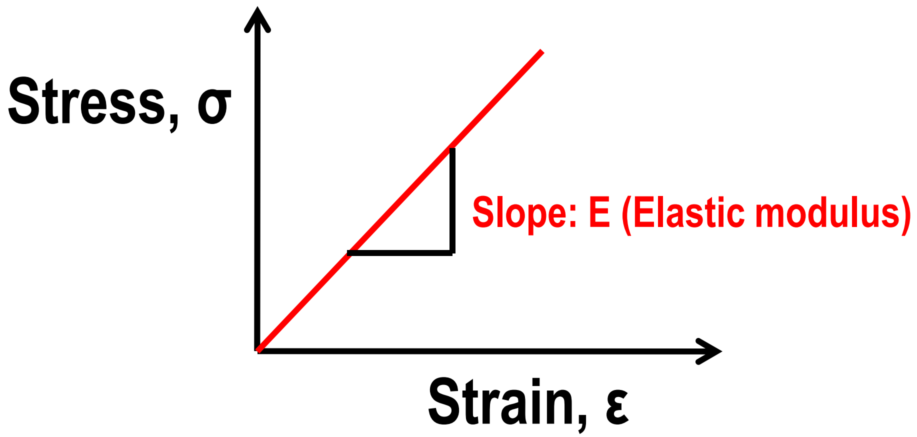
- **Hooke's law**
  - Linear relationship between stress and strain

$$\sigma = E\varepsilon$$

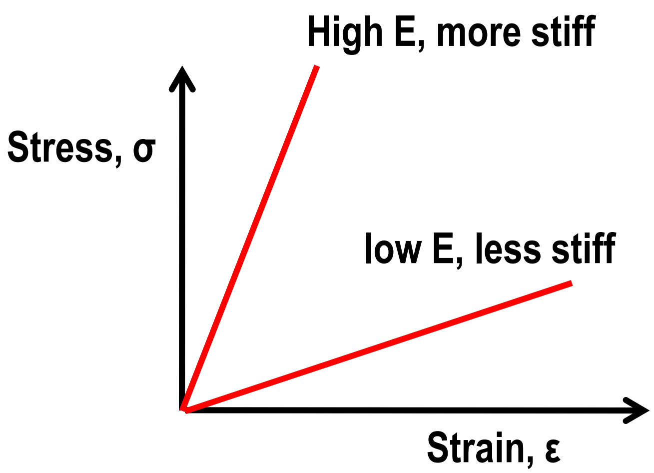
σ: axial stress
ε: axial strain
E: elastic modulus (Young's modulus or modulus of elasticity)





Linear Elasticity, Hooke's Law, and Poisson's Ratio

- **Elastic Modulus**
  - Also called Young's modulus or Modulus of elasticity
  - Unit: same as stress, e.g., MPa, GPa
  - Higher E means more stiff material
  - E is a material property → different value for different materials



Linear Elasticity, Hooke's Law, and Poisson's Ratio

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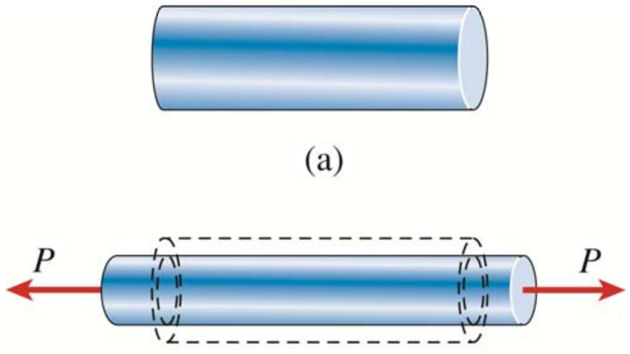

- Loaded in tension, the axial elongation is lateral contraction
- **<u>Poisson's ratio (v)</u>**

$$\nu = -\frac{lateral\ \ strain}{axial\ \ strain} = -\frac{\varepsilon'}{\varepsilon}$$

  - Ratio of lateral strain to axial strain
  - (-) sign needed because lateral and axial strains normally have opposite signs.
  - Upper limit: 0.5
  - Cork: ~0.0, Rubber: ~0.5

Linear Elasticity, Hooke's Law, and Poisson's Ratio

- Assumptions in the textbook (CHILE)
  - Continuous: material does not have discontinuity ←→ discontinous
  - Homogeneous: material has the same composition at every point ←→ inhomogeneous (heterogeneous)
  - Isotropic: materials have the same properties in all directions ←→ Anisotropic
  - Linearly Elastic: ←→ nonlinear elastic or nonlinear
- Complicated problem (DIANE)
  - Discontinous, inhomogeneous, Anisotropic, Nonlinearly Elastic

