Week 14, 31 May
Week 15, 7 June

Mechanics in Energy Resources Engineering - Chapter 10. Statically Indeterminate Beams

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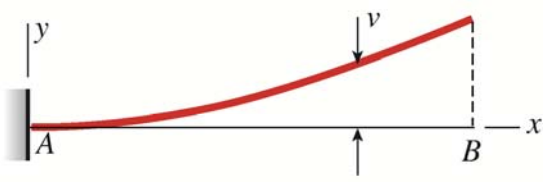
Deflections of Beams



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- Introduction
- Differential Equations of the Deflection Curve (처짐곡선의 미분방정식)
- Deflections by Integration of the Bending-Moment Equation (굽힘모멘트 방정식의 적분에 의한 처짐)
- Deflections by Integration of the Shear-Force and Load Equations (전단력과 하중방정식의 적분에 의한 처짐)
- Method of Superposition (중첩법)
- Moment-Area Method (모멘트-면적법)
- Nonprismatic Beams (불균일단면 보)

$$EI\frac{d^2v}{dx^2} = EIv'' = M$$

$$EI\frac{d^3v}{dx^3} = EIv''' = V$$

$$EI\frac{d^4v}{dx^4} = EIv'''' = -q$$

Chapter 10. Statically Indeterminate Beams

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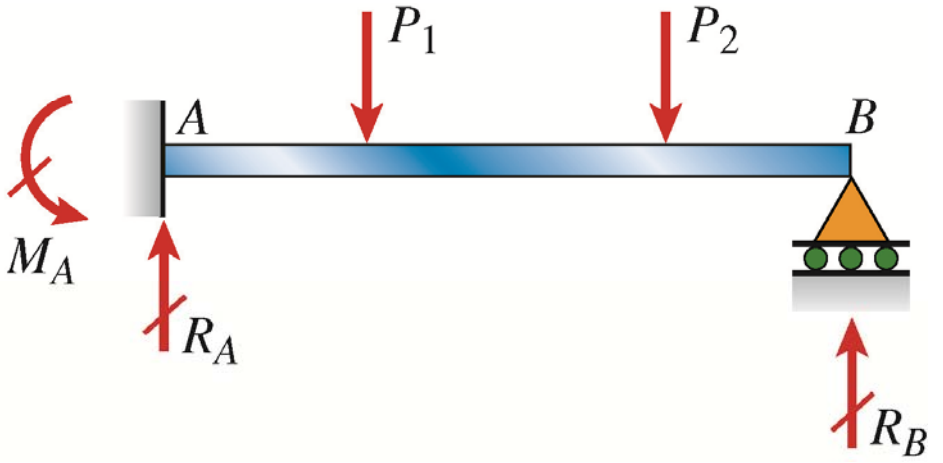
- Introduction
- Types of Statically Indeterminate Beams (부정정보의 형태)
- Analysis by the Differential Equations of the Deflection Curve (처짐곡선의 미분방정식에 의한 해석)
- Method of Superposition (중첩법)



propped cantilever beam

Statically indeterminate structure
March 2010

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Statically indeterminate structure
Definition

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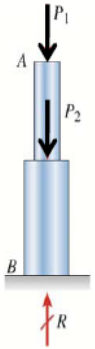
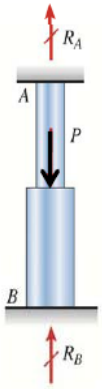


$R = P_1 + P_2$

Unknowns can be solved by Equil. Eq.
→정정



$R_A + R_B = P$

Equil. Eq. is not enough to solve the unknowns
→ 부정정

- Statically determinate (정정, 靜定)
  - reactions and internal forces can be obtained from equilibrium equations alone (via Free Body Diagram)
  - No need to know the properties (e.g., E, v, G) of the materials
- Statically indeterminate (부정정, 不靜定)
  - Equilibrium + additional equations related to the displacement
  - Need to know the properties (e.g., E, v, G) of the materials

Introduction

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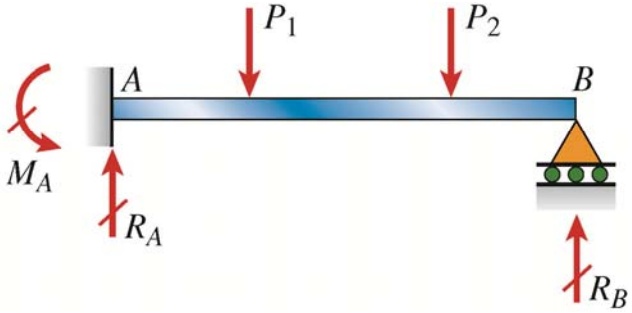
- Statically indeterminate structures:
  - Number of reactions > Number of independent Eqs. of Equil.
  - Method of Analysis

1) Supplement Equil. Eq. with Differential Eq. of deflection curve

$$EI\frac{d^2v}{dx^2} = EIv'' = M$$

2) Method of superposition

- Supplement Equil. Eq. with Compatability Eq. and Force-Displacement (Deflection) Eq.



Types of Statically Indeterminate Beams

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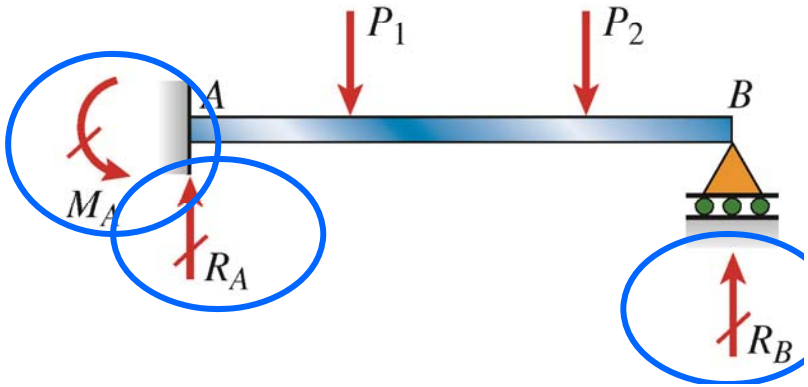
- Most of structures in everyday life are statically indeterminate
  - Automobile frames, buildings & underground structures
  - More sophisticated techniques are needed for analysis
- Degree of static indeterminacy (부정정 차수):
  - Number of unknown reactions - number of independent equilibrium equation
  - Static redundants: excess reactions



