Week 10, 3 May

Mechanics in Energy Resources Engineering - Chapter 7 Analysis of Stress and Strain

Ki-Bok Min, PhD

Assistant Professor
Energy Resources Engineering
Seoul National University

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schedule

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- Ch.7 Analysis of Stress and Strain
  - 3 May, 10 May, 12 May
- Ch.8 Application of Plane Stress
  - 17 May, 19 May
- Ch.9 Deflection of Beams
  - 24 May, 26 May, 31 May
- Ch.10 Statically Indeterminate Beams
  - 2 June, 7 June
- Final Exam: 9 June

Outline

- Introduction
- Plane Stress
- Principal Stresses and Maximum Shear Stresses
- Mohr's Circle for Plane Stress
- Hooke's Law for Plane Stress
- Triaxial Stress
- Plane Strain

Introduction

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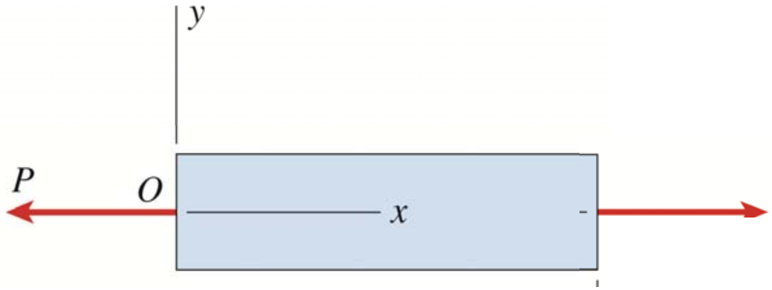
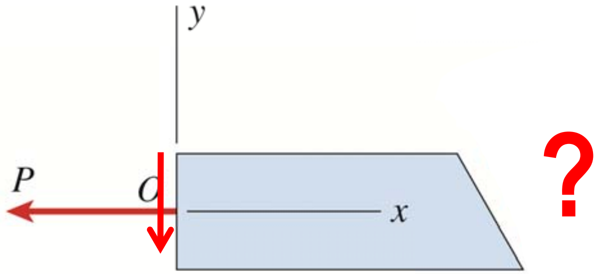
- Stresses in cross section

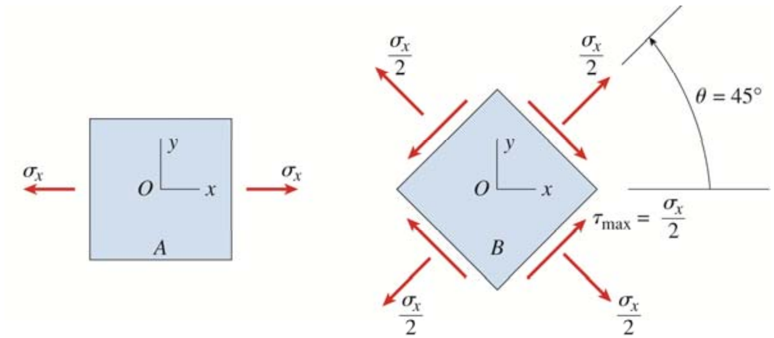


- Stresses in inclined section: larger stresses may occur
  - Finding the normal and shear stresses acting on inclined section is necessary
  - Main content of Ch.5!



Introduction

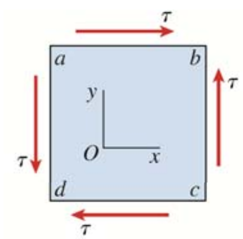
- We have already learned this!
  - Uniaxial Stress & Stresses in inclined section

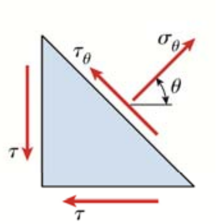


$$\sigma_\theta = \sigma_x \cos^2\theta = \frac{1}{2}\sigma_x\left(1+\cos 2\theta\right)$$

$$\tau_\theta = -\sigma_x \sin\theta\cos\theta = -\frac{\sigma_x}{2}\sin 2\theta$$