Linear Elasticity, Hooke's Law, and Poisson's Ratio

- Example 1-3 Steep pipe in compression
  - Given, L=1.2 m, d1: 110 mm, d2: 150mm, P=620 kN, E=200 GPa, v=0.3
  - Determine the
    - (a) shortening,
    - (b) the lateral strain
    - (c) change in diameters
    - (d) increase in the wall thickness

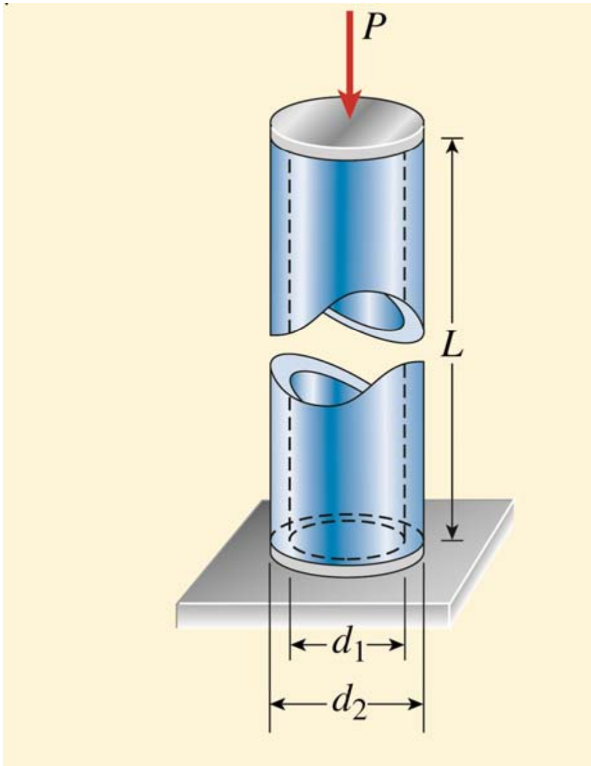

FIG. 1-23 Example 1-3. Steel pipe in compression

Shear Stress and Strain

Shear stress (전단응력)

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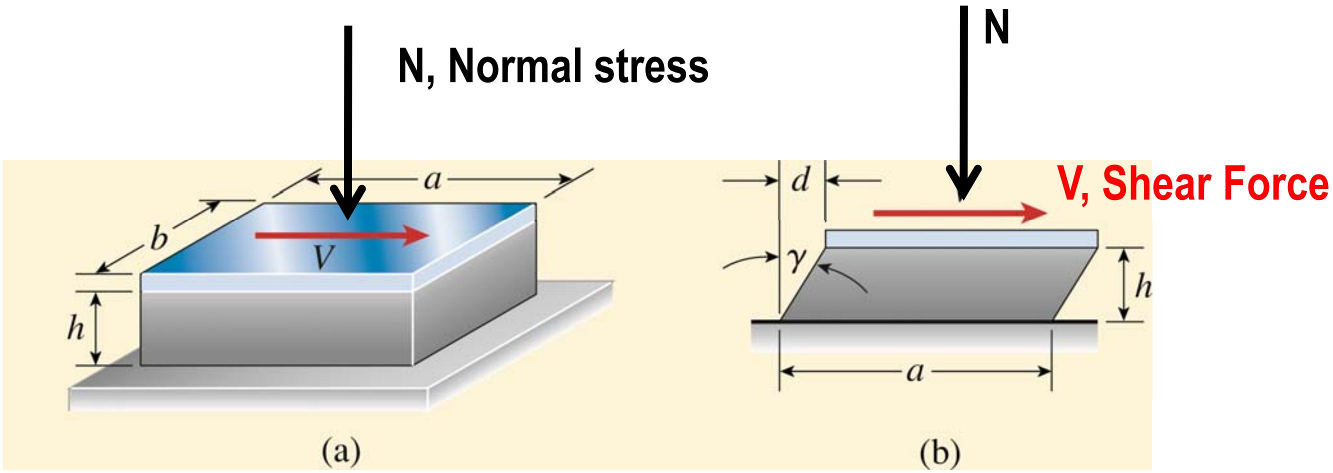
- **<u>Shear stress</u>**: acts *tangential* to the surface of the material



$$\tau = \frac{V}{A}$$

V: Shear Force

A: cross sectional area (a x b)