Types of Statically Indeterminate Beams

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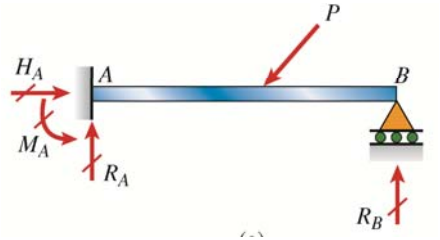
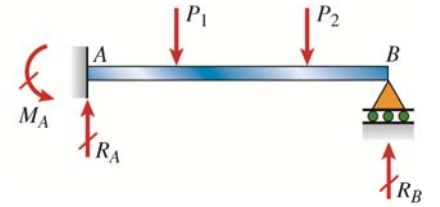
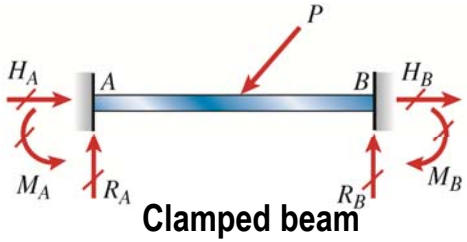
- Degree of static indeterminacy (부정정 차수):



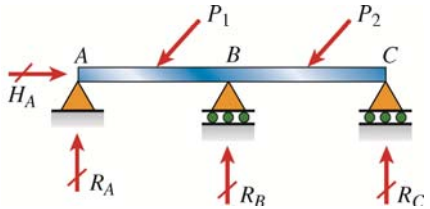
Statically indeterminate to the 1st degree



Statically indeterminate to the 1st degree



Statically indeterminate to the 3rd degree



Statically indeterminate to the 1st degree

Analysis by the differential equations of the deflection curve

Example 10-1

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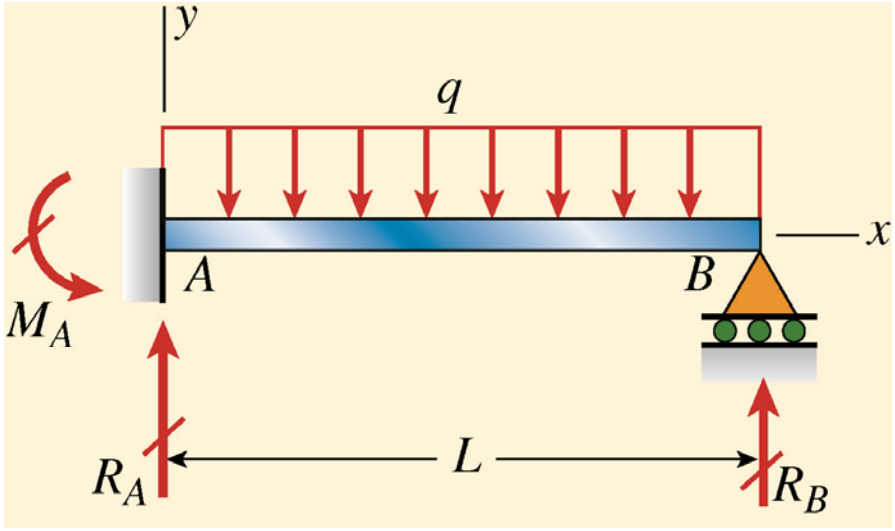
- Reactions, shear forces, bending moments, slopes and deflections?
- 3 Unknown reactions: M_A, R_A, R_B
- 2 independent equilibrium Equations

$$\sum F_v = 0 \longrightarrow R_A + R_B - qL = 0$$

$$\sum M = 0 \longrightarrow M_A - \frac{qL^2}{2} + R_B L = 0$$



Analysis by the differential equations of the deflection curve
Example 10-1

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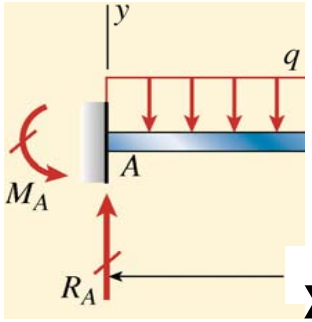
– Bending moment, M at distance x

$$M = R_A x - M_A - \frac{qx^2}{2} \xrightarrow[M_A = \frac{qL^2}{2} - R_B L]{R_A = qL - R_B} M = qLx - R_B x - \frac{qL^2}{2} + R_B L - \frac{qx^2}{2}$$



– Differential equation of the def. curve

$$EIv'' = M = qLx - R_B x - \frac{qL^2}{2} + R_B L - \frac{qx^2}{2}$$

– Two successive integrations

$$EIv' = \frac{qLx^2}{2} - \frac{R_B x^2}{2} - \frac{qL^2 x}{2} + R_B Lx - \frac{qx^3}{6} + C_1$$

$$EIv = \frac{qLx^3}{6} - \frac{R_B x^3}{6} - \frac{qL^2 x^2}{4} + \frac{R_B Lx^2}{2} - \frac{qx^4}{24} + C_1 x + C_2$$

Analysis by the differential equations of the deflection curve

Example 10-1

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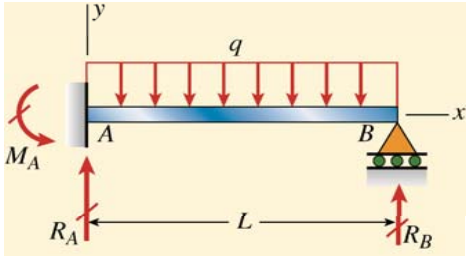
– Boundary Conditions

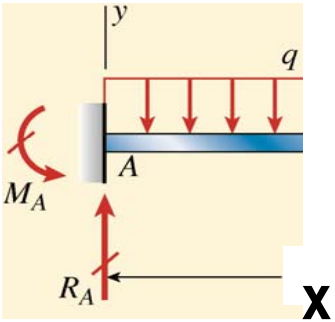


$$v(0) = 0 \longrightarrow C_2 = 0$$

$$v'(0) = 0 \longrightarrow C_1 = 0$$

$$v(L) = 0 \longrightarrow R_B = \frac{3qL}{8}$$

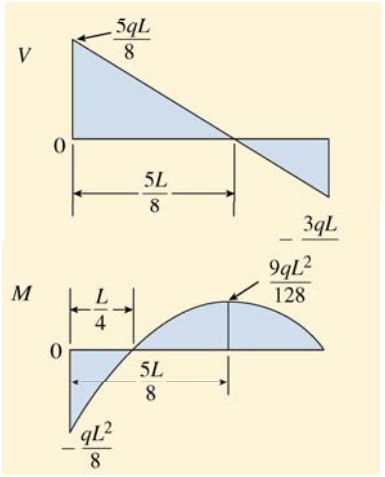
$$\sum F_v = 0, \quad \sum M = 0 \longrightarrow R_A = \frac{5qL}{8}, \quad M_A = \frac{qL^2}{8}$$