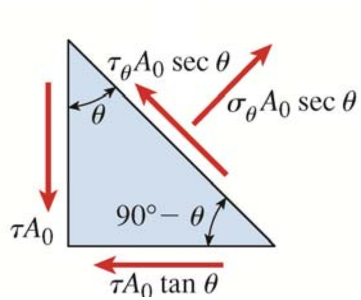
  - Pure Shear & Streses in inclined section





$$\sigma_\theta = \tau \sin 2\theta$$

$$\tau_\theta = \tau \cos 2\theta$$

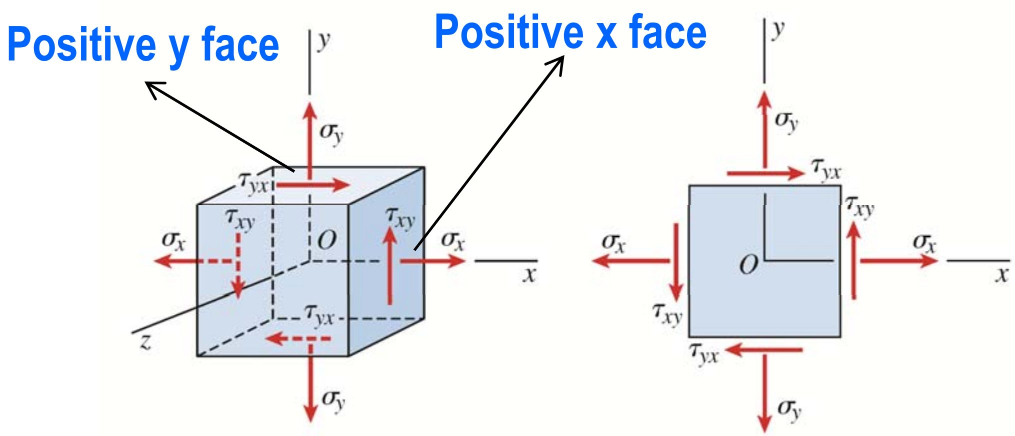


Introduction

- ONE instrinsic state of stress can be expressed in many many different ways depending on the reference axis (or orientation of element).
  - Similarity to force: One intrinsic state of force (vector) can be expressed similarly depending on the reference axis.
  - Difference from force: we use different transformation equations from those of vectors
  - Stress is NOT a vector BUT a (2nd order) tensor → they do not combine according to the parallelogram law of addition

Plane Stress
Definition

- Plane Stress: Stresses in 2D plane
- Normal stress, σ : subscript identify the face on which the stress act. Ex) σ_x
- Shear stress, τ : 1st subscript denotes the face on which the stress acts, and the 2nd gives the direction on that face. Ex) τ_{xy}



Plane Stress
Definition

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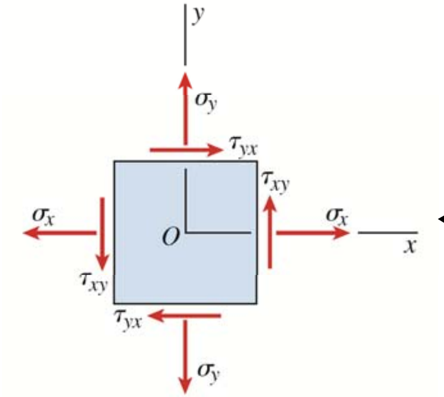
- Sign convention
  - Normal stress: tension (+), compression (-)
  - Shear stress:
    - acts on a positive face of an element in the positive direction of an axis (+) : plus-plus or minus-minus
    - acts on a positive face of an element in the negative direction of an axis (-): plus-minus or minus-plus

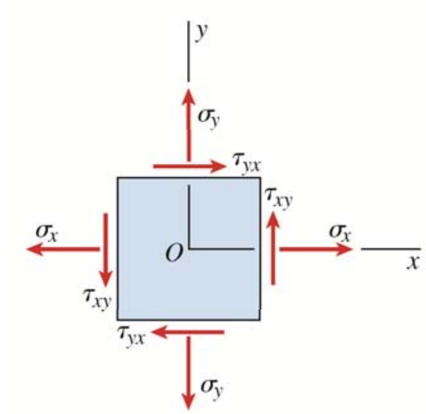


Positive normal & shear stresses

Plane Stress
Definition

- Shear stresses in perpendicular planes are equal in magnitude and directions shown in the below.
  - Derived from the moment equilibrium