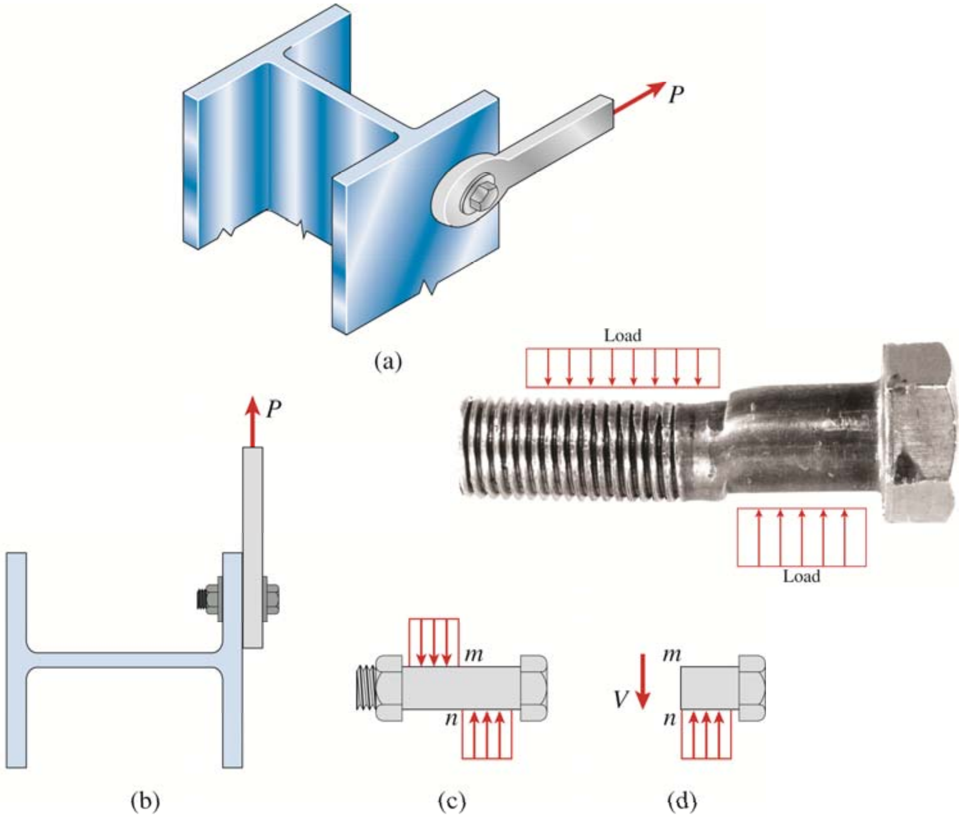
- Unit is equal to normal stress. e.g., MPa

Shear Stress and Strain
Shear stress

- Shear stress

$$\tau = \frac{V}{A}$$

V: Shear Force → P in this case
A: cross sectional area of the bolt



Shear Stress and Strain
Shear stress

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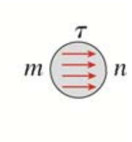
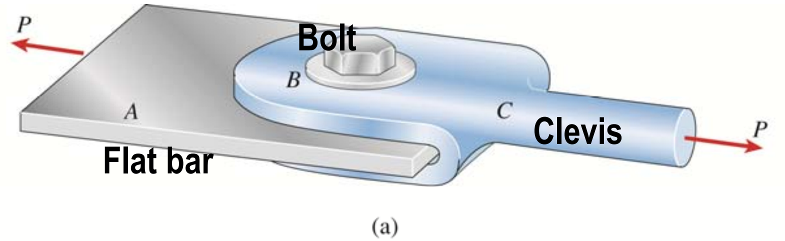
- Shear stress

$$\tau = \frac{V}{A}$$





V: Shear Force → P/2 in this case (double shear)
A: cross sectional area of the bolt

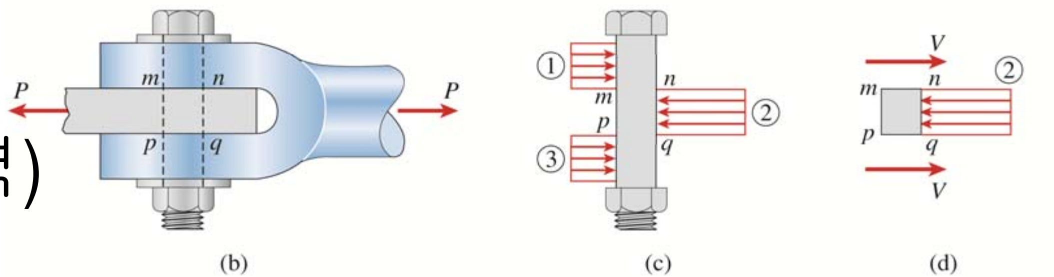


FIG. 1-24 Bolted connection in which the bolt is loaded in double shear

Free Body Diagram Free Body Diagra

- Bearing stress (지압응력)

$$\sigma_b = \frac{F_b}{A_b}$$

F_b: Bearing Force → P/2 in this case
A_b: projected area of curved bearing surface (t x diameter)