– Shear Forces and Bending Moments

$$V = R_A - qx = \frac{5qL}{8} - qx$$

$$M = R_A x - M_A - \frac{qx^2}{2} = \frac{5qLx}{8} - \frac{qL^2}{8} - \frac{qx^2}{2}$$



Analysis by the differential equations of the deflection curve
Example 10-1

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– Slopes and Deflections of the beam

$$v' = \frac{qx}{48EI}\left(-6L^2 + 15Lx - 8x^2\right) \qquad v = -\frac{qx^2}{48EI}\left(3L^2 - 5Lx + 2x^2\right)$$

– Maximum deflection

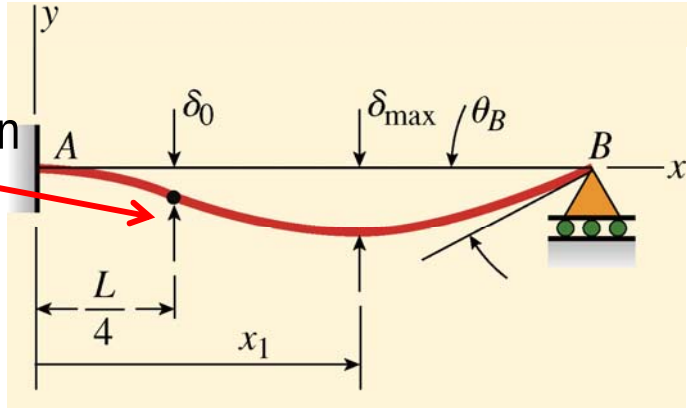
$$v' = 0 = \left(-6L^2 + 15Lx - 8x^2\right)x \qquad x = 0,\ \frac{15 \pm \sqrt{33}}{16}L \qquad x_1 = \frac{15 - \sqrt{33}}{16}L = 0.5785L$$

$$\delta_{\max} = -v_{x=x_1} = \frac{qL^4}{65,536EI}\left(39 + 55\sqrt{33}\right) = \frac{qL^4}{184.6EI}$$

– Point of inflection

Curvature and bending moment change sign

$$\delta_0 = -v_{x=L/4} = \frac{5qL^4}{2048EI}$$



Statically Indeterminate Beams
Example 10-2

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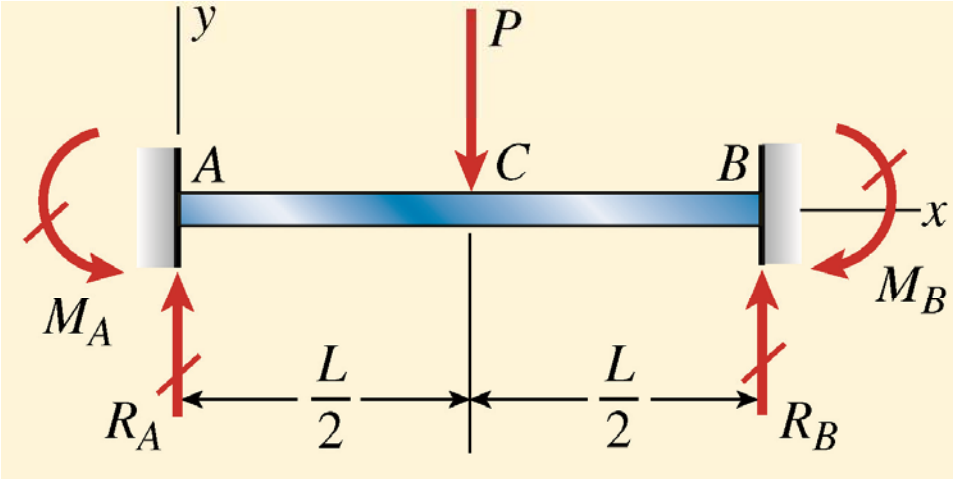
- Reactions, shear forces, bending moments, slopes and deflections?
- 3 Unknown reactions: $M_A, M_{B,} R_A, R_B$
- 2 independent equilibrium Equations

$$\sum F_v = 0 \longrightarrow R_A + R_B - P = 0$$

$$\sum M = 0 \longrightarrow M_A - P\frac{L}{2} + R_B L - M_B = 0$$



Statically Indeterminate Beams
Example 10-2

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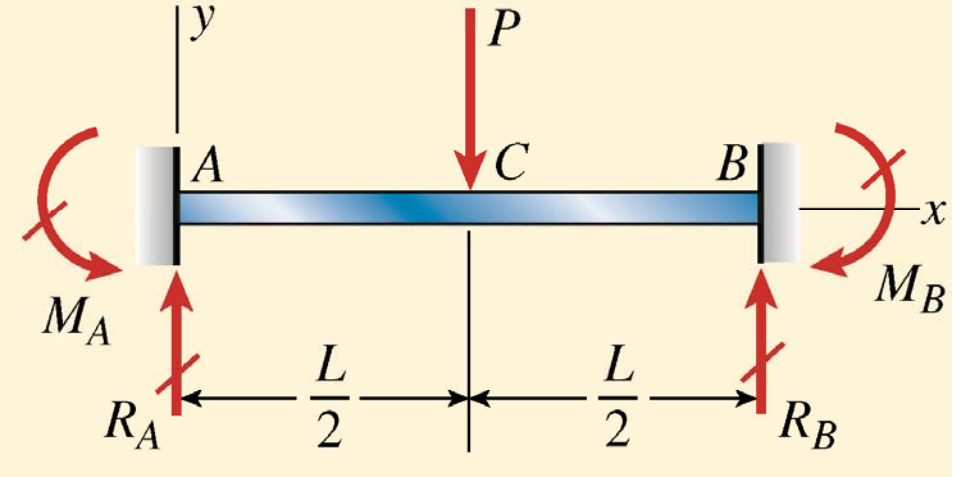
– The number of Boundary Conditions and other conditions is always sufficient to evaluate constants & redundant reactions



