Week 7, 14 March

Mechanics in Energy Resources Engineering - Chapter 5 Stresses in Beams (Basic topics)

Ki-Bok Min, PhD

Assistant Professor
Energy Resources Engineering
Seoul National University

SEOUL NATIONAL UNIVERSITY

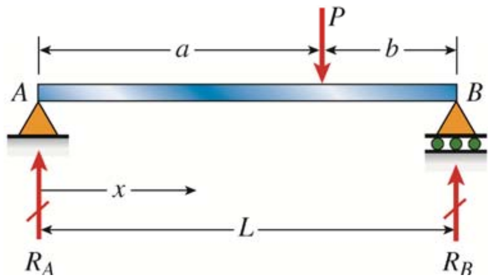
Shear Forces and Bending Moments
Preview

SEOUL NATIONAL UNIVERSITY

- Introduction
- Types of Beams, Loads, and Reactions
- Shear Forces and Bending Moments
- Relationships Between Loads, Shear Forces and Bending Moments
- Shear-Force and Bending-Moment Diagrams

Shear-Force and Bending-Moment Diagrams
Concentrated Load

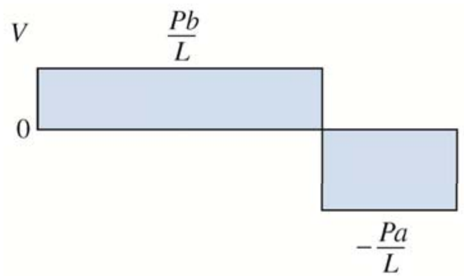
SEOUL NATIONAL UNIVERSITY



- Shear Force Diagram,

$$V = \frac{Pb}{L} \qquad (0 < x < a)$$

$$V = -\frac{Pa}{L} \qquad (0 < x < a)$$

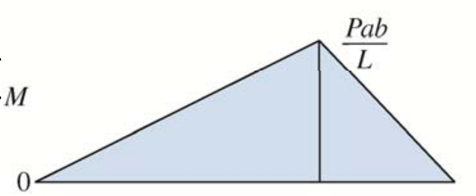


Slope dV/dx =0
→ q=0
Area for x < a
→ increase in M
Area for a<x<b
→ decrease in M

- Bending Moment Diagram

$$M = \frac{Pbx}{L} \qquad (a < x < L)$$

$$M = \frac{Pa}{L}(L - x) \qquad (a < x < L$$



Slope dM/dx = V

Shear-Force and Bending-Moment Diagrams
Uniform Load

SEOUL NATIONAL UNIVERSITY

- From Moment Equilibrium,

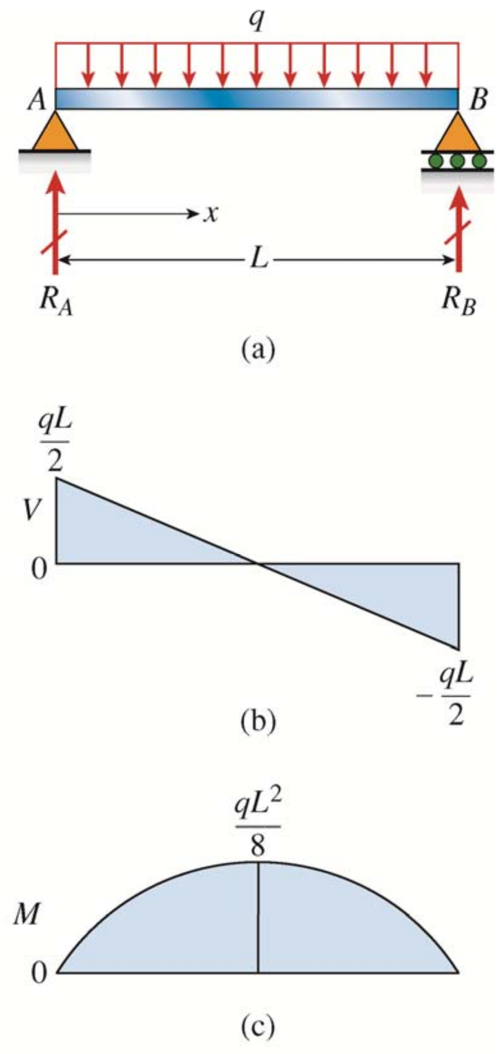
$$R_A = R_B = \frac{qL}{2}$$

- From Free Body Diagram,

$$V = R_A - qx = \frac{qL}{2} - qx$$

$$M = R_A x - qx\left(\frac{x}{2}\right) = \frac{qLx}{2} - \frac{qx^2}{2}$$

- Slope of V?
- Slope of M?