Shear Stress and Strain

Equality of shear stress on perpendicular planes

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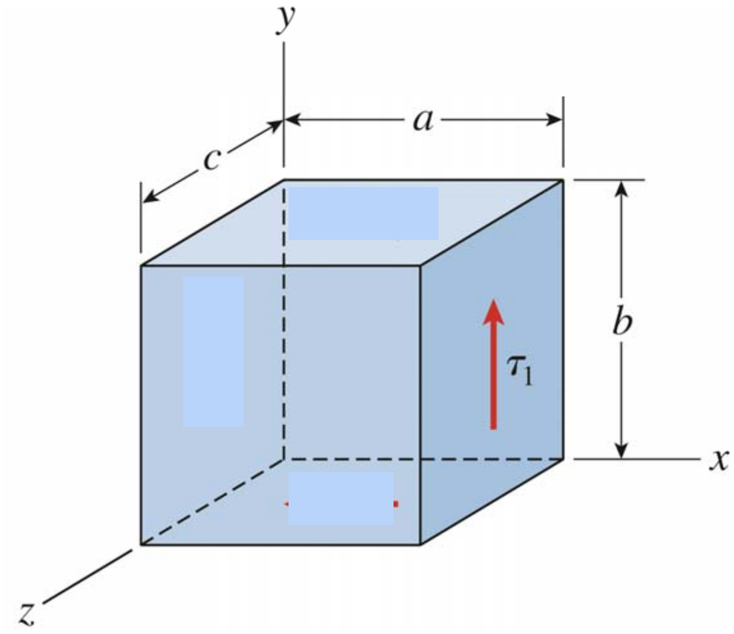

FIG. 1-27 Small element of material subjected to shear stresses

Assume a small element abc

1) Shear stress, τ_1 on area bc → force τ_1 x bc
2) From 'Force Equilibrium'→ same shear stress in opposite side in opposite direction.
   Force τ_1 x bc on left and right-hand sides form a couple (우력)
3) From 'Moment equilibrium' → Force τ_2 x ac on top → τ_1 x abc = τ_2 x abc → $\tau_1 = \tau_2$

Shear Stress and Strain

Equality of shear stress on perpendicular planes

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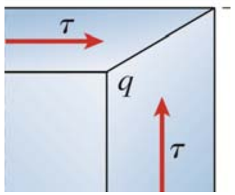
- Shear stresses acting on a rectangular element
- Shear stresses on opposite faces of an element are equal in magnitude and opposite in direction
- Shear stresses on adjacent faces are equal in magnitude and have directions



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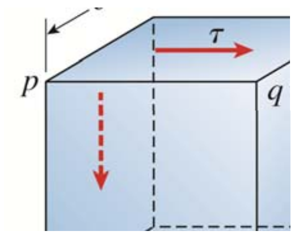