Statically Indeterminate Beams
Example 10-2

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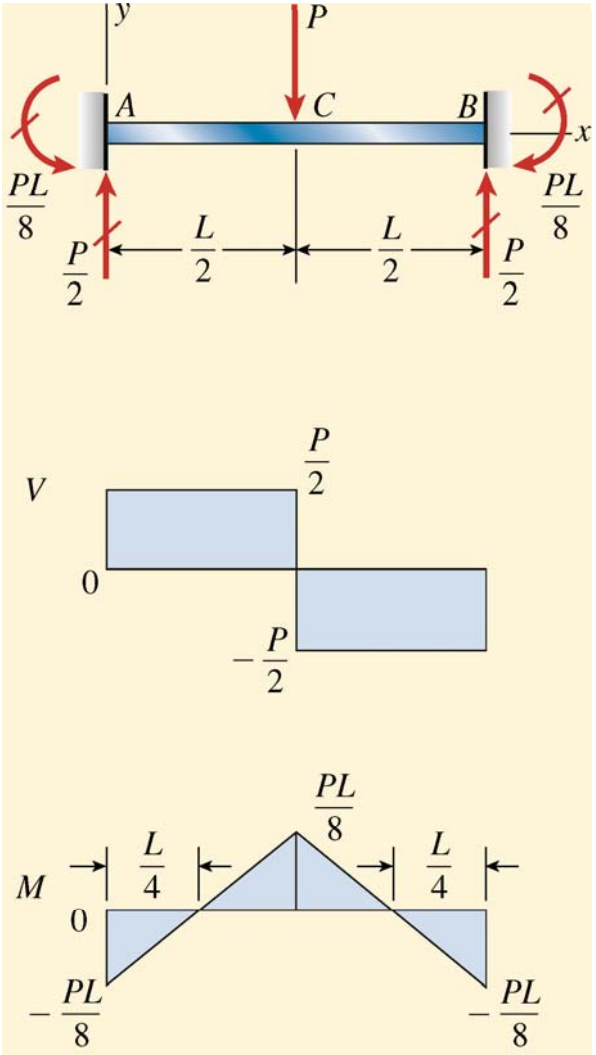

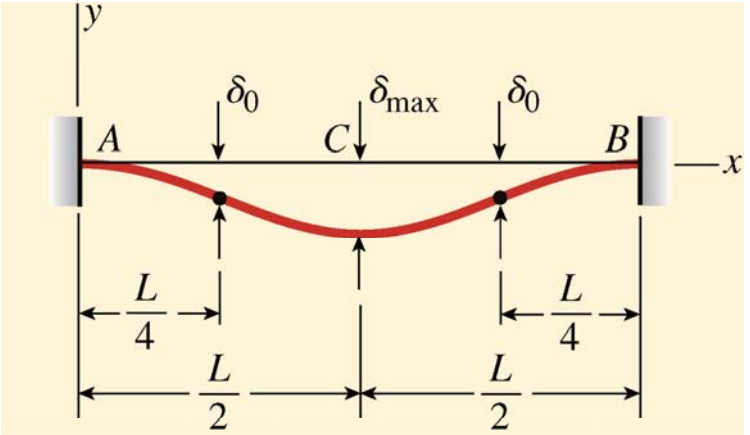

Summary
Chapter 10. Statically Indeterminate Beams

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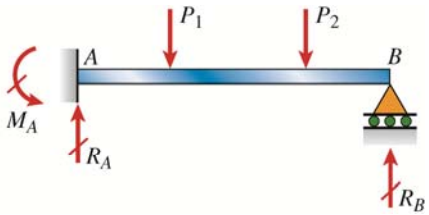
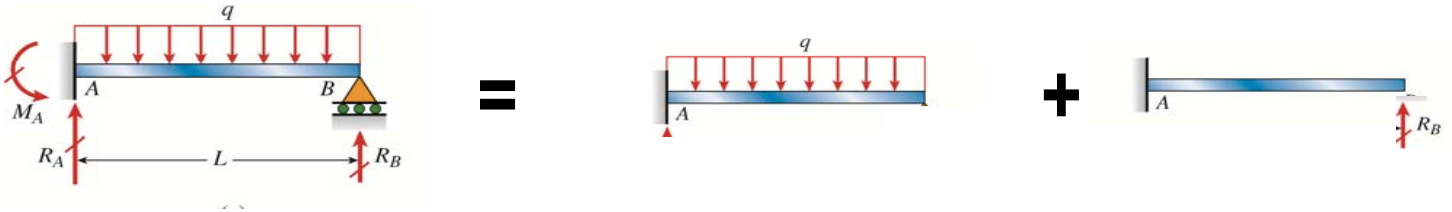
- Key Words/Questions:
  - Statically Indeterminate Beams



  - Can we use $EI\dfrac{d^2v}{dx^2} = EIv'' = M$ to find reactions for statically indeterminate structure?
  - Point of inflection (변곡점)
  - Conditions for Method of Superposition?



Final Exam

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- 14 June 08:30 – 11:00
  - If you can solve the home assignment with confidence, you will do a good job.
  - More than 50% from the home assignments (with some modification).
  - ~90% from the examples and the problems from the textbook.
  - Level of difficulty will be similar to that of the 1st & 2nd exam.
  - Scope: Entire Textbook except ch.6 and ch.11
  - Partial point will be minimized this time (at most 30%)