Shear-Force and Bending-Moment Diagrams
Several Concentrated Loads

- From Moment Equilibrium,

$$R_A + R_B = P_1 + P_2 + P_3$$

- From Free Body Diagram,

$$V = R_A \quad M = R_A x \qquad (0 < x < a_1)$$

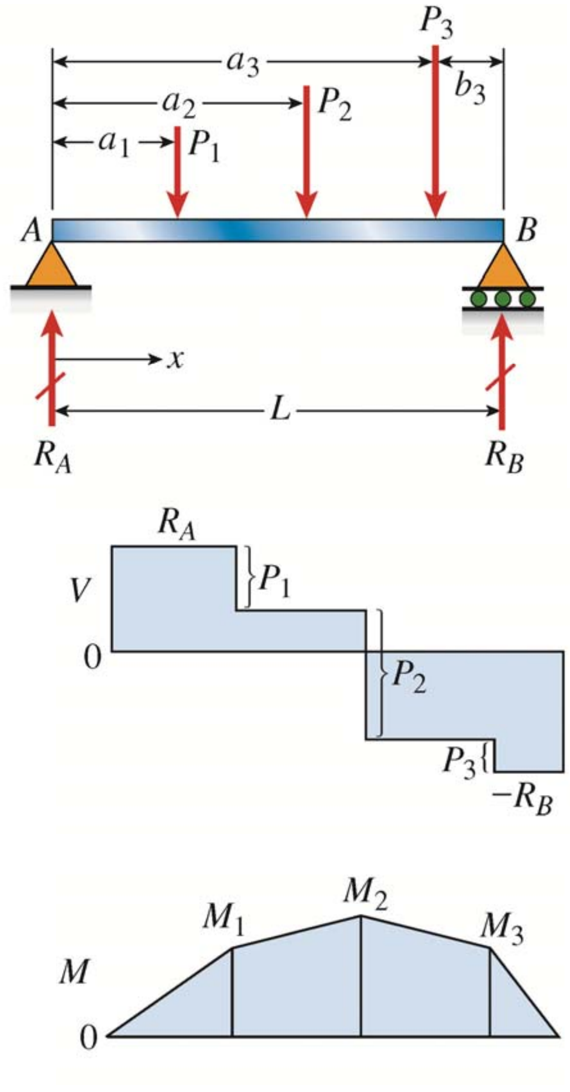
$$V = R_A - P_1 \quad M = R_A x - P_1(x - a_1) \qquad (a_1 < x < a_2)$$

$$V = -R_B + P_3$$

$$M = R_B(L - x) - P_3(L - b_3 - x) \qquad (a_2 < x < a_3)$$

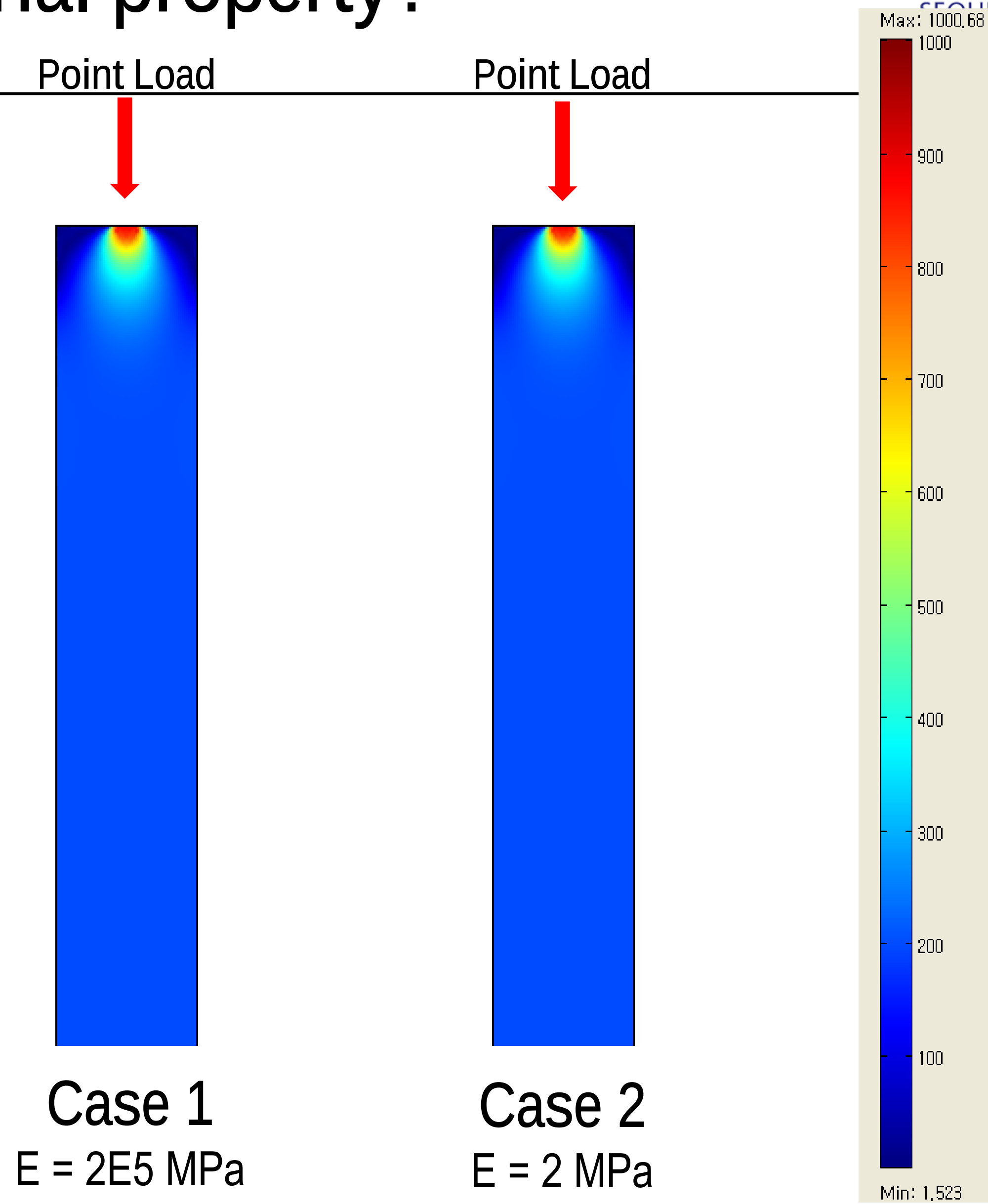
$$V = -R_B$$

$$M = R_B(L - x) \qquad (a_3 < x < L)$$



Saint-Venant Principle
Effect of material property?