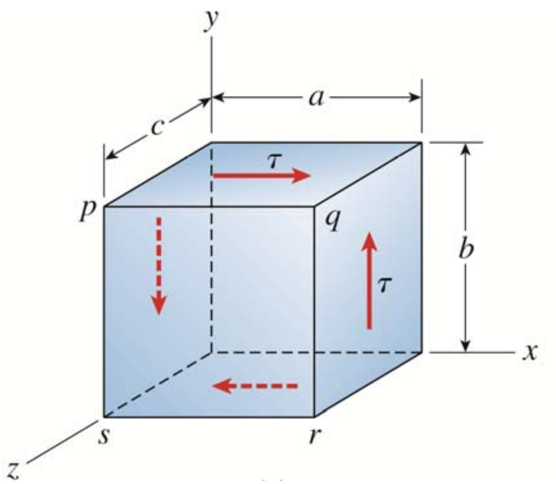

Shear Stress and Strain
Shear Strain

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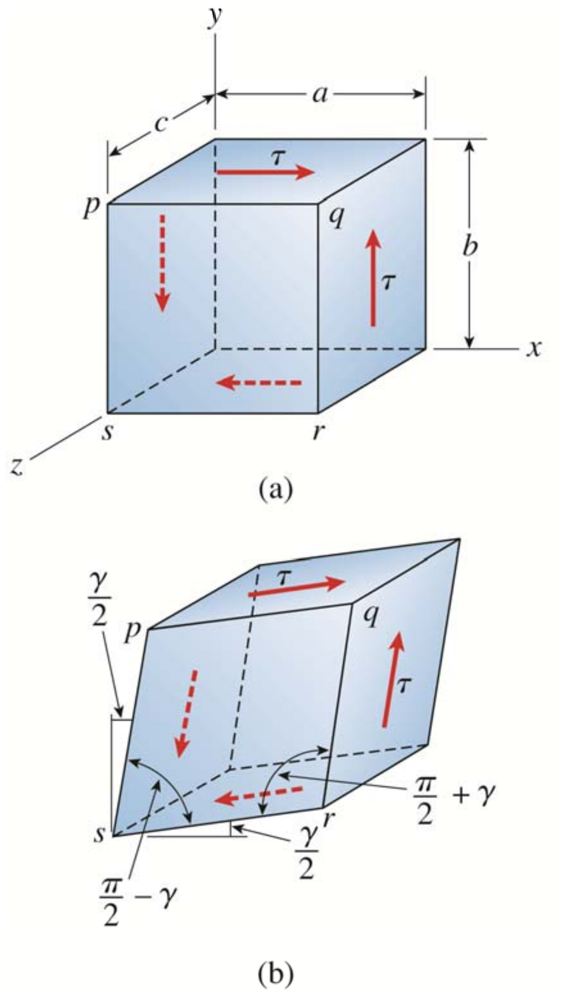
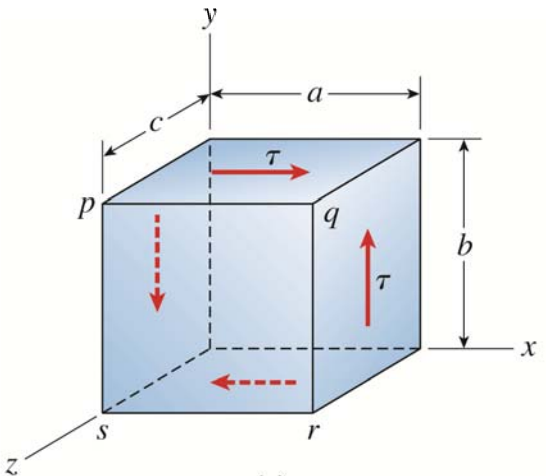
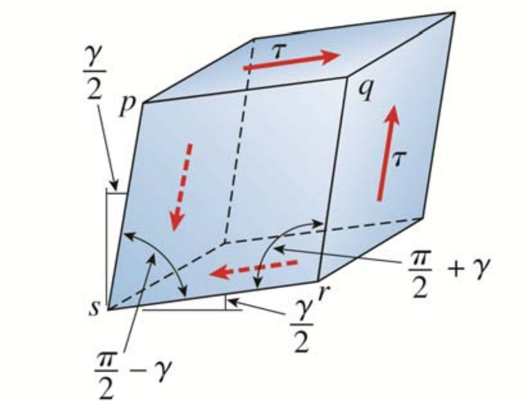


- Shear strain produce a change in the shape - <u>length does not change</u>
- Rectangular parallelepiped (직육면체) → oblique parallelepiped
- The angles between the side faces change. $\pi/2$ → '$\pi/2 - \gamma$' or '$\pi/2 + \gamma$'
- Shear strain is measured in degrees or radians

Shear Stress and Strain
Shear Strain



(a)



(b)

- Sign convention:
  - The angle between two positive faces (or two negative faces) is reduced → (+)
  - The angle between two positive faces (or two negative faces) is increased → (-)

Shear Stress and Strain
Hooke's law in shear

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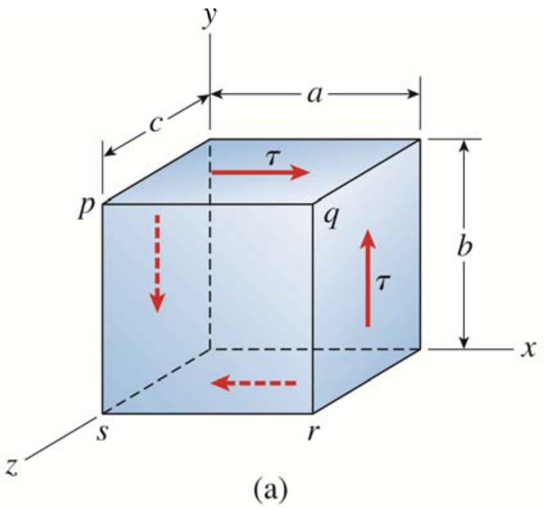
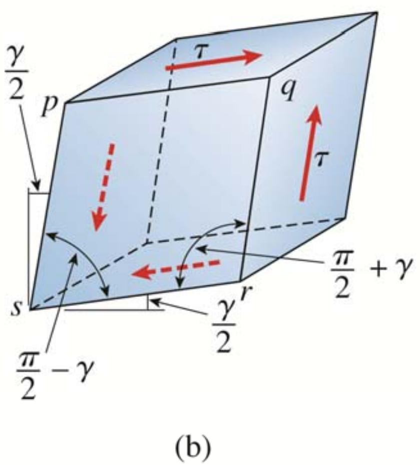