$$\tau_{xy} = \tau_{yx}$$



- In 2D (plane stress), we need three components to describe a complete state of stress

$$\sigma_x \quad \sigma_y \quad \tau_{xy}$$

$$\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$$

Plane Stress
Stresses on inclined sections

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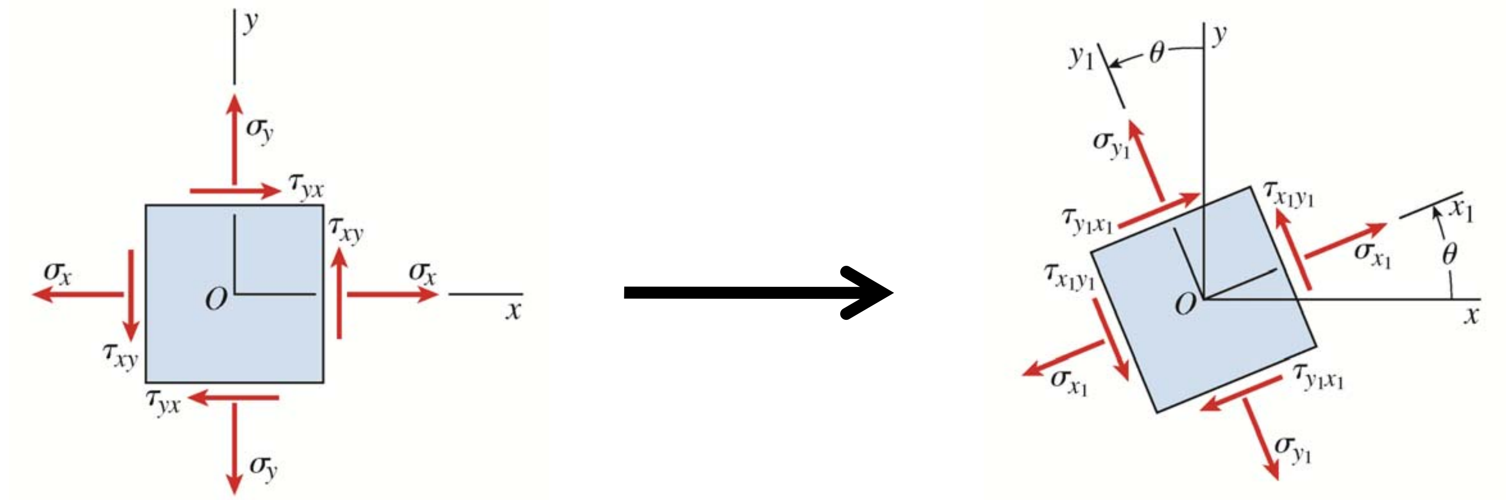
- Stresses acting on inclined sections assuming that σ_x, σ_y, τ_{xy} are known.
  - x_1y_1 axes are rotated counterclockwise through an angle θ
  - Strategy??? →
  - wedge shaped stress element



Plane Stress
Stresses on inclined sections

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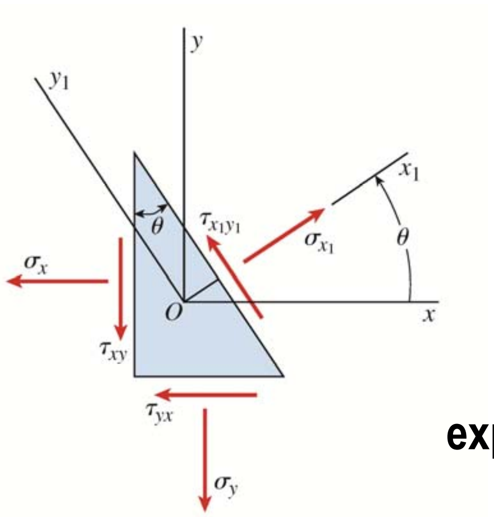
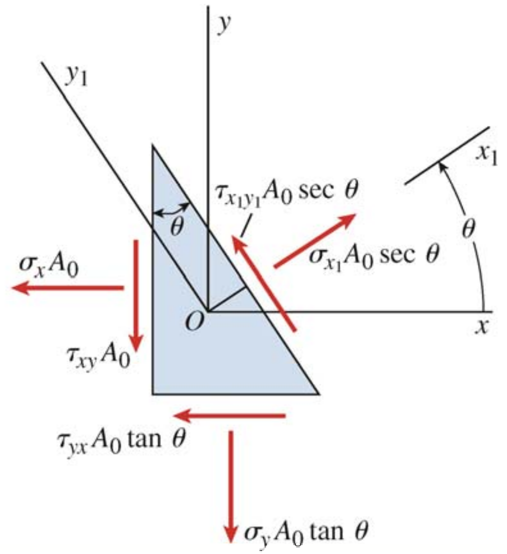