SEOUL NATIONAL UNIVERSITY



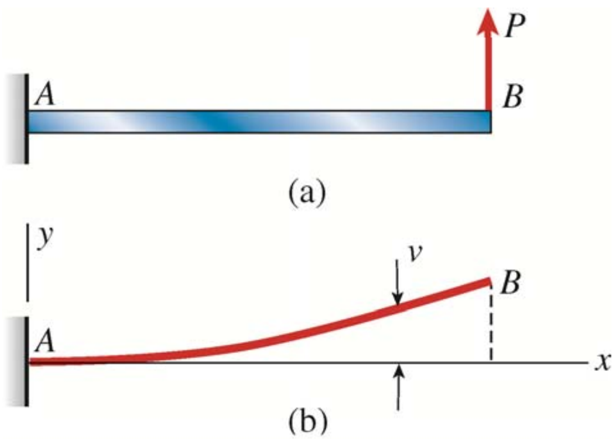
Outline

- Introduction
- Pure Bending and Nonuniform Bending
- Curvature of Beam
- Longitudinal Strains in Beams
- Normal Stress in Beams
- Design of Beams for Bending Stresses
- Nonprismatic Beams
- Shear Stresses in Beams of Rectangular Cross Section
- Shear Stresses in Beams of Circular Cross Section
- Shear Stresses in the Webs of Beams with Flanges

Introduction

SEOUL NATIONAL UNIVERSITY

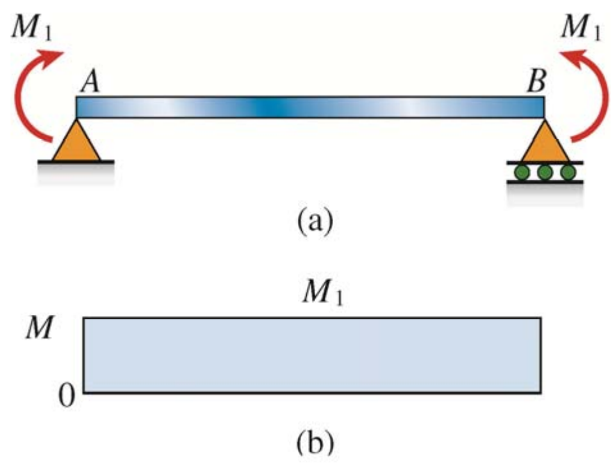
- Chapter 4 → Shear forces (V) & Bending Moments (M).
- How about stresses and strains associated with V & M?
- Assumption:
  - Beams are symmetric about the xy plane.
  - y-axis is an axis of symmetry of the cross section
  - All loads act in this same plane, known as the plane of bending



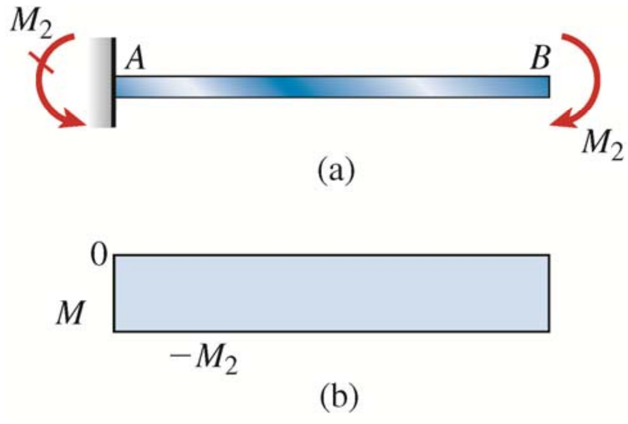
Pure Bending and Nonuniform Bending

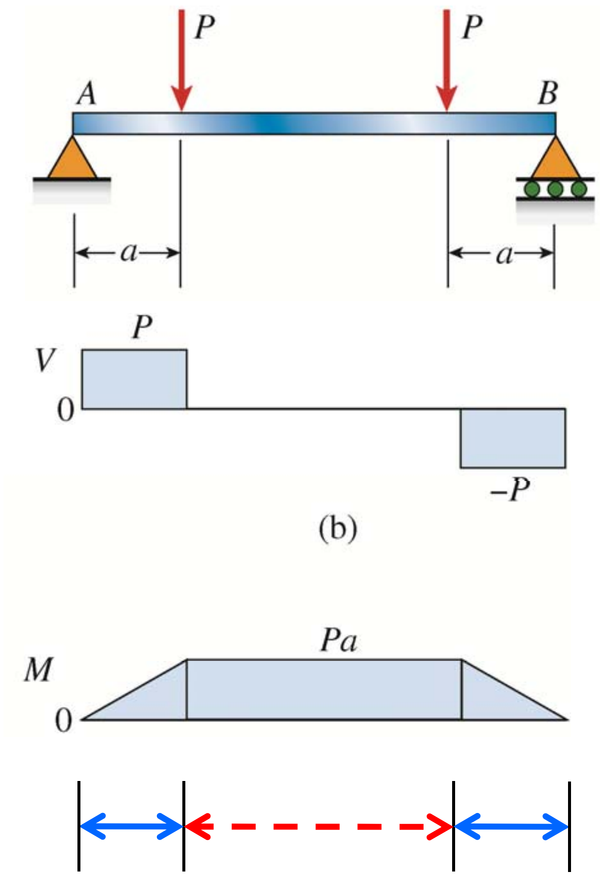
SEOUL NATIONAL UNIVERSITY

- Pure Bending:
  - Flexure of a beam under a constant bending moment.
  - Occurs only in regions with zero shear force
- Nonuniform bending
  - Flexure in the presence of shear forces
  - Bending moment changes
- Simple beam AB loaded by two couples M_1
  - Constant bending moment & shear force 0