(a)



(b)

$$\tau = G\gamma$$

τ: Shear stress
γ: Shear strain
G: shear modulus (전단계수) or shear modulus of elasticity

- Elastic modulus (E), Poisson's ratio (v) and Shear modulus (G) are not independent → if we know E and v, we can calculate G.

$$G = \frac{E}{2(1+\nu)}$$

Derivation will be covered in sec 3.6

Normal and Shear Stress/Strain

	Stress	Strain	Relations
Normal	σ ⊥ A $\sigma = \frac{P}{A}$	$\varepsilon = \frac{\delta}{L}$	$\sigma = E\varepsilon$
Shear	τ // A $\tau = \frac{V}{A}$	γ *angular change*	$\tau = G\gamma$

Allowable Stresses and Allowable loads (허용응력과 허용하중)

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- Structures
  - Any object that must support or transmit loads
- Strength
  - The capacity of the object to support or transmit loads
- To avoid a failure
  - Actual strength of a structure > required strength
- Factor of Safety n

$$n = \frac{actual\ \ strength}{required\ \ strength}$$

Allowable Stresses and Allowable loads
Factor of Safety, n

$$Factor\ of\ \ Safety,\ n = \frac{actual\ \ strength}{required\ \ strength}$$

– N > 1.0 to avoid a failure, ~ 10.0

– Consideration

- Probability of accidental overloading
- Types of loading (static or dynamic)
- Load applied once or repeated
- Possibility of fatigue failure
- Inaccuracy in construction
- Deterioration
- Accuracy of the method of analysis
- Consequence of failure

Allowable Stresses and Allowable Loads

- Allowable stress

$$Allowable\ stress = \frac{Yield\ Strength}{Factor\ of\ Safety}$$

- Allowable load

$$Allowable\ load = (Allowable\ stress)(Area)$$

- In tension or compression

$$P_{allow} = \sigma_{allow} A$$

- In direct shear

$$P_{allow} = \tau_{allow} A$$

- Allowable load upon bearing

$$P_{allow} = \sigma_b A_b$$

A_b: projected area over which the bearing stress act

Design for Axial loads

- Analysis

Loads → Material property & dimension → Response: stress & strain

- Design

Loads → Intended function: load carrying capacity → Material property & Dimension

  – Required areas of members

$$required\ \ area = \frac{Load\ \ to\ \ be\ \ transmitted}{Allowable\ \ stress}$$

Design for Axial loads

- Optimization
  - Task of designing the best structure to meet a particular goal
- General considerations
  - Weight
  - Aesthetic
  - Economic
  - environmental
  - political

Review of statics
Equilibrium

- Equilibrium of a rigid body
- A rigid body is in equilibrium when the resultants of the system of forces acting on the body is zero.
  - Vector sum of all external forces acting on the body is zero

$$\sum \mathbf{F} = 0 \qquad \sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

  - Vector sum of the moments of the external forces about any pont O is zero

$$\sum \mathbf{M}_0 = 0 \qquad \sum M_x = 0, \sum M_y = 0, \sum M_z = 0$$

Review of statics
Free Body Diagram (FBD, 자유물체도)

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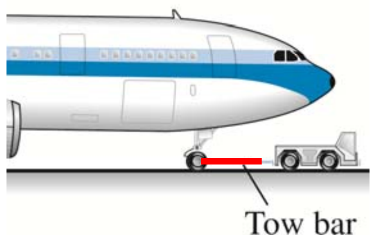
- Drawing that shows
  - 'a body of interest' separated from all other interacting bodies and,
  - all external forces, both known and unknown, that are applied to the body.
  - 'Free' means that external bodies are replaced with by the forces.
- FBD clearly establishes which body or portion of the body is being studied
- *The importance of drawing a correct free-body diagram cannot be overemphasized*



Review of Statics
FBD (Support Reactions)

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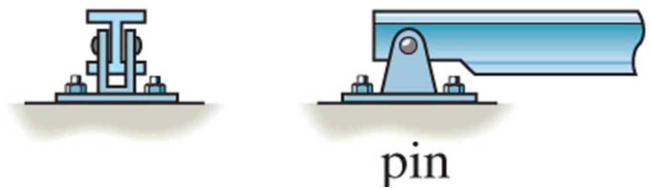
- Rule 1: a support prevents the translation → force developed
- Rule 2: rotation is prevented → a couple moment is exerted
- Three types of support:

  1) Roller: Prevents the translation in vertical direction

  2) Pin: Prevents the translation in any direction



  3) Fixed support: Prevents both translation and rotation

Review of Statics
FBD (Support Reactions)

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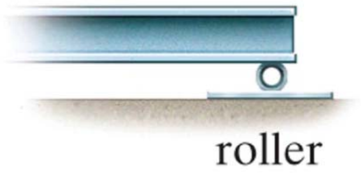
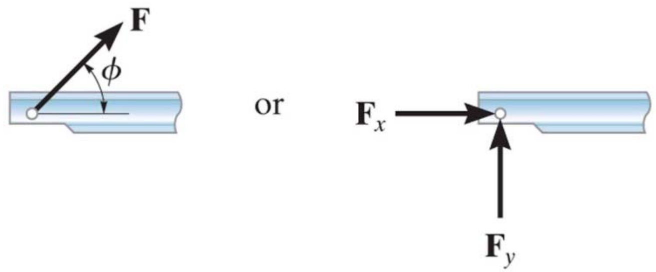
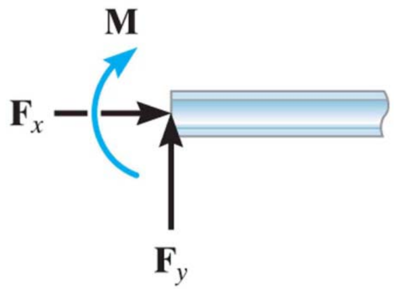


Free body Diagram

Free body Diagram



Free body Diagram



Problem solving
Steps

- Make a clear statement of the problem, identify the what is known and what is to be found and draw clear sketches
- Analysis;
  - Apply principles of mechanics
    - Free Body Diagram
    - Equations of Equilibrium
    - Allowable stress…
  - Interpret the results in terms of the physical behavior (Does the number make sense?)
- Check the errors (calculation, units)
- Present the results in clear, neat fashion.