Free Body Diagram

express in terms of "Force"



- Force Equilibrium Equations in x_1 and y_1 directions

$$
\begin{aligned}
\sum F_{x_1} &= \sigma_{x_1} A_0 \sec\theta - \sigma_x A_0 \cos\theta - \tau_{xy} A_0 \sin\theta \\
&\quad - \sigma_y A_0 \tan\theta \sin\theta - \tau_{yx} A_0 \tan\theta \cos\theta = 0
\end{aligned}
$$

$$
\begin{aligned}
\sum F_{y_1} &= \tau_{x_1 y_1} A_0 \sec\theta + \sigma_x A_0 \sin\theta - \tau_{xy} A_0 \cos\theta \\
&\quad - \sigma_y A_0 \tan\theta \cos\theta + \tau_{yx} A_0 \tan\theta \sin\theta = 0
\end{aligned}
$$

Plane Stress
Stresses on inclined sections

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- Using $\tau_{xy} = \tau_{yx}$ and simplifying

$$\sigma_{x_1} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x_1 y_1} = -\left(\sigma_x - \sigma_y\right) \sin\theta \cos\theta + \tau_{xy} \left(\cos^2\theta - \sin^2\theta\right)$$

  - When θ = 0,

$$\sigma_{x_1} = \sigma_x \qquad \tau_{x_1 y_1} = \tau_{xy}$$

  - When θ = 90,

$$\sigma_{x_1} = \sigma_y \qquad \tau_{x_1 y_1} = -\tau_{xy}$$

Plane Stress
Transformation Equations

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- From half angle and double angle formulas

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

- Transformation equations for plane stress

$$\sigma_{x_1} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \tau_{x_1y_1} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

  – Intrinsic state of stress is the same but the reference axis are different

  – Derived solely from equilibrium→ applicable to stresses in any kind of materials (linear or nonlinear or elastic or inelastic)

Plane Stress
Transformation Equations

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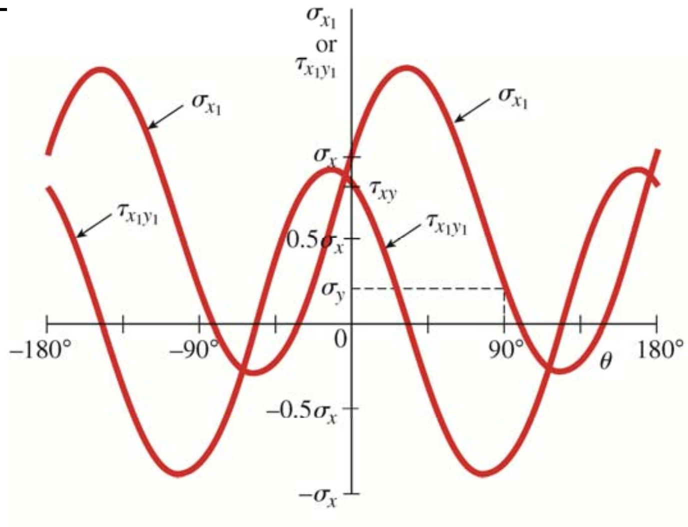

With σ_y=0.2σ_x & τ_{xy}=0.8 σ_x

- For σ_{y1}, $\theta \rightarrow \theta + 90$,

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

  - Making summations

$$\sigma_{x_1} + \sigma_{y_1} = \sigma_x + \sigma_y$$

  - Sum of the normal stresses acting on perpendicular faces of plane stress elements is constant and independent of θ