Pure Bending and Nonuniform Bending
Other examples





nonuniform bending

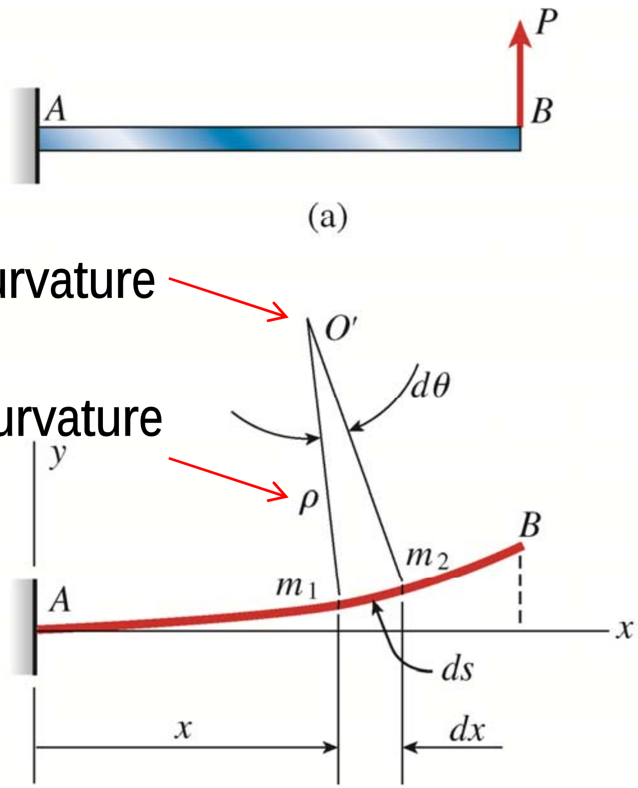
Pure bending

nonuniform bending

Curvature of a Beam
definition

SEOUL NATIONAL UNIVERSITY

- Strains and stresses due to lateral load are directly related to the curvature of deflection curve.
  - Two points m_1 & m_2 on the deflection curve
  - Center of curvature
  - Radius of curvature



- Curvature (κ, 곡률, 曲率): reciprocal of the radius of curvature
  - Measure of how sharply a beam is bent