Method of Superposition

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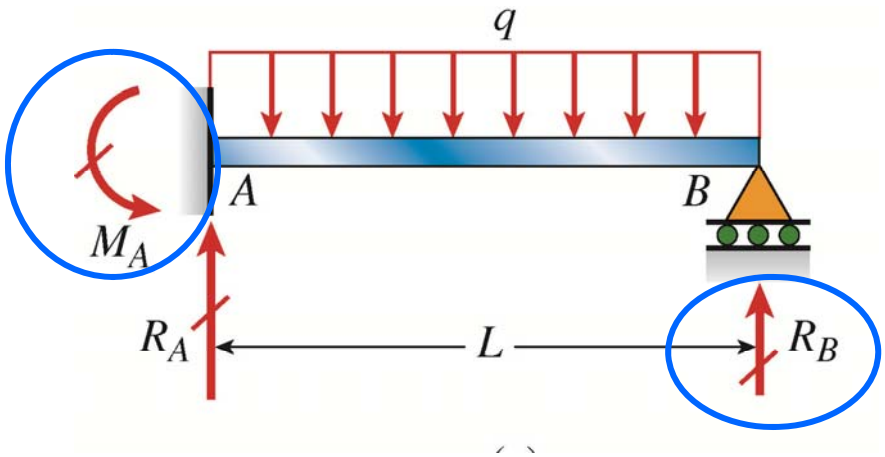
- Separate redundants
  - R_B as redundants or
  - M_A as redundants



Statically Indeterminate Structure

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Statically indeterminate structure example

Statically indeterminate structure

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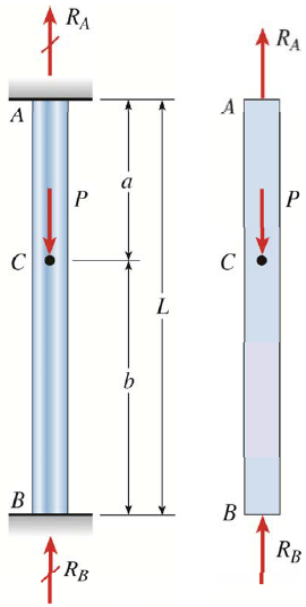

1) Equilibrium Equation $\sum F_{ver} = 0 \longrightarrow R_A - P + R_B = 0$

2) Clue for additional equation:

- A bar with both ends fixed does not change in length

$\delta_{AB} = 0$ ←Compatibility equation: the change in length must be compatible with the conditions at the supports

3) Compat. Eq. in terms of Forces: force-displacement relations

$\delta = \frac{PL}{EA} \longrightarrow \delta_{AC} = \frac{R_A a}{EA} \qquad \delta_{CB} = -\frac{R_B b}{EA}$

Why minus here?

- By combining 2) and 3),

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = \frac{R_A a}{EA} - \frac{R_B b}{EA} = 0$$

- Finally, combining above with 1) Equil. Eq.

$$R_A = \frac{Pb}{L} \qquad R_B = \frac{Pa}{L}$$

- We can also calculate the displacements

$$\delta_C = \delta_{AC} = \frac{R_A a}{EA} = \frac{Pab}{LEA}$$

Method of Superposition
R_B as Redundant (여분력)

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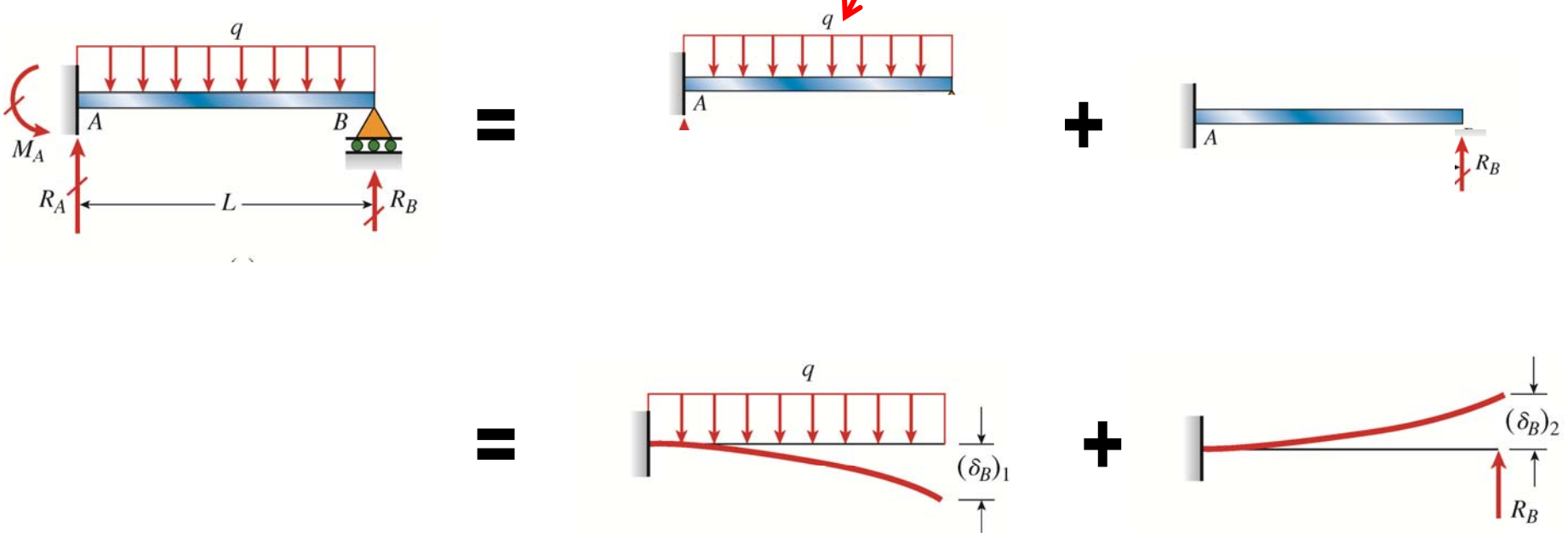
- Separate redundants

Released structure (이완구조물):
remove restraints corresponding to redundants



Method of Superposition

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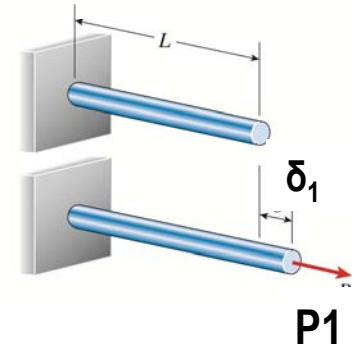
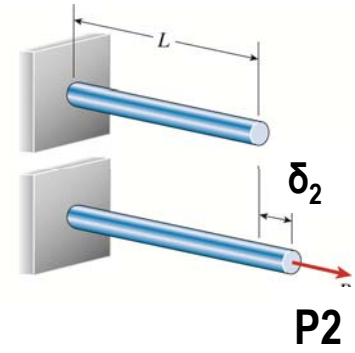
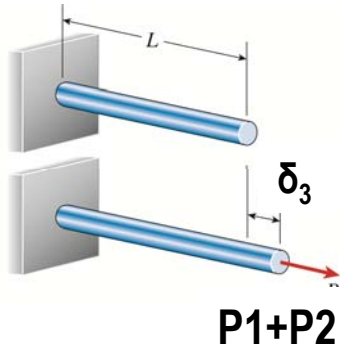
- An example of when Method of Superposition does not apply?
  - Strain energy

$$U = \frac{P^2 L}{2EA} = \frac{EA\delta^2}{2L}$$







$$\frac{P_1^2 L}{EA} + \frac{P_2^2 L}{EA} \neq \frac{(P_1 + P_2)^2 L}{EA}$$

Method of Superposition
R_B as Redundant (여분력)