Plane Stress
Special Cases of Plane Stress

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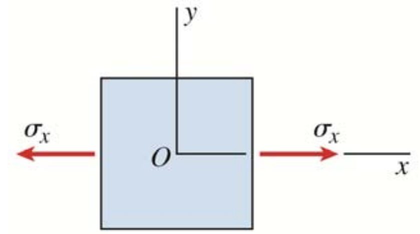
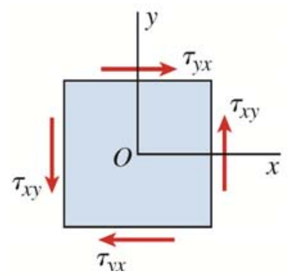
- Uniaxial stress

$$\sigma_{x_1} = \frac{\sigma_x}{2}(1+\cos 2\theta) \qquad \tau_{x_1 y_1} = -\frac{\sigma_x}{2}\sin 2\theta$$



- Pure Shear

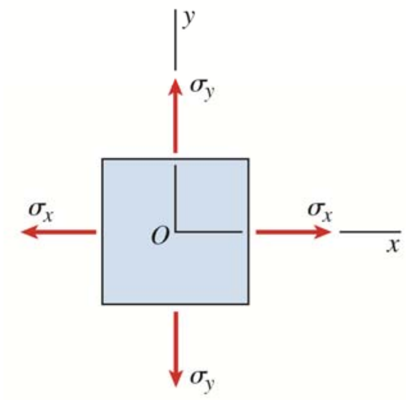
$$\sigma_{x_1} = \tau_{xy}\sin 2\theta \qquad \tau_{x_1 y_1} = \tau_{xy}\cos 2\theta$$



- Biaxial Stress

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$



Plane Stress
Example 7-1

- Determine the stress acting on an element inclined at an angle θ = 45°

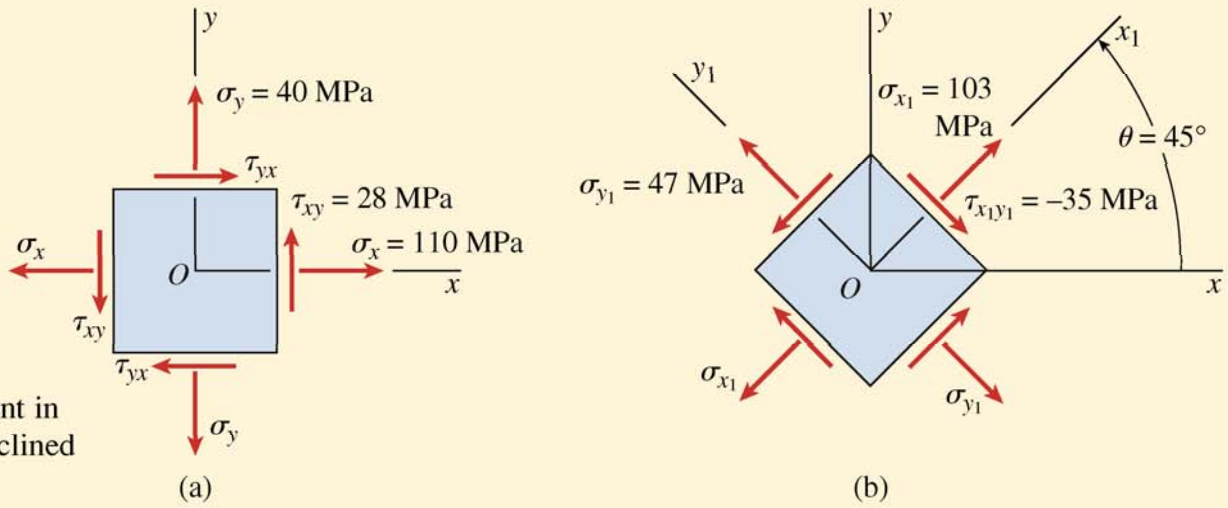


FIG. 7-7 Example 7-1. (a) Element in plane stress, and (b) element inclined at an angle $\theta = 45°$