$$\kappa = \frac{1}{\rho}$$

Curvature of a Beam

- From the geometry of triangle $O'm_1m_2$,

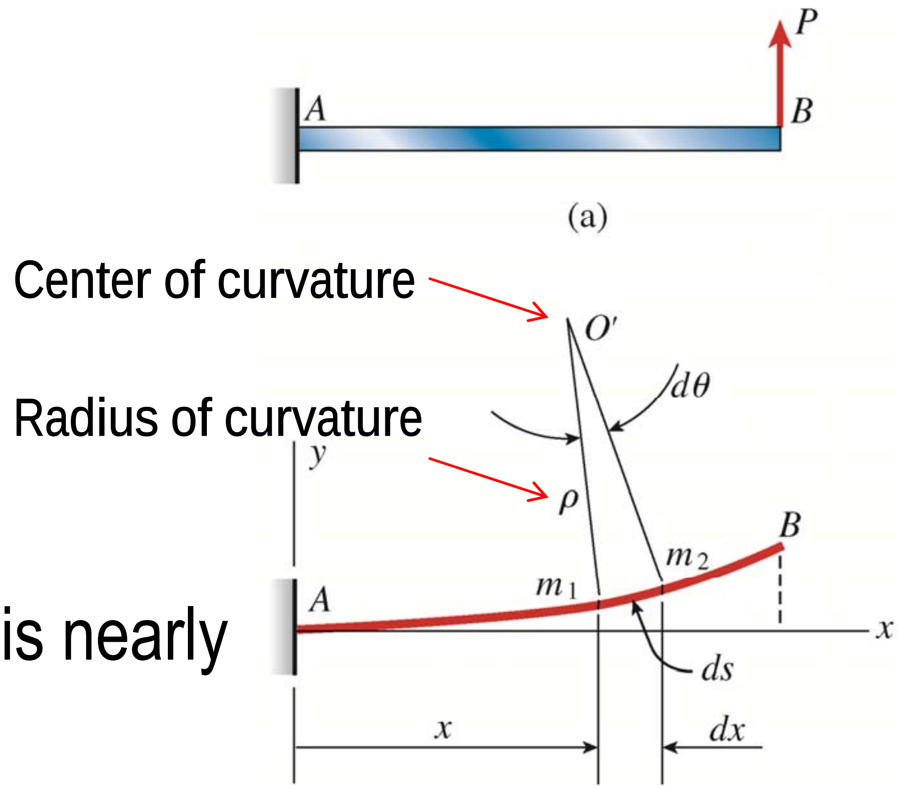
$$\rho d\theta = ds$$

- By rearranging,

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$$

- Under the assumption of small deflections → deflection curve is nearly flat

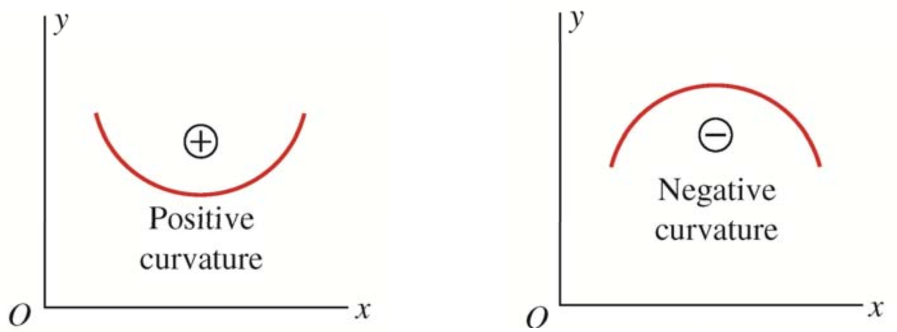
$$\kappa = \frac{1}{\rho} = \frac{d\theta}{dx}$$



Curvature of a Beam
sign convention

SEOUL NATIONAL UNIVERSITY

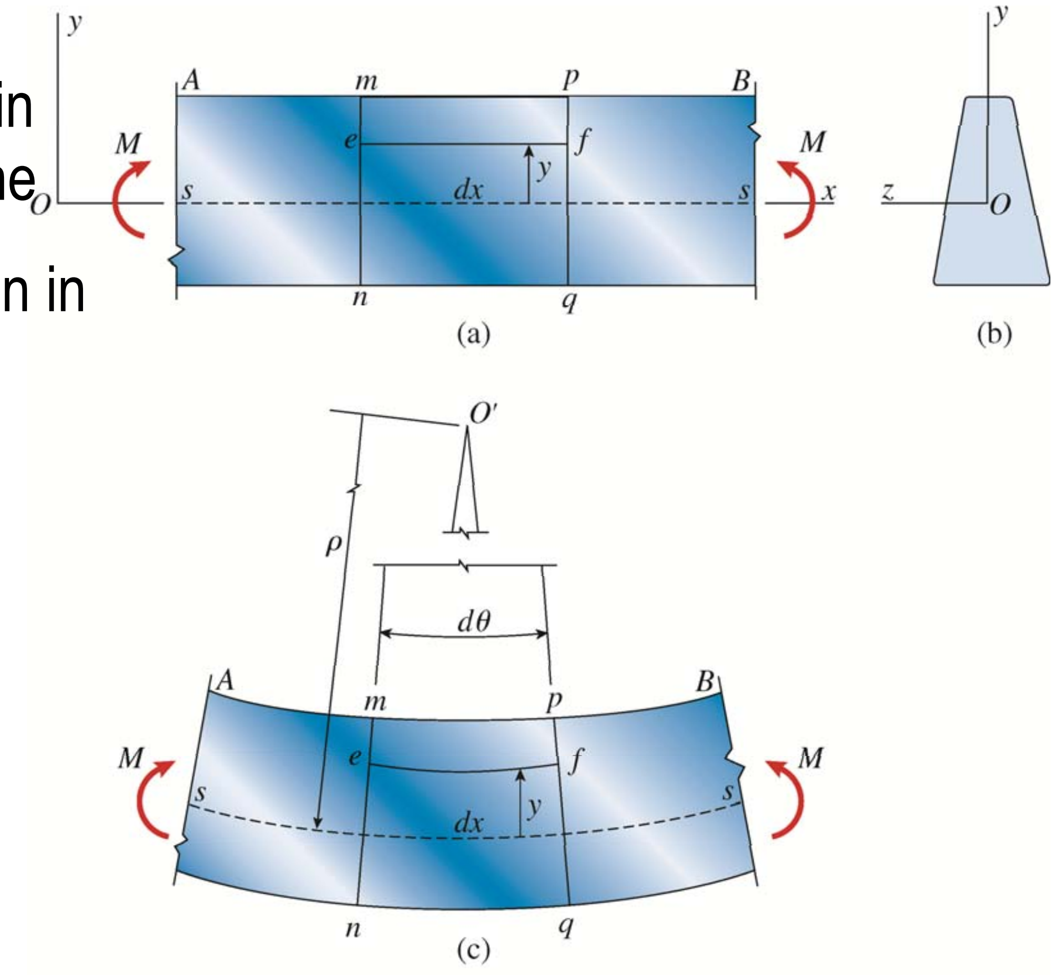
- Sign convention of curvature
  - (+) : beam is bent concave upward (위로 오목)
  - (-): beam is bent concave downward (아래로 오목)



Longitudinal Strains in Beams

SEOUL NATIONAL UNIVERSITY

- Basic assumption:
  - Cross section of a beam in pure bending remain plane
  - (There can be deformation in the plane itself)
- Upper part: shorten→ compression
- Lower part: elongate→ tension