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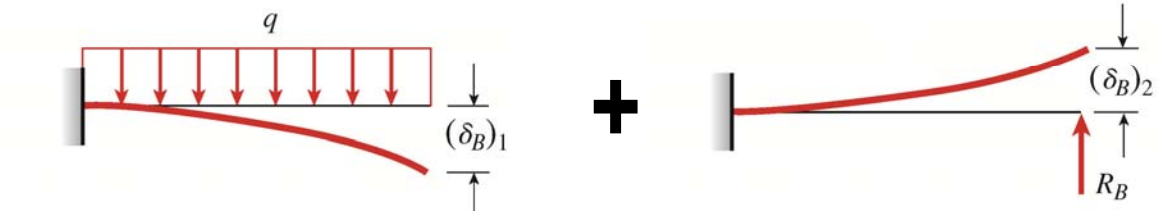
- Total defl. = defl. due to uniform load + defl due to redundant
- Equation of compatibility
  - Deflection due to uniform load - deflection due to redundant

$$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$$

$$(\delta_B)_1 = \frac{qL^4}{8EI} \qquad (\delta_B)_2 = \frac{R_B L^3}{3EI}$$

$$\delta_B = \frac{qL^4}{8EI} - \frac{R_B L^3}{3EI} = 0 \longrightarrow R_B = \frac{3qL}{8} \xrightarrow{\text{Equil. Eq.}} R_A = \frac{5qL}{8} \qquad M_A = \frac{qL^2}{8}$$



Method of Superposition
R_B as Redundant (여분력)

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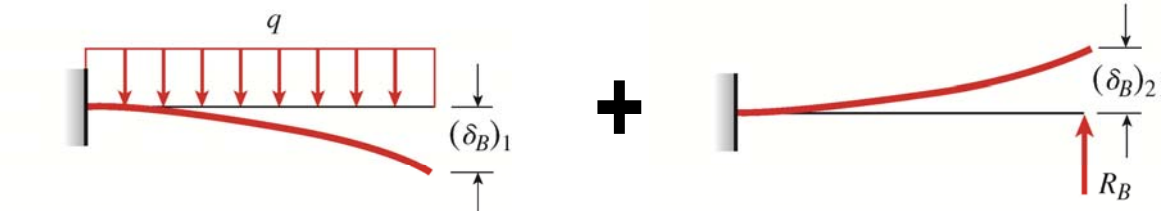
- Total defl. = defl. due to uniform load + defl due to redundant

$$v_1 = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v_2 = \frac{R_B x^2}{6EI}(3L - x)$$

$$v = v_1 + v_2 = -\frac{qx^2}{48EI}(3L^2 - 5Lx + 2x^2)$$



Method of Superposition
M_A as Redundant (여분력)

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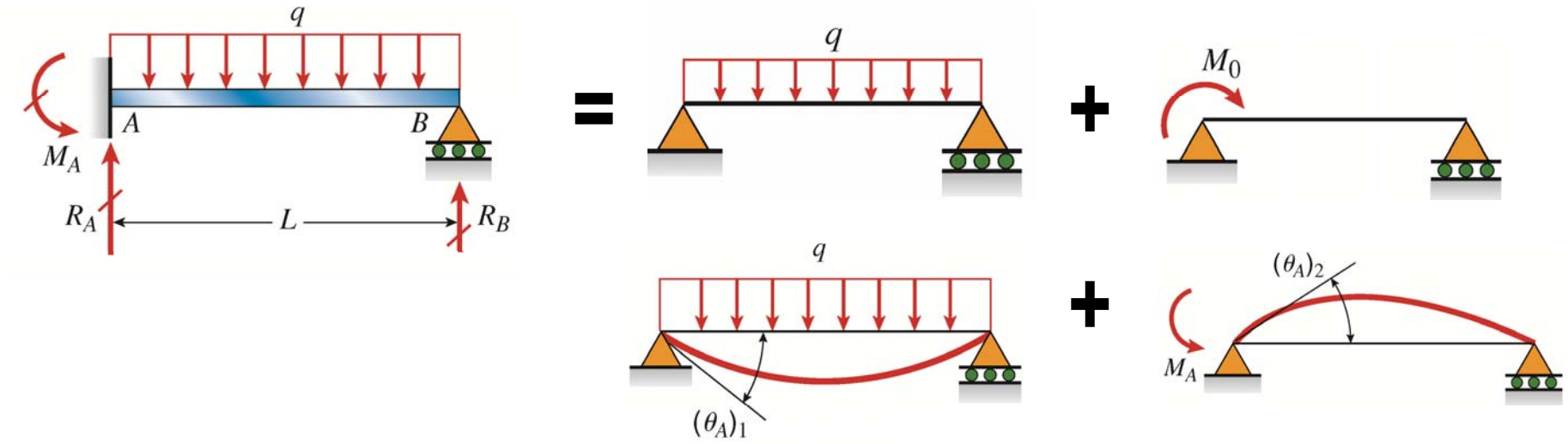

$$(\theta_A)_1 = \frac{qL^3}{24EI} \qquad (\theta_A)_2 = \frac{M_A L}{3EI}$$

$$\theta = (\theta_A)_1 - (\theta_A)_2 = \frac{qL^3}{24EI} - \frac{M_A L}{3EI} = 0 \longrightarrow M_A = \frac{qL^2}{8}$$