Plane Stress
Example 7-1

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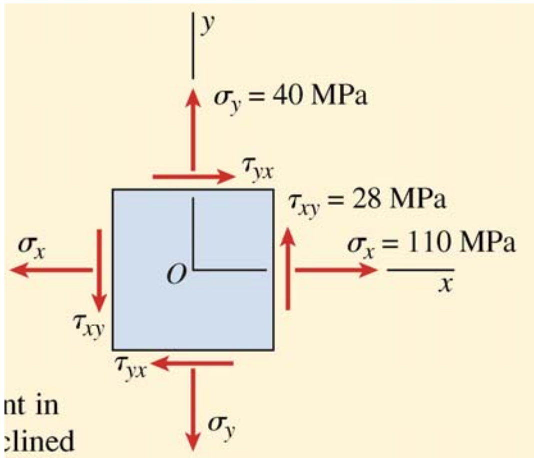

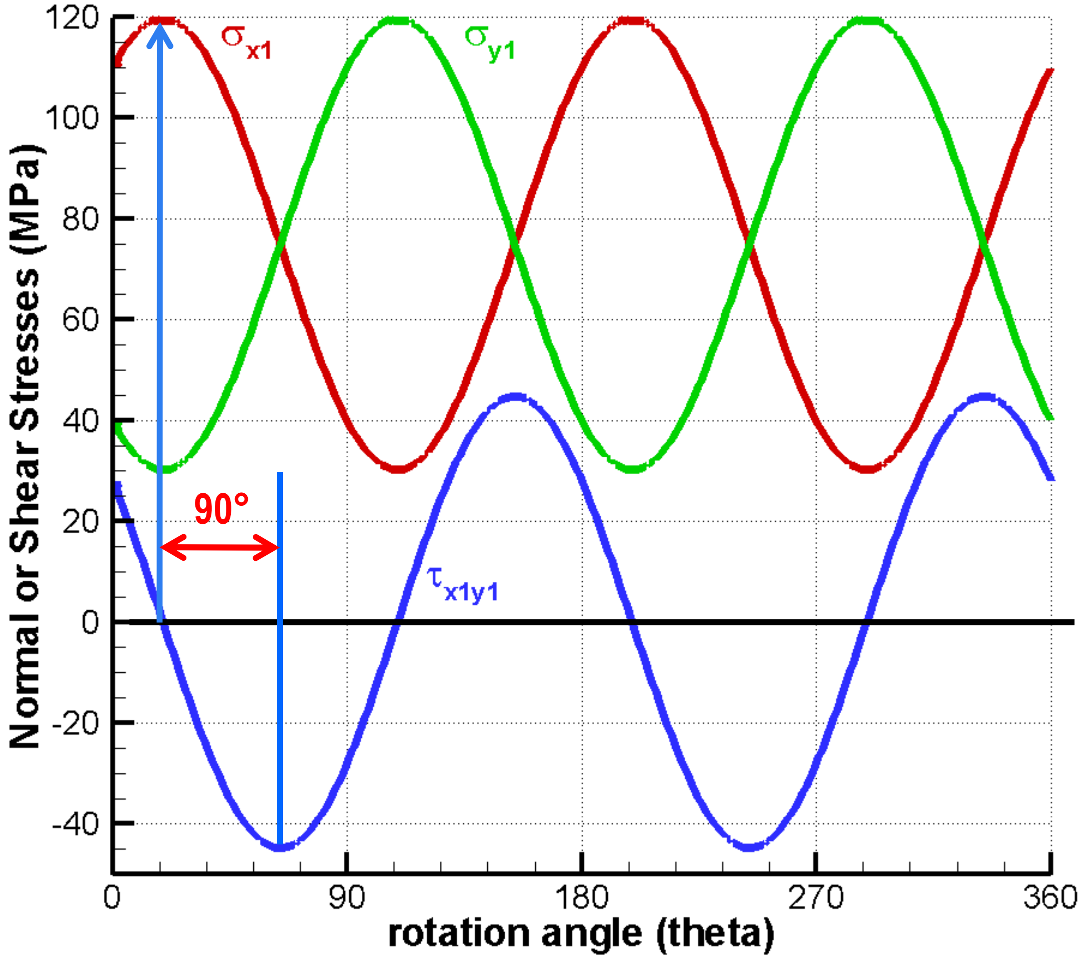

Principal Stresses and Maximum Shear Stresses

Mohr's Circle for Plane Stress

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Hooke's Law for Plane Stress

Triaxial Stress

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Plane Strain

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