Longitudinal Strains in Beams

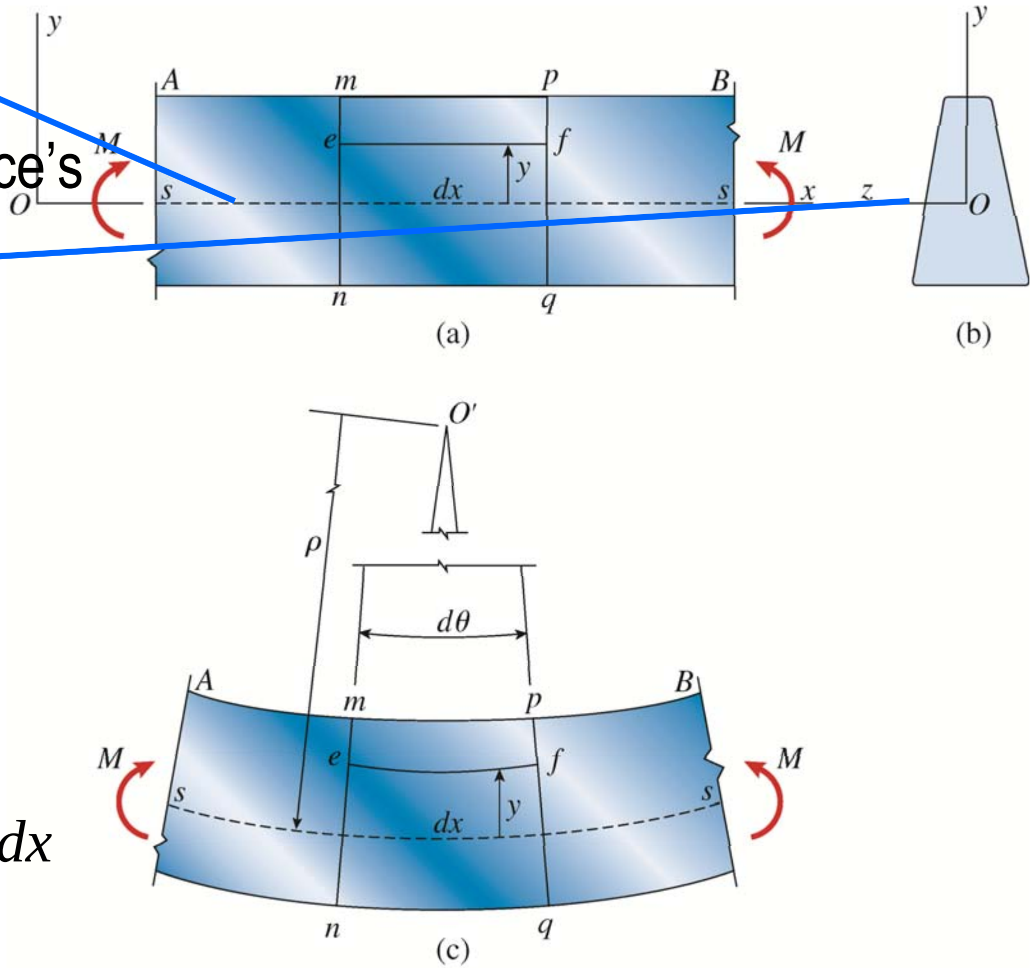
SEOUL NATIONAL UNIVERSITY

- Neutral surface: no longitudinal strain
- Neutral axis: neutral surface's intersection with cross-sectional plane
- At neutral surface:

$$dx = \rho d\theta$$

- Length L1 of line *ef* after bending

$$L_1 = (\rho - y)d\theta = dx - \frac{y}{\rho}dx$$



Longitudinal Strains in Beams

SEOUL NATIONAL UNIVERSITY

- Longitudinal strain

$$\varepsilon_x = -\frac{y}{\rho} = -\kappa y$$

  - Strain-curvature relation
  - Longitudinal strain is proportional to the curvature & distance y from the neutral surface (regardless of the material)
  - Longitudinal stress expected
  - Transverse strains due to Poisson's ratio → does not induce transverse stress, why?

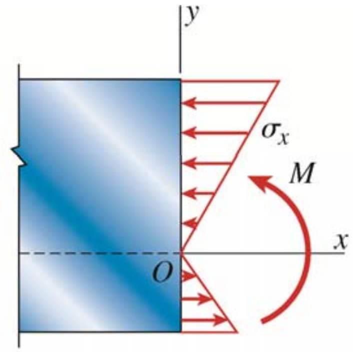
Normal Stress in Beams

SEOUL NATIONAL UNIVERSITY

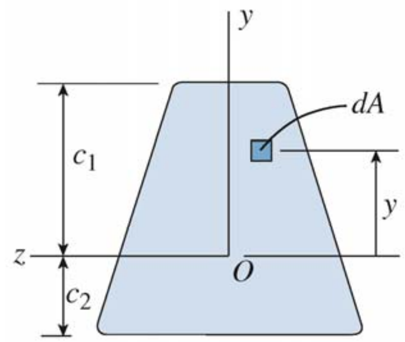
- From Hooke's Law,

$$\sigma_x = E\varepsilon_x = -\frac{Ey}{\rho} = -E\kappa y$$

  - Stresses are compression above the neutral surface with positive curvature
  - Still not practical. Why?
  - Determine y & relationship between κ (curvature) and M (Bending Moment)



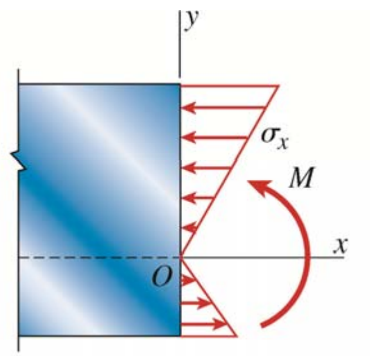
(a)