Equil. Eq. $\longrightarrow$ $R_A = \frac{5qL}{8}$ $\quad R_B = \frac{3qL}{8}$

Method of Superposition
Example 10-3

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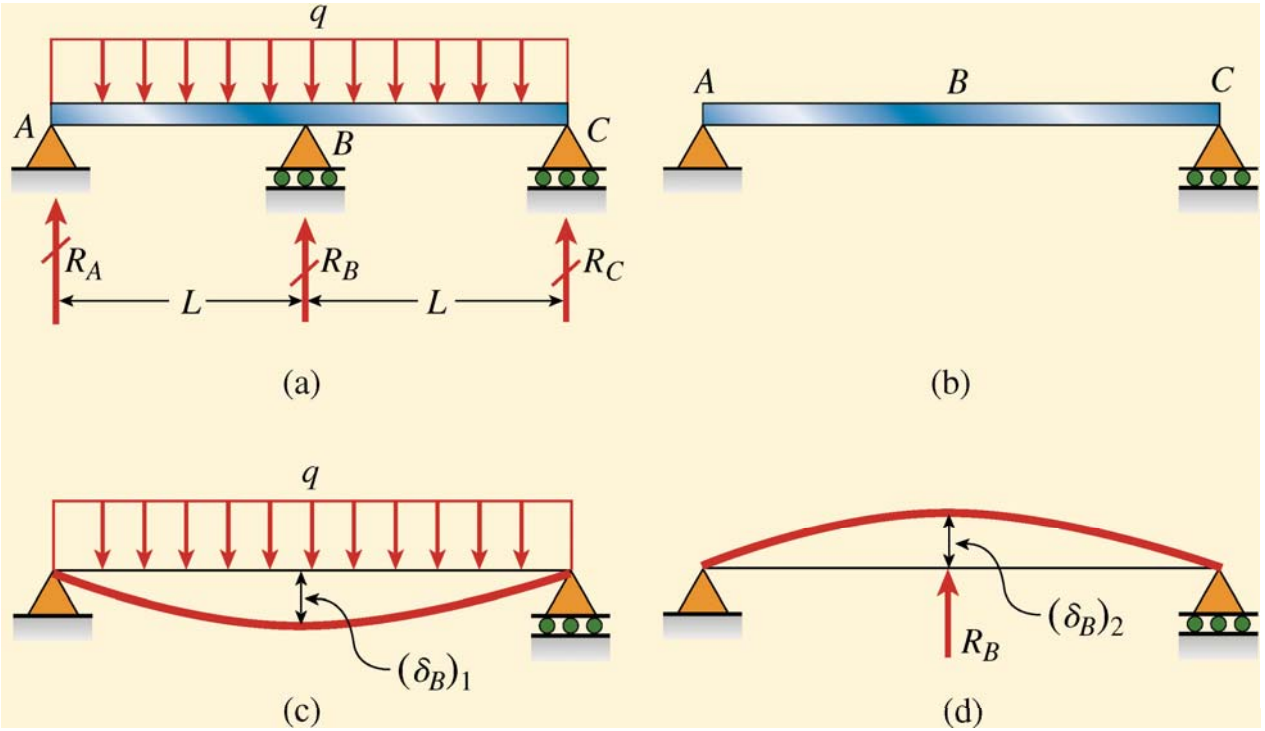
– Reactions?



Method of Superposition
Example 10-4

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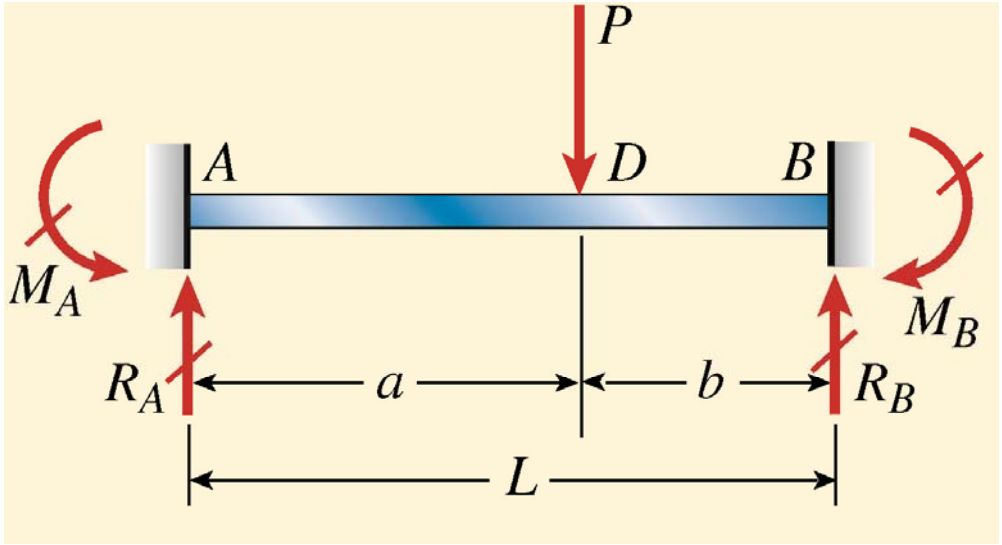
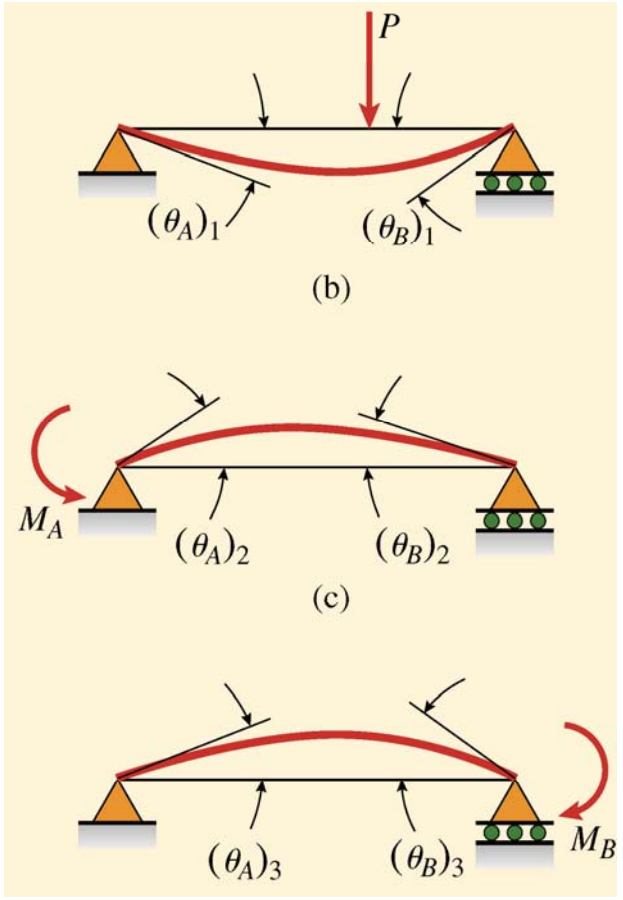
– Reactions & deflection at point D?