Example 1-8
Two-bar truss ABC on pin supports

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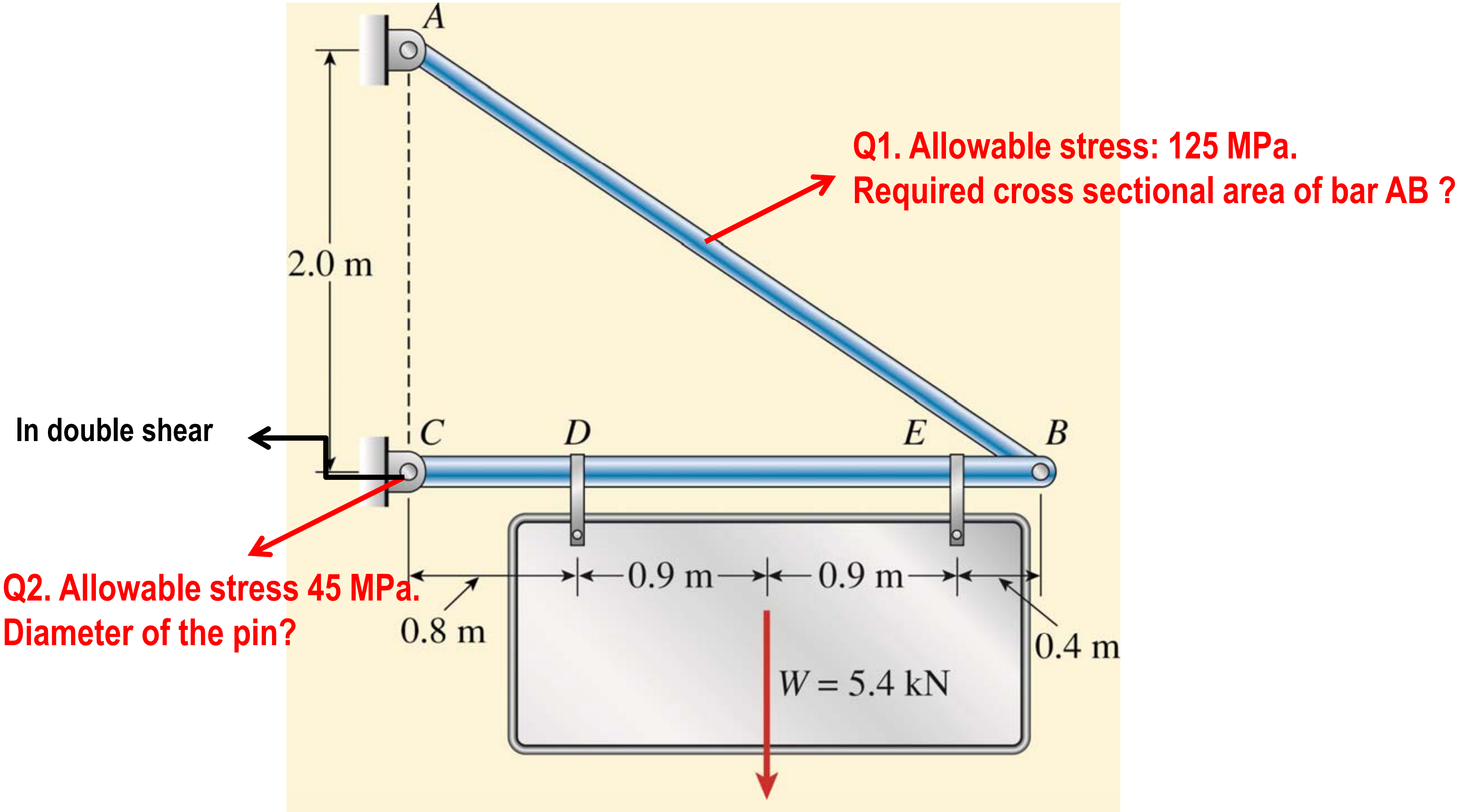
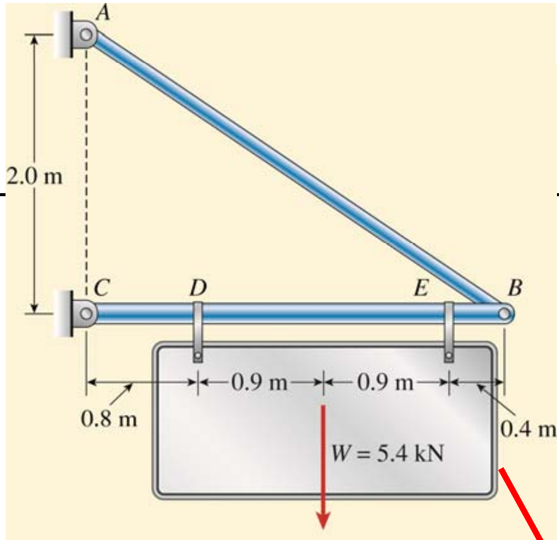
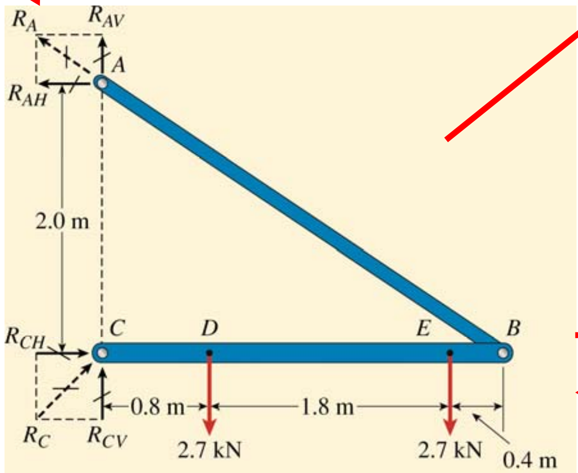


FIG. 1-33 Example 1-8. Two-bar truss *ABC* supporting a sign of weight *W*



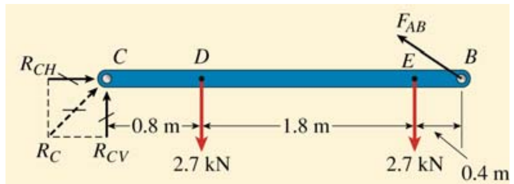
Free body Diagram



Reactions calculated

$$Area = \frac{Load\ to\ be\ transmitted}{Allowable\ stress}$$

Free body Diagram



- **Loads** : *active forces* **that are applied to the structure by some external cause**

* **Reactions:** *passive forces* **that are induced at the supports of the structure**

Summary

- Introduction to Mechanics of Materials
- Normal Stress and Normal Strain
- Mechanical Properties of Materials
- Elasticity, Plasticity, and Creep
- Linear Elasticity, Hooke's Law, and Poisson's Ratio
- Shear Stress and Shear Strain
- Allowable Stresses and Allowable Loads
- Design for Axial Loads and Direct Shear
- Review of statics

Next week

- Introduction
- Changes in Lengths of Axially Loaded Members
- Changes in Lengths Under Nonuniform Conditions
- Statically Indeterminate Structures
- Thermal Effects, Misfits, and Prestrains
- Stresses on Inclined Sections
- Strain Energy
- Stress Concentrations*

- All the figures are taken from Gere and Goodno (2009) unless otherwise stated.