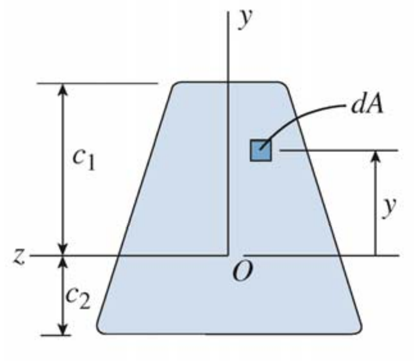
Normal Stress in Beams

SEOUL NATIONAL UNIVERSITY

- Resultant of the normal stresses
  - Resultant force in x direction is zero
  - The resultant moment is equal to the bending moment M



(a)



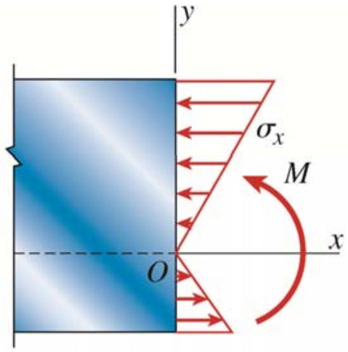
Normal Stress in Beams
Location of Neutral Axis

SEOUL NATIONAL UNIVERSITY

- Because there is no resultant force acting on the cross section

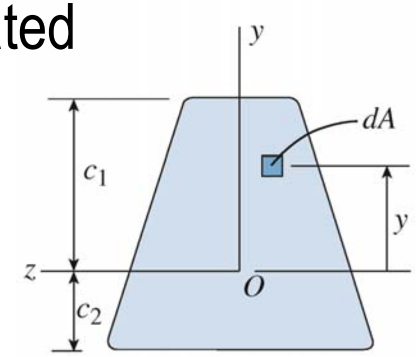
$$\int_A \sigma_x dA = -\int_A E\kappa y dA = 0$$

$$\int_A y dA = 0$$



(a)

  - First moment of the area of the cross section evaluated with respect to z-axis is zero. → z-axis must pass through the centroid.
  - Y axis is also axis of symmetry



- The origin O of coordinates is located at the centroid of the cross sectional area

Normal Stress in Beams
Moment-Curvature Relationship

SEOUL NATIONAL UNIVERSITY

- Elemental moment

$$dM = -\sigma_x y dA$$

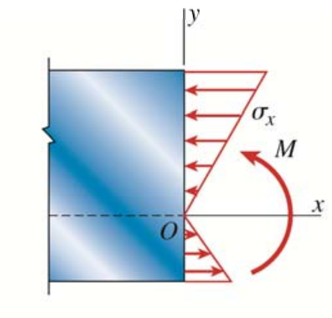
$$M = \int_A \kappa E y^2 dA = \kappa E \int_A y^2 dA$$

$$M = \kappa EI \qquad I = \int_A y^2 dA$$

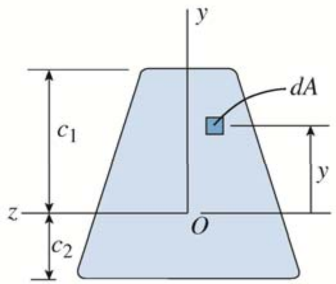
- Moment-Curvature Equation

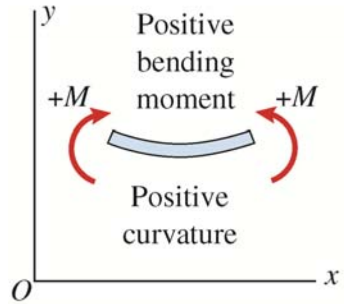
$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

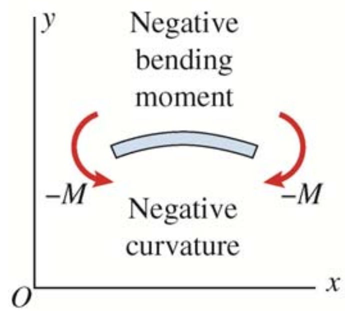
Flexural rigidity: a measure of the resistance of a beam to bending



(a)







Normal Stress in Beams
Flexure Formula (굽힘 공식)

- Finally, bending stress due to bending moment is:

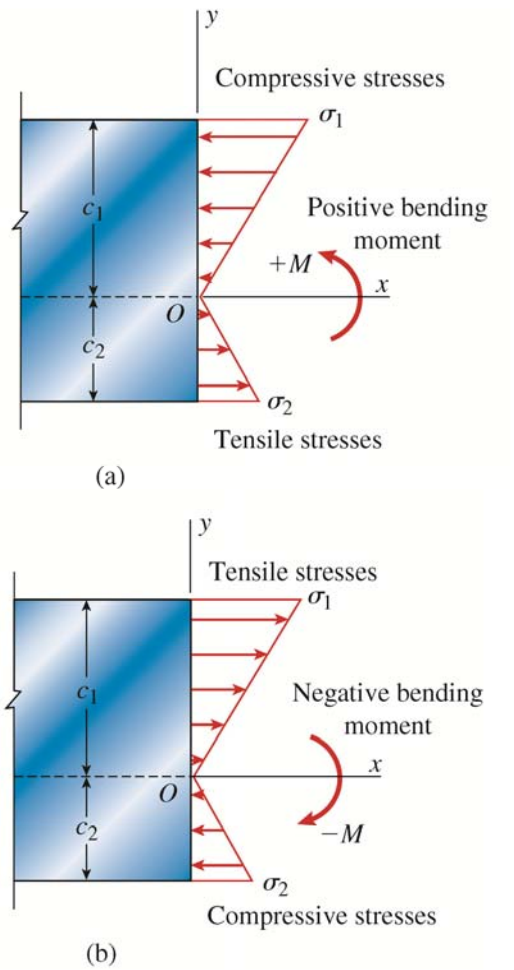
$$\sigma_x = -\frac{My}{I}$$

Bending stress

- Maximum tensile and compressive bending stresses occur at points located farthest from the neutral axis.