Method of Superposition
Example 10-5

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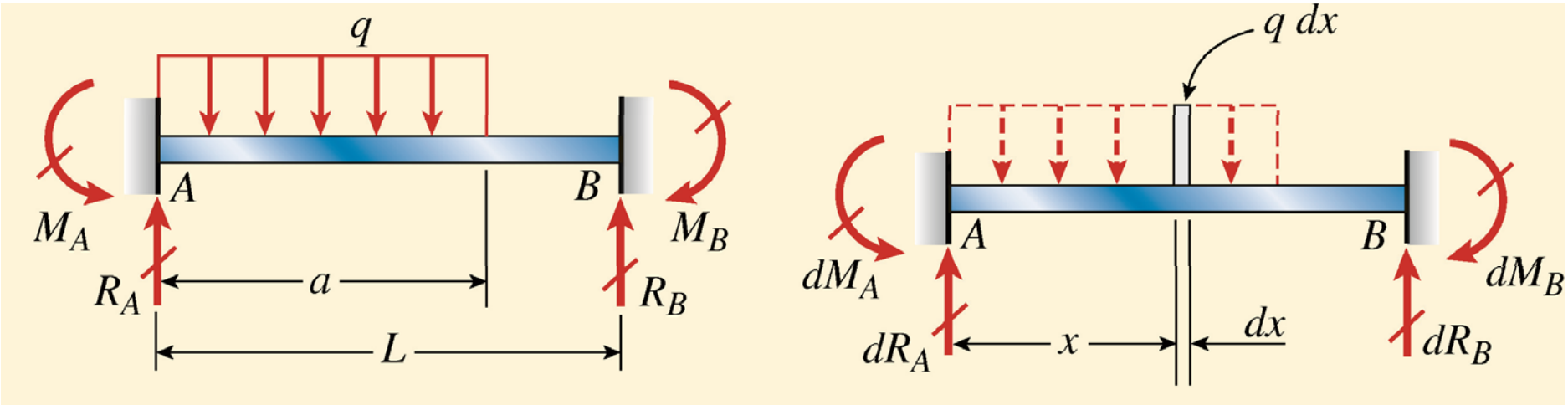
– Reactions?



Method of Superposition
Example 10-6

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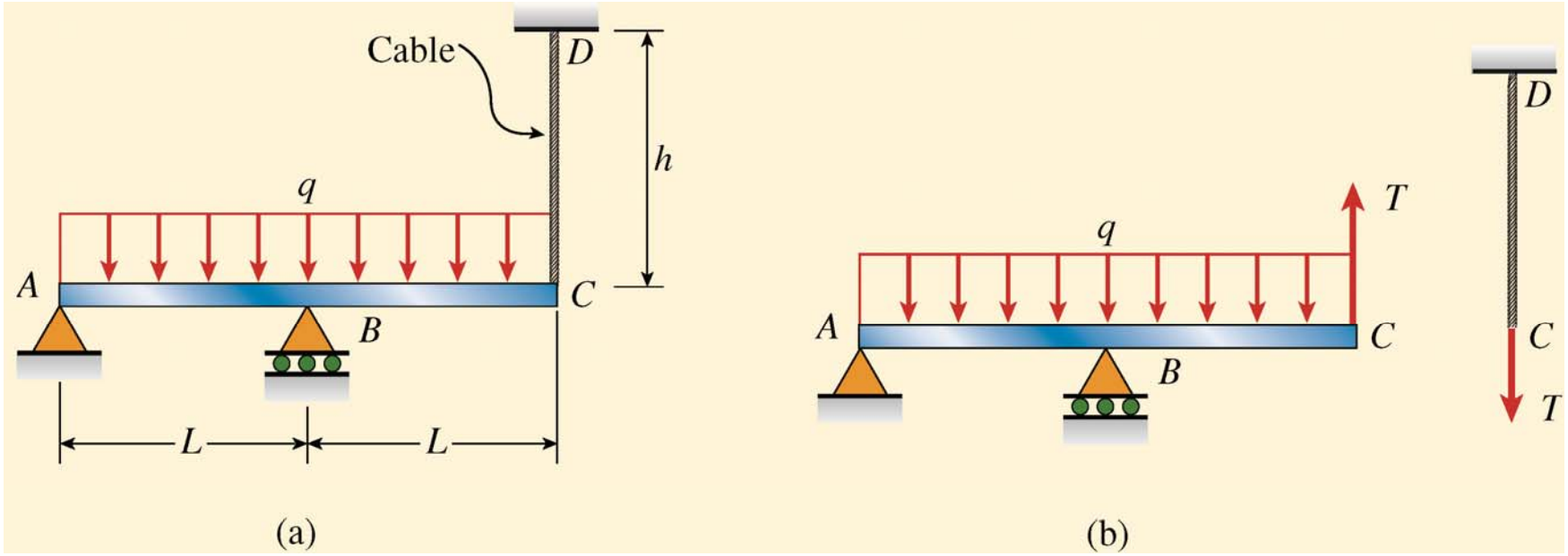
- Magnitude of tensile force T?



Summary
Chapter 10. Statically Indeterminate Beams

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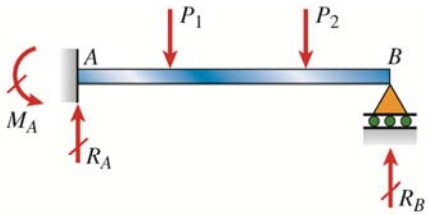
- Key Words/Questions:
  - Statically Indeterminate Beams



  - Can we use $EI\dfrac{d^2v}{dx^2} = EIv'' = M$ to find reactions for statically indeterminate structure?
  - Point of inflection (변곡점)
  - Conditions for Method of Superposition?