$$\sigma_1 = -\frac{Mc_1}{I} = -\frac{M}{S_1} \qquad \sigma_2 = \frac{Mc_2}{I} = \frac{M}{S_2}$$

$$S_1 = \frac{I}{c_1} \qquad S_2 = \frac{I}{c_2}$$



Normal Stress in Beams
Flexure Formula (굽힘 공식)

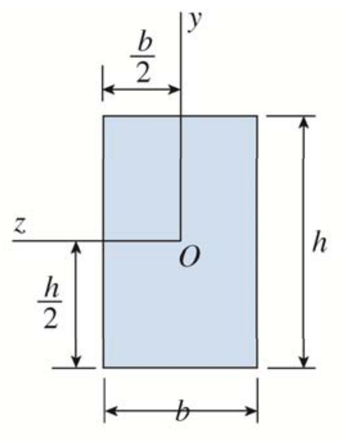
SEOUL NATIONAL UNIVERSITY

- Section modulus: combines properties into a single quantity.
- Doubly symmetric shapes: when $c_1 = c_2 = c$
- Maximum tensile and maximum compressive stresses are equal numerically

$$\sigma_1 = -\sigma_2 = -\frac{Mc}{I} = -\frac{M}{S}$$

  - A beam of rectangular cross section

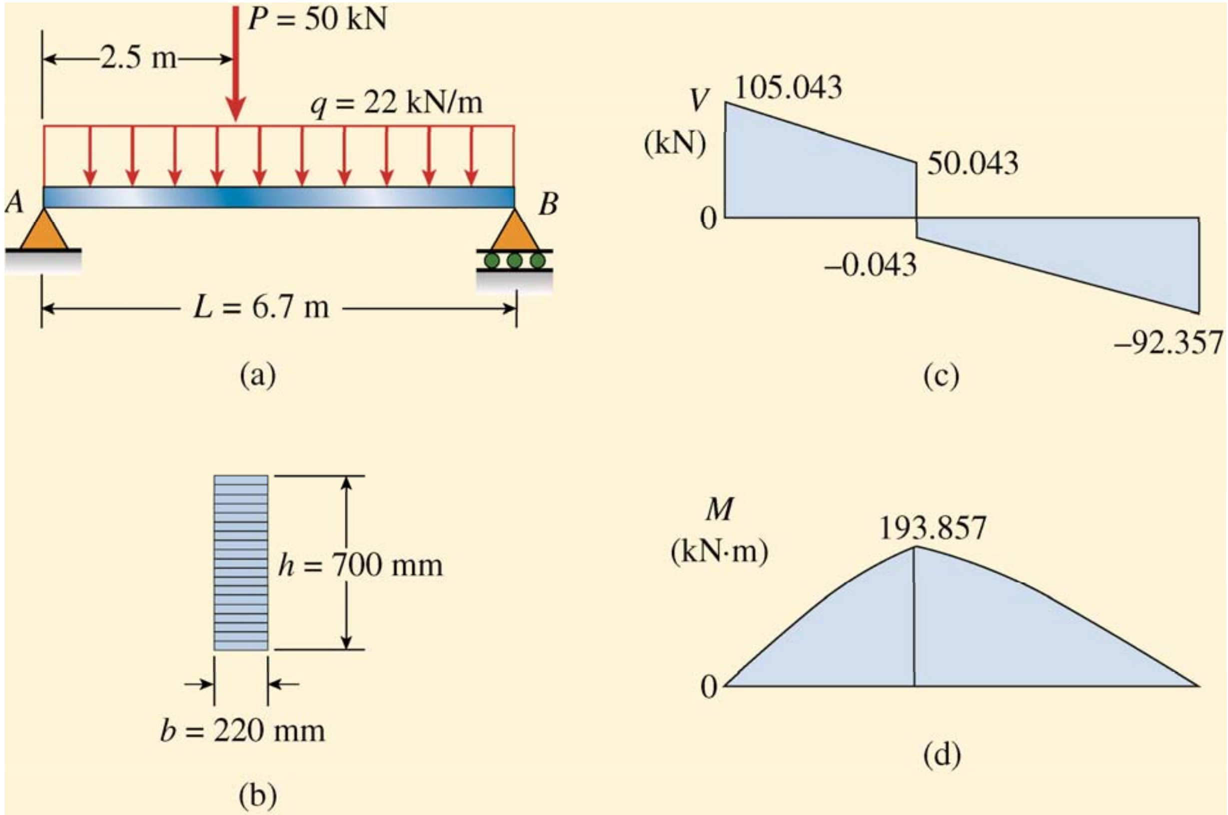
$$I = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$



Example 5-3

SEOUL NATIONAL UNIVERSITY

- Maximum tensile and compressive stress in the beam due to bending?



Summary

- Introduction
- Pure Bending and Nonuniform Bending
- Curvature of Beam
- Longitudinal Strains in Beams
- Normal Stress in Beams
- Design of Beams for Bending Stresses
- Nonprismatic Beams
- Shear Stresses in Beams of Rectangular Cross Section
- Shear Stresses in Beams of Circular Cross Section
- Shear Stresses in the Webs of Beams with Flanges